Lectures on Gravitational-Wave Astronomy

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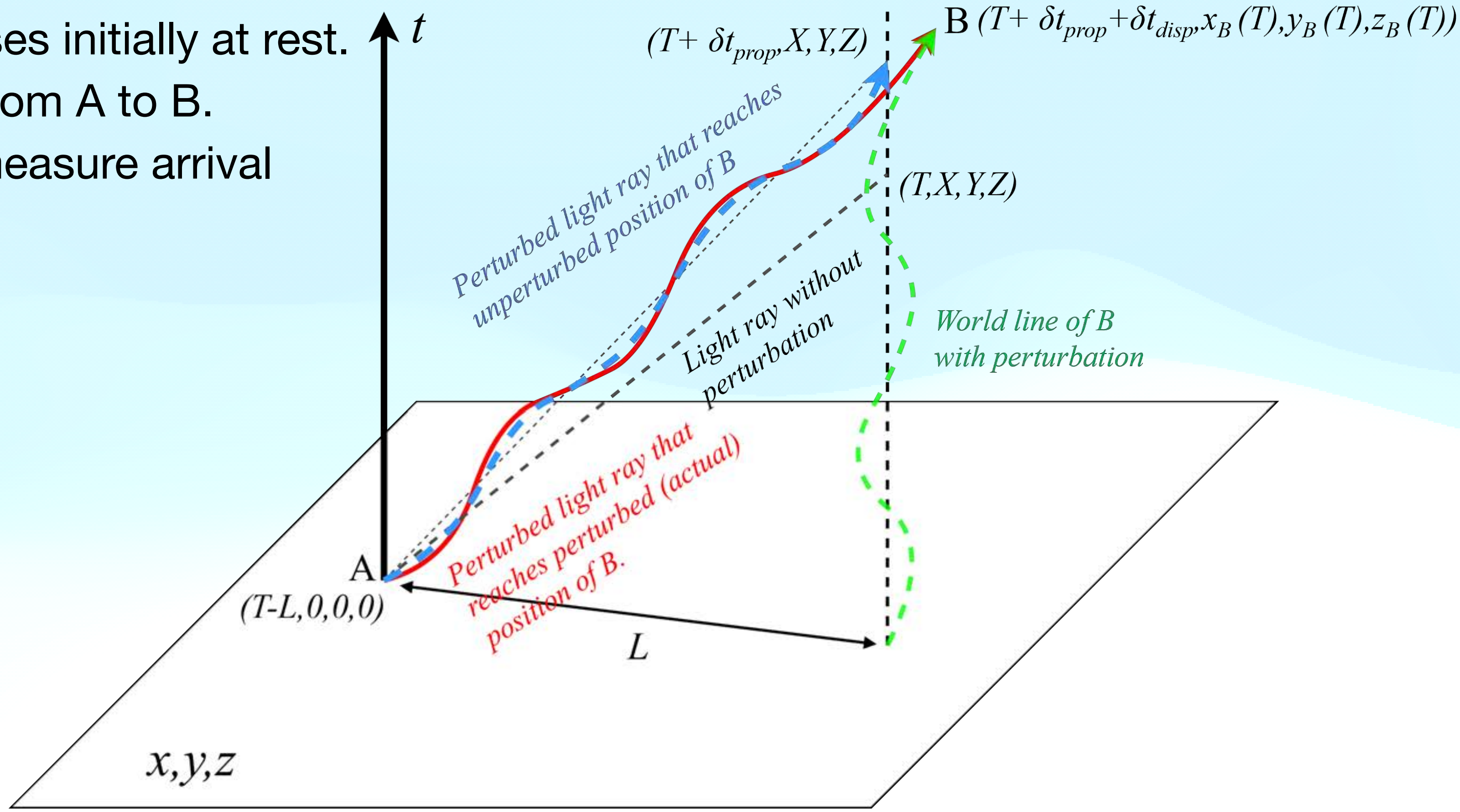
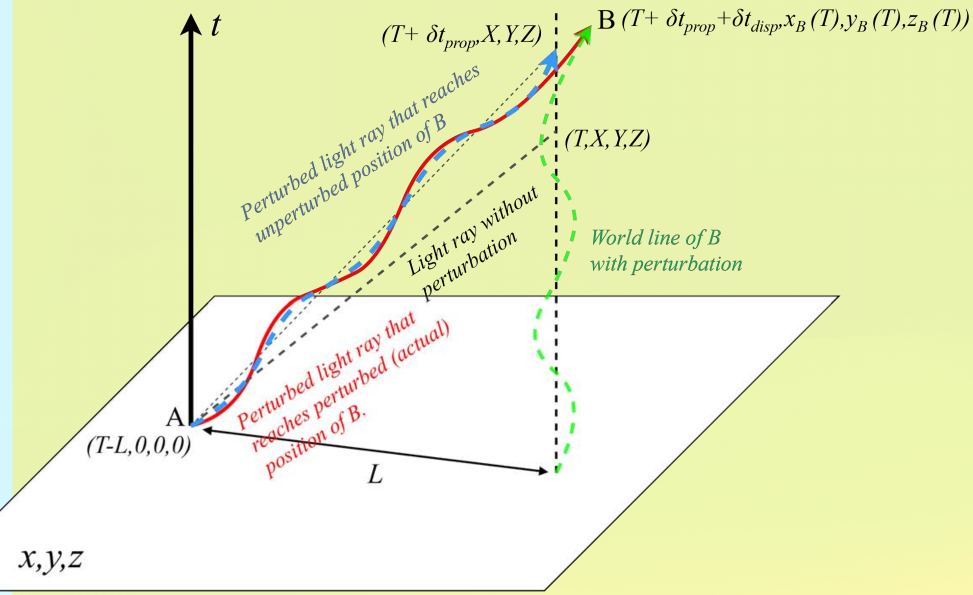
Interaction between GW and Detectors

- Two free masses initially at rest.
- Light travels from A to B.
- Ideal clocks measure arrival time



- Three different "*physical* effects" (which are in fact **gauge dependent**):
    1. Proper time τ differ from t (gravitational redshift)
    2. Test masses move under gravity force
    3. Light rays distorted due to metric perturbation.

Interaction between GW and Detectors



$$\|h_{\mu\nu}\| = \begin{pmatrix} h_{00} & h_{0j} \\ h_{i0} & h_{ij} \end{pmatrix}$$

gravitational redshift

$$\delta t_{\rm redshift} = \int \left(\frac{d\tau}{dt_D} - 1 \right) dt_D = -\frac{1}{2} \int^t h_{00}(t, x^j_D) dt$$

tidal displacement

$$\frac{d^2 x^j_D}{dt^2} + \Gamma^j_{00} = 0 \,. \qquad \Gamma^j_{00} = \frac{1}{2} \left[2h_{j0,0} - h_{00,j} \right]$$

$$n^j = (X, Y, Z)/L, \qquad \delta t_{\rm disp} = n^j \delta x^j_D$$

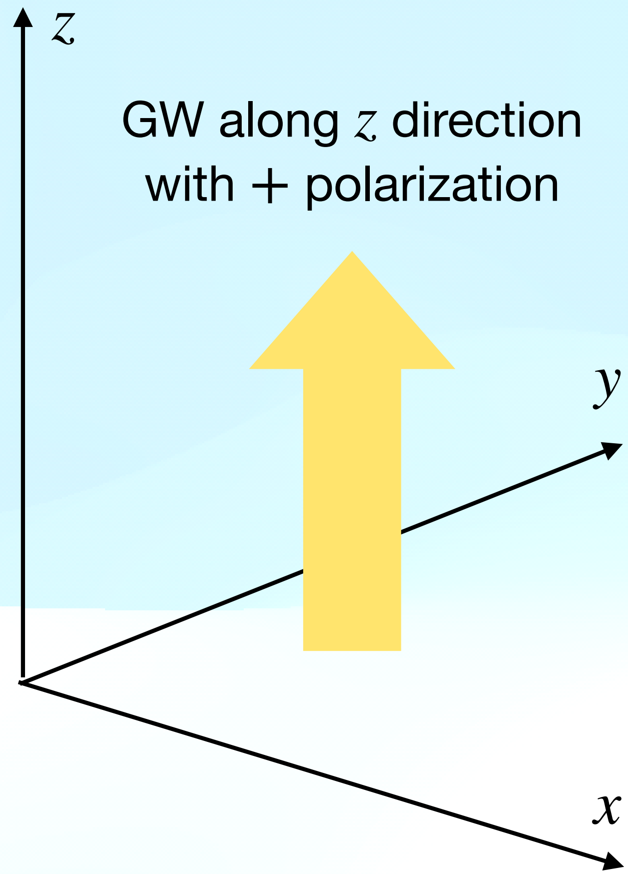
distortion of light propagation

$$\delta t_{\rm prop} = \frac{1}{2L} \int h_{\mu\nu}[x^\rho_{\rm ray}(\lambda)] \frac{dx^\mu_{\rm ray}(\lambda)}{d\lambda} \frac{dx^\nu_{\rm ray}(\lambda)}{d\lambda} = \frac{L}{2} \int_0^1 d\lambda \left[h_{00} + h_{0j} n^j + h_{ij} n^i n^j \right]$$

Gauge transformation: $h_{\mu\nu} \to h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$

TT Gauge

GW along z direction
with + polarization

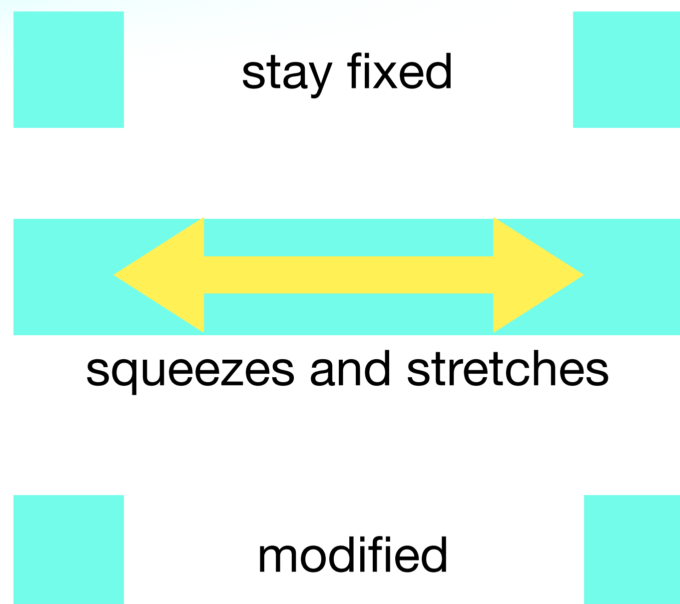


$$\|h_{\alpha\beta}(t,x,y,z)\| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t-z) & h_\times(t-z) & 0 \\ 0 & h_\times(t-z) & -h_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

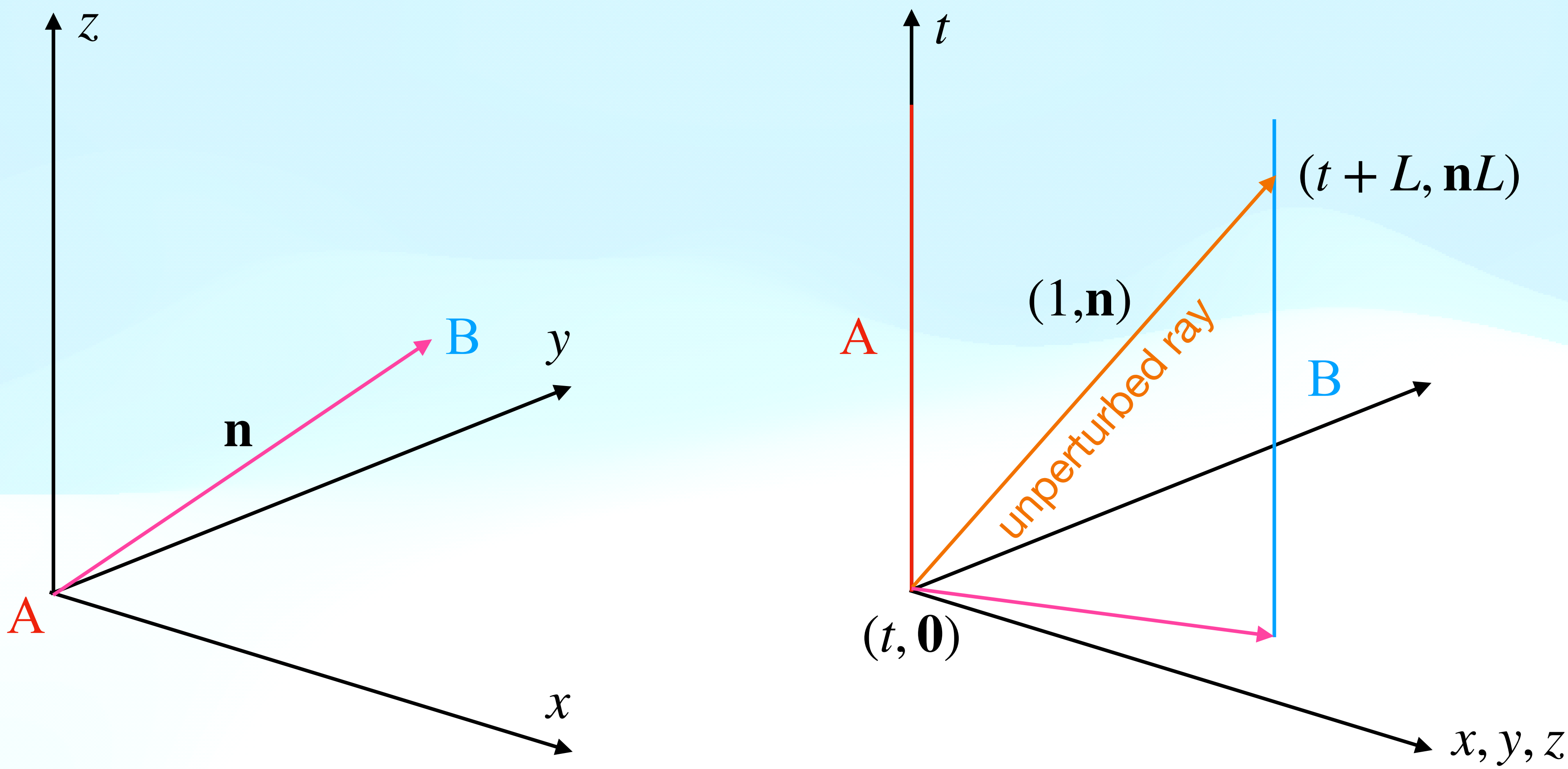
free masses
(initially at rest)

rigid object

light propagatin?



Response to GW in TT gauge



$$\Delta t = \frac{L}{2}\int_0^1 d\zeta n^i n^j h_{ij}^{TT}(t + \zeta L, \mathbf{x}_A + \zeta \mathbf{n} L)$$

integral of the projection of h_{ij} along light propagation direction

Response to GW in TT gauge

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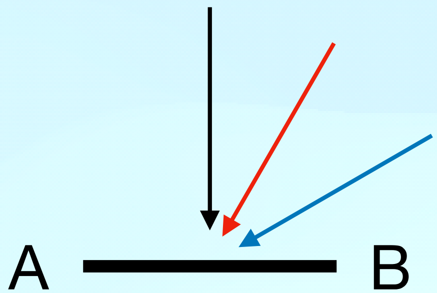
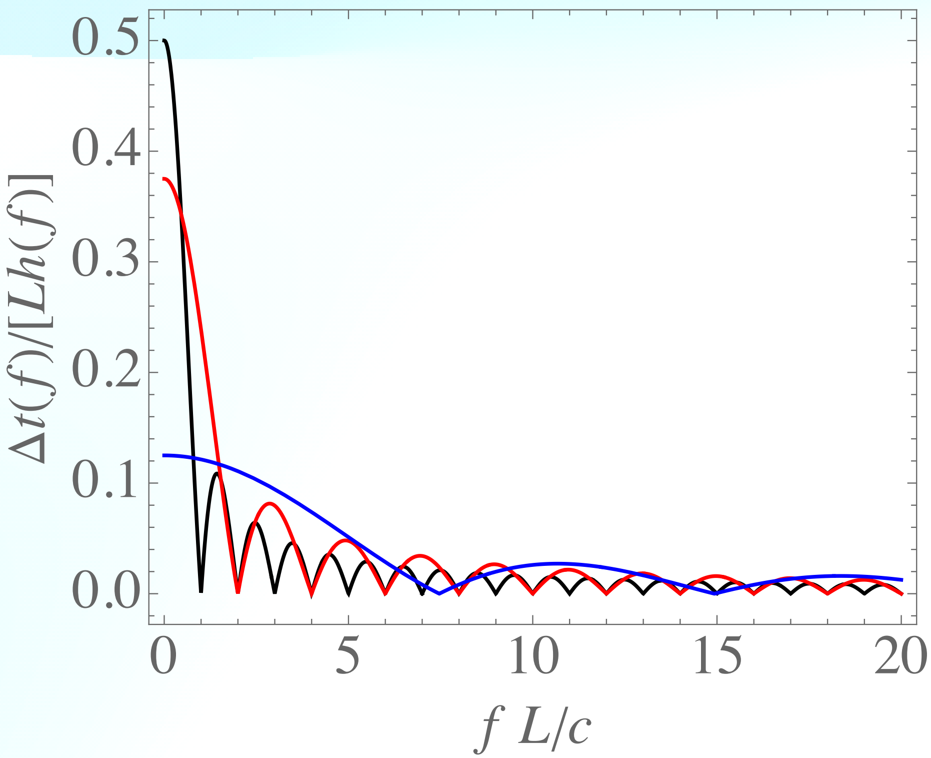


Long wavelength
Low frequency

$$\Delta t \approx \frac{L h_{ij} n^i n^j}{2}$$



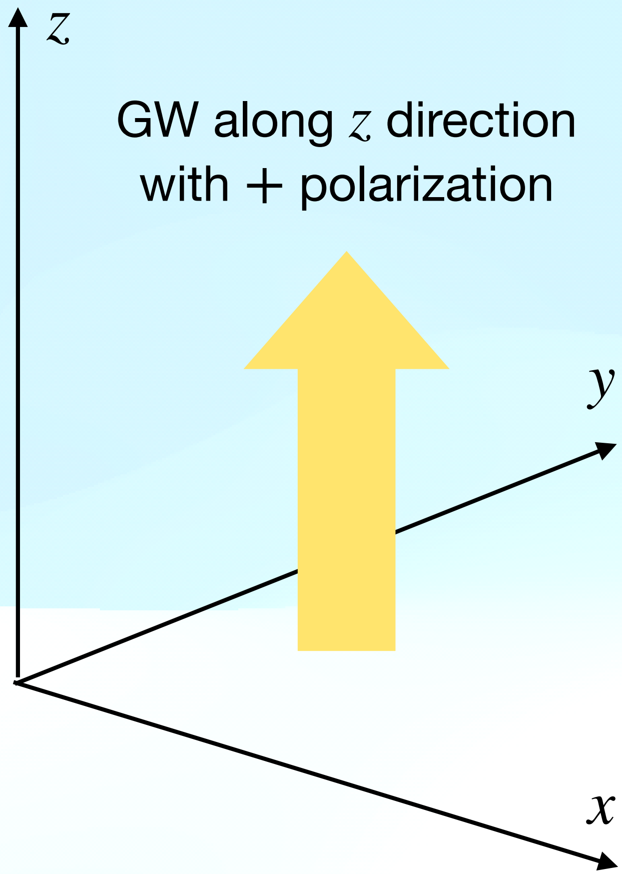
in general, for $h_{ij} = \sum_{p=+,\times} H_p e_{p,ij}(\mathbf{k}) e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$

$$\Delta t = \frac{H_p e_{p\,ij}(\mathbf{k}) n^i n^j}{2} \frac{e^{-i\omega(t+L)+i\mathbf{k}\cdot\mathbf{x}_B} - e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}_A}}{-i(\omega - \mathbf{k}\cdot\mathbf{n})}$$

polarization matching
$\mathbf{n} \perp \mathbf{k}$

phase matching
$\mathbf{n} \propto \mathbf{k}$

Local Lorentz Frame



$$C_{00} = \frac{x^2 - y^2}{2}$$

$$C_{0k} = \frac{1}{3}(-xz, yz, x^2 - y^2)$$

$$C_{jk} = \frac{1}{6}\begin{pmatrix} z^2 & 0 & -xz \\ 0 & -z^2 & yz \\ -xz & yz & -(x^2 - y^2) \end{pmatrix}$$

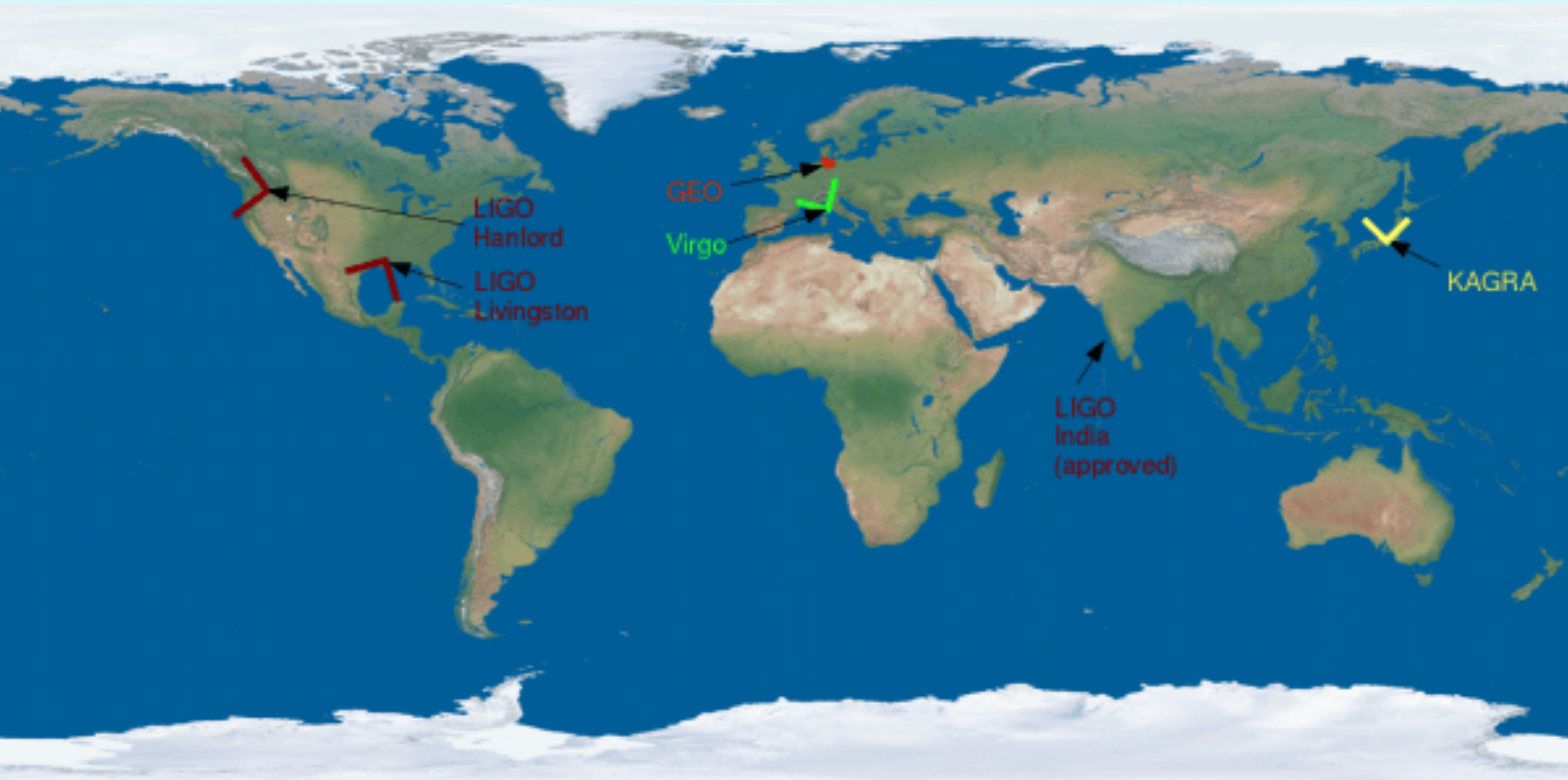
$$\|h_{\mu\nu}^{\mathrm{LLF,LW}}\| = \begin{pmatrix} C_{00} & C_{0k} \\ C_{j0} & C_{jk} \end{pmatrix} \ddot{h}_+(t)$$

free masses
(initially at rest)

rigid object

light propagation?

Ground-Based Laser Interferometers



Ground-Based Laser Interferometers

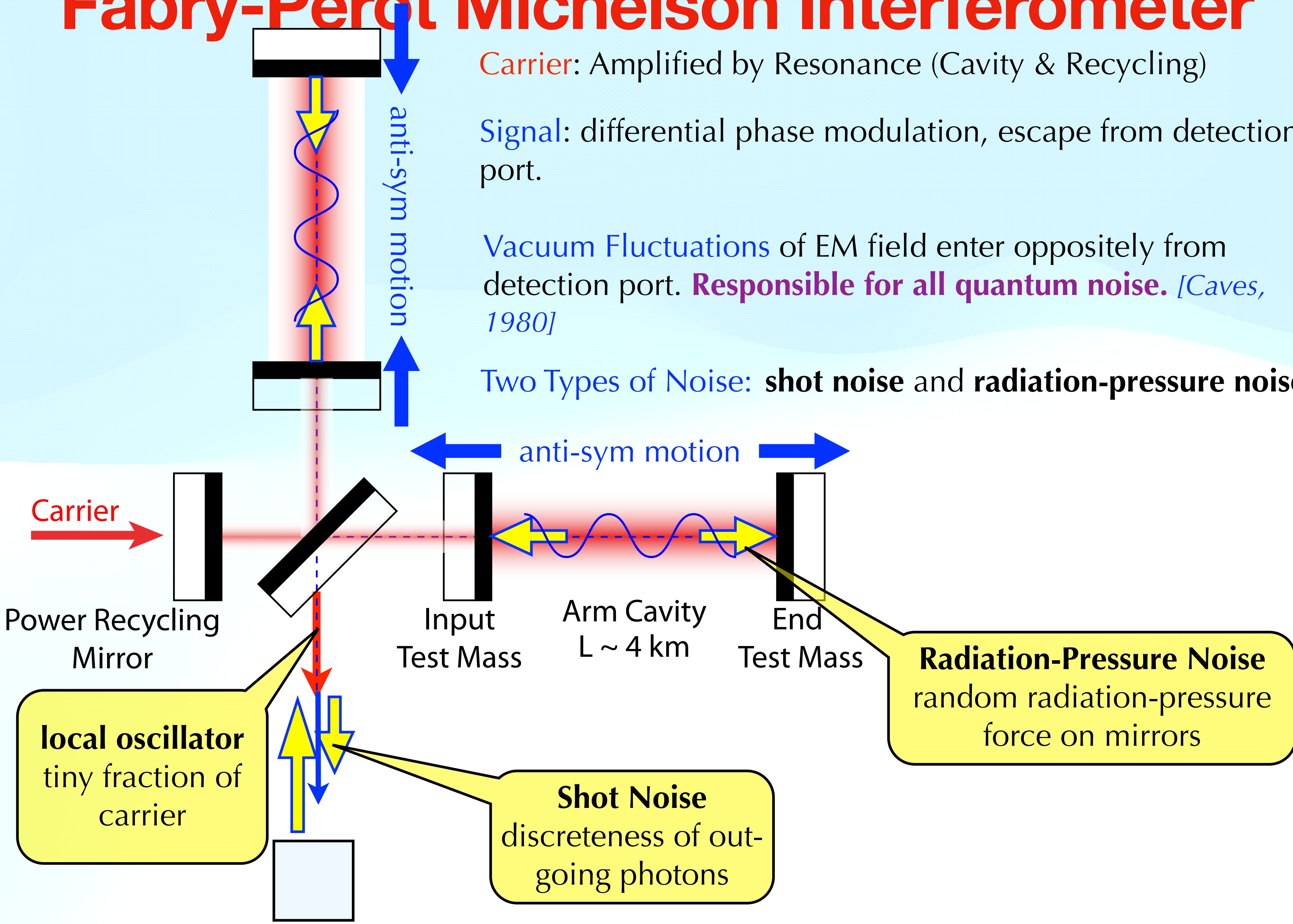
Fabry-Perot Michelson Interferometer

Carrier: Amplified by Resonance (Cavity & Recycling)

Signal: differential phase modulation, escape from detection port.

Vacuum Fluctuations of EM field enter oppositely from detection port. **Responsible for all quantum noise.** *[Caves, 1980]*

Two Types of Noise: **shot noise** and **radiation-pressure noise**



Quantum Limit: Sensing versus Back Action

Must introduce light field to fully describe measurement process

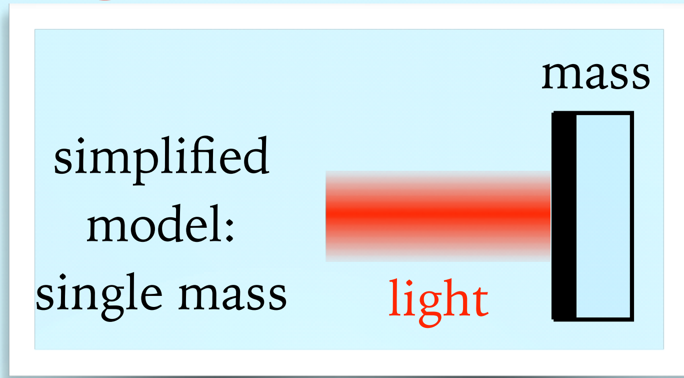
Sensing Noise: Position Uncertainty from N photons: $\Delta x \sim \frac{1}{\sqrt{2N}} \frac{\lambda}{2\pi}$

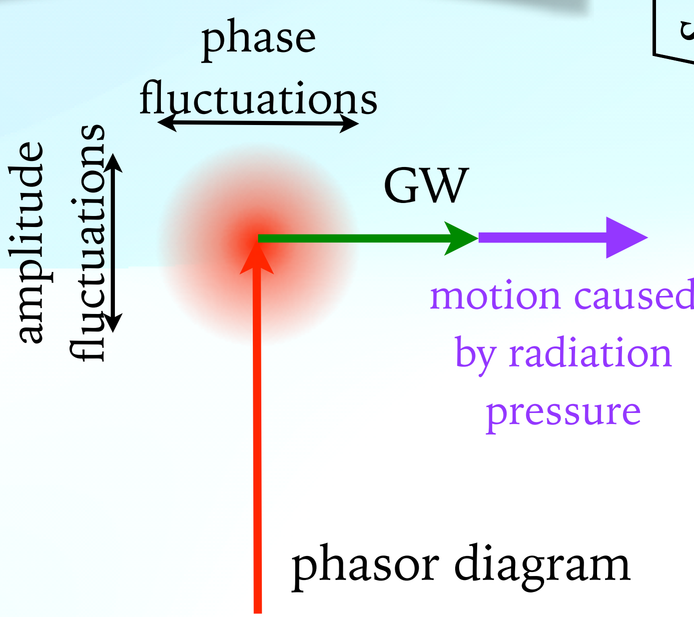
Back-Action Noise: Momentum Uncertainty from N photons: $\Delta p \sim \sqrt{\frac{N}{2}} \hbar \frac{2\pi}{\lambda}$

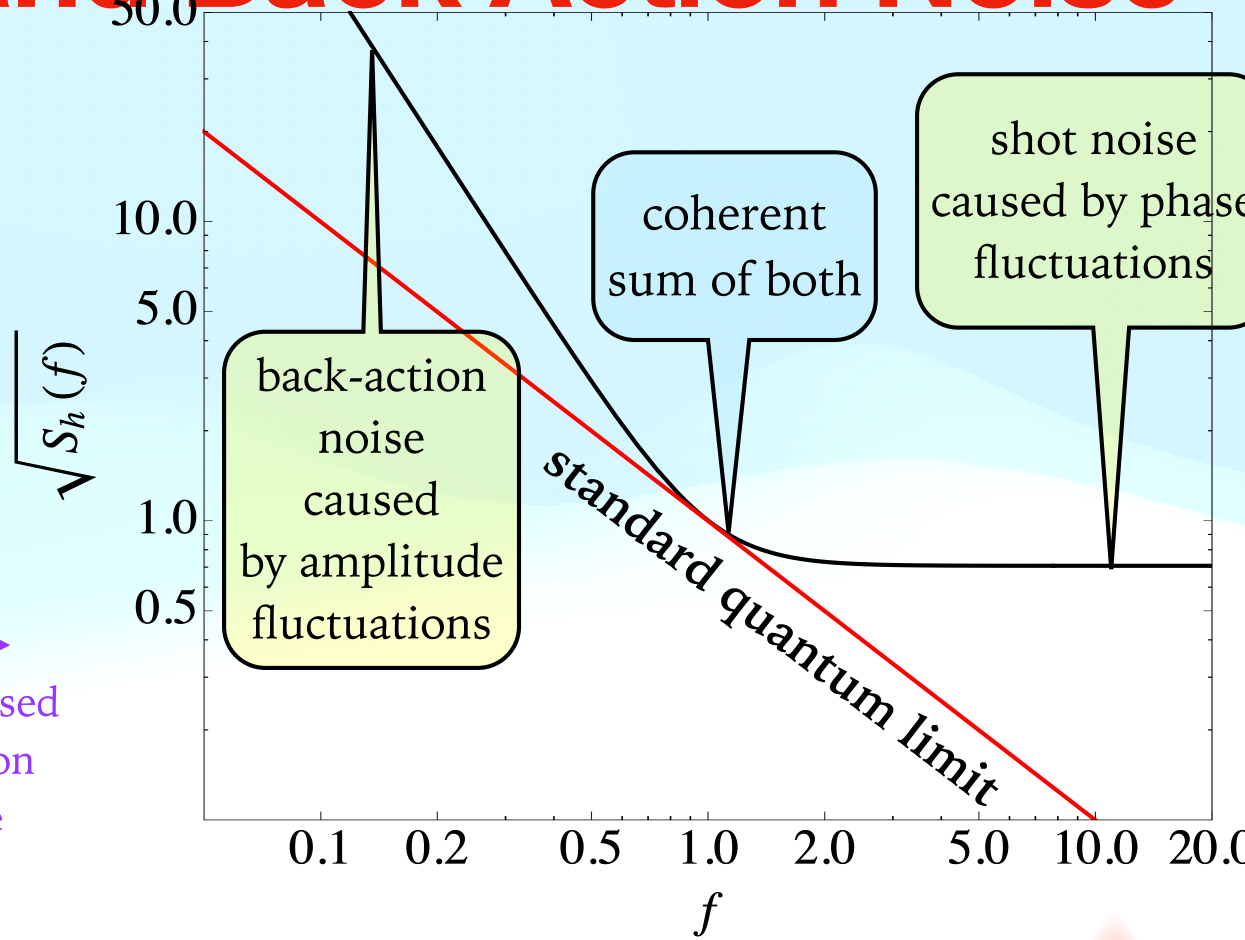
Total Uncertainty: $\Delta x_{\text{tot}} \sim \sqrt{\Delta x^2 + \frac{\Delta p^2 T^2}{M^2}} = \sqrt{\frac{1}{2N}\left(\frac{\lambda}{2\pi}\right)^2 + \frac{\hbar^2 T^2}{M^2}\frac{N}{2}\left(\frac{2\pi}{\lambda}\right)^2}$

Photon's fluctuations also lead to the Standard Quantum Limit: $\Delta x_{\text{tot}} \geq \Delta x_{\text{SQL}}$

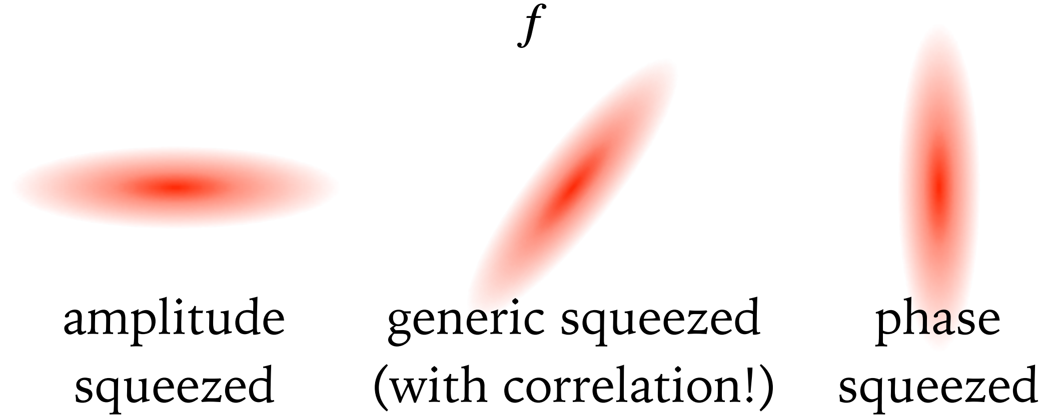
Shot Noise and Back Action Noise



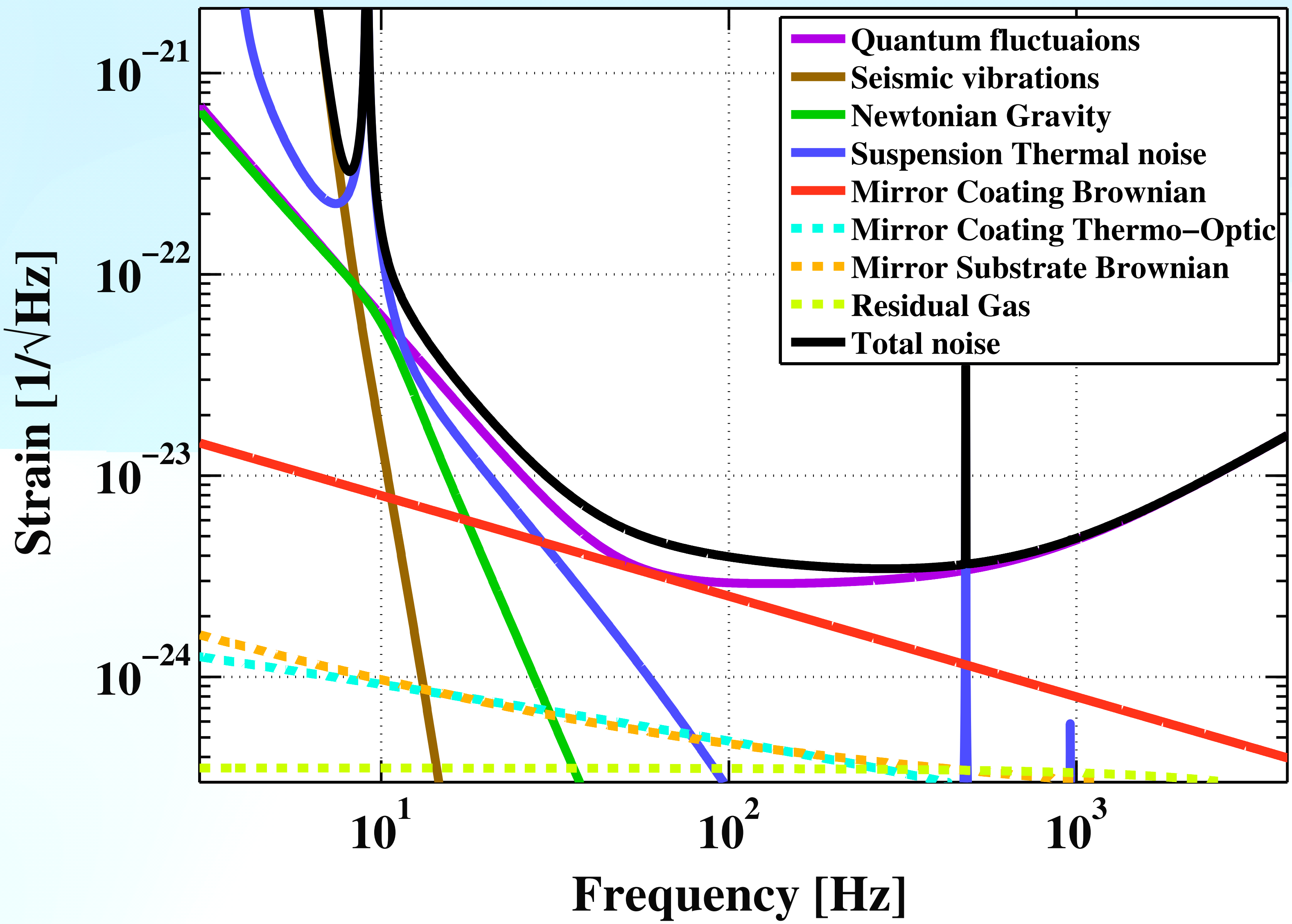


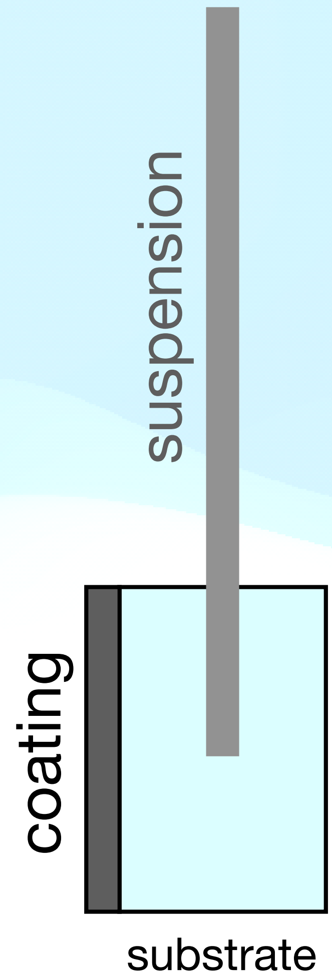


EM field can be prepared
into "squeezed states"



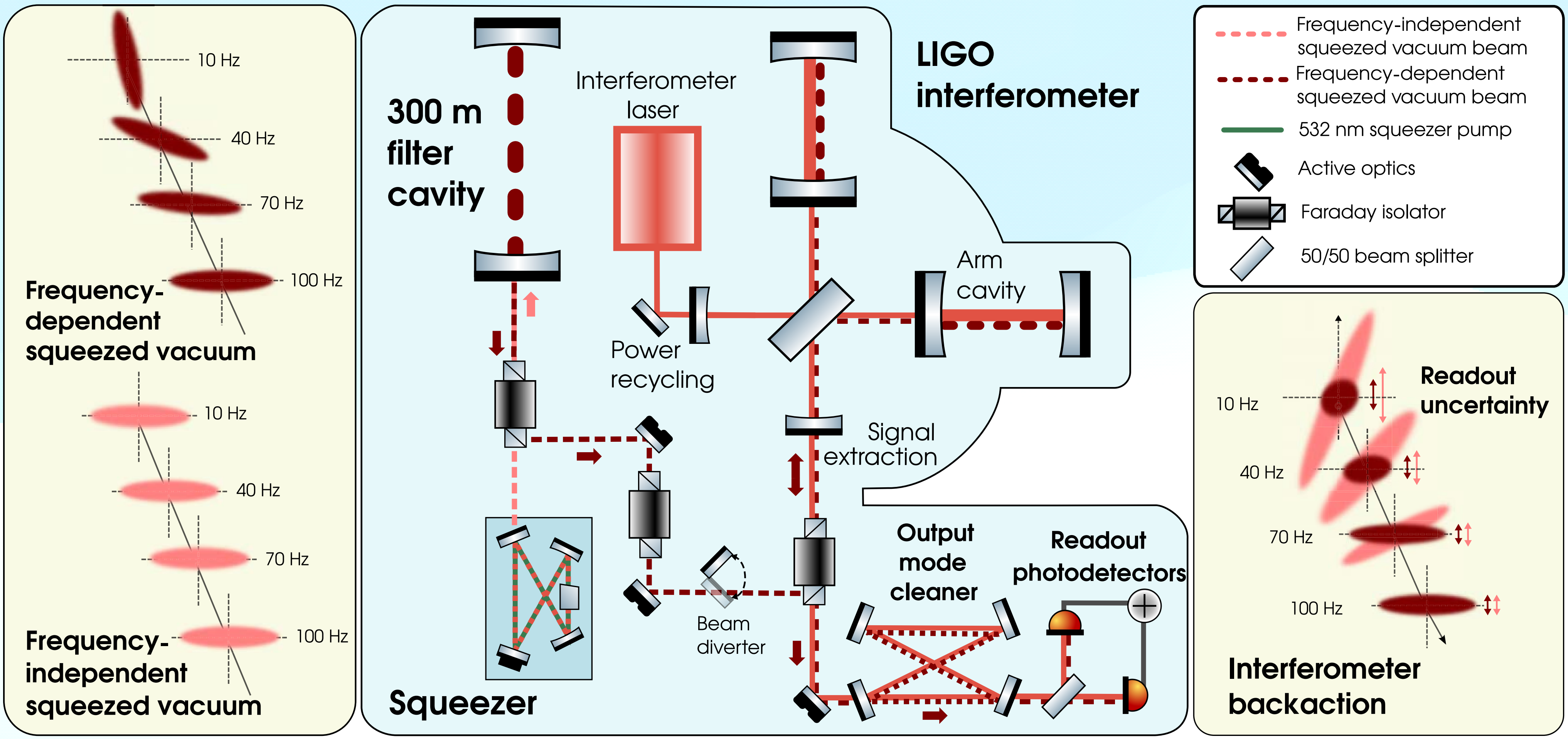
Other Types of Noise



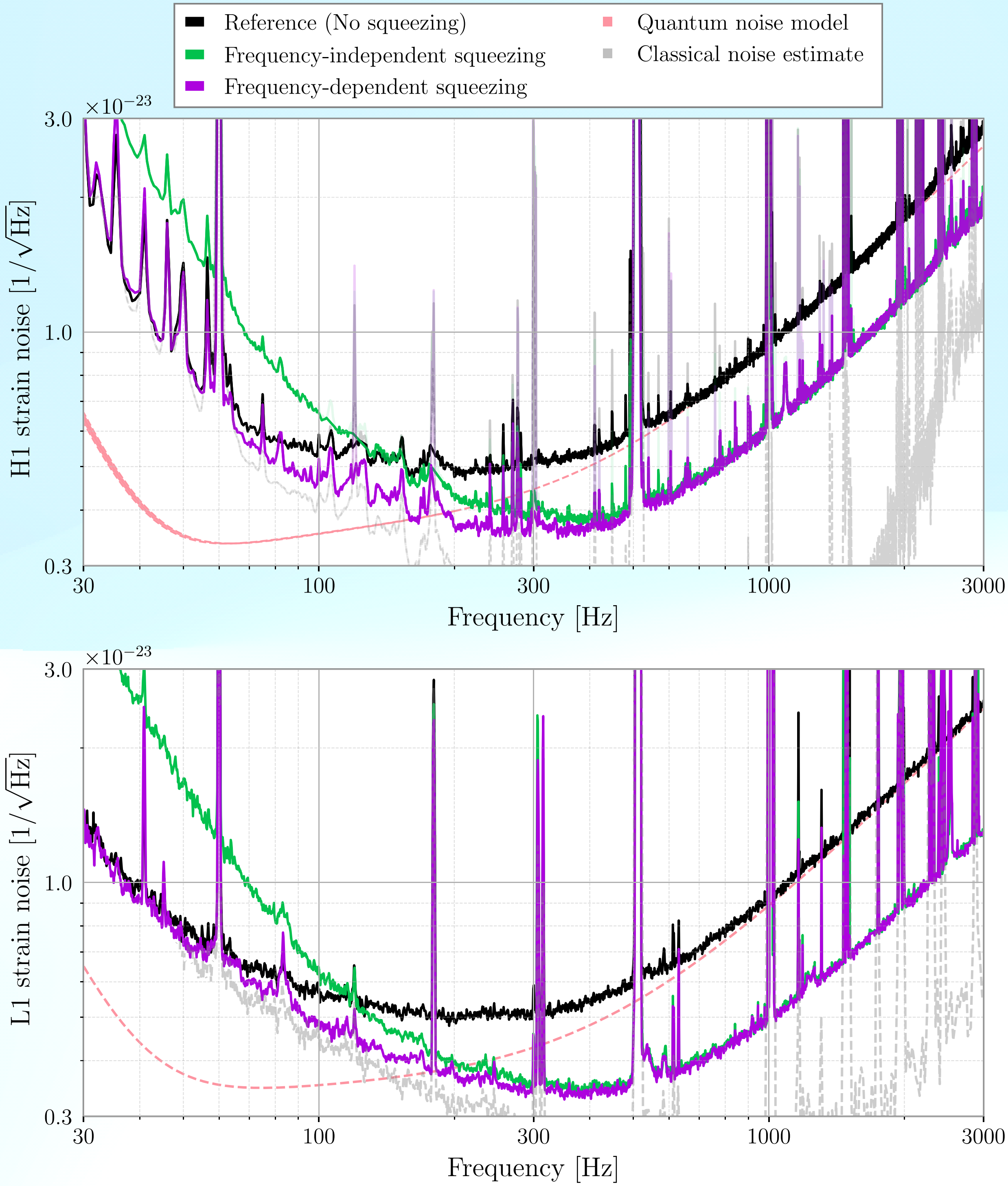


[LIGO Lab]

Frequency-Dependent Squeezing with LIGO



[Ganapathy et al., 2023]

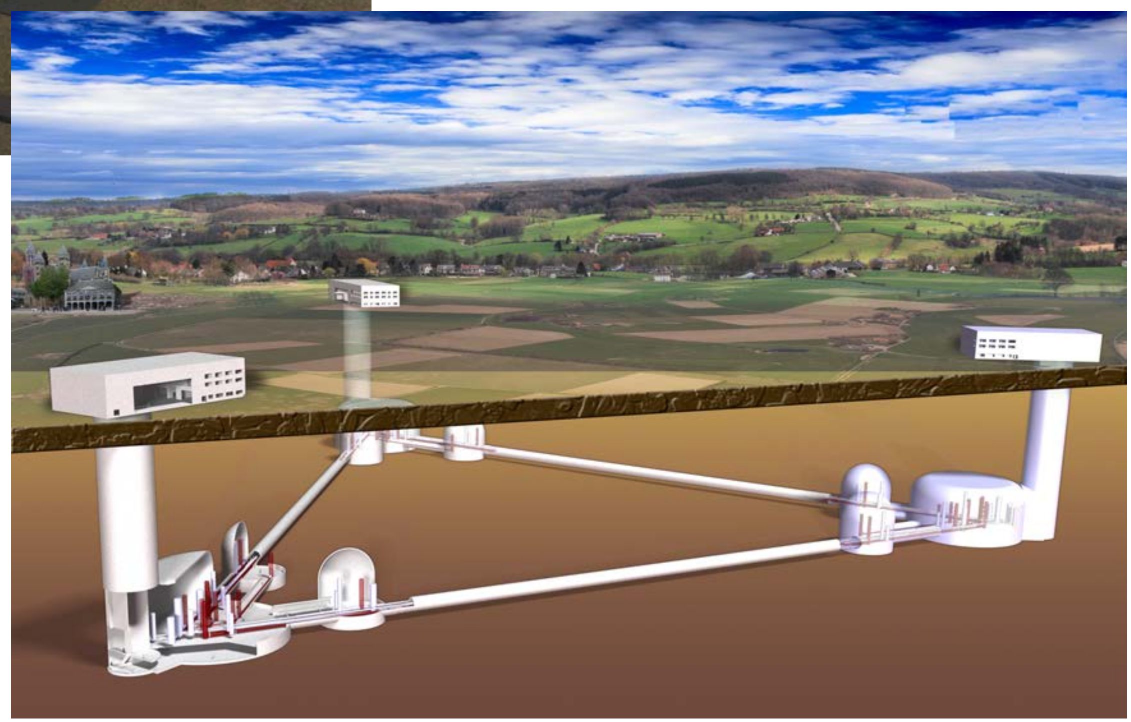


[Ganapathy et al., 2023]

Future Detectors

Cosmic Explorer

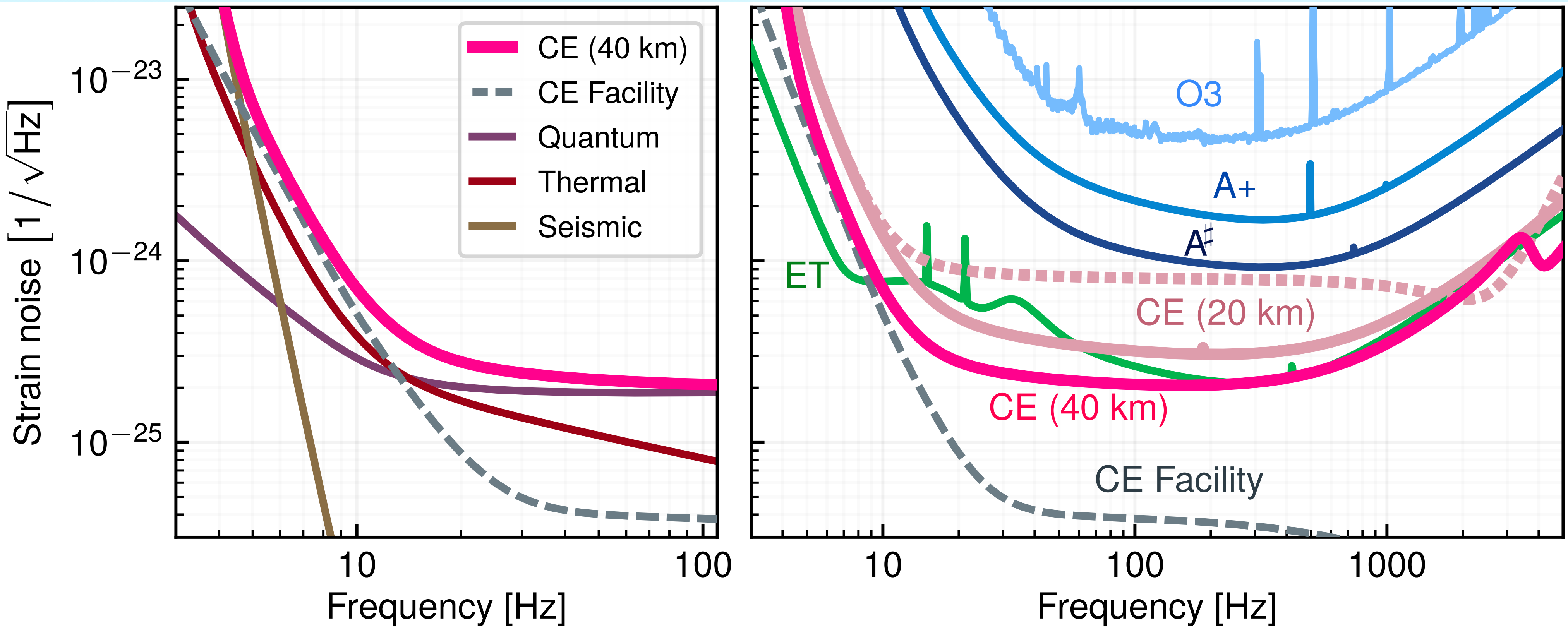
www.cosmicexplorer.org



Einstein Telescope

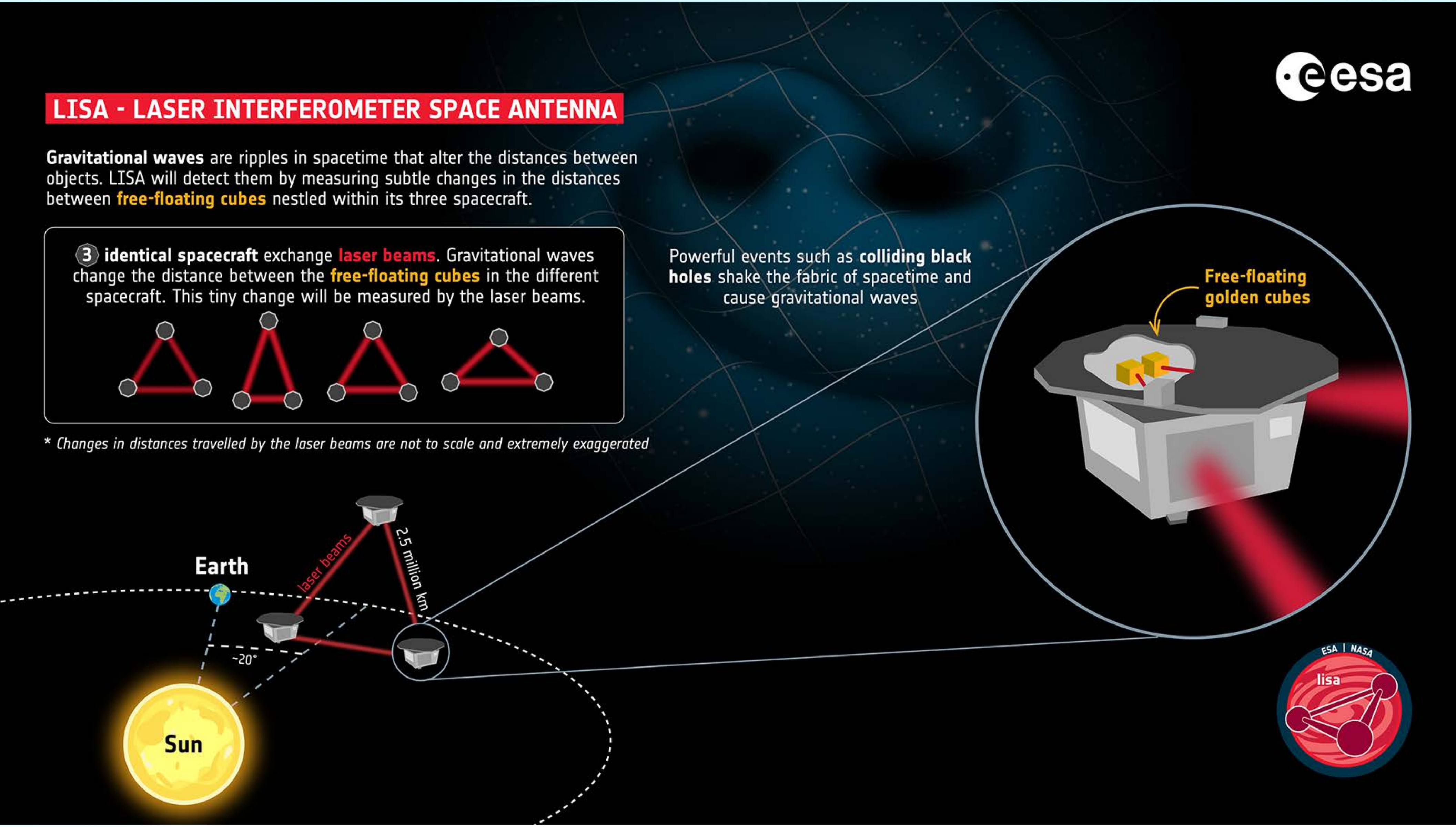
www.et-gw.eu

Future Detectors



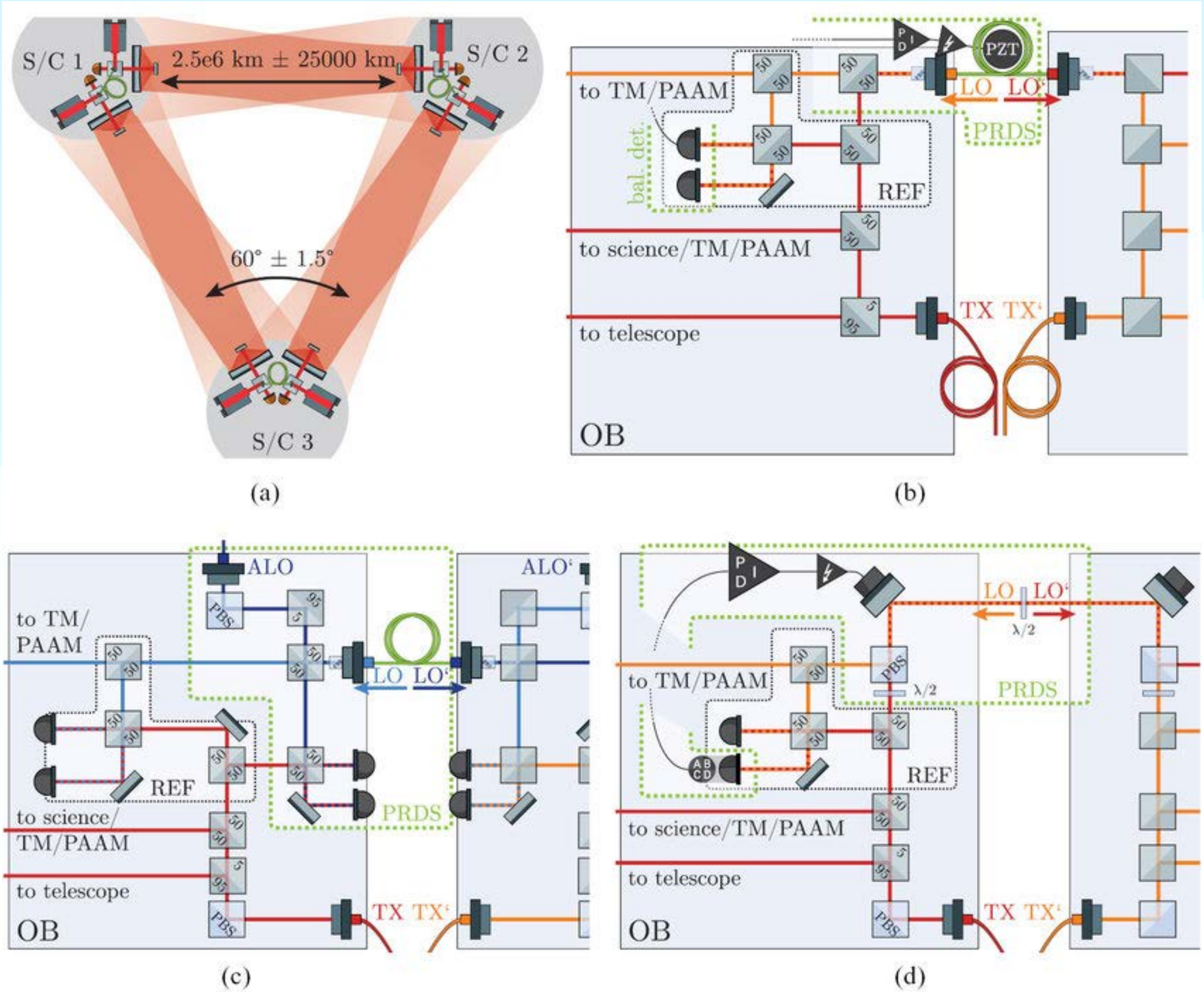
www.cosmicexplorer.org

Laser Interferometer Space Antenna



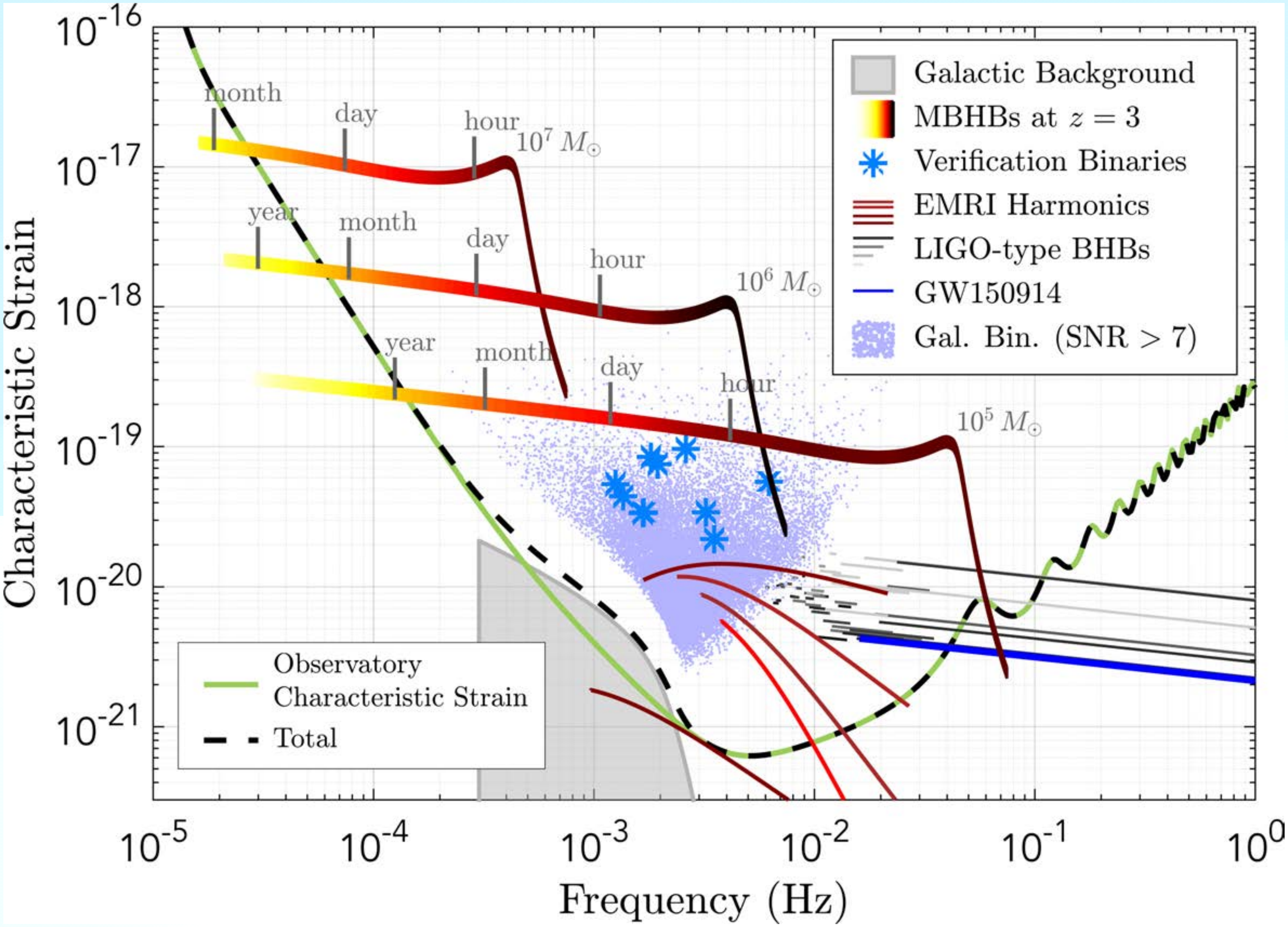
[ESA]

Laser Interferometer Space Antenna



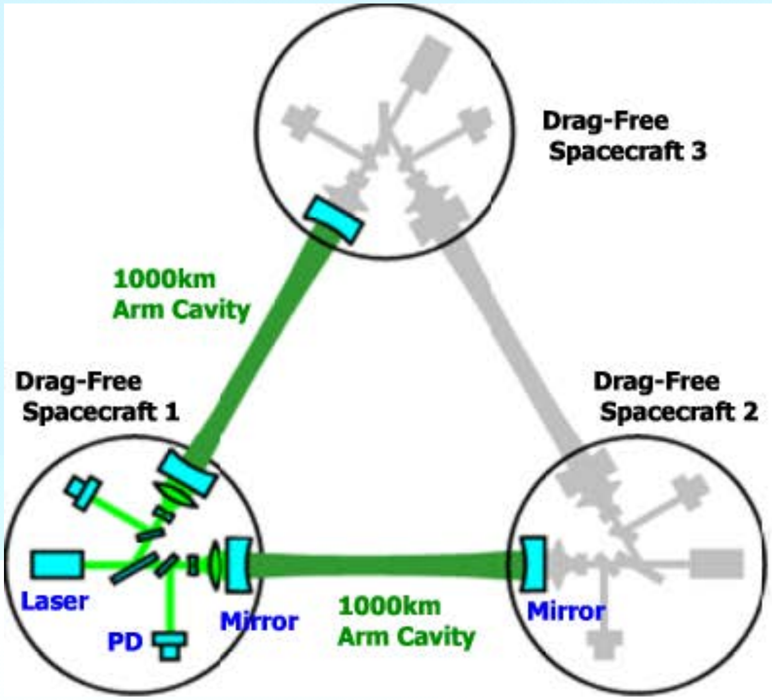
[ESA]

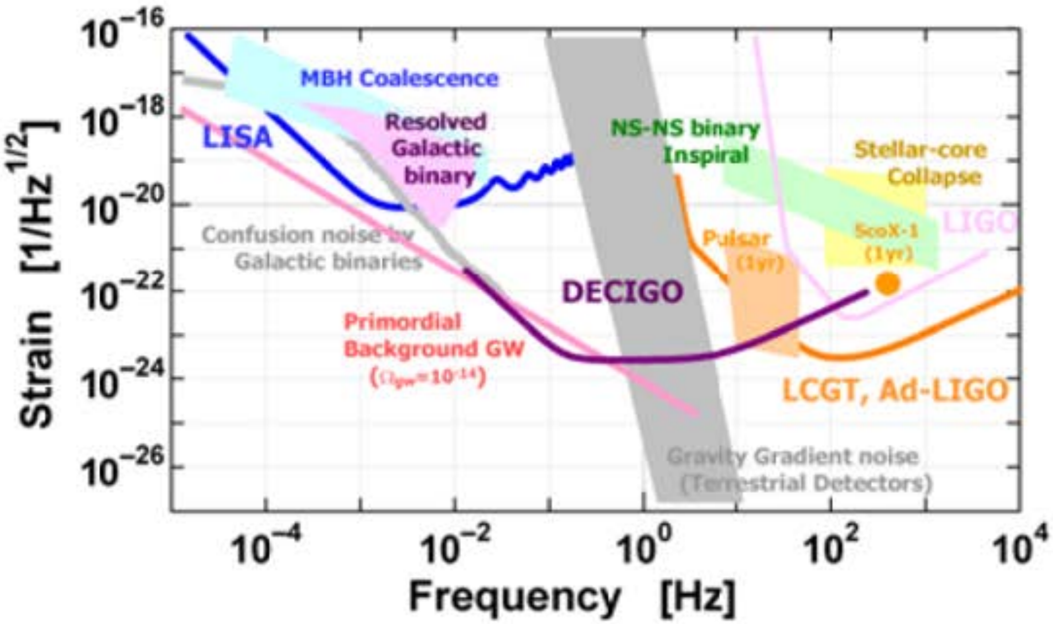
Laser Interferometer Space Antenna



$$\Delta t = \frac{H_p e_{p\,ij}(\mathbf{k}) n^i n^j}{2} \frac{e^{-i\omega(t+L)+i\mathbf{k}\cdot\mathbf{x}_B} - e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}_A}}{-i(\omega - \mathbf{k}\cdot\mathbf{n})}$$

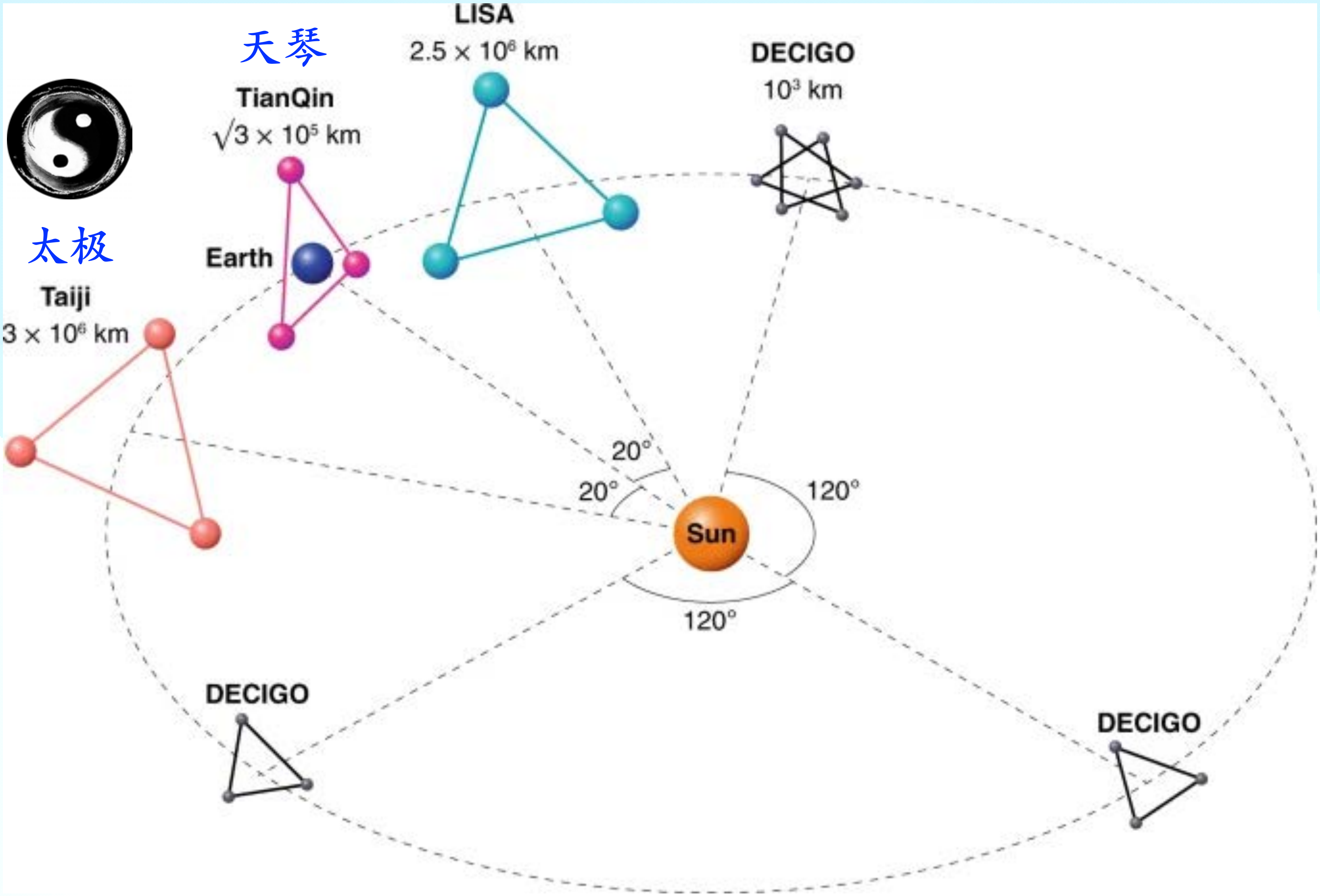
DECIGO



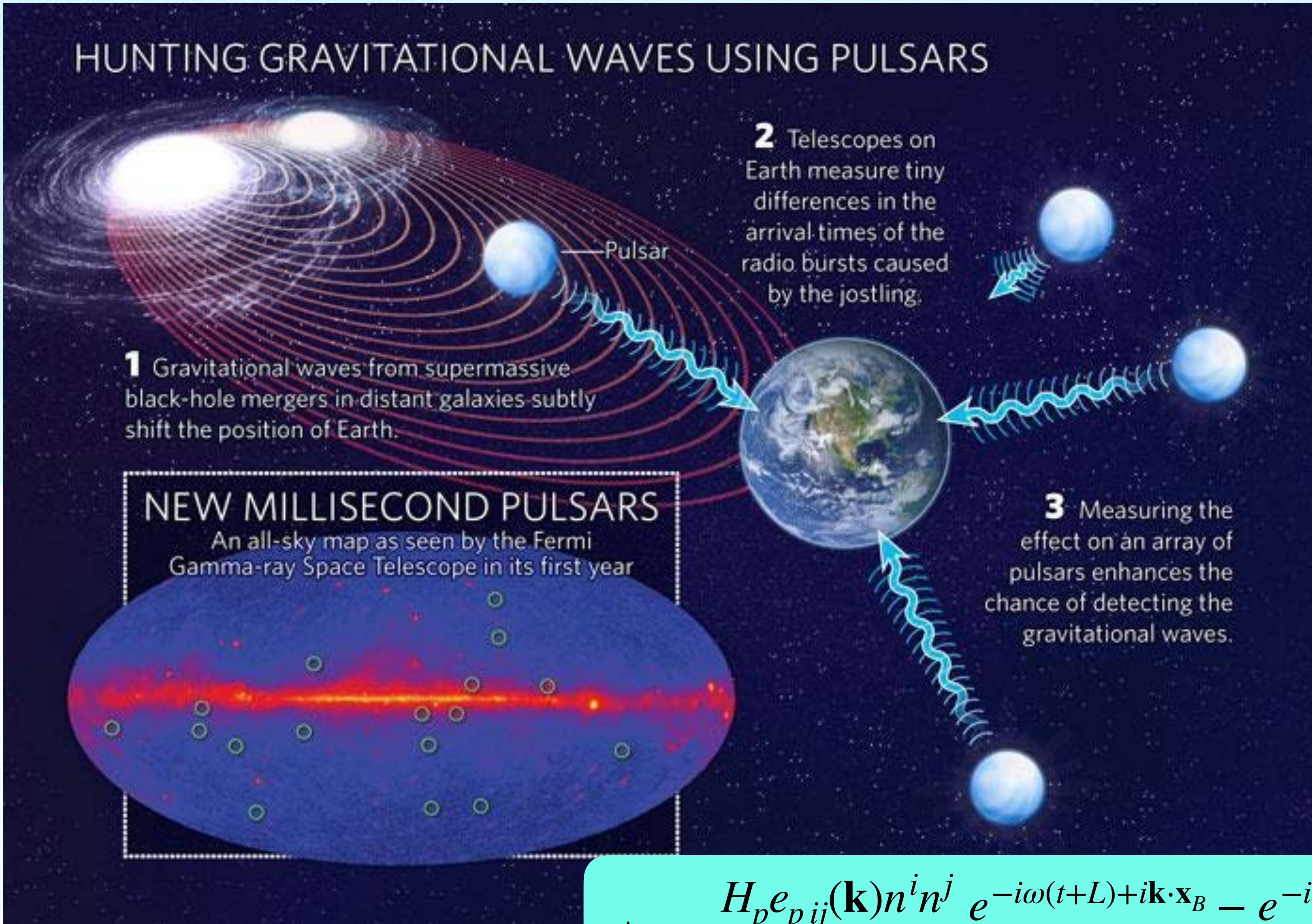


[Shuichi Sato and Seiji Kawamura]

Space Based Detectors



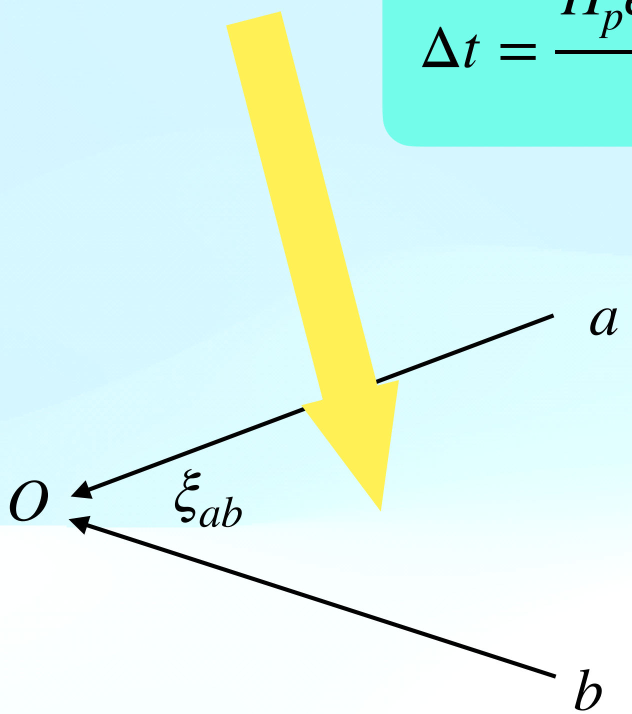
Pulsar Timing Array



$$\Delta t = \frac{H_p e_{p\,ij}(\mathbf{k}) n^i n^j}{2} \frac{e^{-i\omega(t+L)+i\mathbf{k}\cdot\mathbf{x}_B} - e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}_A}}{-i(\omega - \mathbf{k}\cdot\mathbf{n})}$$

Stochastic Background From PTA

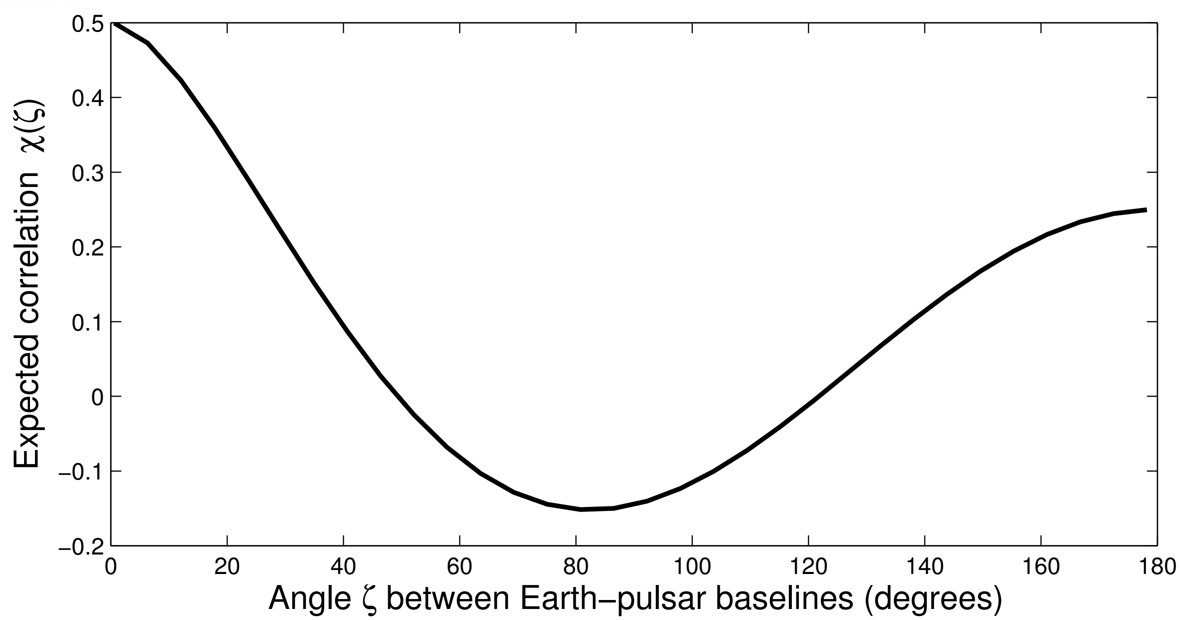
$$\Delta t = \frac{H_p e_{p\,ij}(\mathbf{k}) n^i n^j}{2} \frac{e^{-i\omega(t+L)+i\mathbf{k}\cdot\mathbf{x}_B} - e^{-i\omega t+i\mathbf{k}\cdot\mathbf{x}_A}}{-i(\omega - \mathbf{k}\cdot\mathbf{n})}$$



$$\Gamma(\theta) = \frac{1}{3} - \frac{1}{6}\left(\frac{1-\cos\theta}{2}\right) + \left(\frac{1-\cos\theta}{2}\right)\log\left(\frac{1-\cos\theta}{2}\right)$$

Hellings-Downs Curve [Hellings & Downs, 1983]

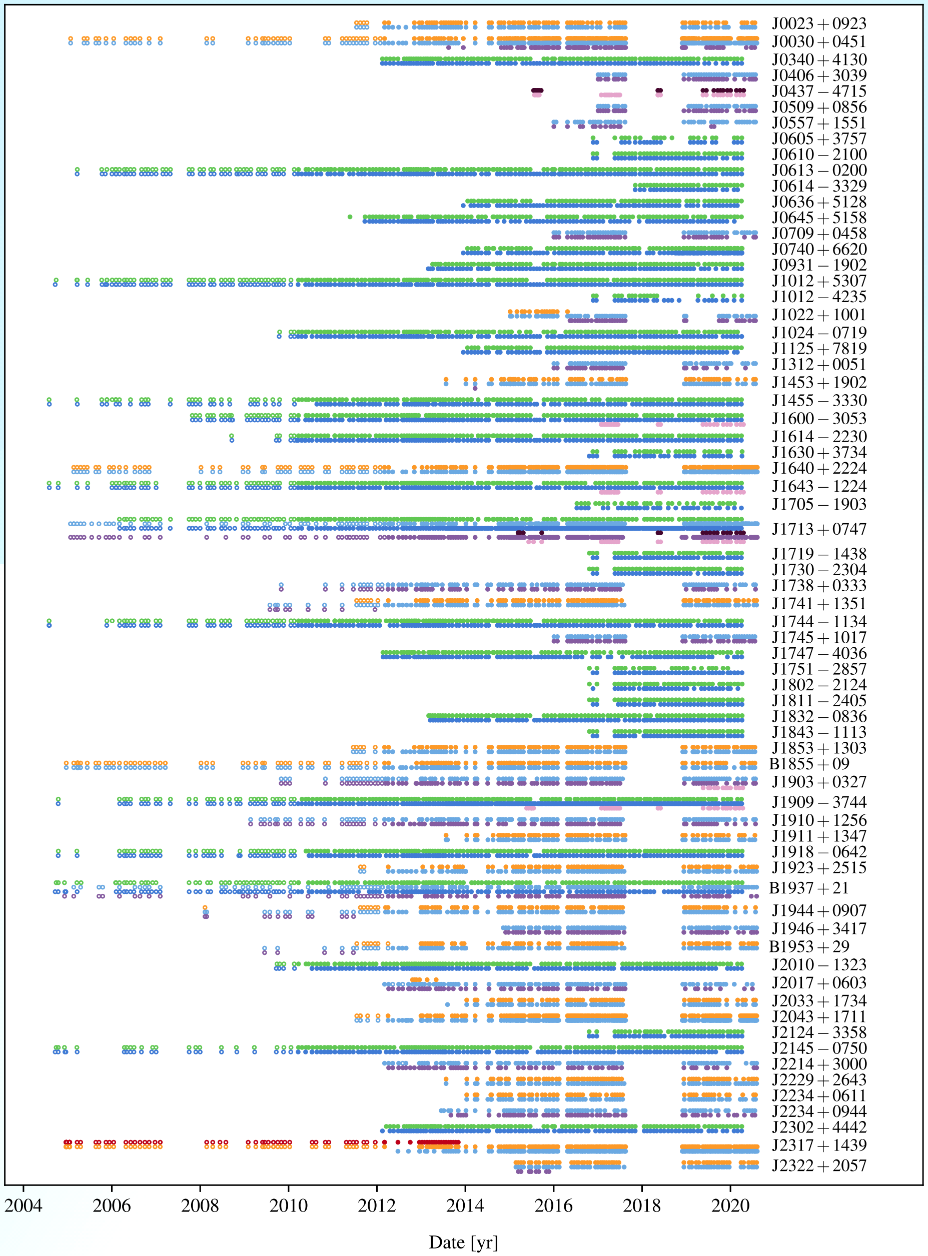
$$\frac{S_{\Delta t_a \Delta t_b}}{\sqrt{S_{\Delta t_a} S_{\Delta t_b}}} \xrightarrow[\text{fixing } \xi_{ab}]{\text{randomize over } h \text{ or } ab} \Gamma(\xi_{ab})$$

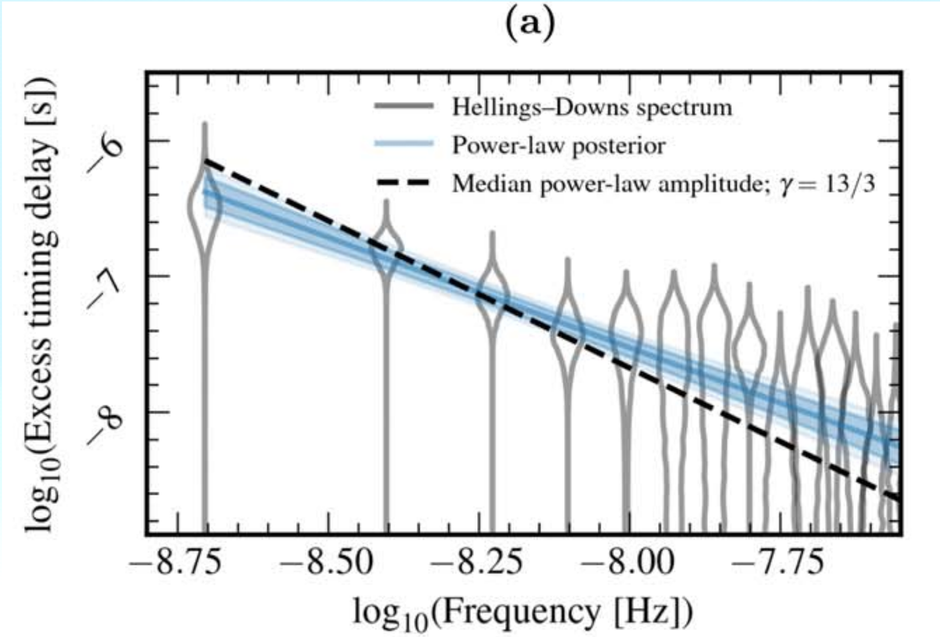


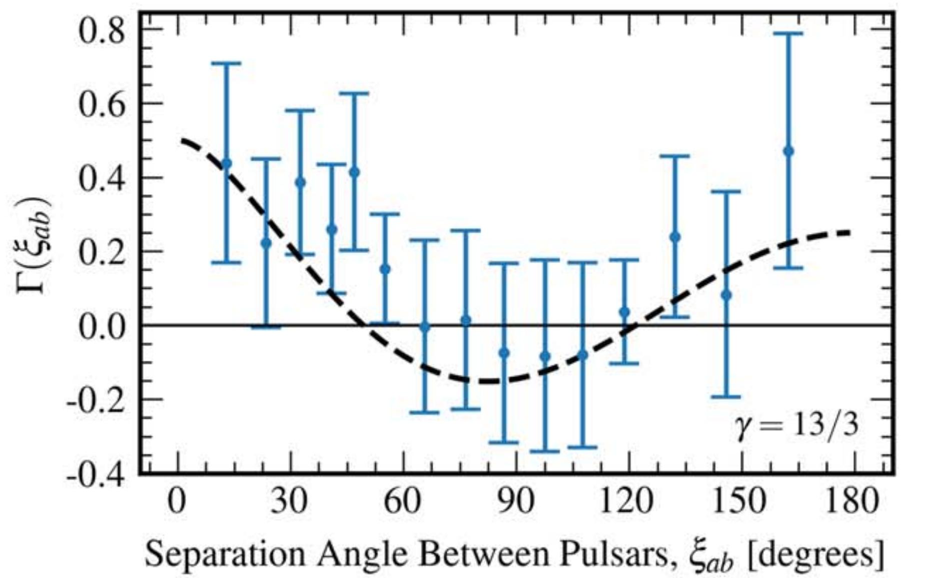
[Jenet & Romano, 2014]

NanoGrav 15 Year Result

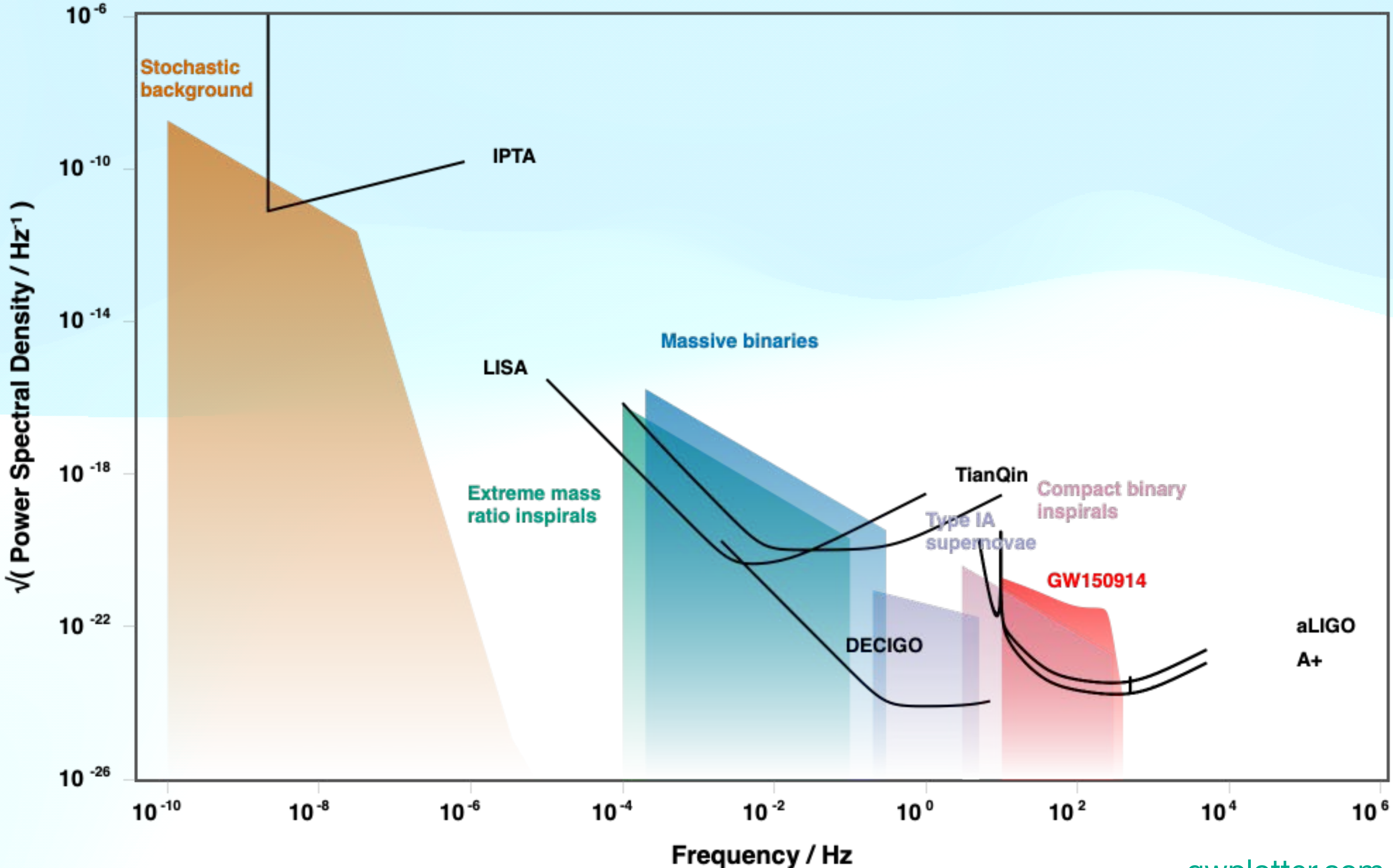
Gabriella Agazie *et al* 2023 *ApJL* **951**







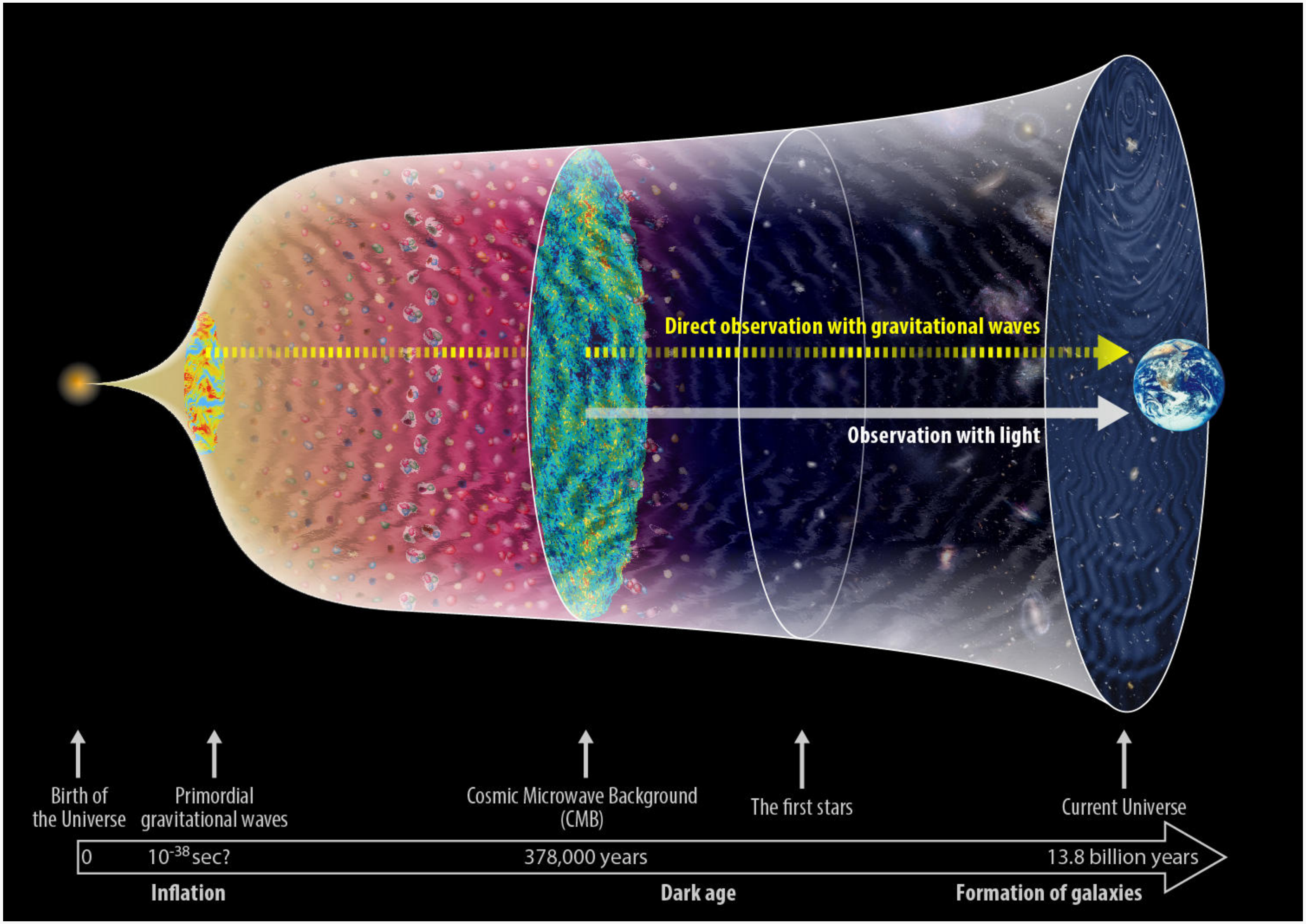
The Gravitational-Wave Spectrum



gwplotter.com

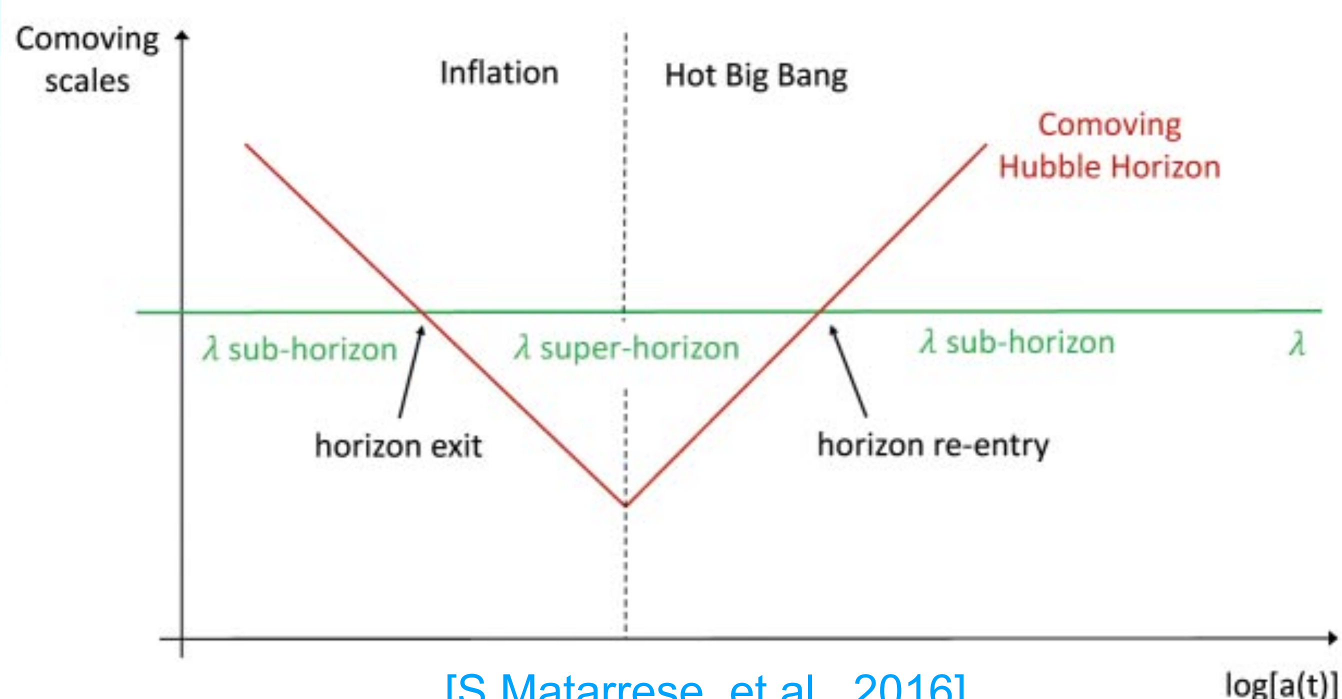
Sources of Gravitational Waves

[National Astronomical Observatory of Japan, gwpo.nao.ac.jp]



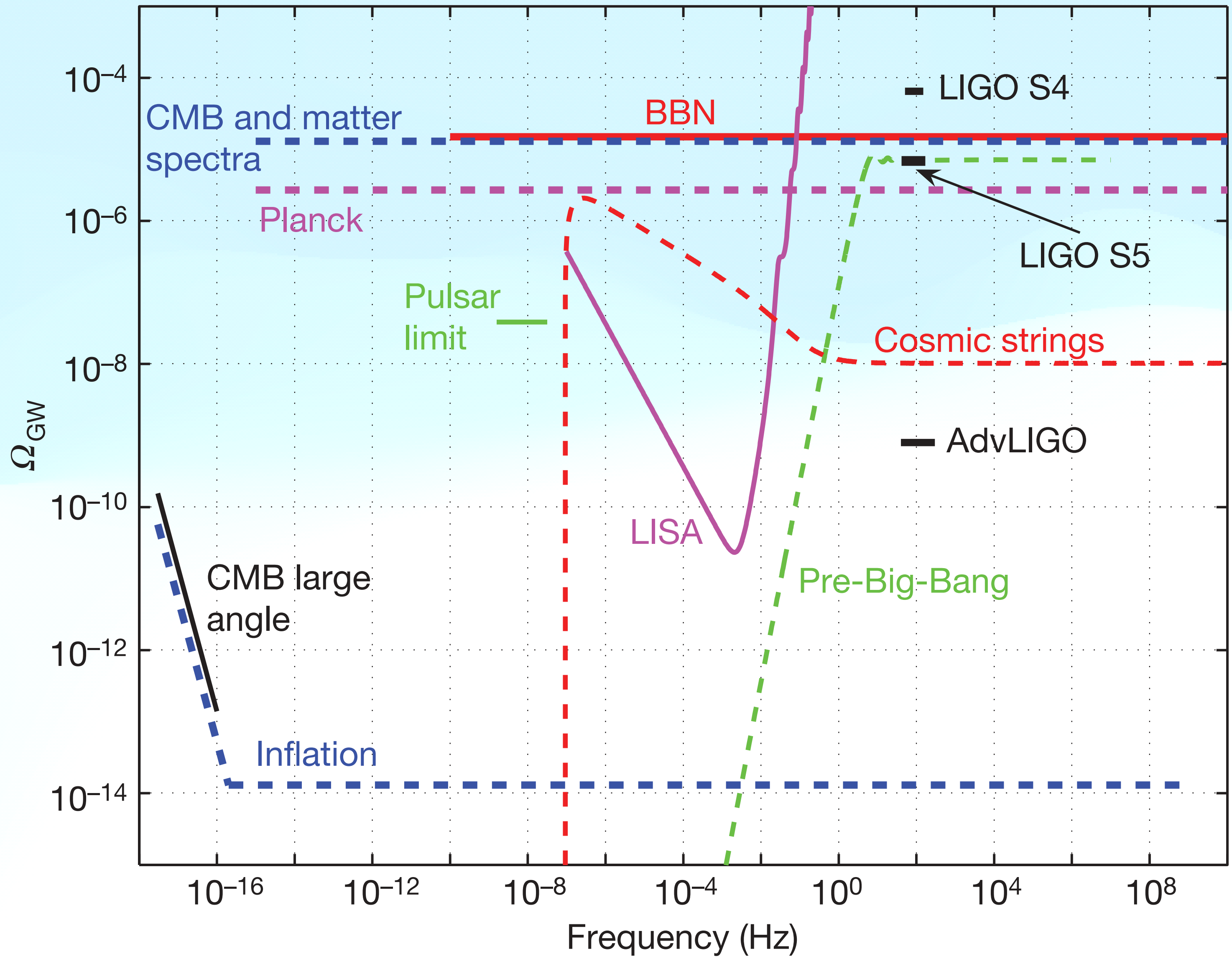
Inflation and Stochastic GW background

- Motivation for inflation
  - Universe is nearly spatially flat right now
  - the CMB is very homogeneous
- Inflation
  - The universe expanded very fast
  - Slow-roll inflation: a period where expansion rate is constant
  - Drives fluctuations in energy density and curvature of the universe.
  - Generates stochastic gravitational waves



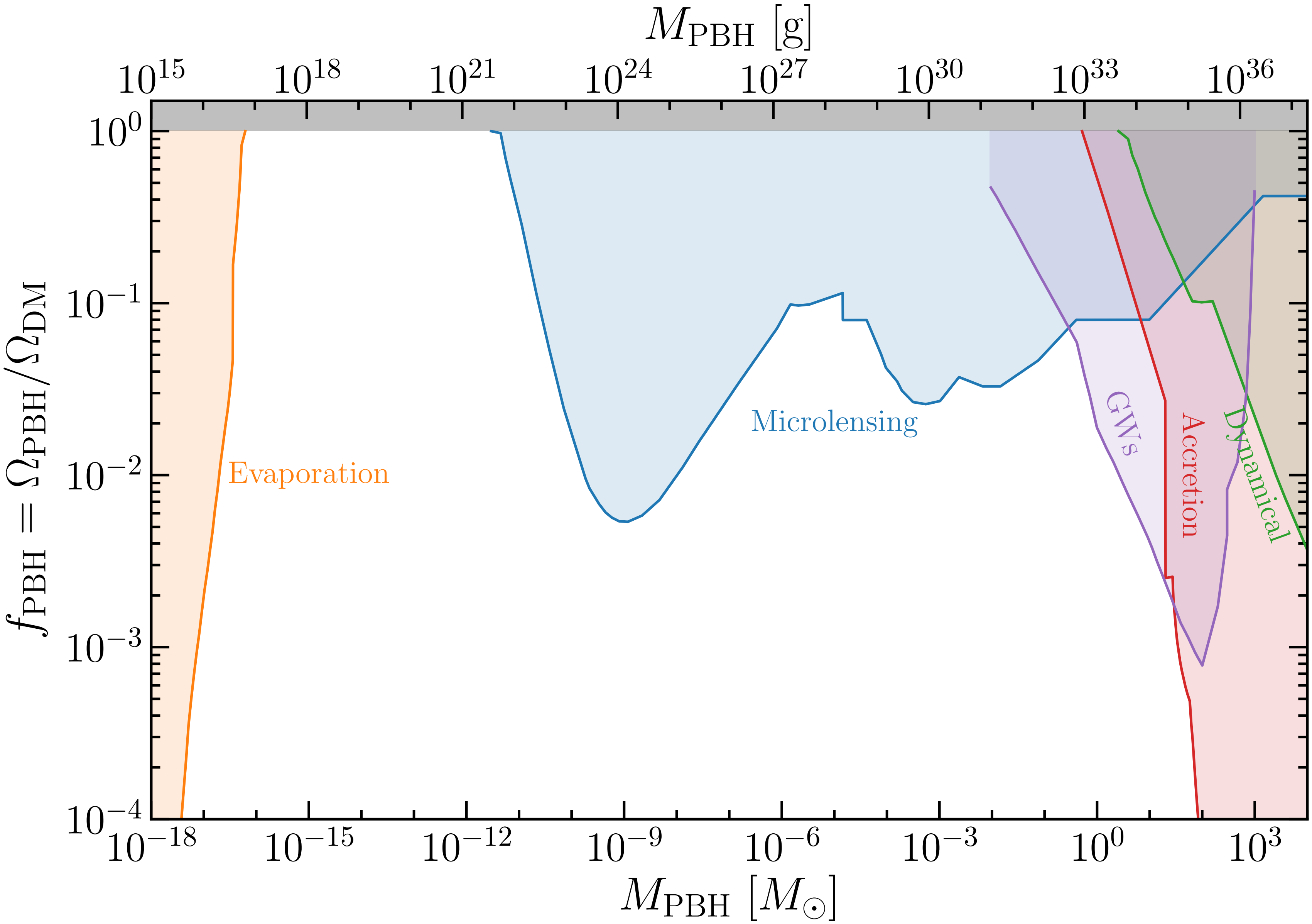
[S Matarrese, et al., 2016]

Inflation and Stochastic GW background



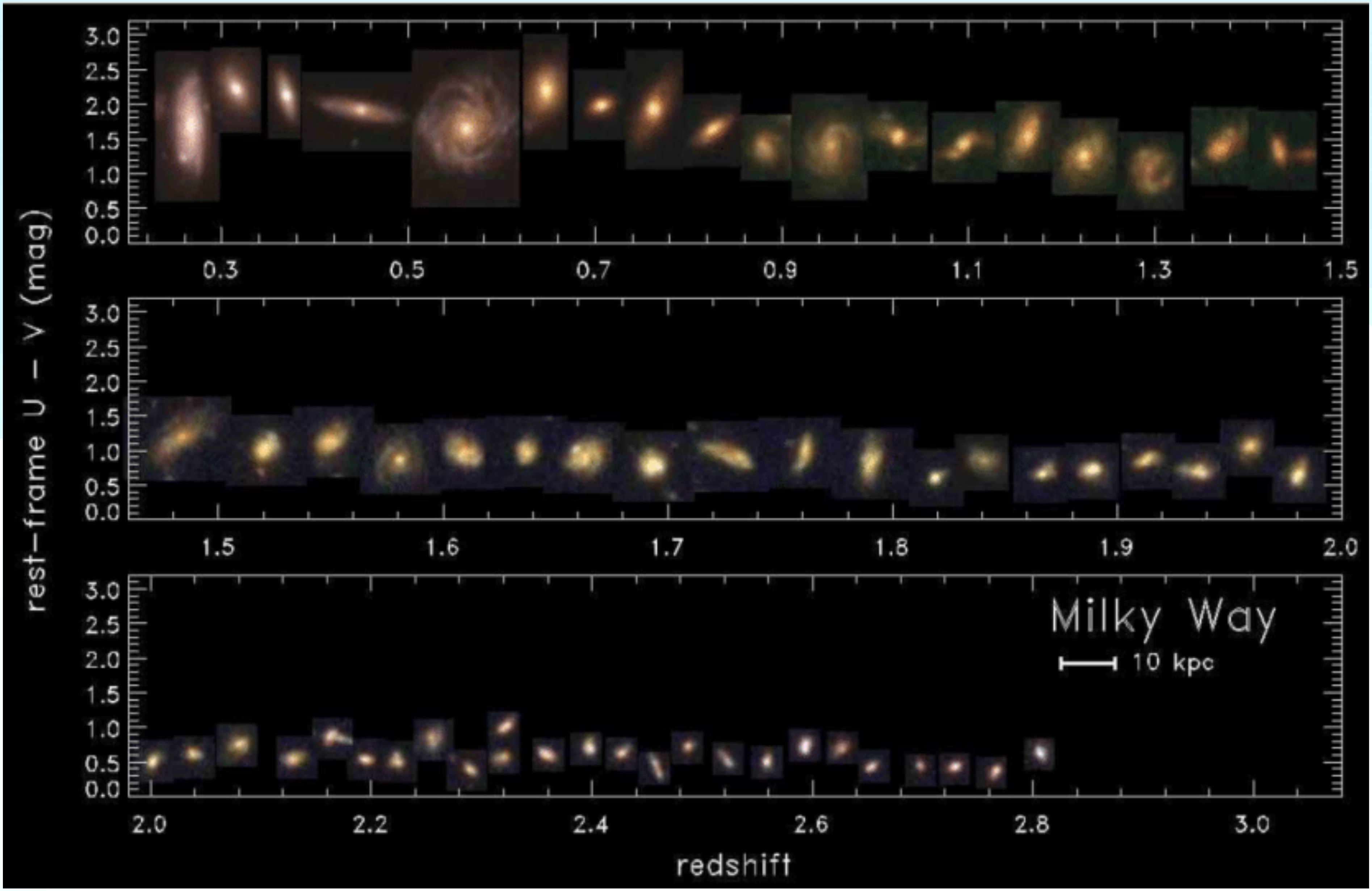
[LIGO-VIRGO Collaboration, 2009]

Constraints on Primordial Black Holes

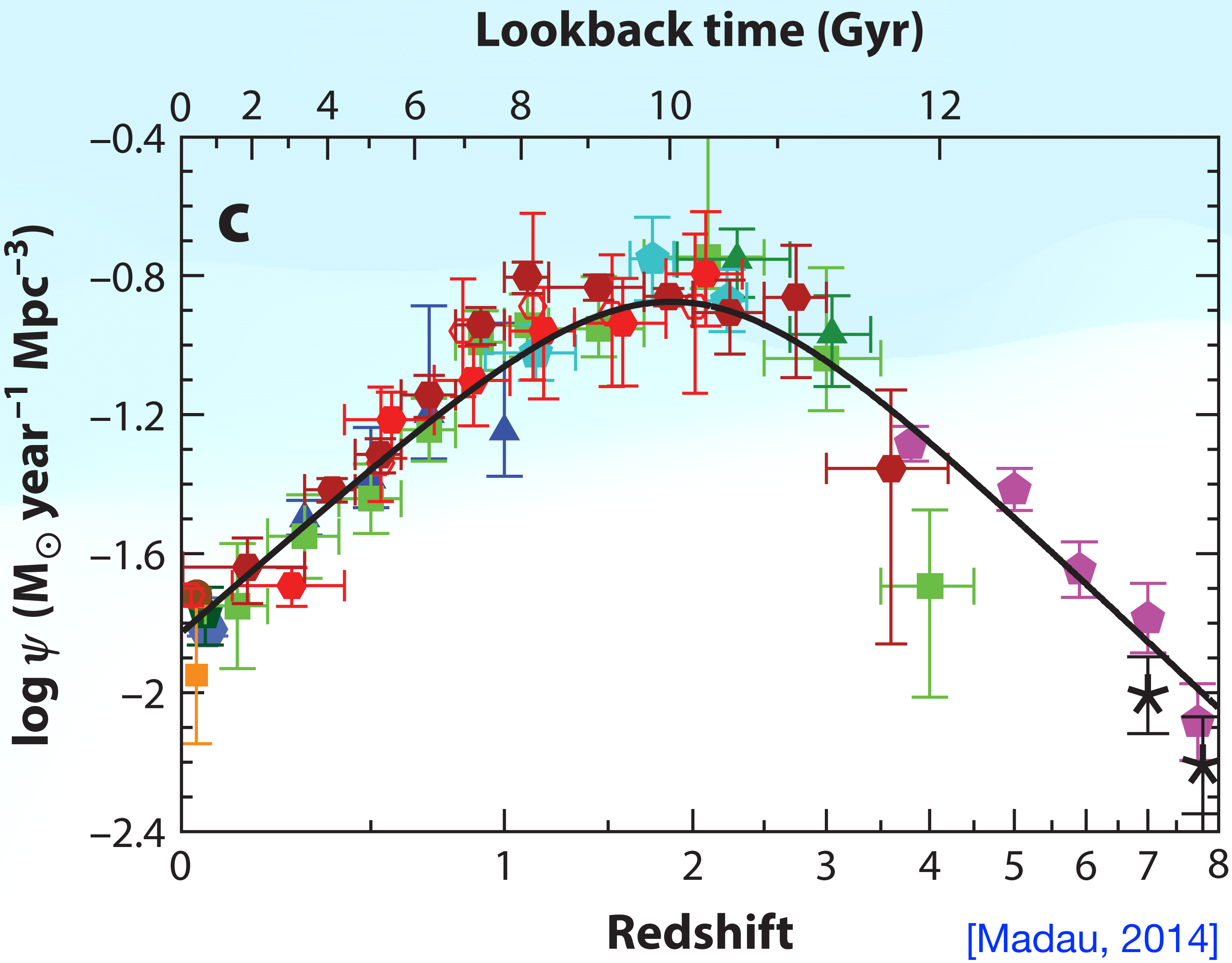


[Green & Kavanagh, 2021]

Formations of Galaxies

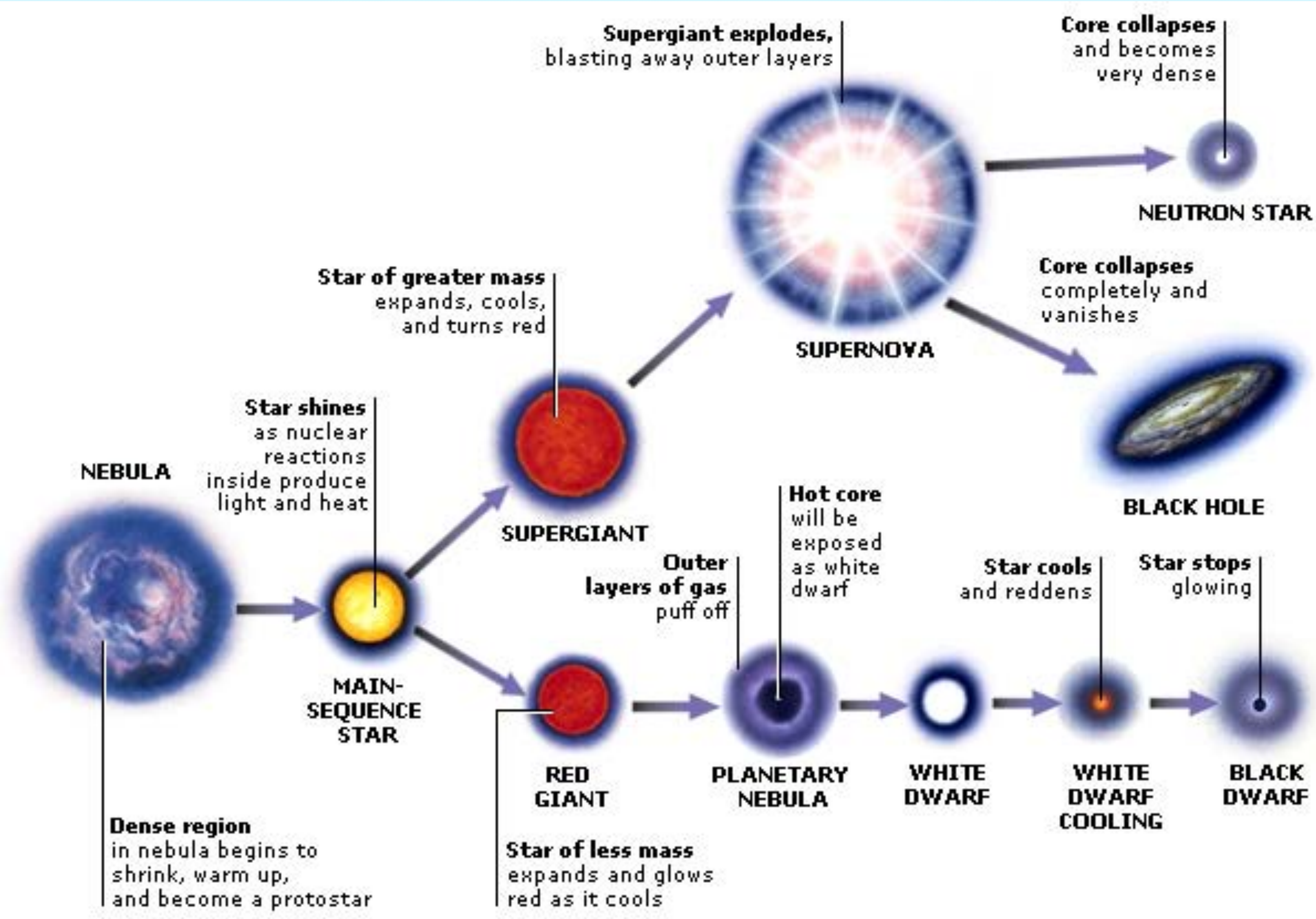


Stellar formation history

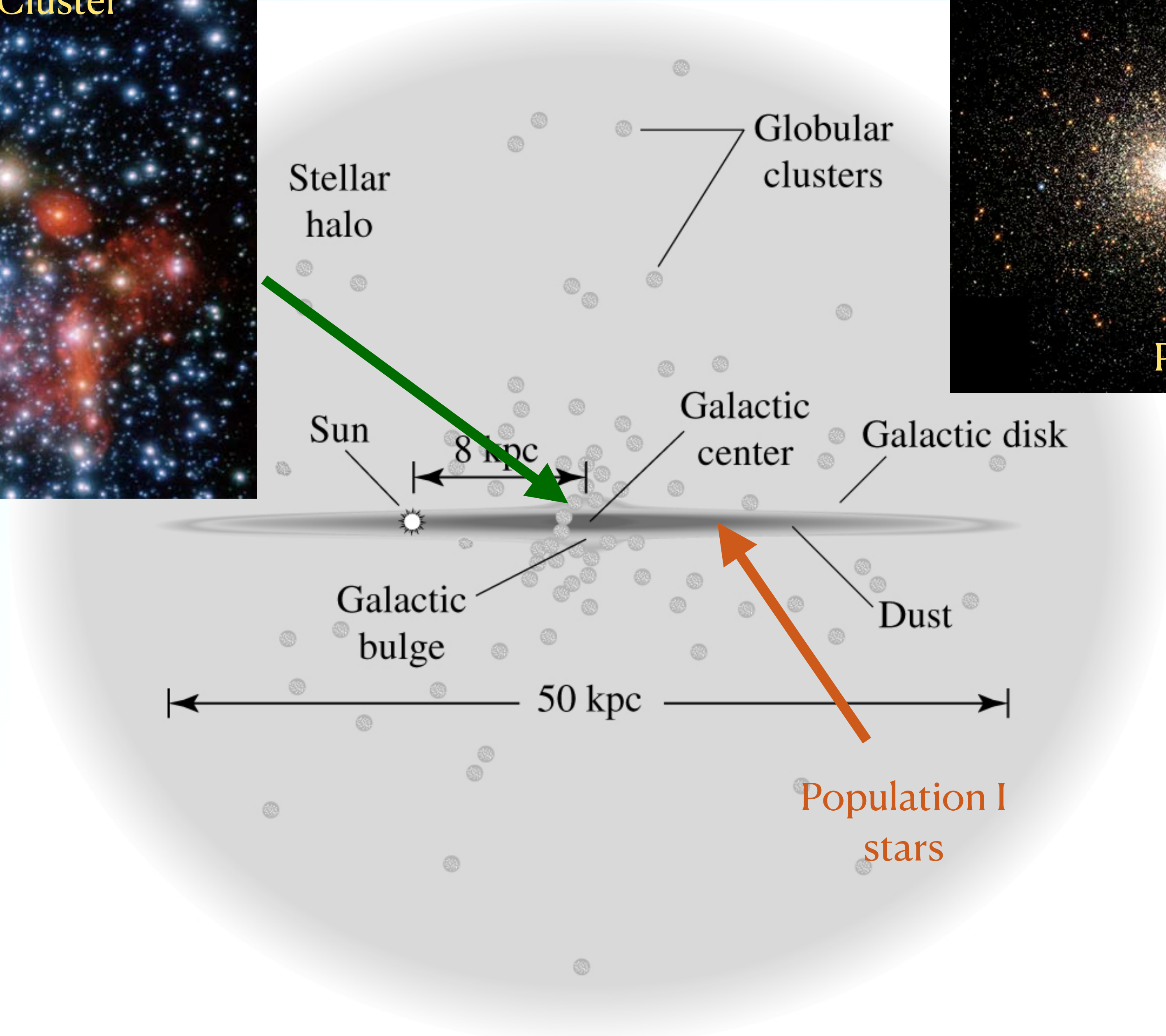


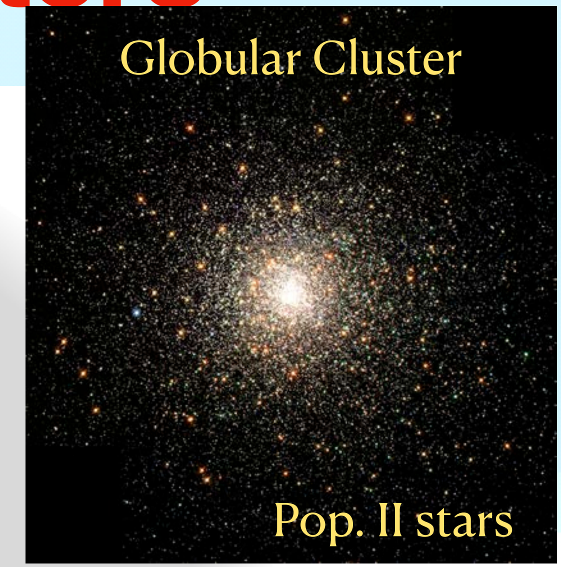
[Madau, 2014]

Stellar Evolution



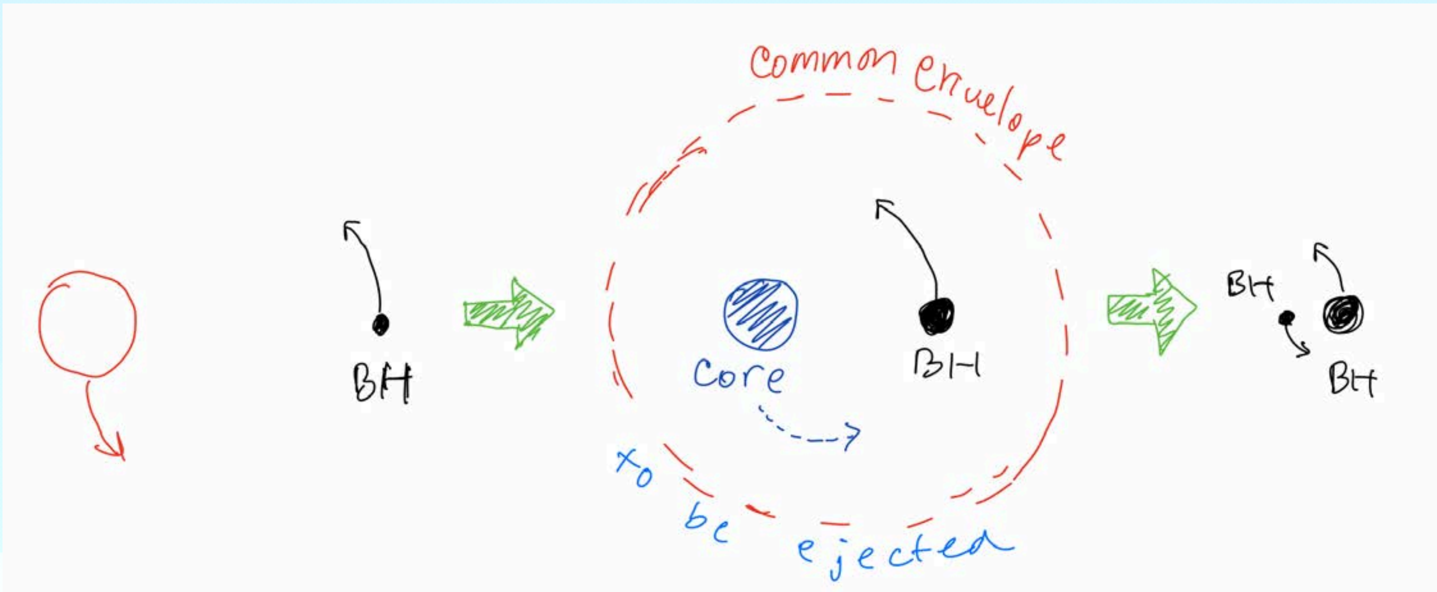
Galactic field versus clusters



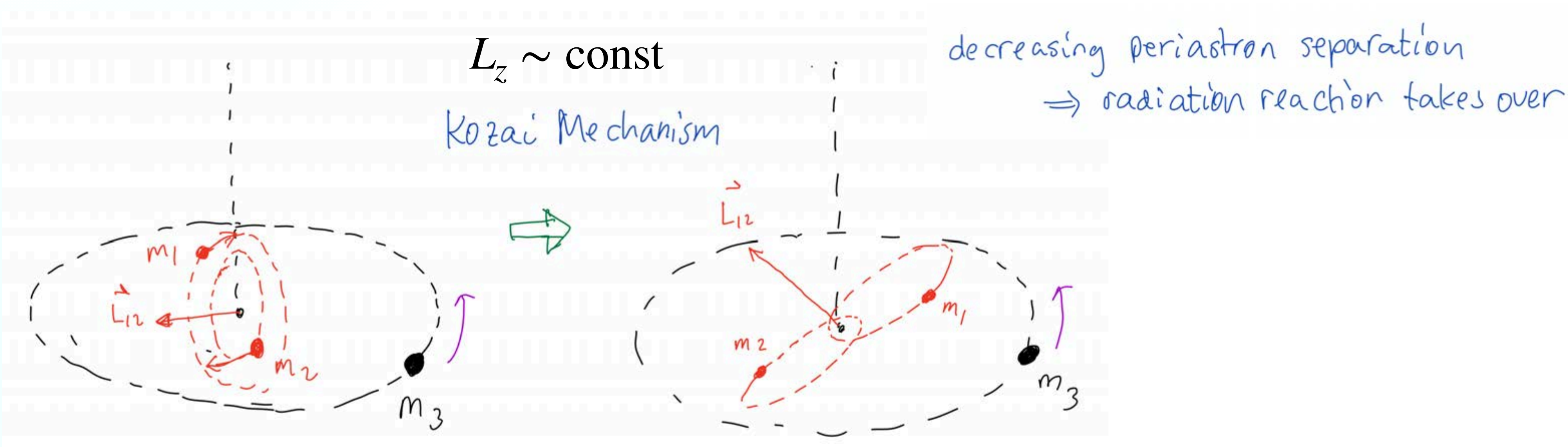


Formation of Merging Binaries

Binary stars that evolve into compact objects, and then become very close to each other

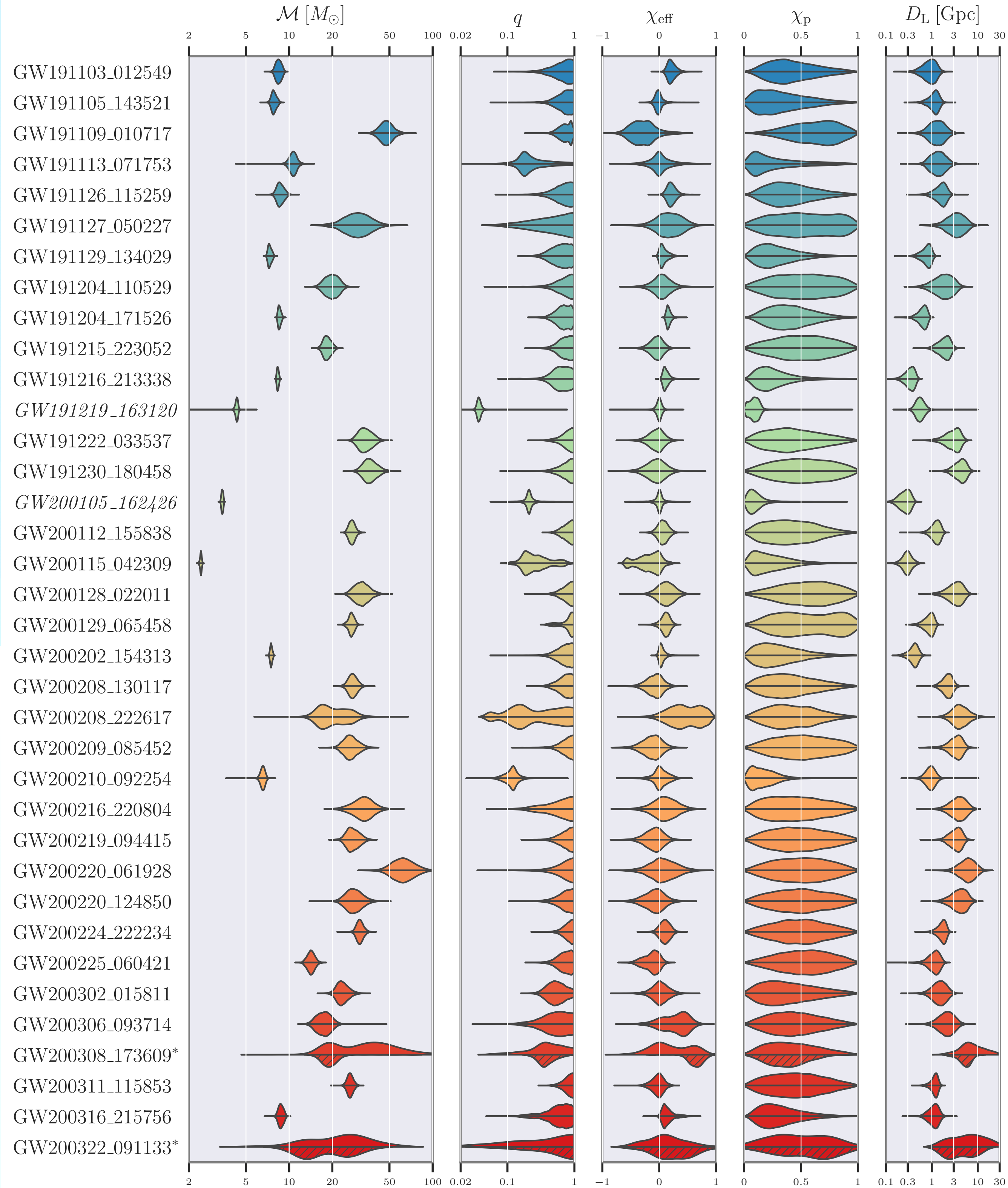


Binaries that form via triple or quadrupole interaction



Galactic field versus clusters

Galactic Field	Globular Clusters	Nuclear Clusters
Core-Collapse Supernova Mass Gap between 50—65 and 130—160, due to Pair Instability. Low spins?	Dynamical formation due to close encounters Substantial Spin? [Equal-mass non-spinning binaries create a/M ~ 0.7] BH easily escape?	Dynamical formation due to close encounters, esp in migration traps Substantial Spin? BH may not escape? *Multiple Generations?* Signatures interacting with SMBH? Disk?



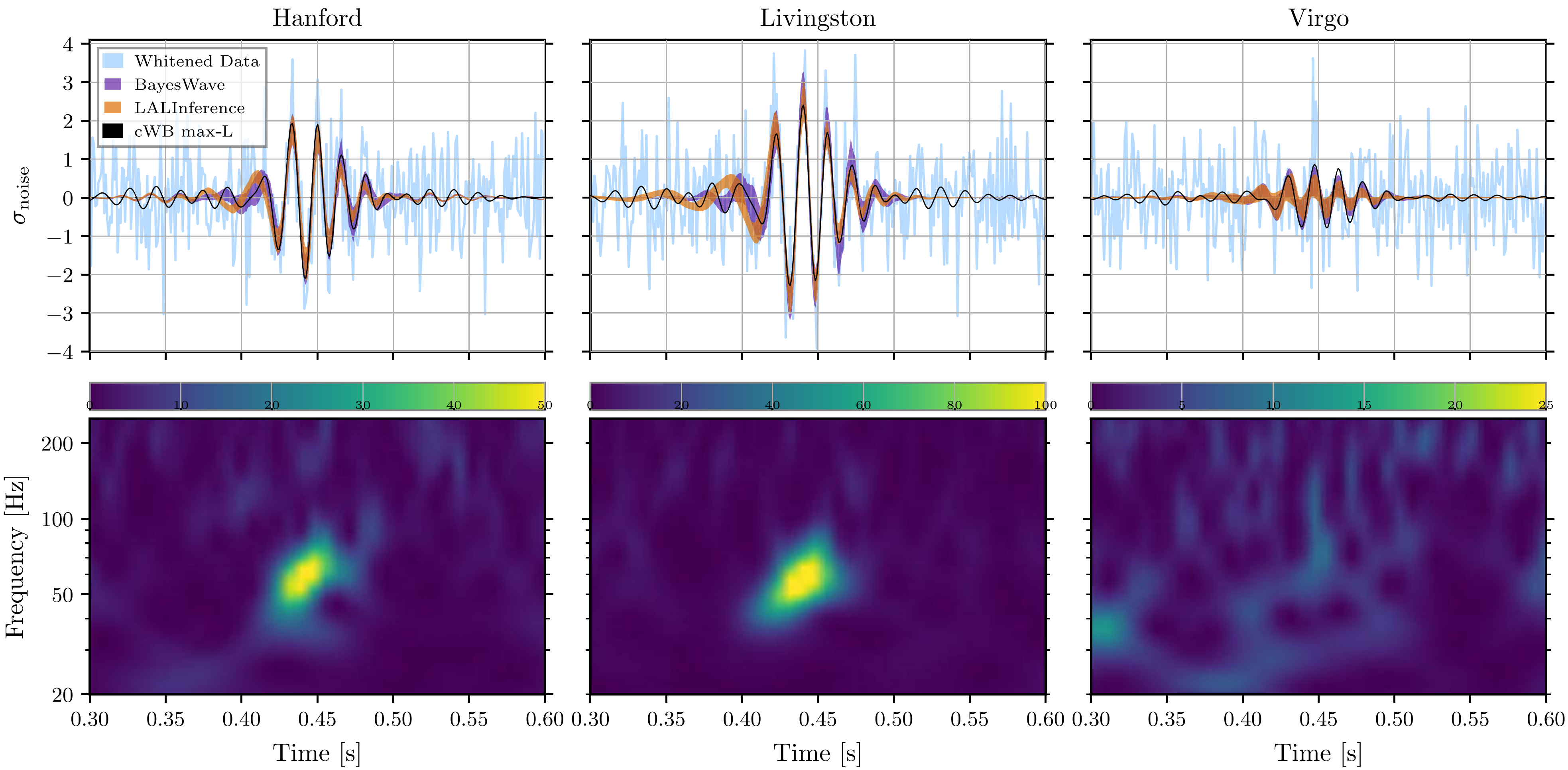
Effect of phase cumulation

$$\chi_{\rm eff} = \frac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{L}_{\rm N}}{M},$$

Effect of precessions

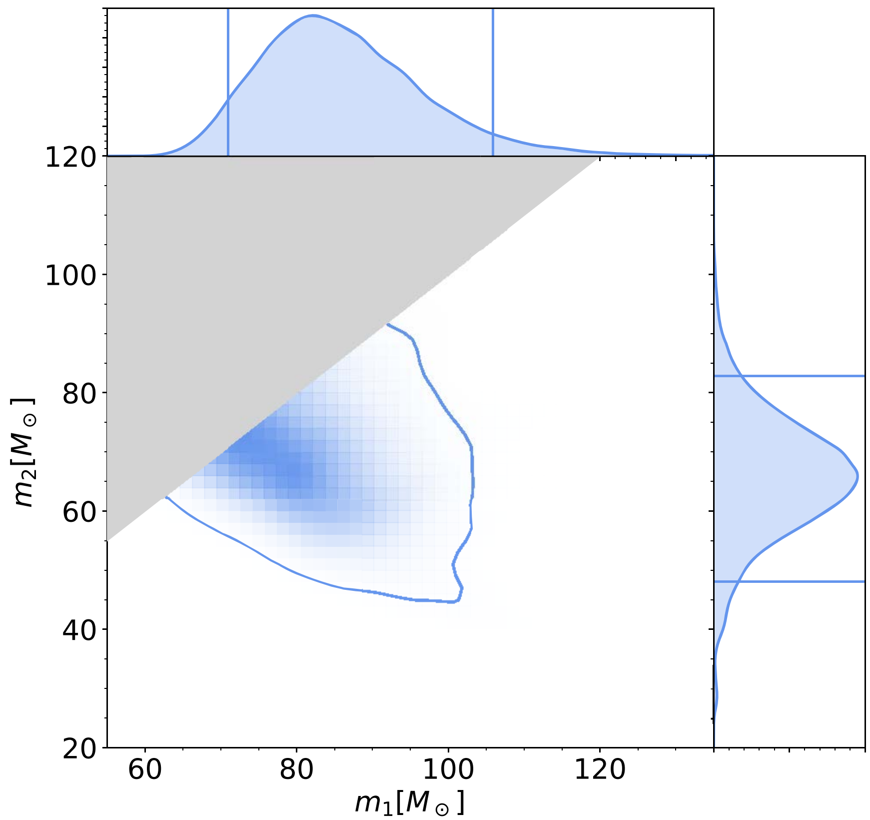
$$\chi_{\rm p} = \max\left\{\chi_{1,\perp}, \frac{q(4q+3)}{4+3q}\chi_{2,\perp}\right\},$$

GW190521


Hanford
Livingston
Virgo
Whitened Data
BayesWave
LALInference
cWB max-L
$\sigma_{\rm noise}$
Frequency [Hz]
Time [s]

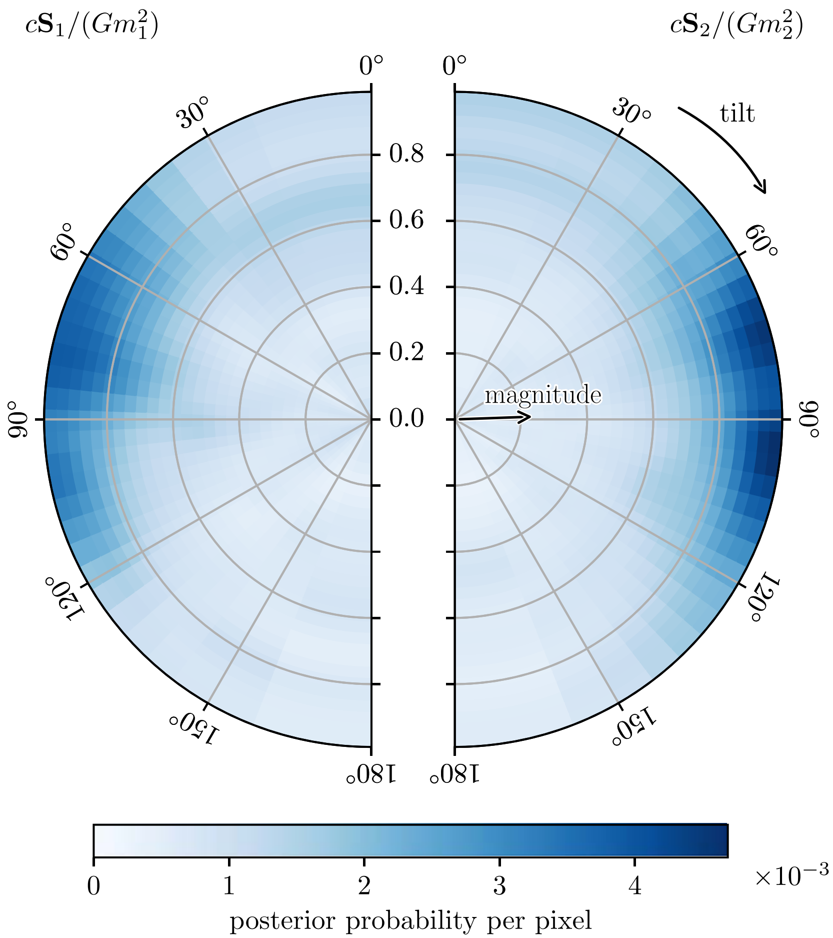
GW190521

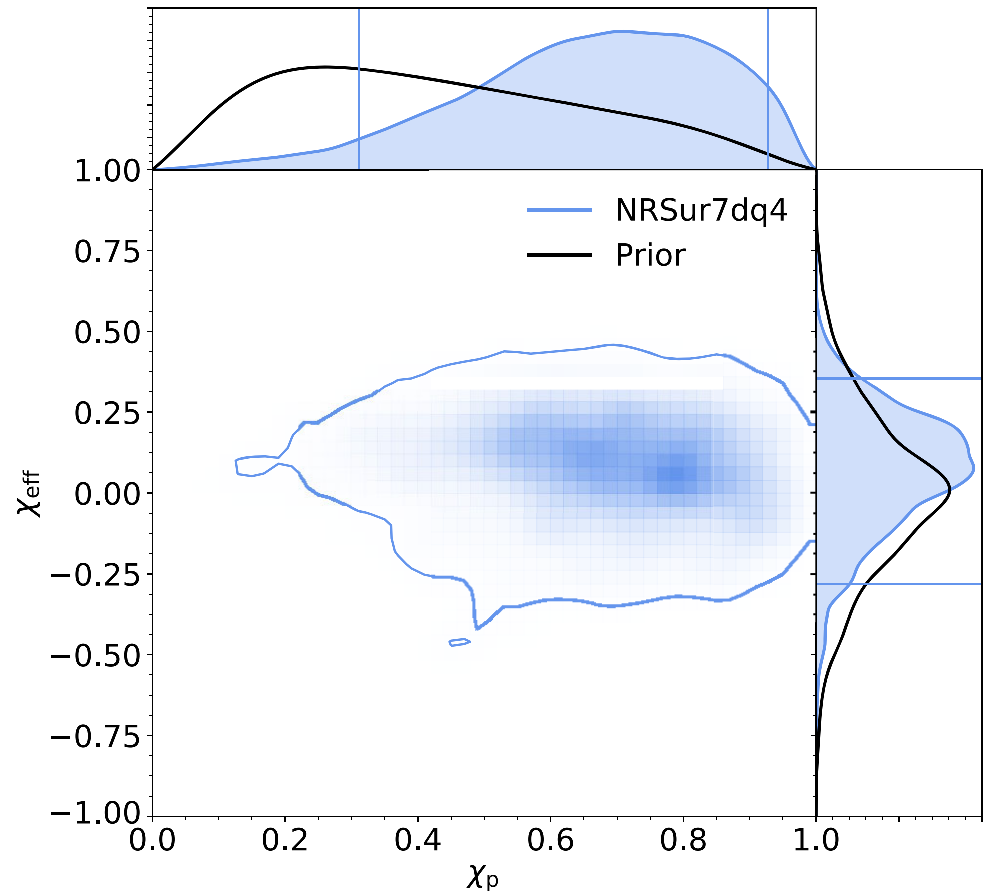
Parameter	
Primary mass	$85^{+21}_{-14}\ M_\odot$
Secondary mass	$66^{+17}_{-18}\ M_\odot$
Primary spin magnitude	$0.69^{+0.27}_{-0.62}$
Secondary spin magnitude	$0.73^{+0.24}_{-0.64}$
Total mass	$150^{+29}_{-17}\ M_\odot$
Mass ratio ($m_2/m_1 \leq 1$)	$0.79^{+0.19}_{-0.29}$
Effective inspiral spin parameter ($\chi_{\rm eff}$)	$0.08^{+0.27}_{-0.36}$
Effective precession spin parameter ($\chi_{\rm p}$)	$0.68^{+0.25}_{-0.37}$
Luminosity Distance	$5.3^{+2.4}_{-2.6}$ Gpc
Redshift	$0.82^{+0.28}_{-0.34}$
Final mass	$142^{+28}_{-16}\ M_\odot$
Final spin	$0.72^{+0.09}_{-0.12}$
$P\ (m_1 < 65\ M_\odot)$	0.32%
$\log_{10}$ Bayes factor for orbital precession	$1.06^{+0.06}_{-0.06}$
$\log_{10}$ Bayes factor for nonzero spins	$0.92^{+0.06}_{-0.06}$
$\log_{10}$ Bayes factor for higher harmonics	$-0.38^{+0.06}_{-0.06}$



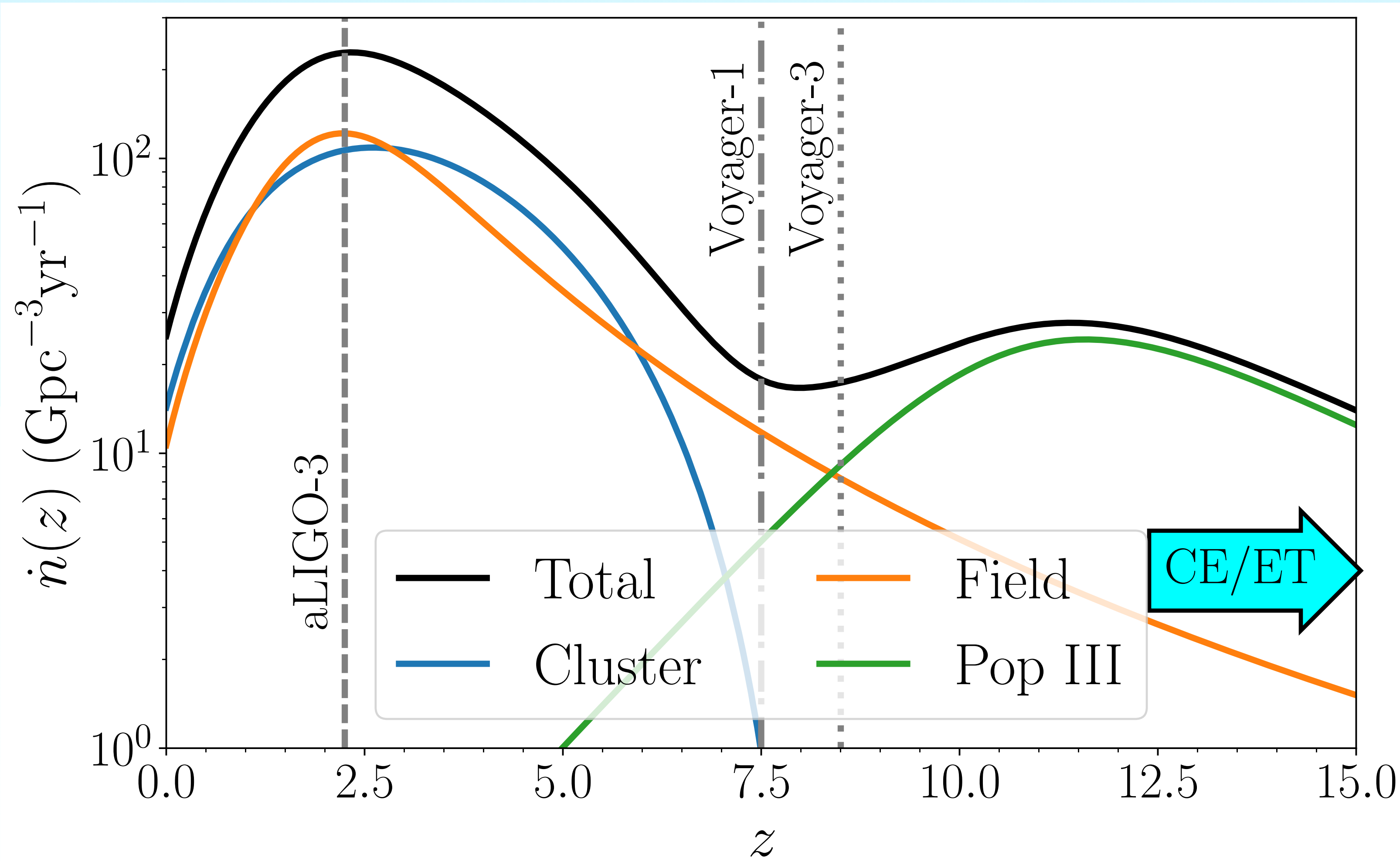
- Massive
- May have orbital precessions.

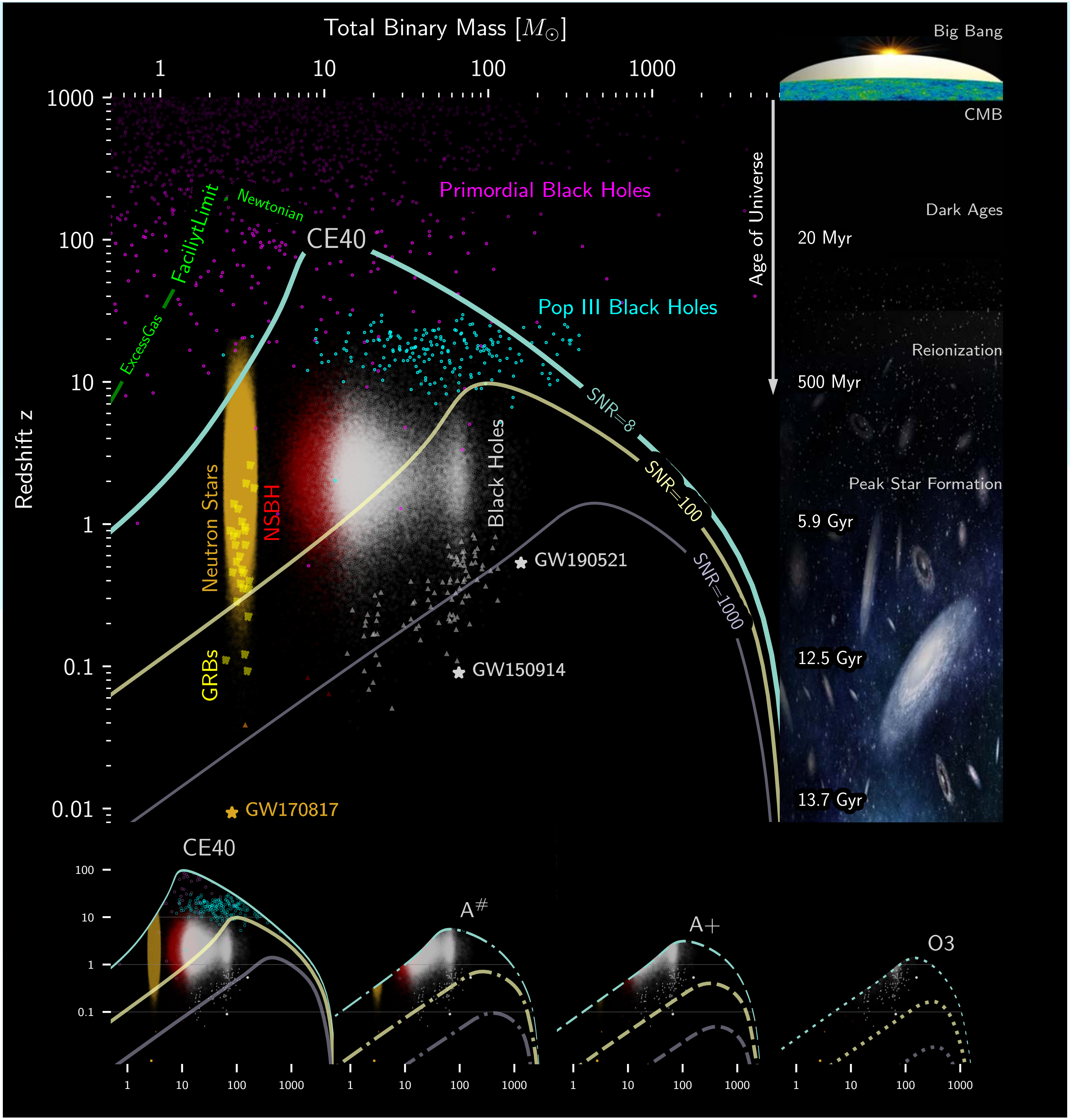
GW190521


$cS_1/(Gm_1^2)$
$cS_2/(Gm_2^2)$
tilt
magnitude
0°
30°
60°
90°
120°
150°
180°
0.0
0.2
0.4
0.6
0.8
0
1
2
3
4
$\times 10^{-3}$
posterior probability per pixel

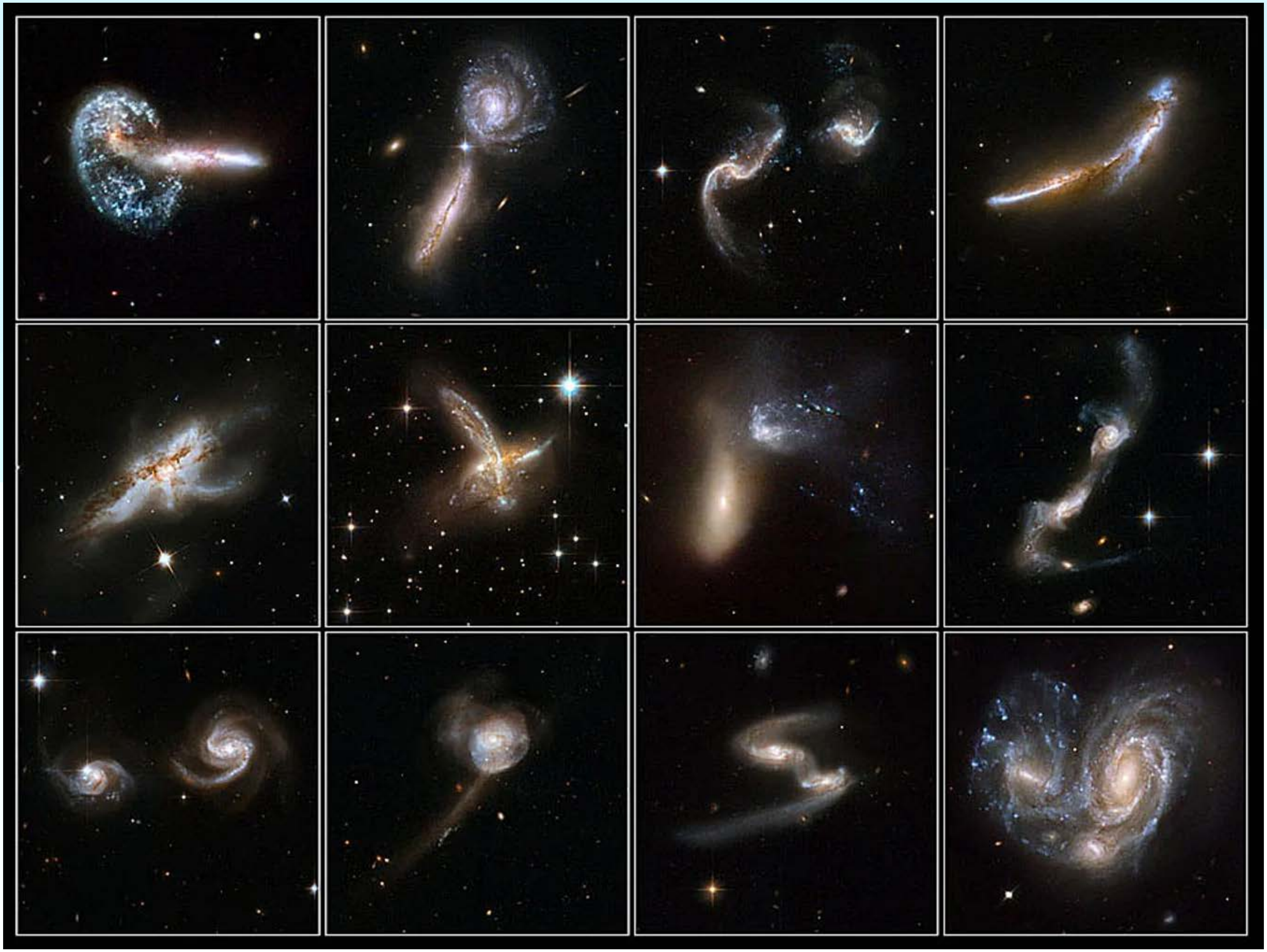

NRSur7dq4
Prior
χ_{eff}
χ_p
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

Population III stars



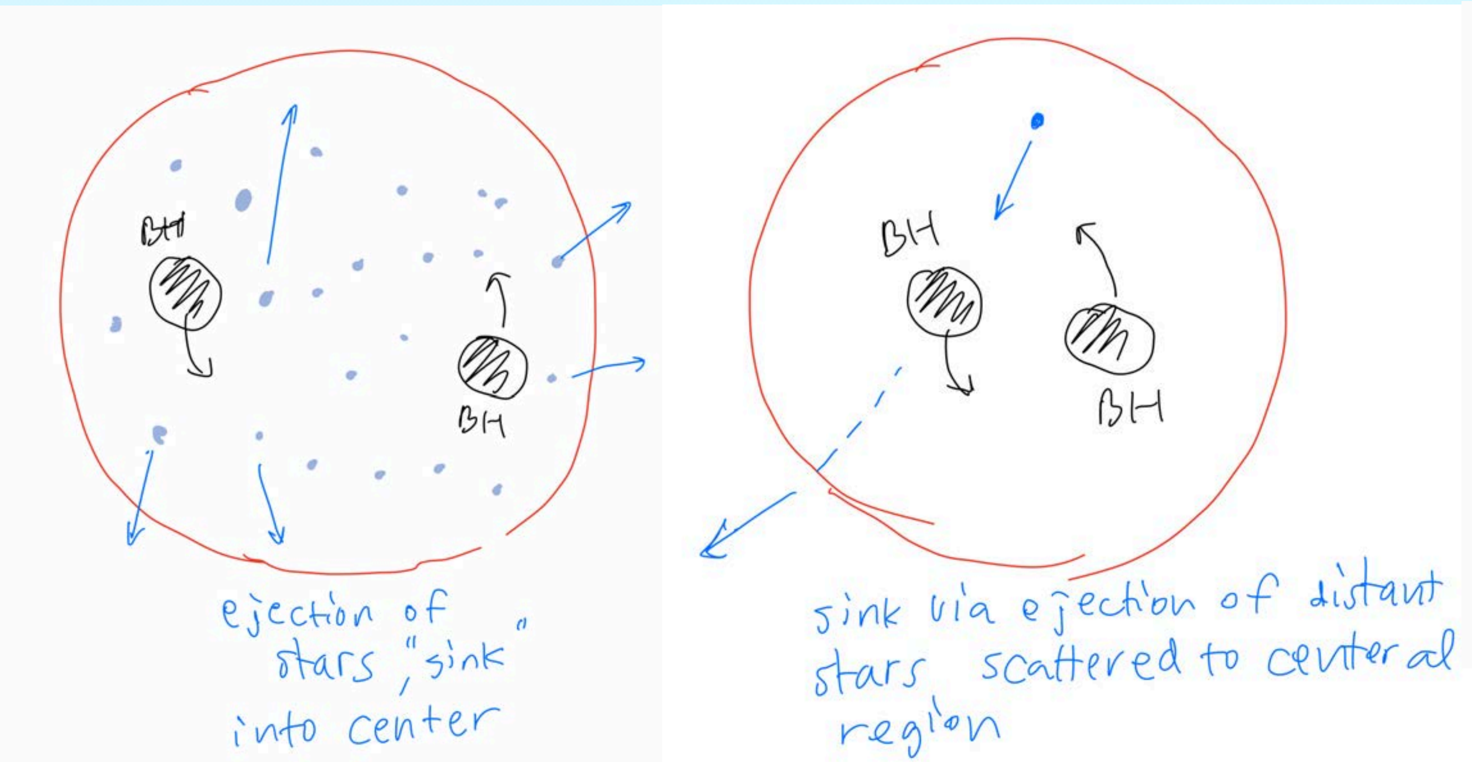

Total Binary Mass [$M_\odot$]
Big Bang
CMB
Dark Ages
20 Myr
Reionization
500 Myr
Peak Star Formation
5.9 Gyr
12.5 Gyr
13.7 Gyr
Age of Universe
Redshift z
Primordial Black Holes
Pop III Black Holes
CE40
Newtonian
FaciliytLimit
ExcessGas
Neutron Stars
NSBH
Black Holes
GRBs
SNR=8
SNR=100
SNR=1000
GW190521
GW150914
GW170817
A#
A+
O3

Collisions of Galaxies and their BHs



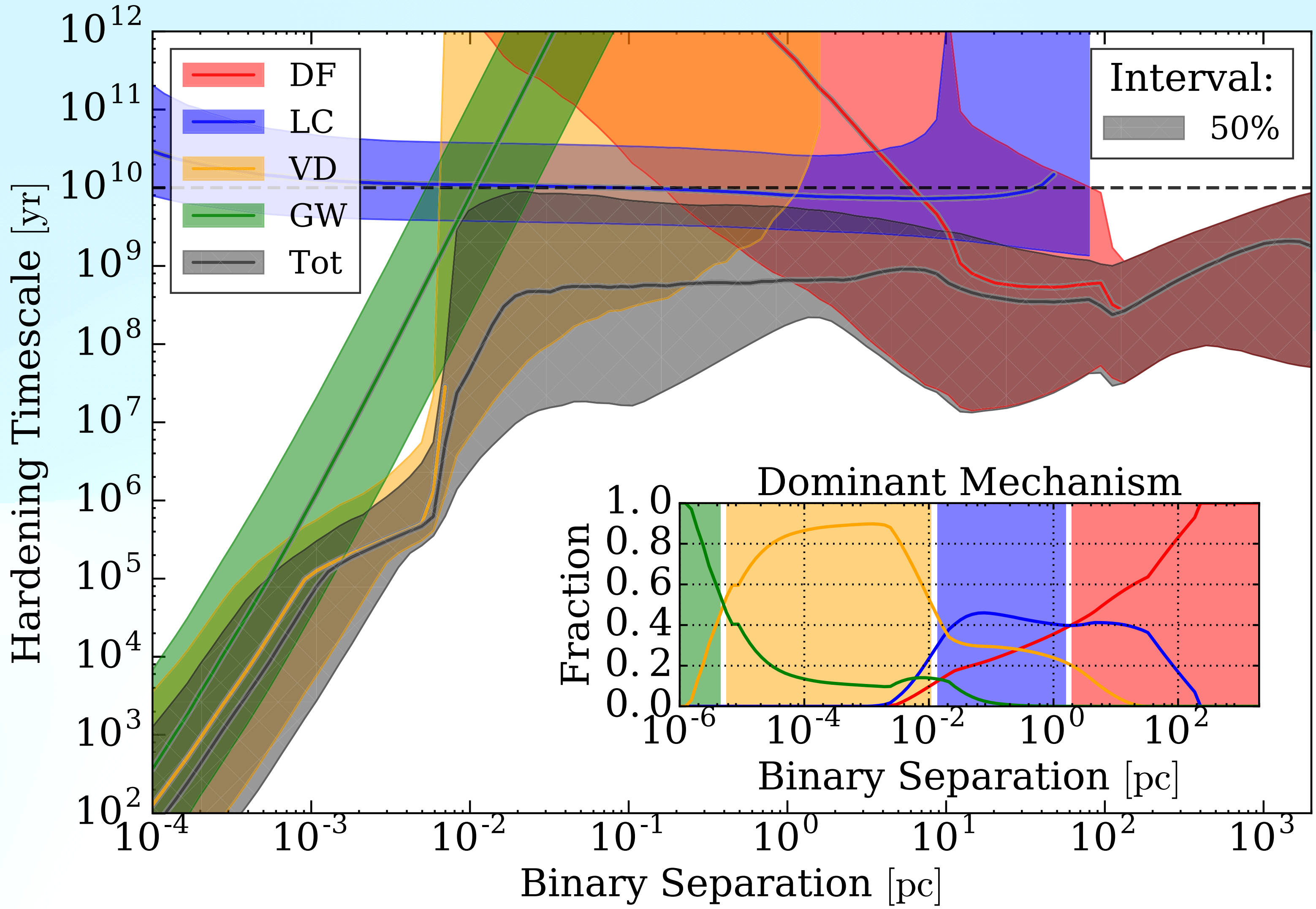
NASA, ESA, the Hubble Heritage Team (STScI/AURA)-ESA/Hubble Collaboration and A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University), K. Noll (STScI), and J. Westphal (Caltech)

Collisions of Galaxies and their BHs



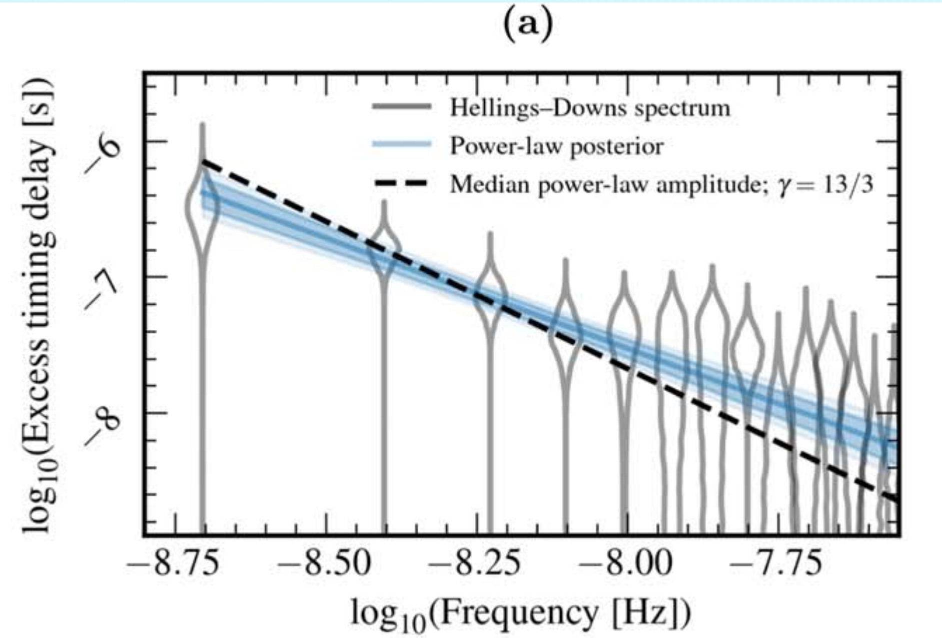
"Hardening" of Super-Massive Black Holes after a galaxy merger [Begelman, Blandford, Rees, 1980]

Hardening of SMBH Binaries



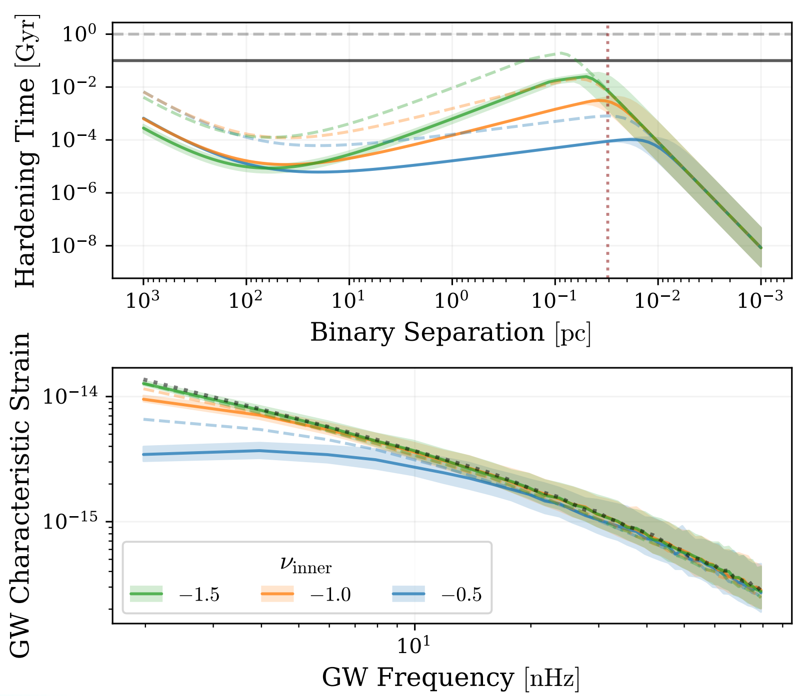
[Kelly, Blechia and Hernquist, 2017]

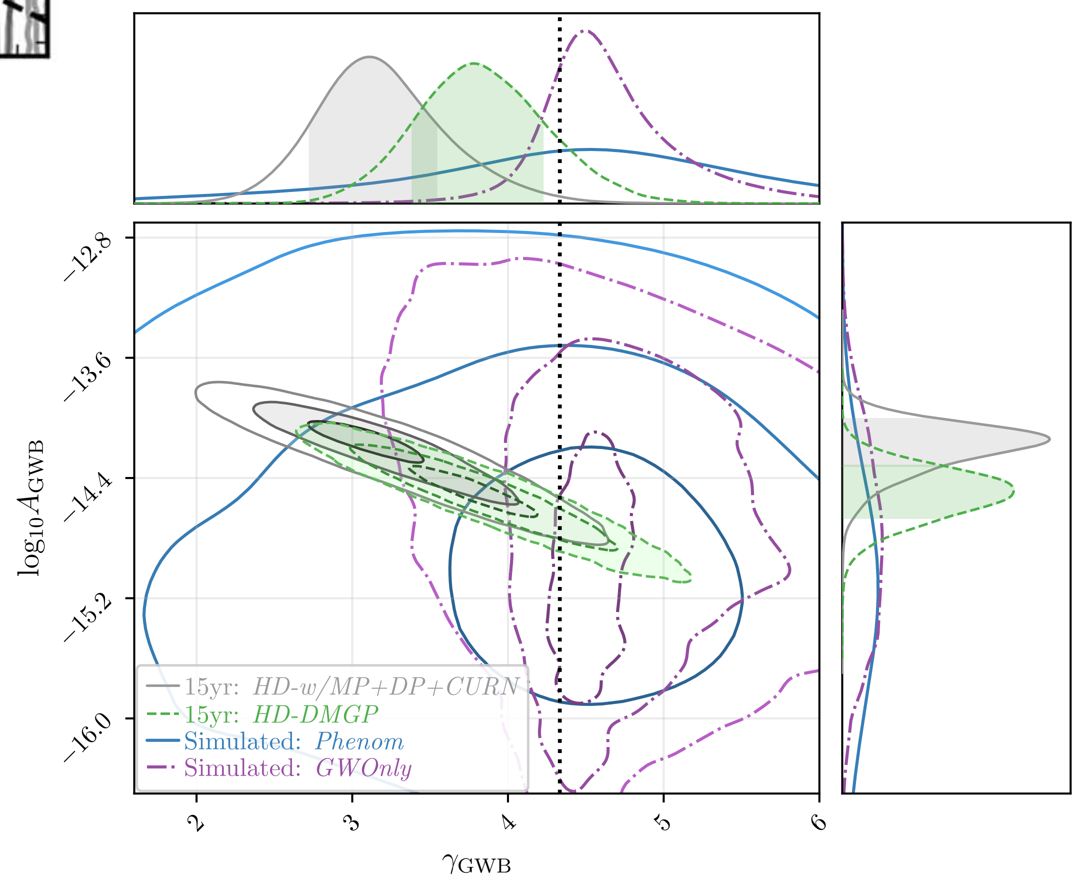
Stochastic Background from SMBH Mergers



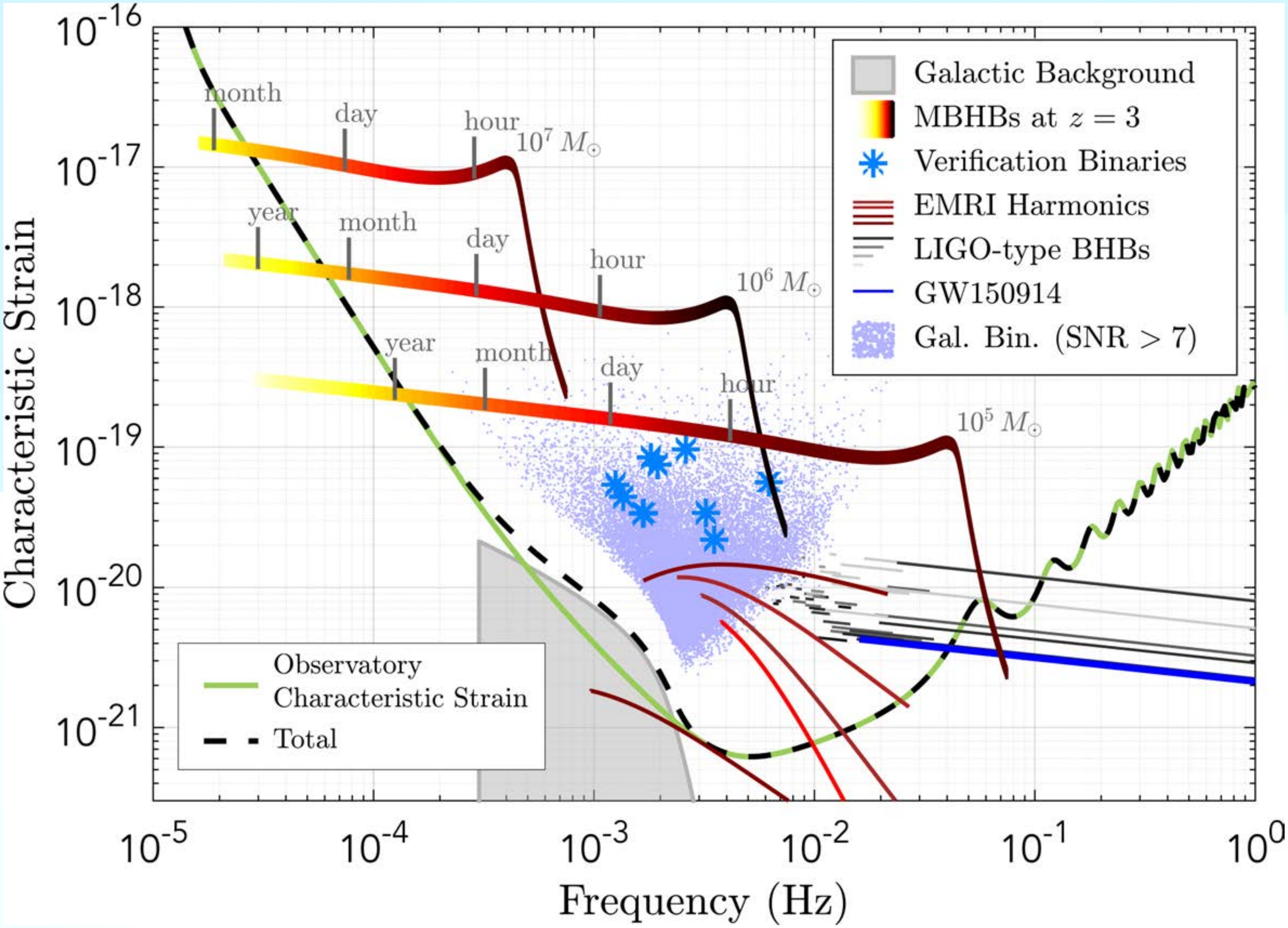
Mechanisms for hardening below parsec scale can be uncertain.

[Gabriella Agazie， 2023]

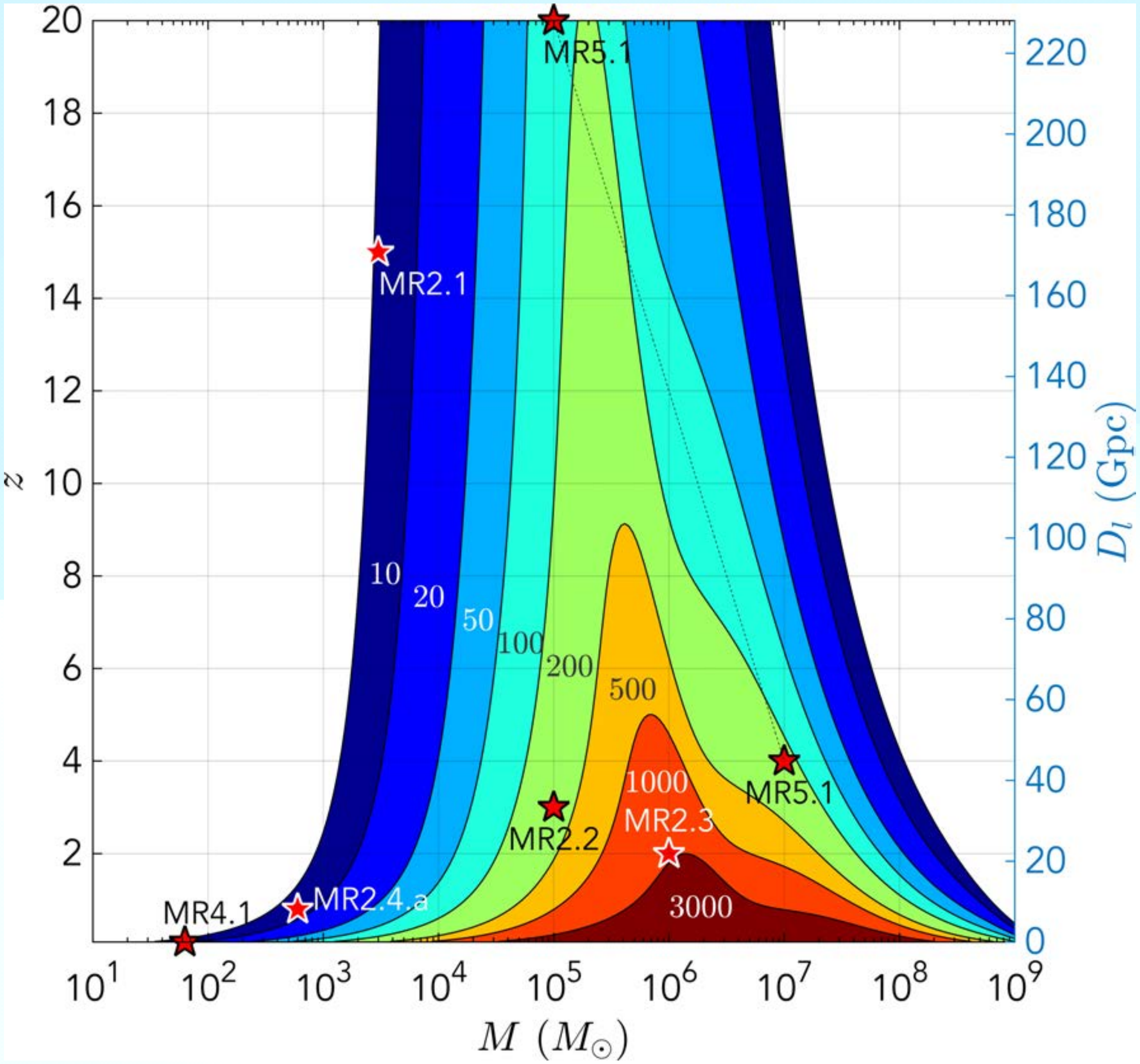




LISA Sources



SMBH Binaries for LISA

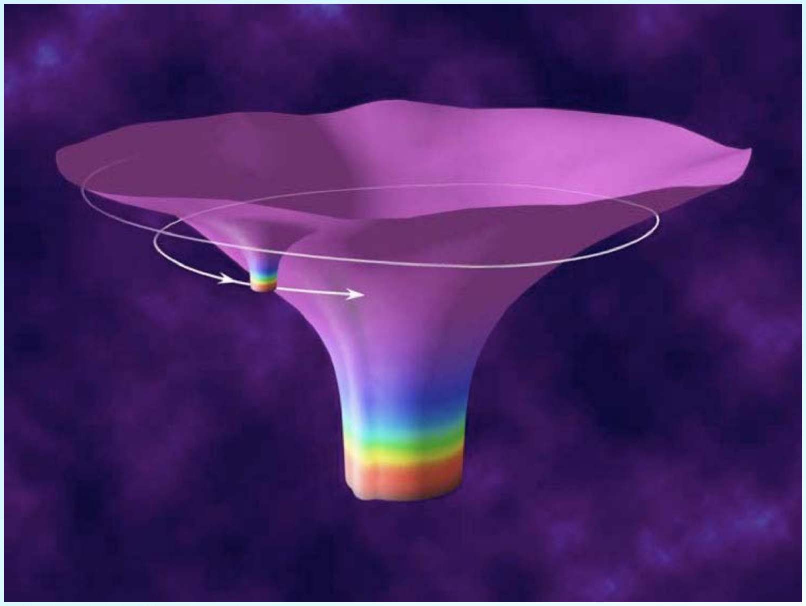


SNR for $q = 0.2$ binaries with total mass M.
[K. Danzmann et al., LISA Proposal]

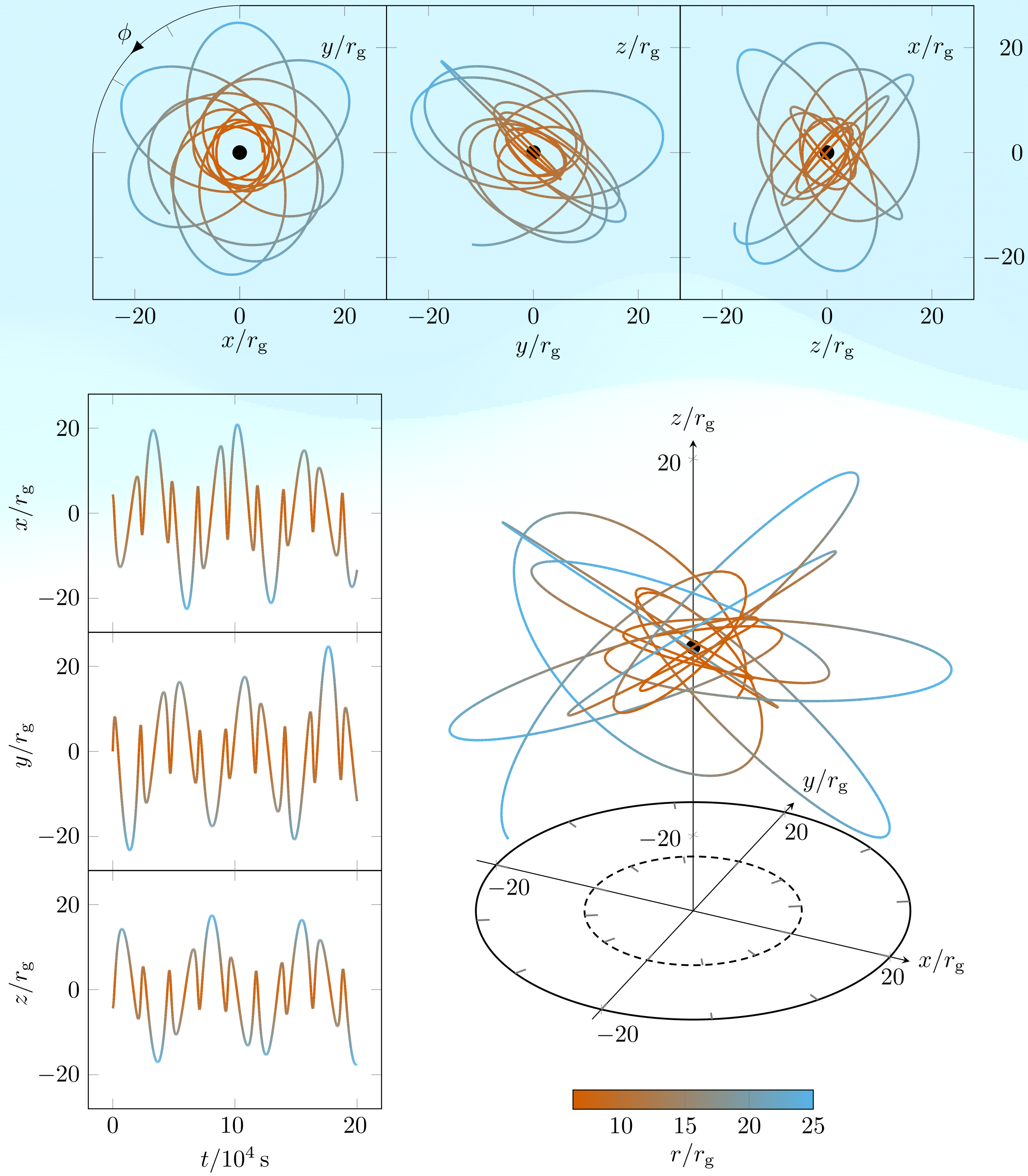
… from LISA Proposal

- "Trace the origin, growth and merger history of massive black holes across cosmic ages
- Search for seed black holes at cosmic dawn
- Study the growth mechanism of MBHs from the epoch of the earliest quasars
- Observation of EM counterparts to unveil the astrophysical environment around merging binaries
- Test the existence of Intermediate Mass Black Hole Binaries (IMBHBs)"

Extreme Mass Ratio Inspirals (EMRIs)

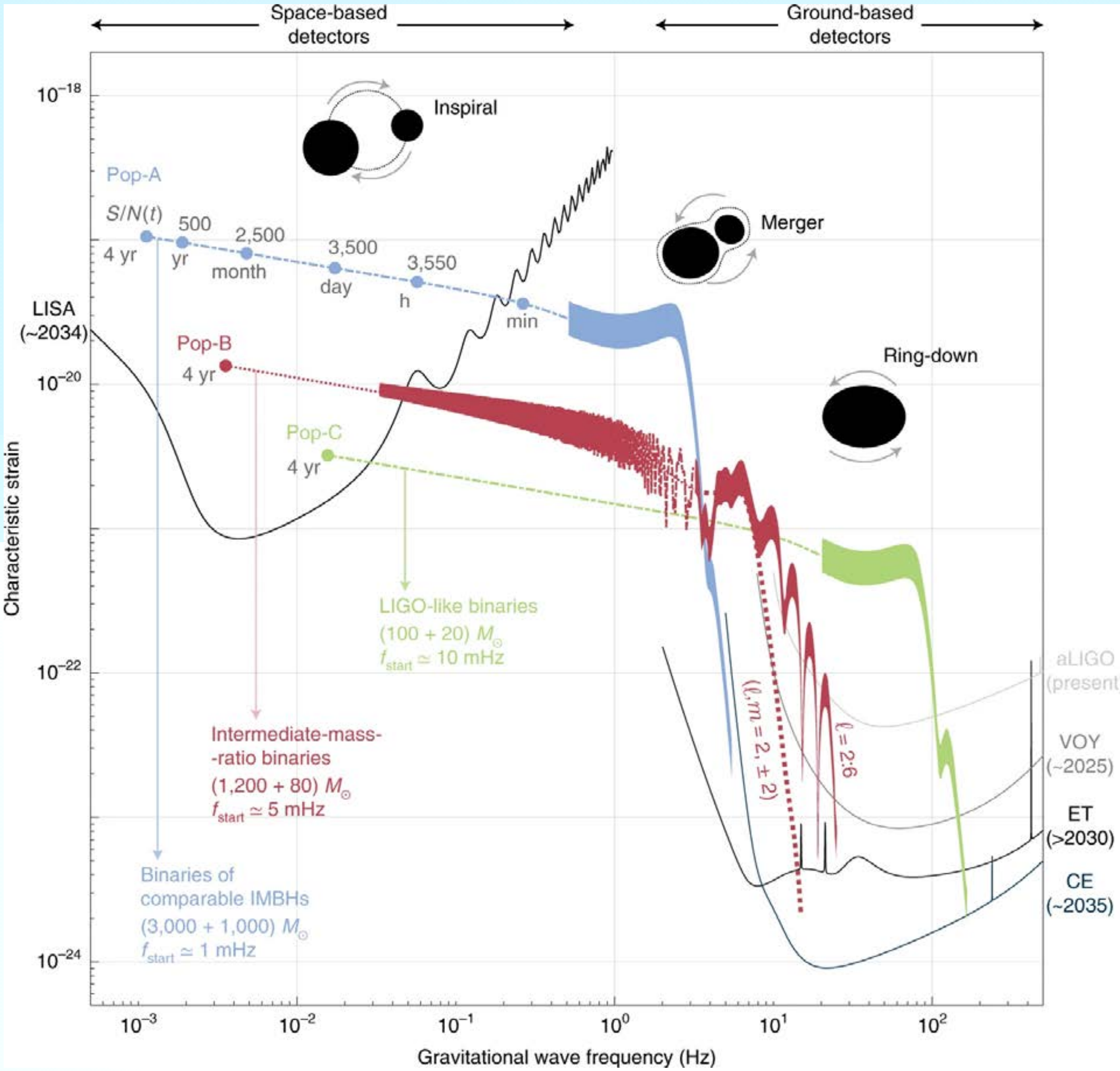


- Stellar-mass object circles SMBH for long many cycles.
- Can be treated with black-hole perturbation theory (second order): interesting theoretical problem.
- Maps space-time geometry around and astrophysical environments around SMBH .
- Form via direct capture & tidal disruption of stellar-mass binaries. Enhanced for SMBH binaries.



[C. Berry et al., 2020 White Paper]

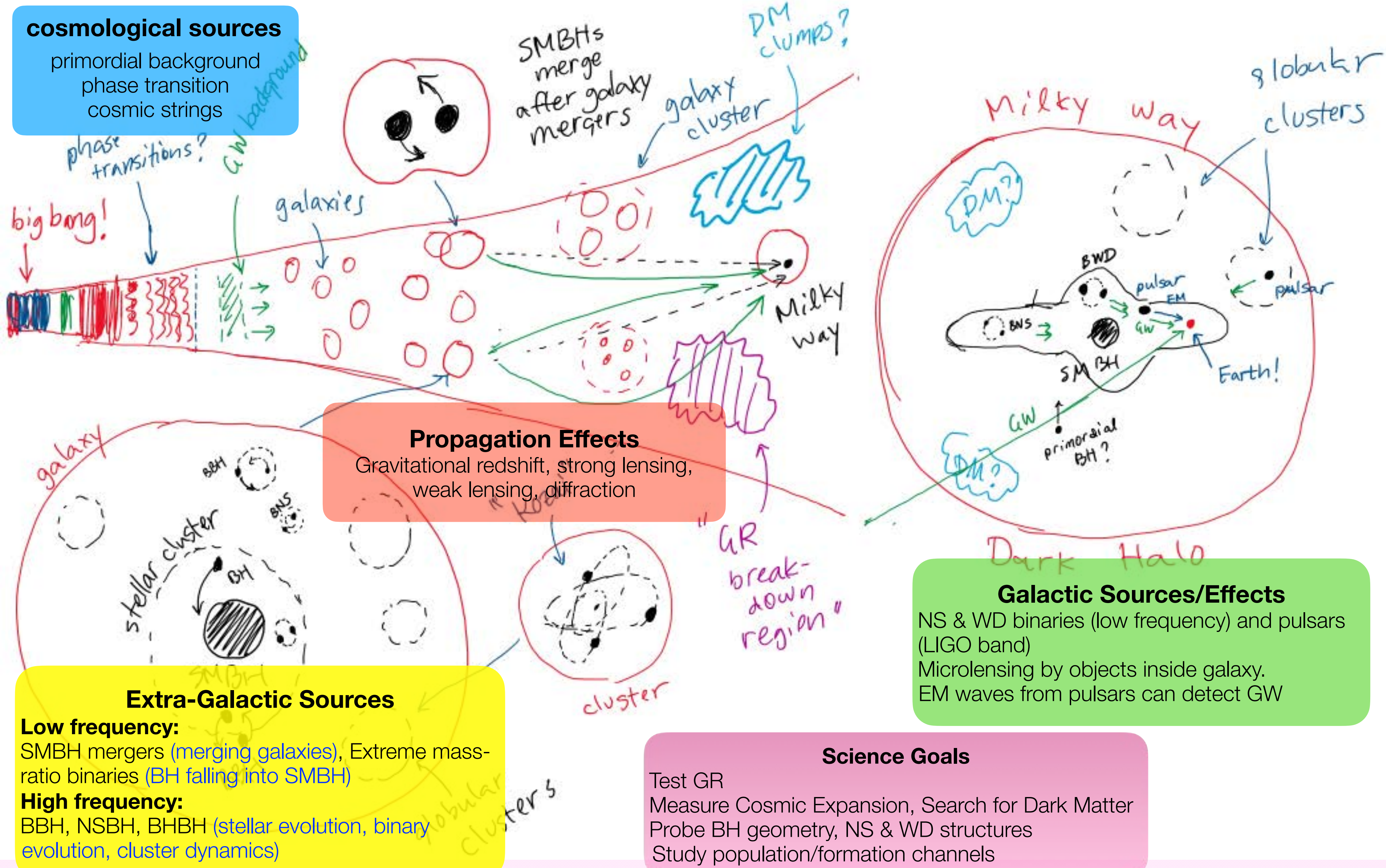
Multi-Band Observations



[Jani, Shoemaker and Cutler, 2020]

- Adding Deci-Hertz detector (e.g., DECIGO, TOBA) will be more productive!
- **Triple systems offer more opportunities!**

Summary of Lecture 2



cosmological sources
primordial background
phase transition
cosmic strings

Propagation Effects
Gravitational redshift, strong lensing, weak lensing, diffraction

Galactic Sources/Effects
NS & WD binaries (low frequency) and pulsars (LIGO band)
Microlensing by objects inside galaxy.
EM waves from pulsars can detect GW

Extra-Galactic Sources
Low frequency:
SMBH mergers (merging galaxies), Extreme mass-ratio binaries (BH falling into SMBH)
High frequency:
BBH, NSBH, BHBH (stellar evolution, binary evolution, cluster dynamics)

Science Goals
Test GR
Measure Cosmic Expansion, Search for Dark Matter
Probe BH geometry, NS & WD structures
Study population/formation channels