Lectures on Gravitational-Wave Astronomy

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Contents

- **Lecture 1**
  - Brief Introduction; gravitational waves from binary stars.
  - Black holes and tests of General Relativity.
  - Neutron stars.

- **Lecture 2**
  - Interaction between gravitational waves and measuring devices.
  - Ground- and space-based detectors; pulsar timing arrays.
  - Gravitational-wave sources from the early universe.
  - Gravitational-wave sources within galaxies.

General Relativity

- Space-time geometry described by metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- Non-gravitational physics uses "**covariant derivative**", adapted to curved spacetime.

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

$$\nabla_\mu F^{\mu\nu} = J^\nu$$

- Geometry is **curved** by energy/momentum content of spacetime.

a lot of calculus

$$g_{\mu\nu} \longrightarrow G_{\mu\nu} \qquad G_{\mu\nu} \sim R_{\mu\nu} \sim R_{\mu\nu\alpha\beta} \sim \frac{1}{\mathscr{R}^2}$$

Einstein's Equation

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

- **Gauge freedom**: different $g_{\mu\nu}(x^\alpha)$ may correspond to the same spacetime since they may be related by coordinate transformations.

Gravitational Waves are Linear Perturbations

- Linear perturbations of Minkowski geometry

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu}\,, \quad h = h_{\mu\nu}\eta^{\mu\nu}$$

- Gauge Freedom

$$x^\alpha \to x^\alpha + \xi^\alpha \qquad\qquad h_{\alpha\beta} \to h_{\alpha\beta} - \xi_{\alpha,\beta} - \xi_{\beta,\alpha}$$

- Obtain Einstein Tensor

$$G_{\alpha\beta} = \bar{R}_{\alpha\beta} = \frac{1}{2}\left[-\bar{h}_{\alpha\beta,\mu}{}^{,\mu} - \eta_{\alpha\beta}\bar{h}_{\mu\nu}{}^{,\mu\nu} + \bar{h}_{\alpha\mu}{}^{,\mu}{}_{,\beta} + \bar{h}_{\beta\mu}{}^{,\mu}{}_{,\alpha}\right] = 8\pi T_{\alpha\beta}$$

(one can choose Lorenz gauge) $\bar{h}^{\alpha\beta}{}_{,\beta} = 0$

- Linearized Einstein's Equation

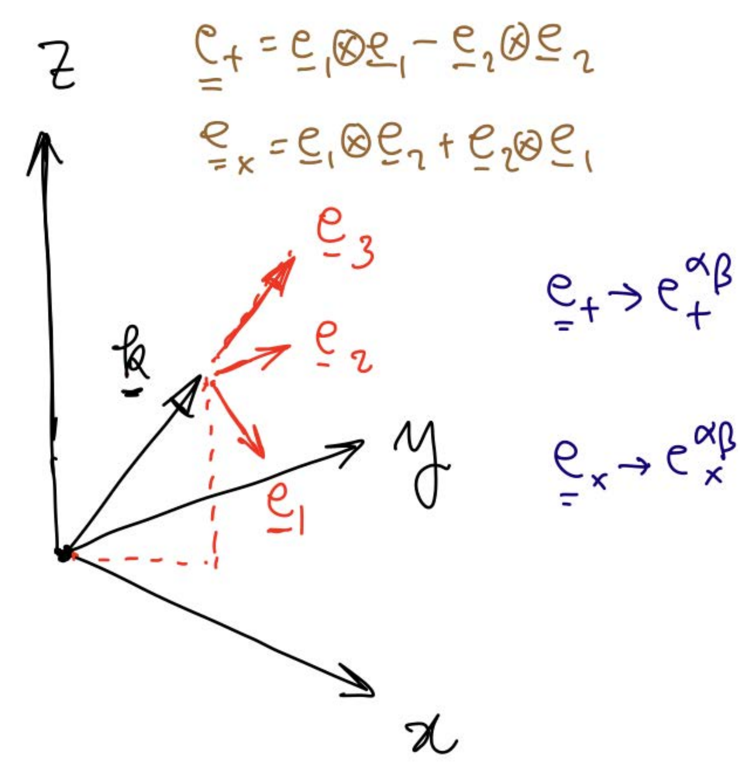
$$\Box\,\bar{h}_{\alpha\beta} = -\,16\pi T_{\alpha\beta}$$

The TT Gauge

- For a plane wave along the z direction, we can choose a gauge like

$$\|h_{\alpha\beta}(t,x,y,z)\| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t-z) & h_\times(t-z) & 0 \\ 0 & h_\times(t-z) & -h_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- In general, if wave vector is $\mathbf{k}$
  - Find orthonormal basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ with $\mathbf{e}_3 = \mathbf{k}$
  - Define
    - $\mathbf{e}_+ = \mathbf{e}_1 \otimes \mathbf{e}_1 - \mathbf{e}_2 \otimes \mathbf{e}_2$
    - $\mathbf{e}_\times = \mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1$



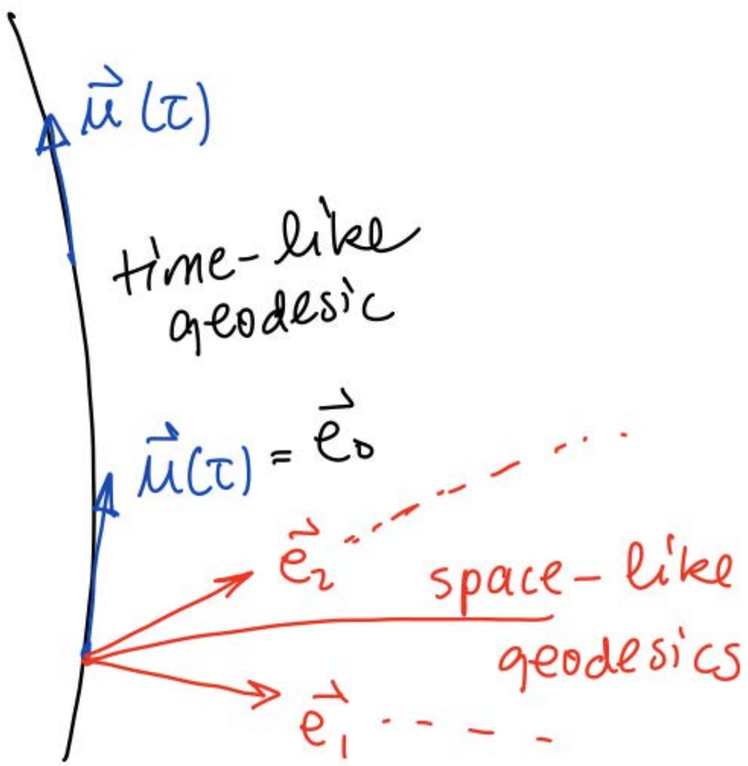
$$h_+ = \frac{1}{2} h_{ij} e_+^{ij}, \quad h_\times = \frac{1}{2} h_{ij} e_\times^{ij}$$

[only keep transverse traceless components]

Effect of GW on Freely Falling Objects

$$ds^2 = -(1 + R_{0l0m}x^l x^m)dt^2 - \frac{4}{3}R_{0ljm}x^l x^m dt^j dt + \left(\delta_{jk} - \frac{1}{3}R_{jlkm}x^l x^m\right) dx^j dx^k$$

$$R_{0l0m} = -\frac{1}{2}\ddot{h}^{\rm TT}_{lm} \sim \frac{h}{\lambda^2}, \quad R_{0jlm} \sim R_{jlkm} \sim \frac{h}{\lambda^2}$$



Fermi Normal Coordinates

- **In this gauge (coordinate system):**
  - **Local Lorentz Frame** [metric is flat up to $O(x/\lambda)^2$]
  - For $L \ll \lambda_{\rm GW}/(2\pi)$, main effect is objects feel tidal gravity force:

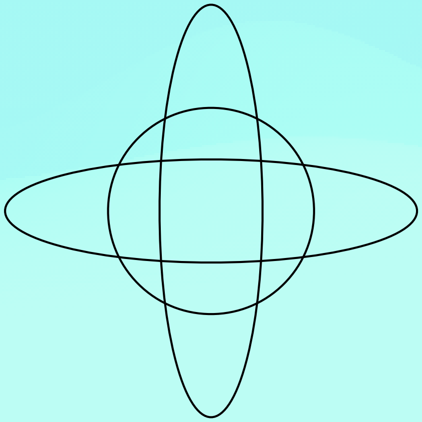
$$\ddot{x}^j = \frac{F^j}{M} - R_{0l0j}x^l = \frac{F^j}{M} + \frac{\ddot{h}^{\rm TT}_{lj}}{2}x^l$$

$$\delta x = \frac{h^{\rm TT}_{lj}}{2}x^j$$

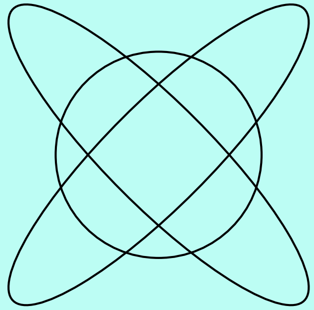
Polarizations of Gravitational Waves

$$\ddot{x}^j = \frac{F^j}{M} - R_{0l0j}x^l = \frac{F^j}{M} + \frac{\ddot{h}_{lj}^{\mathrm{TT}}}{2}x^l \qquad \delta x = \frac{h_{lj}^{\mathrm{TT}}}{2}x^j$$

+ polarization $h_{xx} = -h_{yy} = h_+$

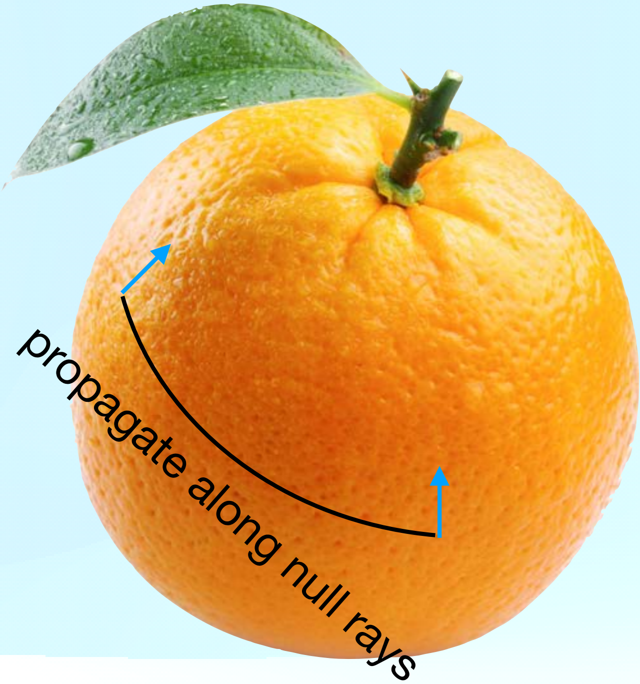


× polarization $h_{xy} = h_{yx} = h_\times$



plane perpendicular to propagation direction

Wave Propagation on Curved Background (I)



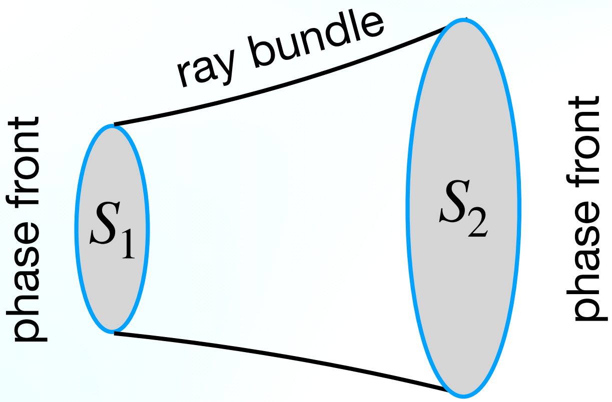
$$g_{\mu\nu} = g^{B}_{\mu\nu} + h_{\mu\nu} \qquad \nabla^{\alpha}\nabla_{\alpha}h_{\mu\nu} + 2\,{}^{\mathrm{B}}R^{\alpha}{}_{\mu\nu}{}^{\beta}h_{\alpha\beta} = 0$$

Short-wavelength Approximation:

$$h_{\mu\nu} = A_{\mu\nu}e^{i\Phi/\epsilon}$$

$$k_{\alpha} = \nabla_{\alpha}\Phi \qquad k_{\alpha}k^{\alpha} = 0 \qquad k^{\alpha}\nabla_{\alpha}k^{\beta} = 0$$

phase Φ constant along **null rays**, with tangent vector field k^{α}



$$A_2 = A_1\frac{\sqrt{S_1}}{\sqrt{S_2}}$$

$$A_{\mu\nu} = e_{\mu\nu}A\,, \quad e_{\mu\nu}e^{\mu\nu} = 2\,, \quad e_{\mu\nu} = e_{\nu\mu}\,, \quad e_{\mu\nu}k^{\mu} = 0$$

decomposed into **amplitude** A and **polarization tensor** $e_{\mu\nu}$

$$k^{\alpha}\nabla_{\alpha}e_{\mu\nu} = 0\,, \quad k^{\alpha}\nabla_{\alpha}A = -\frac{1}{2}A\,\nabla_{\alpha}k^{\alpha}$$

polarization tensor **parallel transported**; amplitude decays ~ $1/\sqrt{S}$

Wave Propagation on Curved Background (II)

$$g_{\mu\nu} = g^B_{\mu\nu} + h_{\mu\nu} \qquad \nabla^\alpha \nabla_\alpha h_{\mu\nu} + 2\, {}^{\mathrm{B}}R^{\alpha}{}_{\mu\nu}{}^{\beta} h_{\alpha\beta} = 0$$

- Existence of wave cause second-order correction

$$g_{\alpha\beta} = g^B_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta}$$

- … because Einstein's equation is nonlinear

$$G_{\mu\nu} = G^B_{\mu\nu} + \epsilon G^{(1)}_{\mu\nu}[h^{(1)}_{\alpha\beta}] + \epsilon^2 G^{(2)}_{\mu\nu}[h^{(1)}_{\alpha\beta}] + \epsilon^2 G^{(1)}_{\mu\nu}[h^{(2)}_{\alpha\beta}]$$

- Second-order perturbation is **quadratic** in first-order perturbation

$$G^{(1)}_{\mu\nu}[h^{(2)}_{\alpha\beta}] = -G^{(2)}_{\mu\nu}[h^{(1)}_{\alpha\beta}] \qquad h^{(2)} \sim h^{(1)} h^{(1)}$$

double frequency or **zero frequency**

- Average over wavelength provides "energy-momentum content of GW".

$$T^{\mathrm{GW}}_{\mu\nu} = \frac{1}{32\pi} \left\langle h^{\mathrm{TT}}_{\alpha\beta|\mu} h^{\alpha\beta}_{\mathrm{TT}|\nu} \right\rangle$$

$$\frac{1}{\mathcal{R}^2_{\mathrm{GW}}} \sim \frac{h^2}{\lambda_{\mathrm{GW}}}$$

GW from Binary Stars

$$\Box \bar{h}_{\alpha\beta} = -16\pi T_{\alpha\beta}$$

Using **retarded potential**: gravitational radiation is caused by **stress**

$$h_{jk}^{\mathrm{TT}}(t, \mathbf{x}) = 4\left[\int \frac{T_{jk}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'\right]^{\mathrm{TT}}$$

… and energy-momentum conservation, within mass-quadrupole approximation, radiation caused by change in mass-quadrupole moment

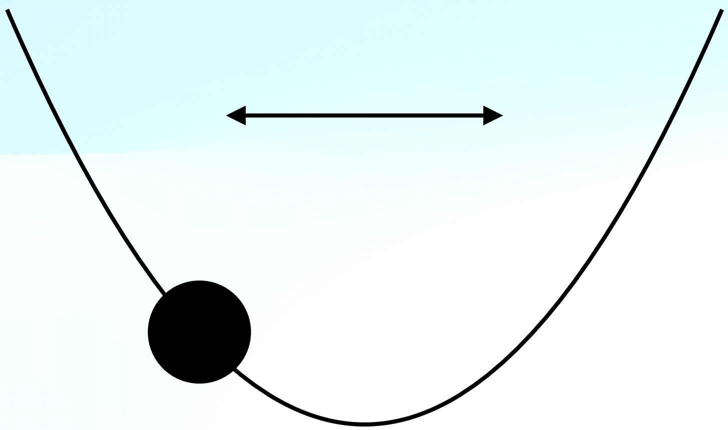
$$h_{jk}^{\mathrm{TT}}(t, \mathbf{x}) = \left[\frac{2\ddot{I}_{jk}(t - r)}{r}\right]^{\mathrm{TT}}$$

$$I_{jk} = \int d^3\mathbf{x}\, \rho(\mathbf{x})\, x_j x_k$$

Harmonic Oscillator?

$$h_{jk}^{\mathrm{TT}}(t,\mathbf{x}) = 4\left[\int \frac{T_{jk}(t-|\mathbf{x}-\mathbf{x}'|,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'\right]^{\mathrm{TT}} \Rightarrow h_{jk}^{\mathrm{TT}}(t,\mathbf{x}) = \left[\frac{2\ddot{I}_{jk}(t-r)}{r}\right]^{\mathrm{TT}}$$

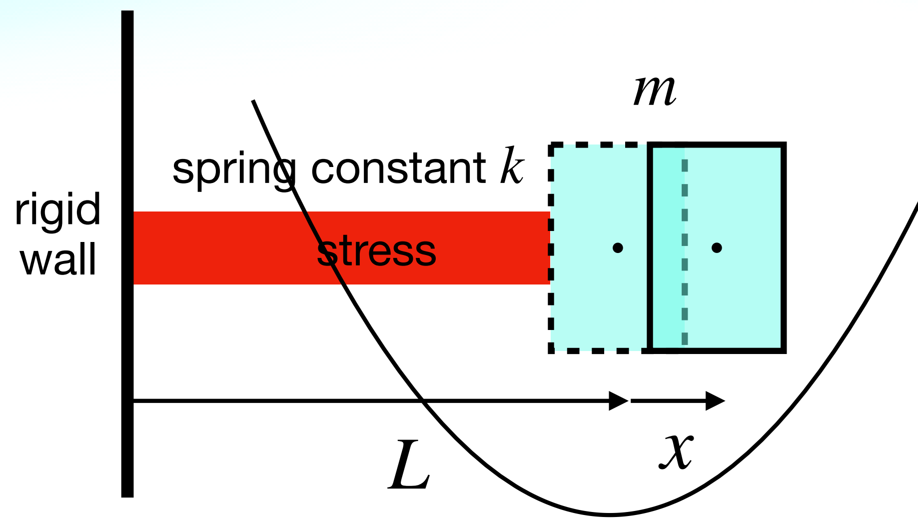
… what about this harmonic oscillator?



$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

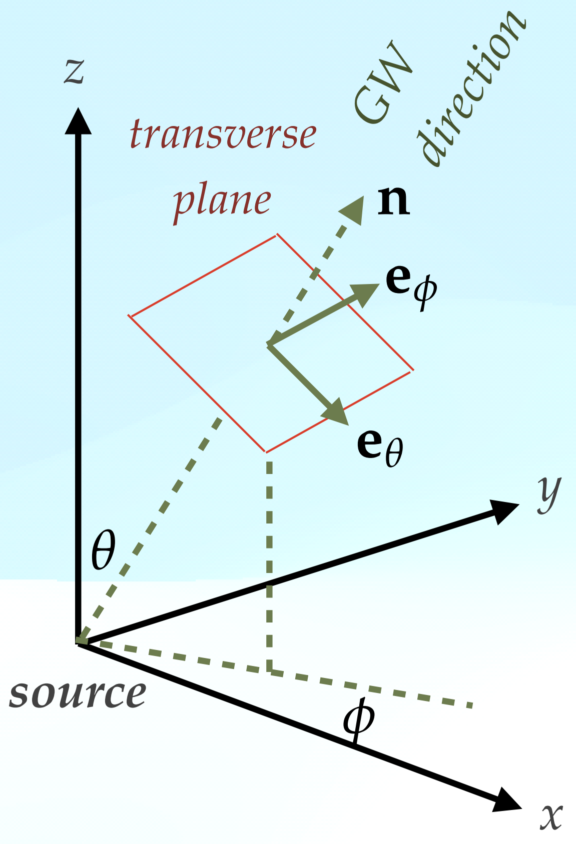
$$I_{xx} = mx^2\,?$$

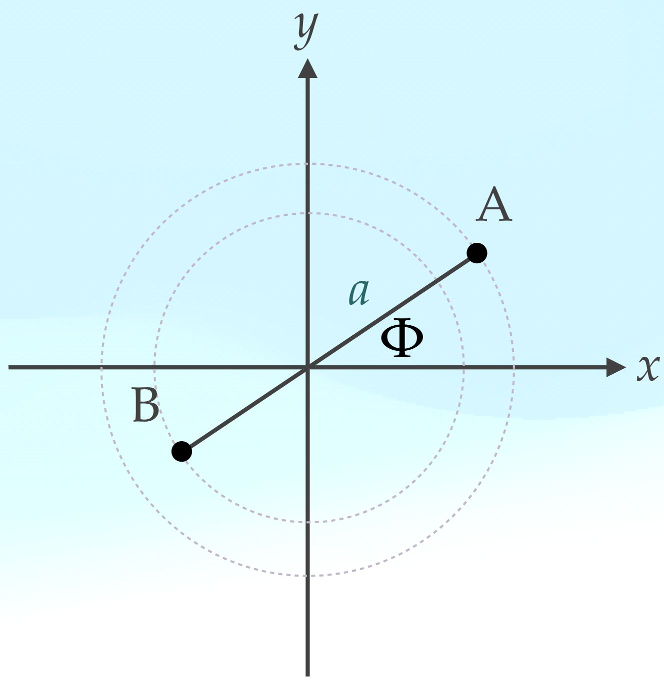
… actually depends on how the “potential” is created



$$I_{xx} = m(L+x)^2$$

Binary Stars in Circular Orbits





$$M = M_A + M_B\,,$$
$$\mu = \frac{M_A M_B}{M}\,,$$
$$\eta \equiv \frac{\mu}{M}\,.$$

Keplerian Motion

$$\dot{\Phi} = \Omega = \sqrt{\frac{M}{a^3}}$$

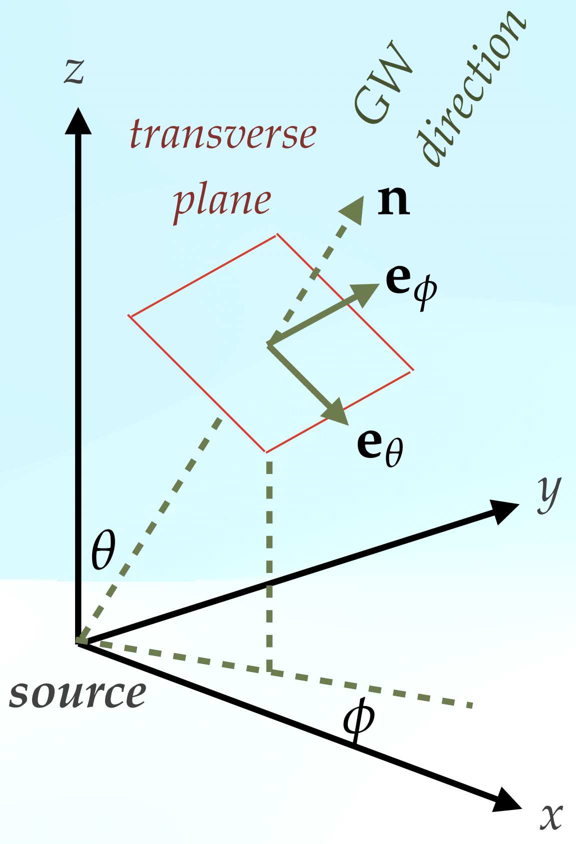
$$\bar{h}^{jk} = +2\left[\frac{\ddot{\mathcal{I}}^{jk}(t-r)}{r}\right]$$

$$h_+ = \frac{1}{2}\bar{h}^{jk}e^+_{jk}\,, \quad h_\times = \frac{1}{2}\bar{h}^{jk}e^\times_{jk}\,.$$

$$I_{xx} = \mu a^2 \cos^2\Phi,\ I_{yy} = \mu a^2 \sin^2\Phi,$$
$$I_{xy} = I_{yx} = \mu a^2 \cos\Phi \sin\Phi$$

$$h_+ = -2(1+\cos^2\theta)\frac{\mu\Omega^2 a^2 \cos 2\Phi}{r}$$
$$h_\times = -4\cos\theta\frac{\mu\Omega^2 a^2 \sin 2\Phi}{r}$$

Binary Stars in Circular Orbits



$$\bar{h}^{jk} = +2\left[\frac{\ddot{\mathcal{I}}^{jk}(t-r)}{r}\right]$$

$$h_+ = \frac{1}{2}\bar{h}^{jk}e^+_{jk}\,,\quad h_\times = \frac{1}{2}\bar{h}^{jk}e^\times_{jk}\,.$$

$$h_+ = -2(1+\cos^2\theta)\frac{\mu\Omega^2a^2\cos 2\Phi}{r}$$

$$h_\times = -4\cos\theta\frac{\mu\Omega^2a^2\sin 2\Phi}{r}$$

Optimally Oriented: $\theta = 0$

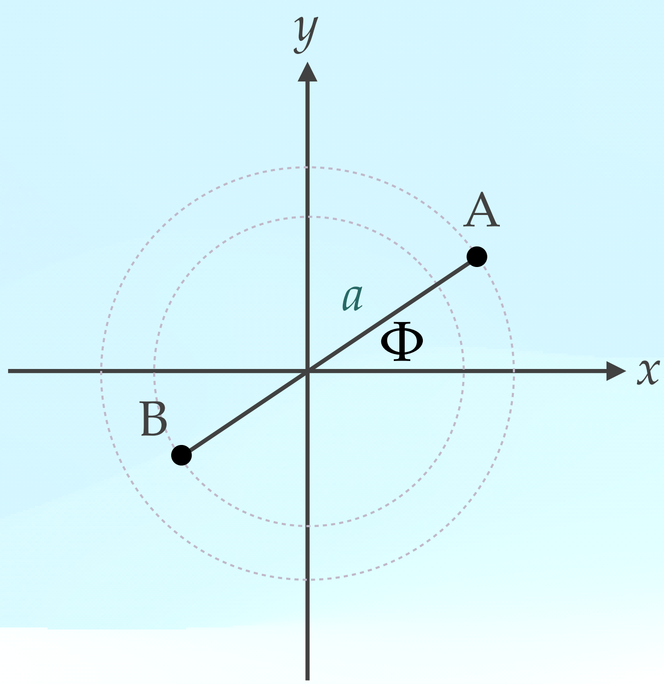
$$h_+ \sim h_+ \sim 4\frac{\mu\Omega^2a^2}{r} \sim 4\frac{\mu v^2}{r} \sim \frac{Mv^2}{r}$$

$$M_\odot \sim 1.5\times 10^3\,\mathrm{m}$$

$$\mathrm{Mpc} \sim 3\times 10^{22}\,\mathrm{m}$$

$$h \sim 5\times 10^{-21}\left(\frac{M}{100\,M_\odot}\right)\left(\frac{100\,\mathrm{Mpc}}{r}\right)\left(\frac{v^2}{0.1}\right)$$

Back Reaction!



Keplerian Motion

$$\dot{\Phi} = \Omega = \sqrt{\frac{M}{a^3}}$$

$$v = \sqrt{M/a} = (M\Omega)^{1/3}$$

$$E_{\rm motion} = -\frac{\mu M}{2a}$$

$$\left(\frac{dE}{dt}\right)_{\rm GW} = \frac{1}{5}\left\langle \dddot{I}_{jk}\dddot{I}^{jk}\right\rangle = \frac{32}{\pi}\frac{\mu^2}{M^2}(M\Omega)^{10/3}$$

Energy carried by GW per unit time

Adiabatic Approach: assuming that orbit remains nearly Keplerian, but with slowly varying parameters.

$$\frac{dE_{\rm motion}}{dt} = -\left(\frac{dE}{dt}\right)_{\rm GW}$$

$$\boxed{\frac{\dot{\Omega}}{\Omega^2} = \frac{96}{5}\left(\frac{\mu}{M}\right)(M\Omega)^{5/3}}$$

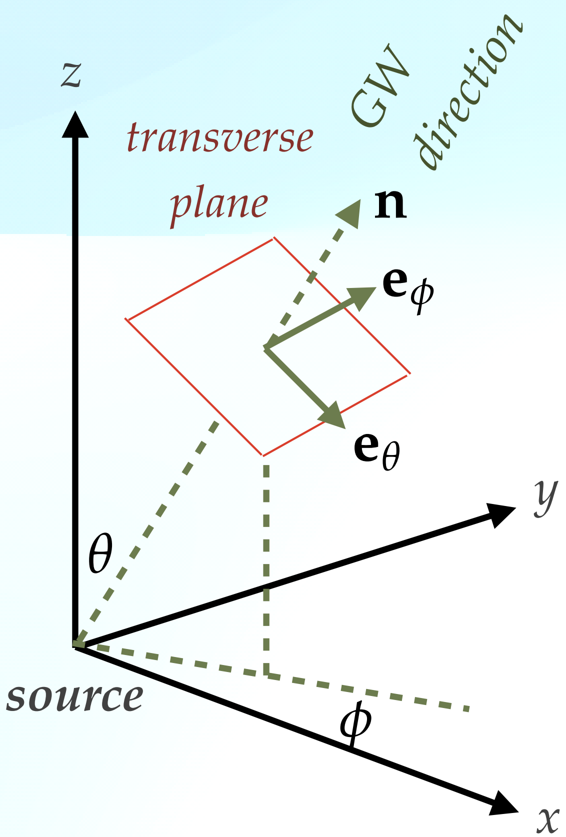
$$\dot{\Omega} \equiv \frac{\Omega}{\tau_{\rm R}} \Rightarrow \frac{\dot{\Omega}}{\Omega^2} = \frac{1}{\Omega\tau_{\rm R}} = \frac{\tau_{\rm orb}}{\tau_{\rm R}} \sim v^5$$

The Newtonian Chirp

$$\frac{\dot{\Omega}}{\Omega^2} = \frac{96}{5}\left(\frac{\mu}{M}\right)(M\Omega)^{5/3} \Rightarrow \Omega(t) = \left[\frac{5}{256\mu M^{2/3}(t_0 - t)}\right]^{3/8}$$

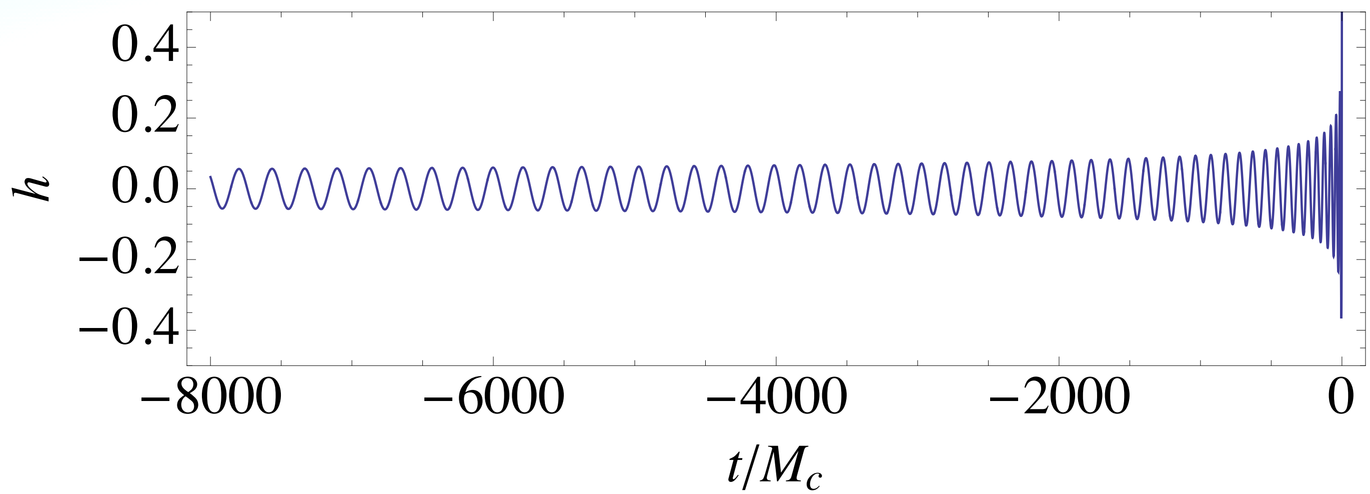
frequency becomes infinite within finite amount of time

$$\Phi(t) = \int^t dt'\Omega(t')$$



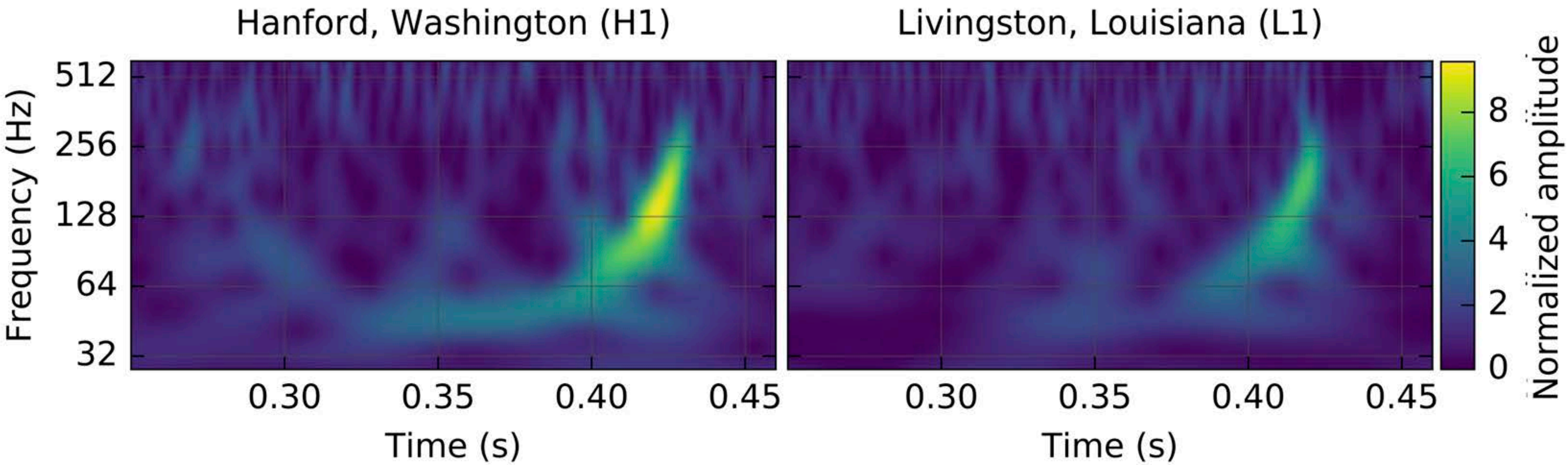
$$h_+ = -2(1+\cos^2\theta)\frac{\mu(M\Omega)^{2/3}\cos 2\Phi}{r}$$

$$h_\times = -4\cos\theta\frac{\mu(M\Omega)^{2/3}\sin 2\Phi}{r}$$



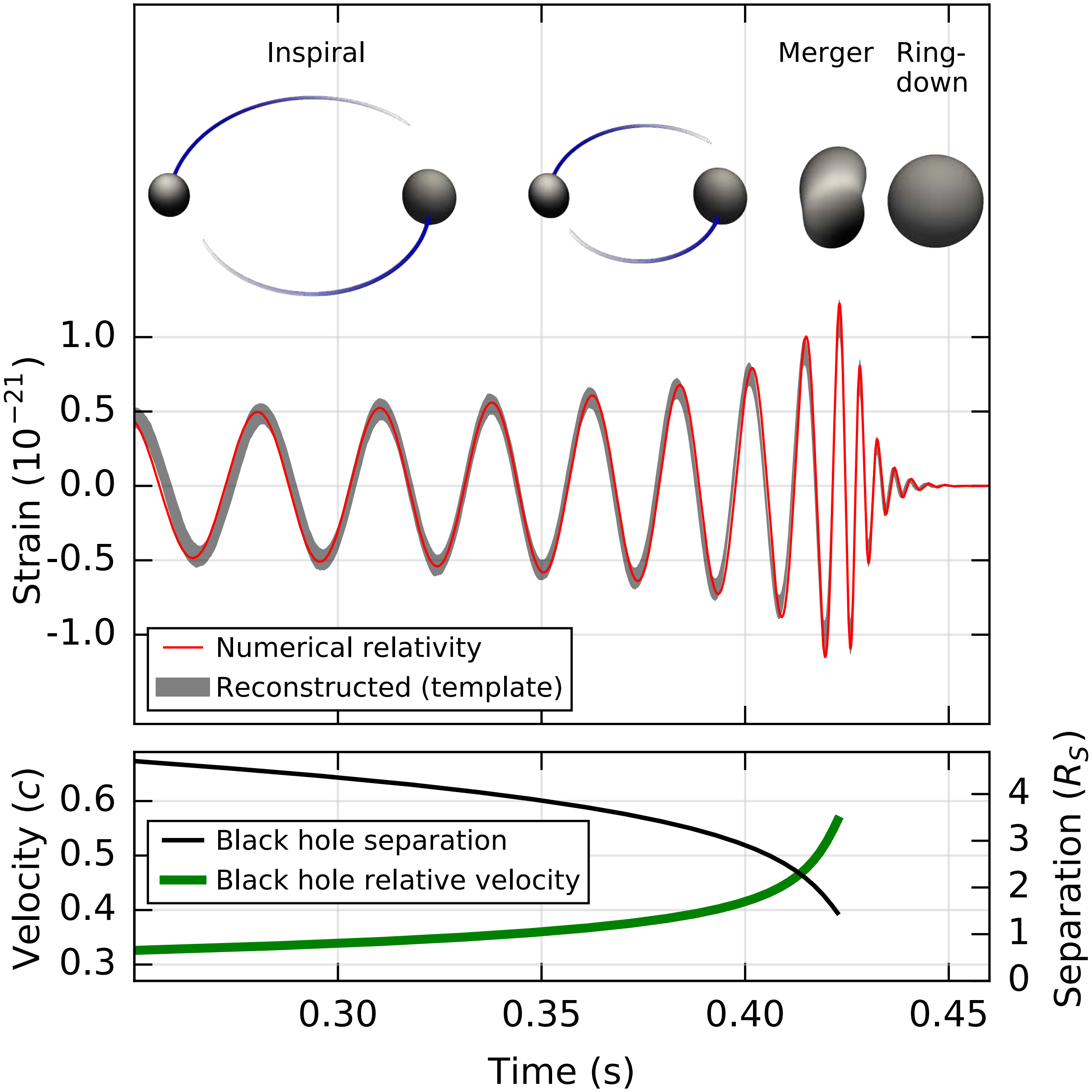
$$M_c \equiv M\eta^{3/5} \equiv M(\mu/M)^{3/5}$$ the "Chirp Mass"

GW150914

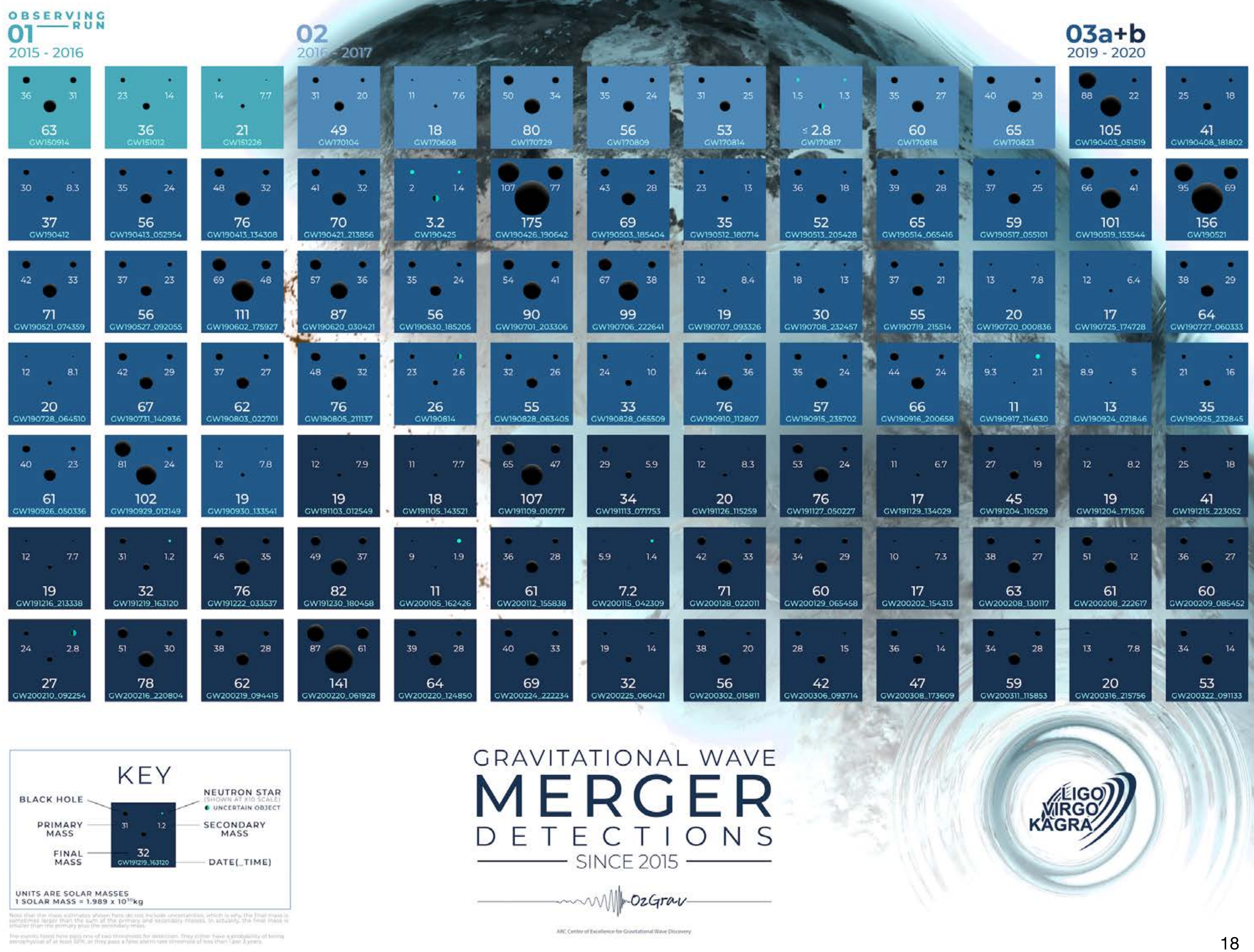


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Frequency (Hz)
512
256
128
64
32
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
0
2
4
6
8

GW150914



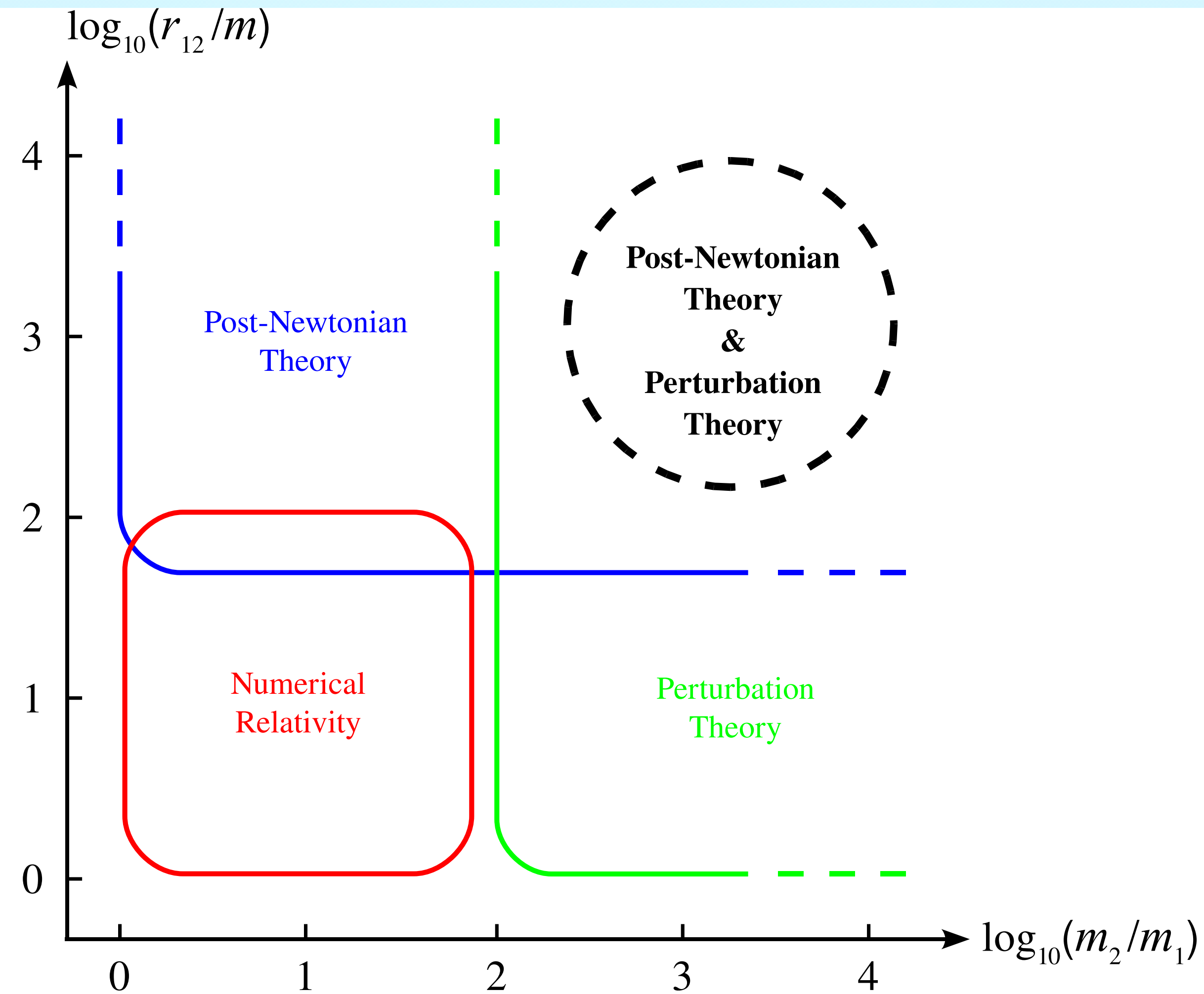
Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$


OBSERVING RUN
01 2015 - 2016
02 2016 - 2017
03a+b 2019 - 2020
36 31 63 GW150914
23 14 36 GW151012
14 7.7 21 GW151226
31 20 49 GW170104
11 7.6 18 GW170608
50 34 80 GW170729
35 24 56 GW170809
31 25 53 GW170814
1.5 1.3 ≤ 2.8 GW170817
35 27 60 GW170818
40 29 65 GW170823
88 22 105 GW190403_051519
25 18 41 GW190408_181802
30 8.3 37 GW190412
35 24 56 GW190413_052954
48 32 76 GW190413_134308
41 32 70 GW190421_213856
2 1.4 3.2 GW190425
107 77 175 GW190426_190642
43 28 69 GW190503_185404
23 13 35 GW190512_180714
36 18 52 GW190513_205428
39 28 65 GW190514_065416
37 25 59 GW190517_055101
66 41 101 GW190519_153544
95 69 156 GW190521
42 33 71 GW190521_074359
37 23 56 GW190527_092055
69 48 111 GW190602_175927
57 36 87 GW190620_030421
35 24 56 GW190630_185205
54 41 90 GW190701_203306
67 38 99 GW190706_222641
12 8.4 19 GW190707_093326
18 13 30 GW190708_232457
37 21 55 GW190719_215514
13 7.8 20 GW190720_000836
12 6.4 17 GW190725_174728
38 29 64 GW190727_060333
12 8.1 20 GW190728_064510
42 29 67 GW190731_140936
37 27 62 GW190803_022701
48 32 76 GW190805_211137
23 2.6 26 GW190814
32 26 55 GW190828_063405
24 10 33 GW190828_065509
44 36 76 GW190910_112807
35 24 57 GW190915_235702
44 24 66 GW190916_200658
9.3 2.1 11 GW190917_114630
8.9 5 13 GW190924_021846
21 16 35 GW190925_232845
40 23 61 GW190926_050336
81 24 102 GW190929_012149
12 7.8 19 GW190930_133541
12 7.9 19 GW191103_012549
11 7.7 18 GW191105_143521
65 47 107 GW191109_010717
29 5.9 34 GW191113_071753
12 8.3 20 GW191126_115259
53 24 76 GW191127_050227
11 6.7 17 GW191129_134029
27 19 45 GW191204_110529
12 8.2 19 GW191204_171526
25 18 41 GW191215_223052
12 7.7 19 GW191216_213338
31 1.2 32 GW191219_163120
45 35 76 GW191222_033537
49 37 82 GW191230_180458
9 1.9 11 GW200105_162426
36 28 61 GW200112_155838
5.9 1.4 7.2 GW200115_042309
42 33 71 GW200128_022011
34 29 60 GW200129_065458
10 7.3 17 GW200202_154313
38 27 63 GW200208_130117
51 12 61 GW200208_222617
36 27 60 GW200209_085452
24 2.8 27 GW200210_092254
51 30 78 GW200216_220804
38 28 62 GW200219_094415
87 61 141 GW200220_061928
39 28 64 GW200220_124850
40 33 69 GW200224_222234
19 14 32 GW200225_060421
38 20 56 GW200302_015811
28 15 42 GW200306_093714
36 14 47 GW200308_173609
34 28 59 GW200311_115853
13 7.8 20 GW200316_215756
34 14 53 GW200322_091133
KEY
BLACK HOLE
PRIMARY MASS
FINAL MASS
NEUTRON STAR (SHOWN AT X10 SCALE)
UNCERTAIN OBJECT
SECONDARY MASS
DATE(_TIME)
31 1.2 32 GW191219_163120
UNITS ARE SOLAR MASSES
1 SOLAR MASS = 1.989 x 10³⁰kg
GRAVITATIONAL WAVE
MERGER
DETECTIONS
SINCE 2015
OzGrav
LIGO VIRGO KAGRA

Extensions to the Newtonian Chirp

- Eccentric orbits.
- Higher v/c and $GM/(rc^2)$ corrections to Newtonian physics.
- Finite-Size effects related to spins, tidal effects, and natures of neutron stars and black holes
- Mergers of the two objects to a final neutron star or black hole

Extensions to the Newtonian Chirp



[L. Blanchet, Living Reviews of Relativity]

Post-Newtonian Dynamics

Full Einstein's equation in "wave equation form"

$$\mathfrak{g}^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\beta}\,, \quad \bar{h}^{\mu\nu} \equiv \eta^{\mu\nu} - \mathfrak{g}^{\mu\nu}$$

Choose Harmonic gauge (Lorenz gauge): $\mathfrak{g}^{\alpha\beta}{}_{,\beta} = 0$

$$\mathfrak{g}^{\alpha\beta}\bar{h}^{\mu\nu}{}_{,\alpha\beta} = (-16\pi)\left[(-g)T^{\mu\nu} + (-g)t^{\mu\nu}_{\rm LL}\right] - \bar{h}^{\alpha\nu}{}_{,\beta}\bar{h}^{\mu\beta}{}_{,\alpha}$$

Landau-Lifshitz pseudotensor

$$\begin{aligned}(-g)\, t^{\alpha\beta}_{\text{L-L}} = \frac{1}{16\pi}\Big\{ & \mathfrak{g}^{\alpha\beta}{}_{,\lambda}\mathfrak{g}^{\lambda\mu}{}_{,\mu} - \mathfrak{g}^{\alpha\lambda}{}_{,\lambda}\mathfrak{g}^{\beta\mu}{}_{,\mu} + \frac{1}{2} g^{\alpha\beta} g_{\lambda\mu}\mathfrak{g}^{\lambda\nu}{}_{,\rho}\mathfrak{g}^{\rho\mu}{}_{,\nu} \\ & - (g^{\alpha\lambda} g_{\mu\nu}\mathfrak{g}^{\beta\nu}{}_{,\rho}\mathfrak{g}^{\mu\rho}{}_{,\lambda} + g^{\beta\lambda} g_{\mu\nu}\mathfrak{g}^{\alpha\nu}{}_{,\rho}\mathfrak{g}^{\mu\rho}{}_{,\lambda}) + g_{\lambda\mu} g^{\nu\rho}\mathfrak{g}^{\alpha\lambda}{}_{,\nu}\mathfrak{g}^{\beta\mu}{}_{,\rho} \\ & + \frac{1}{8}(2g^{\alpha\lambda}g^{\beta\mu} - g^{\alpha\beta}g^{\lambda\mu})(2g_{\nu\rho}g_{\sigma\tau} - g_{\rho\sigma}g_{\nu\tau})\mathfrak{g}^{\nu\tau}{}_{,\lambda}\mathfrak{g}^{\rho\sigma}{}_{,\mu}\Big\} \end{aligned} \qquad (20.22)$$

[Misner, Thorne and Wheeler; Landau and Lifshitz]

PN Binding Energy (Non-Spinning)

- Evolution of circular orbits can be obtained from adiabatic matching:

$$\frac{dx}{dt} = \frac{dE/dt}{dE/dx} = -\frac{\mathcal{F}(x)}{dE(x)/dx}$$

- Multipole waveform can also be obtained.

$$\begin{aligned}
\mathrm{E} = -\frac{\mu c^2 x}{2}\Bigg\{ & 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \\
& + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3 \\
& + \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_\mathrm{E} + \frac{448}{15}\ln(16x)\right]\nu \right. \\
& \left. + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4 \\
& + \mathcal{O}\left(\frac{1}{c^{10}}\right)\Bigg\}.
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}^{\mathrm{log}}_{\mathrm{tail}} = -\frac{\mu c^2 x}{2}\Bigg\{ & \frac{448}{15}\nu x^4 \ln x + \left(-\frac{4988}{35} - \frac{656}{5}\nu\right)\nu x^5 \ln x \\
& + \left(-\frac{1967284}{8505} + \frac{914782}{945}\nu + \frac{32384}{135}\nu^2\right)\nu x^6 \ln x + \mathcal{O}\left(\frac{1}{c^{14}}\right)\Bigg\}.
\end{aligned}$$

$$x \equiv \left(\frac{Gm\Omega}{c^3}\right)^{2/3} = \mathcal{O}\left(\frac{1}{c^2}\right).$$

[L. Blanchet, Living Reviews of Relativity]

PN Energy Flux (Non-Spinning)

- Evolution of circular orbits can be obtained from adiabatic matching:

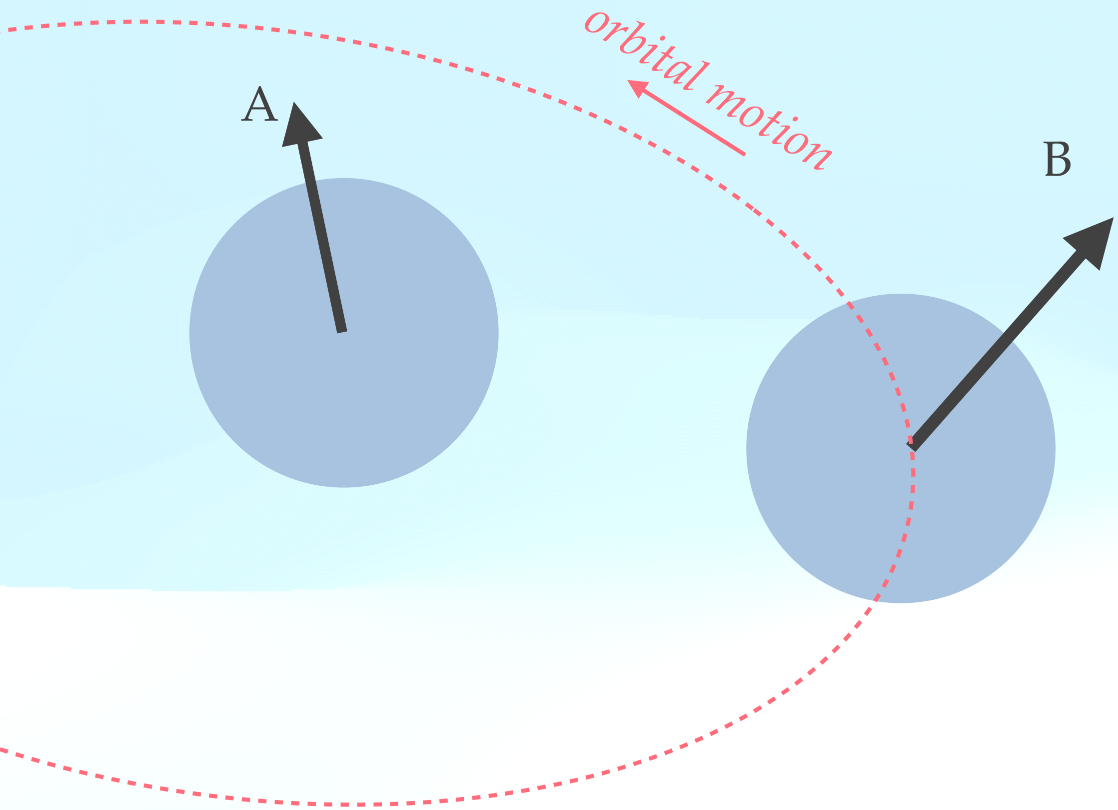
$$\frac{dx}{dt} = \frac{dE/dt}{dE/dx} = -\frac{\mathscr{F}(x)}{dE(x)/dx}$$

- Multipole waveform can also be obtained.

$$\begin{aligned}
\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} \\
& + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\
& + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16\,x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu \right. \\
& \left. - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3 \\
& + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} \\
& + \left[-\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma_E - \frac{1369}{126}\pi^2 + \frac{39931}{294}\ln 2 - \frac{47385}{1568}\ln 3 \right. \\
& + \frac{232597}{8820}\ln x \\
& + \left(-\frac{1452202403629}{1466942400} + \frac{41478}{245}\gamma_E - \frac{267127}{4608}\pi^2 + \frac{479062}{2205}\ln 2 + \frac{47385}{392}\ln 3 \right. \\
& \left. + \frac{20739}{245}\ln x\right)\nu \\
& \left. + \left(\frac{1607125}{6804} - \frac{3157}{384}\pi^2\right)\nu^2 + \frac{6875}{504}\nu^3 + \frac{5}{6}\nu^4\right]x^4 \\
& + \left[\frac{265978667519}{745113600} - \frac{6848}{105}\gamma_E - \frac{3424}{105}\ln(16\,x) + \left(\frac{2062241}{22176} + \frac{41}{12}\pi^2\right)\nu \right. \\
& \left. - \frac{133112905}{290304}\nu^2 - \frac{3719141}{38016}\nu^3\right]\pi x^{9/2} + \mathcal{O}\left(\frac{1}{c^{10}}\right)\Bigg\} .
\end{aligned}$$

[L. Blanchet, Living Reviews of Relativity]

Effects of Spins



Leading-order dynamical effects superimpose with each other.

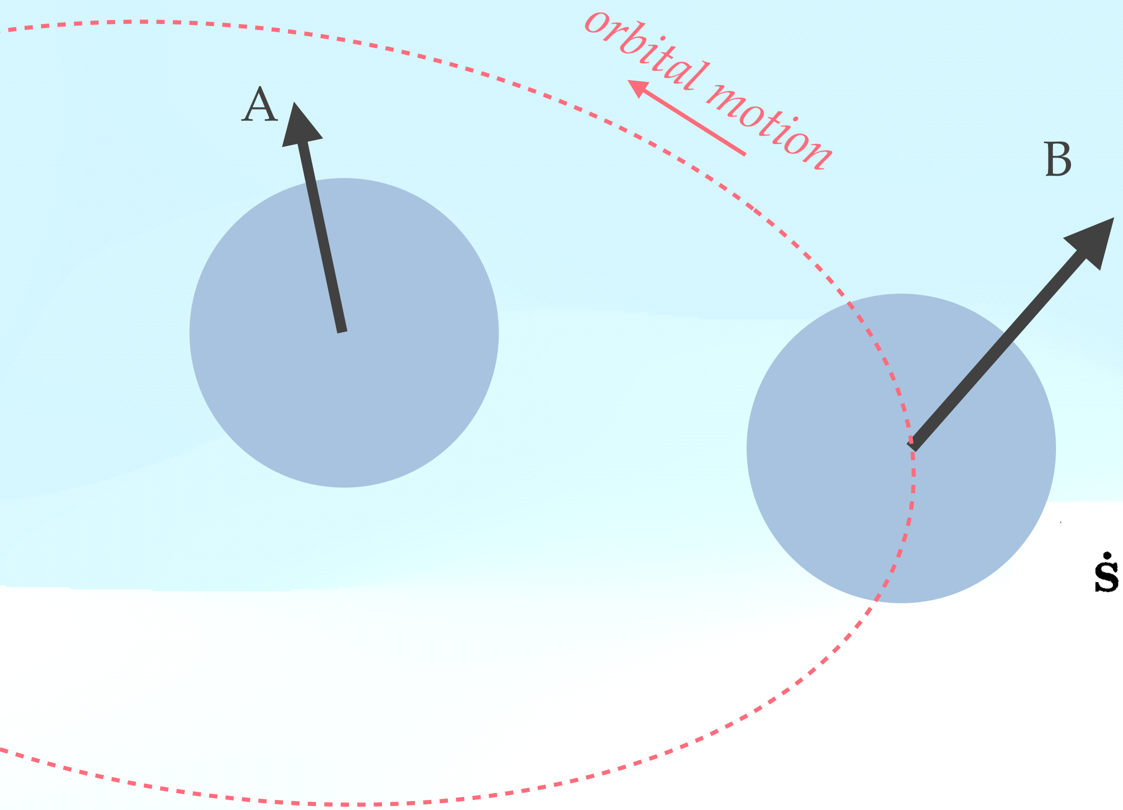
described by **gravito-magnetism**

For aligned spins, both binding energy and radiation flux are affected by spin-orbit and spin-spin coupling.

for non-aligned spins

1. **Orbital motion** *of B around A causes* **S_B** *to* **precess** *[spin-orbit]*
2. **S_A** *causes* **S_B** to **precess** *[spin-spin]*
3. **S_A** *causes* **orbit of B** to **precess** *[spin-orbit]*
4. **S_B** *causes* **orbit of B** to **precess** *[spin-orbit]*

Effects of Spins



$$\frac{d\mathbf{L}_B}{dt} = \underbrace{\frac{2}{r^3}\mathbf{S}_A \times \mathbf{L}_B}_{\sim \mathbf{H}\times\mathbf{v}_B} + \underbrace{\frac{3}{2}\frac{m_A}{m_B}\frac{\mathbf{S}_B \times \mathbf{L}_B}{r_{AB}^3}}$$

gravitomagnetic precession of orbit

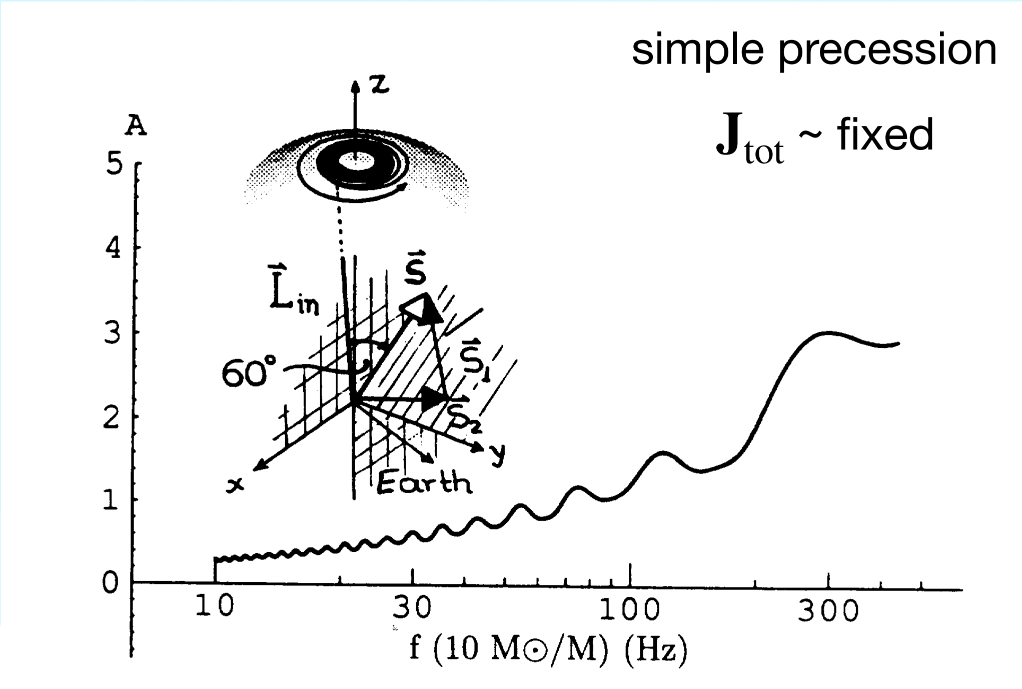
spin-curvature coupling

$$\dot{\mathbf{S}}_1 = \frac{1}{r^3}\left\{(\mathbf{L}_\mathrm{N} \times \mathbf{S}_1)\left(2 + \frac{3}{2}\frac{m_2}{m_1}\right) - \mathbf{S}_2 \times \mathbf{S}_1 + 3(\hat{\mathbf{n}} \cdot \mathbf{S}_2)\hat{\mathbf{n}} \times \mathbf{S}_1\right\}, \tag{2.4a}$$

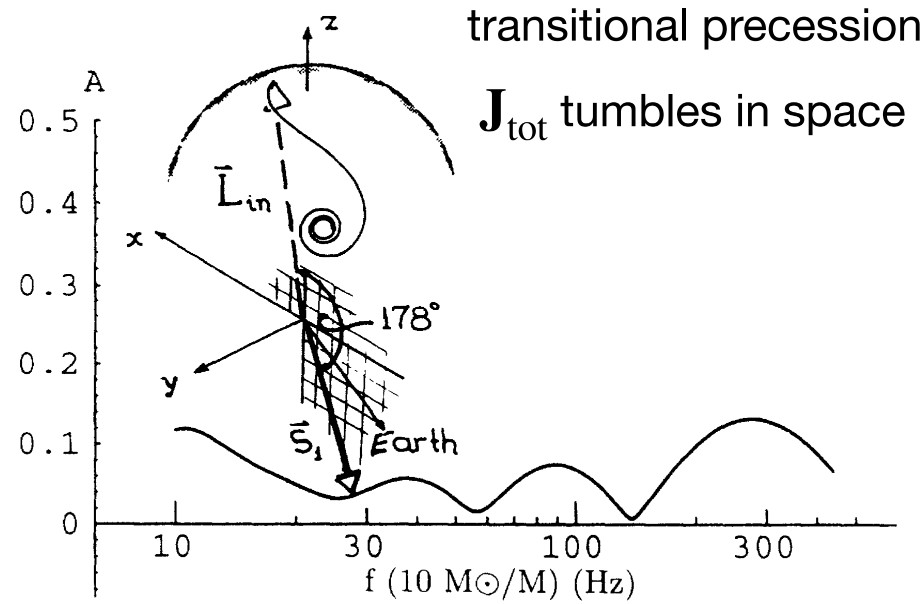
$$\dot{\mathbf{S}}_2 = \frac{1}{r^3}\left\{(\mathbf{L}_\mathrm{N} \times \mathbf{S}_2)\left(2 + \frac{3}{2}\frac{m_1}{m_2}\right) - \mathbf{S}_1 \times \mathbf{S}_2 + 3(\hat{\mathbf{n}} \cdot \mathbf{S}_1)\hat{\mathbf{n}} \times \mathbf{S}_2\right\}, \tag{2.4b}$$

Kidder, 1995

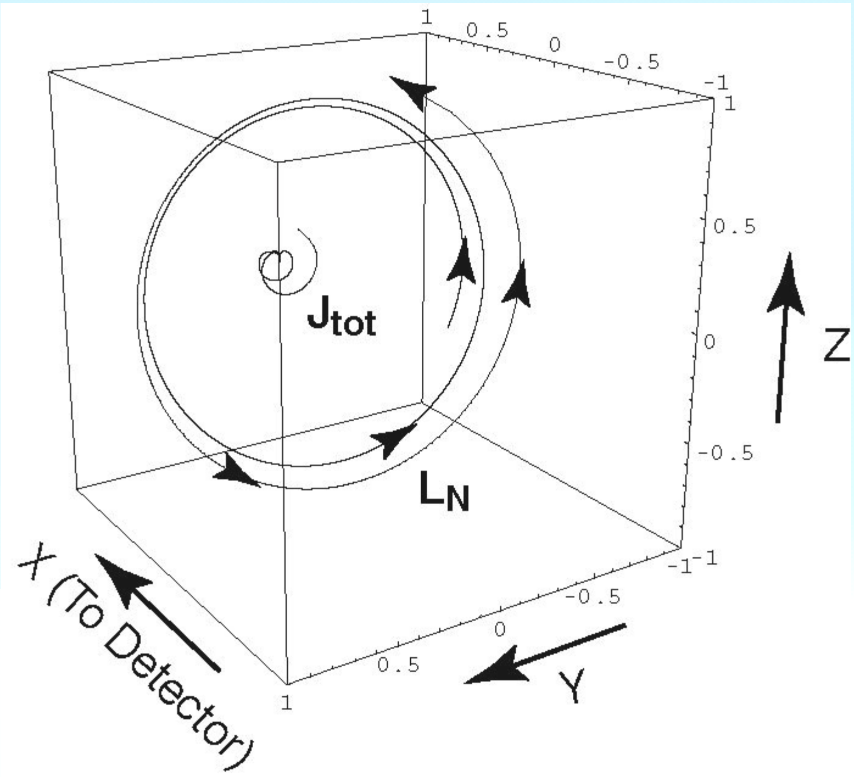
Waveforms from Precessing Binaries

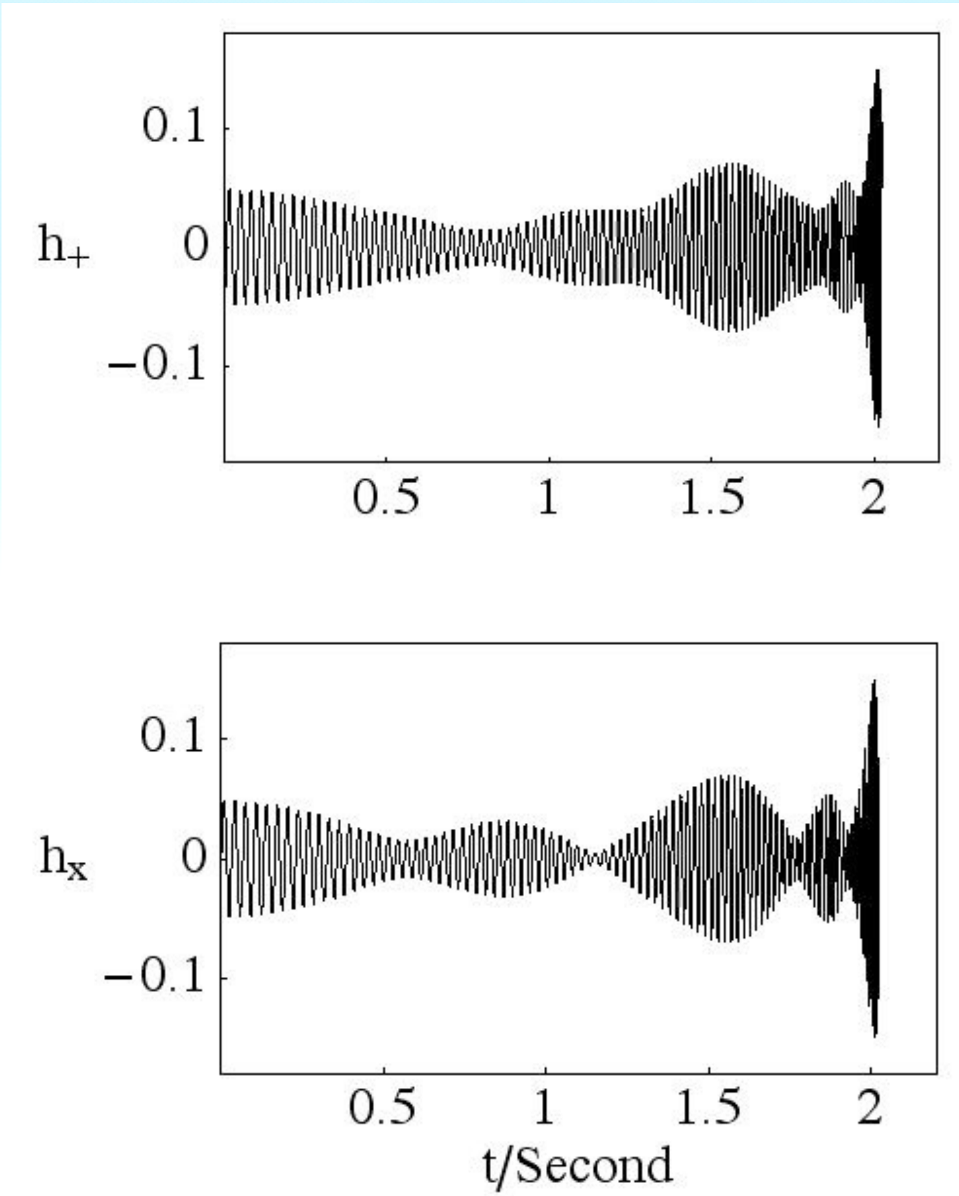


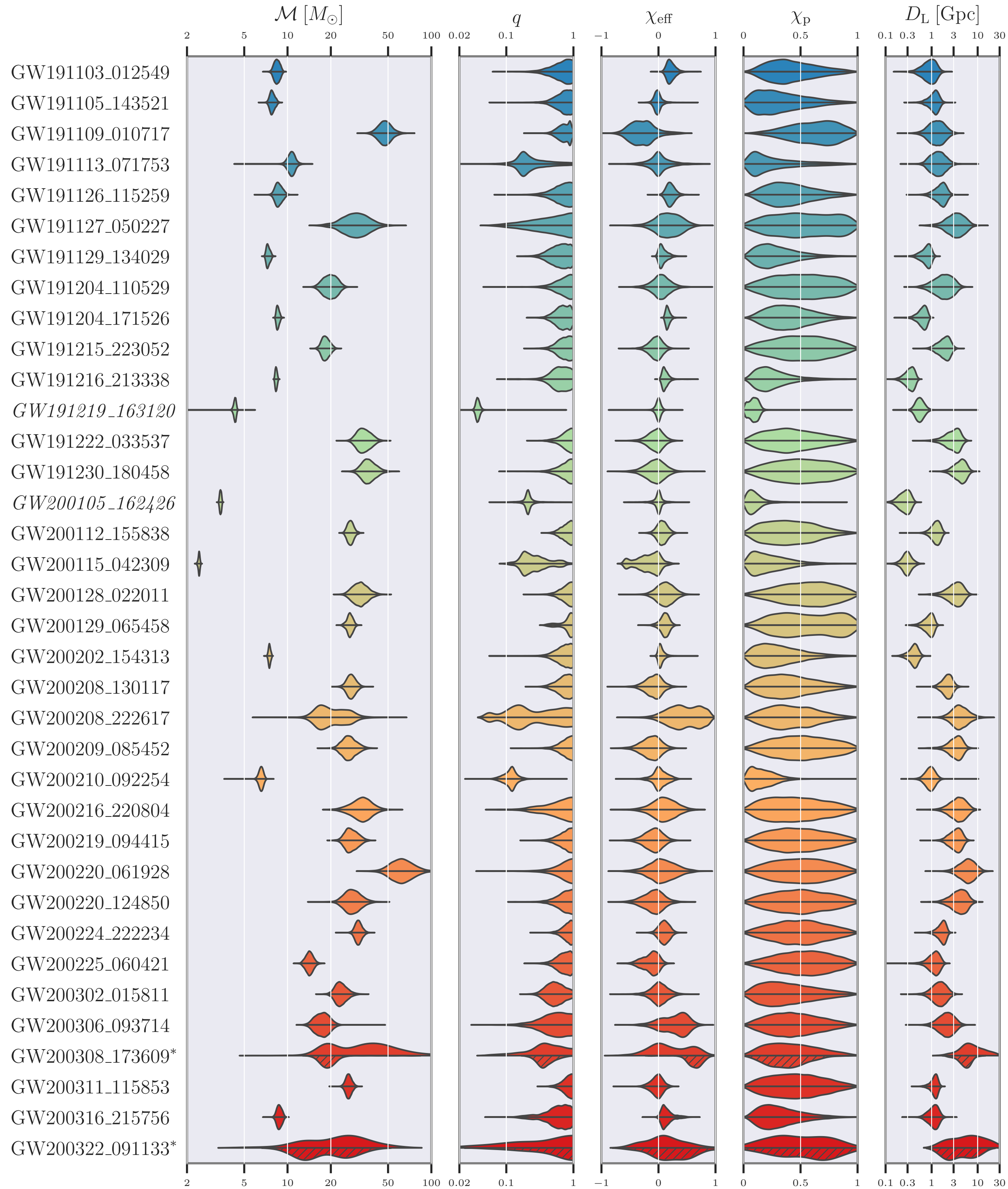
[Apostolatos et al., 1994]



Waveforms from Precessing Binaries







Effect of phase cumulation

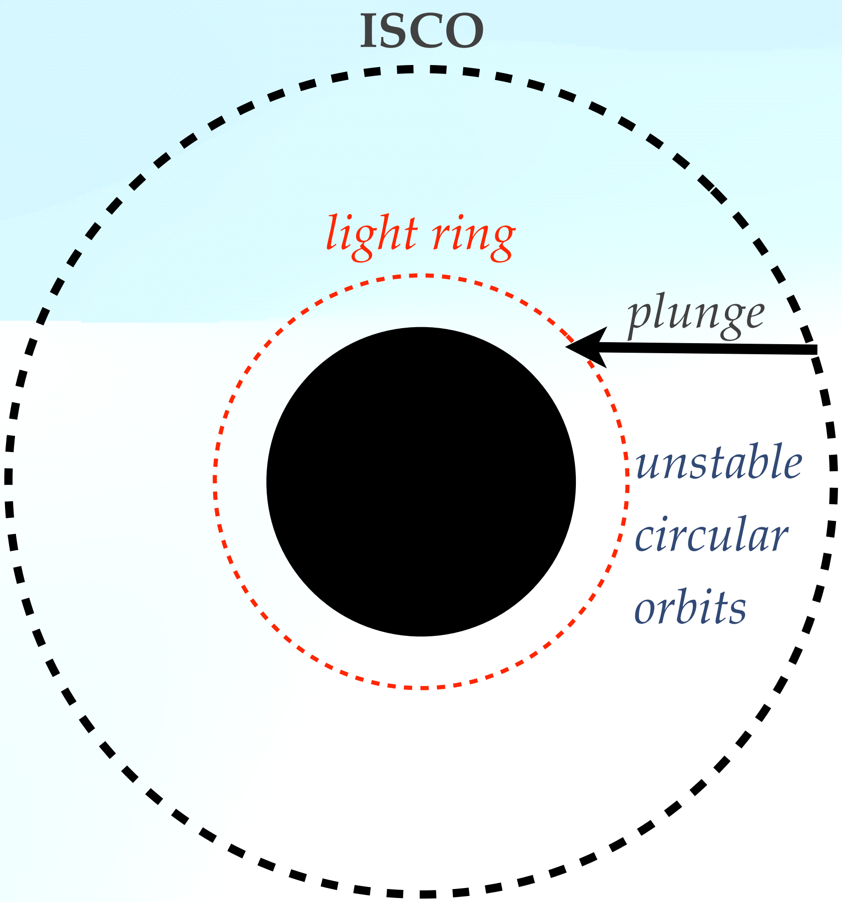
$$\chi_{\rm eff} = \frac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{L}_{\rm N}}{M},$$

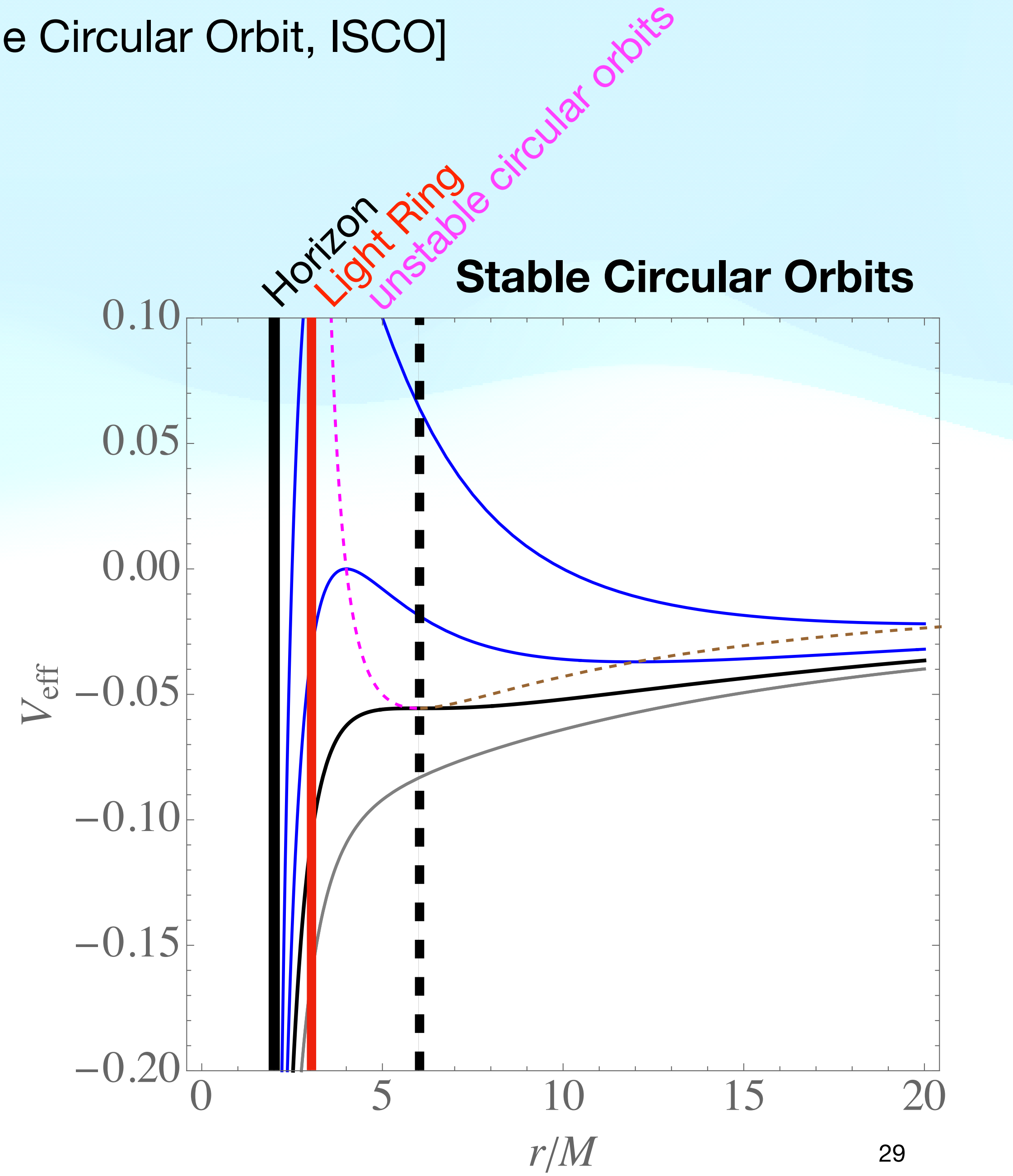
Effect of precessions

$$\chi_{\rm p} = \max\left\{\chi_{1,\perp}, \frac{q(4q+3)}{4+3q}\chi_{2,\perp}\right\},$$

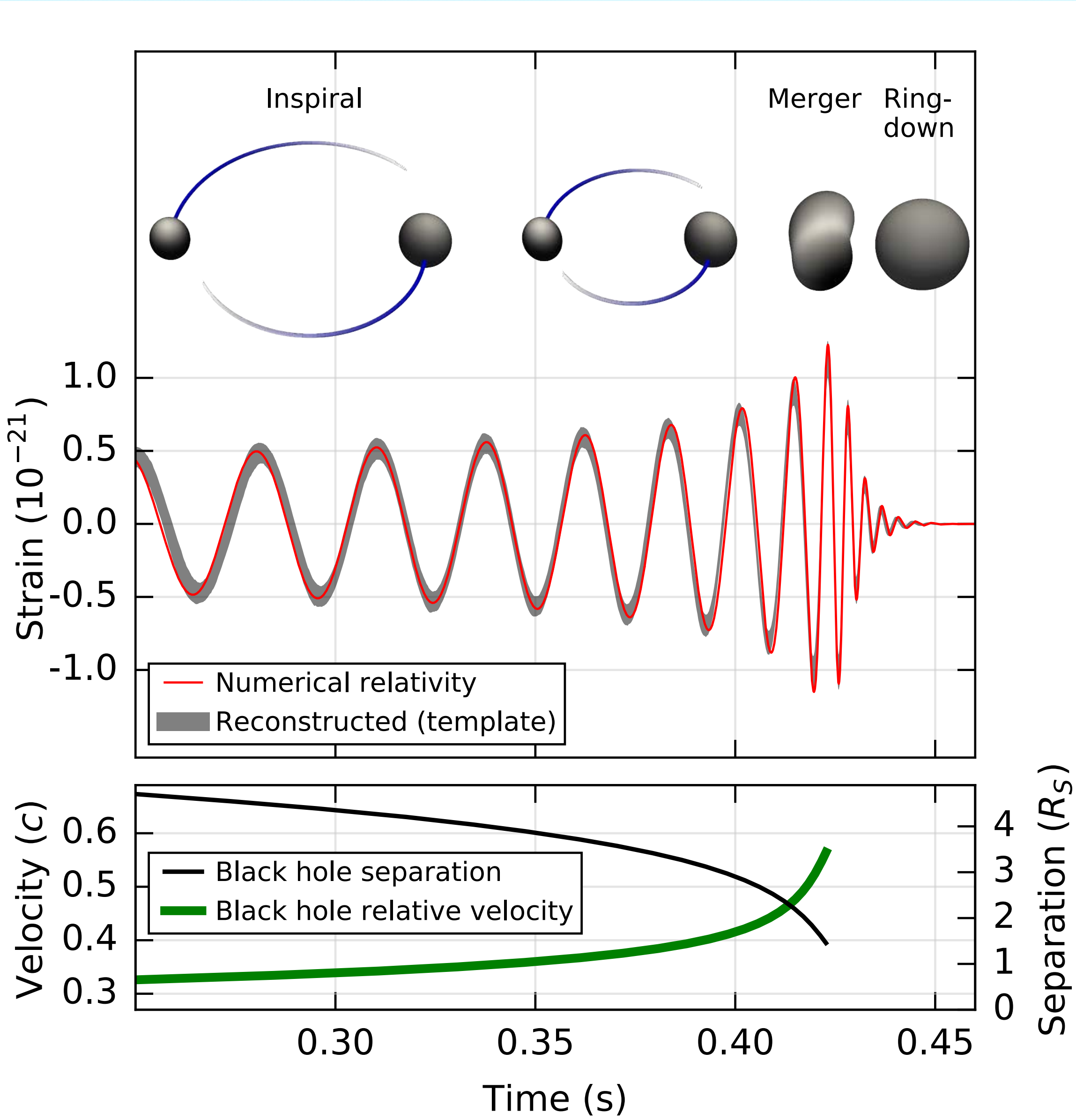
Transition from Inspiral to Plunge

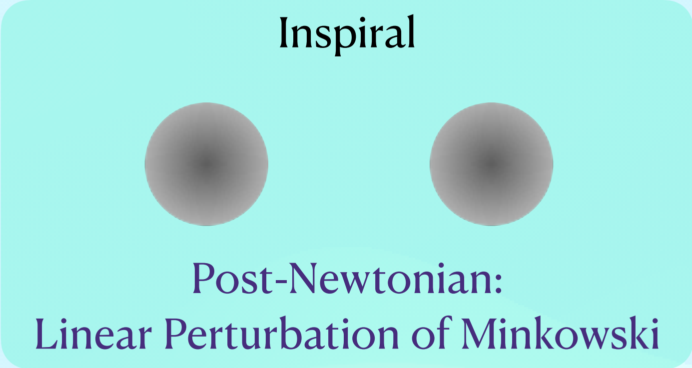
[Innermost Stable Circular Orbit, ISCO]

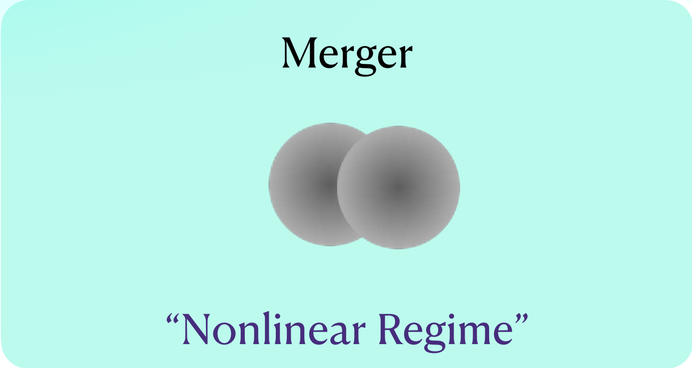




Inspiral, Merger and Ringdown







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Binary black-hole surrogate waveform catalog

Scott E. Field; Chad R. Galley; Jan S. Hesthaven; Jason Kaye; Manuel Tiglio; Jonathan Blackman; Béla Szilágyi; Mark A. Scheel; Daniel A. Hemberger; Patricia Schmidt; Rory Smith; Christian D. Ott; Michael Boyle; Lawrence E. Kidder; Harald P. Pfeiffer; Vijay Varma

This repository contains all publicly available numerical relativity surrogate data for waveforms produced by the Spectral Einstein Code. The base method for building surrogate models can be found in Field et al., PRX 4, 031006 (2014).

Several numerical relativity surrogate models are currently available in this catalog:

- Current models
  1. NRSur7dq4.h5 — This is a surrogate model for binary black hole mergers with generic spins and mass ratios up to 4. A paper describing it can be found at Varma et al., arxiv:1905.09300. It is evaluated with the gwsurrogate Python package, which can be found on PyPI. Instructions for evaluating this surrogate can be found at this example IPython code.
  2. NRHybSur3dq8.h5 — This is a surrogate model for binary black hole systems with generic mass ratios but restricted to nonprecessing spins. Before constructing the surrogate, the NR waveforms are hybridized with post-Newtonian waveforms to include the early inspiral. Therefore this model covers the full stellar mass range for ground-based detectors. A paper describing it can be found at Varma et al., PRD 99, 064045 (2019). It is evaluated with the gwsurrogate Python package, which can be found on PyPI. Instructions for evaluating this surrogate can be found this example IPython code.
  3. NRSur7dq4Remnant — This is a surrogate model for mass, spin, and recoil kick velocity of the remnant BH left behind in generically precessing binary black hole mergers, with mass ratios up to 4. A paper describing it can be found at Varma et al., arxiv:1905.09300. It is evaluated with the surfinBH Python package, which can be found on PyPI. Installation instructions and an ipython help notebook can be found in the same link.
- Older models
  1. SpEC_q1_10_NoSpin_nu5thDegPoly_exclude_2_0.h5 — A surrogate model for binary black hole mergers with non-spinning black holes. This is described in Blackman et al., PRL 115, 121102 (2015). It is evaluated with the gwsurrogate python package, which can be found on PyPI. Instructions for evaluating this surrogate can be found in tutorials included with the gwsurrogate package and in this example IPython code.
  2. NRSur4d2s_FDROM_grid12.h5 and NRSur4d2s_TDROM_grid12.h5 — These are fast frequency-domain and time-domain (respectively) surrogate models for binary black hole mergers where the black holes may be spinning, but the spins are restricted to a parameter subspace which includes some but not all precessing configurations. NRSur4d2s_FDROM_grid12.h5 is the NRSur4d2s_FDROM model described in Blackman et al., PRD 95, 104023, (2017), and NRSur4d2s_TDROM_grid12.h5 is built from the underlying (slower) NRSur4d2s time-domain model in the same way but without the FFTs. These surrogates are also evaluated using gwsurrogate, and a tutorial can be found in this example IPython code.
  3. NRSur7dq2.h5 — This is a surrogate model for binary black hole mergers with generic spins. A paper

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arXiv:1809.09125
10.1103/PhysRevD.99.064045
arXiv:1905.09300

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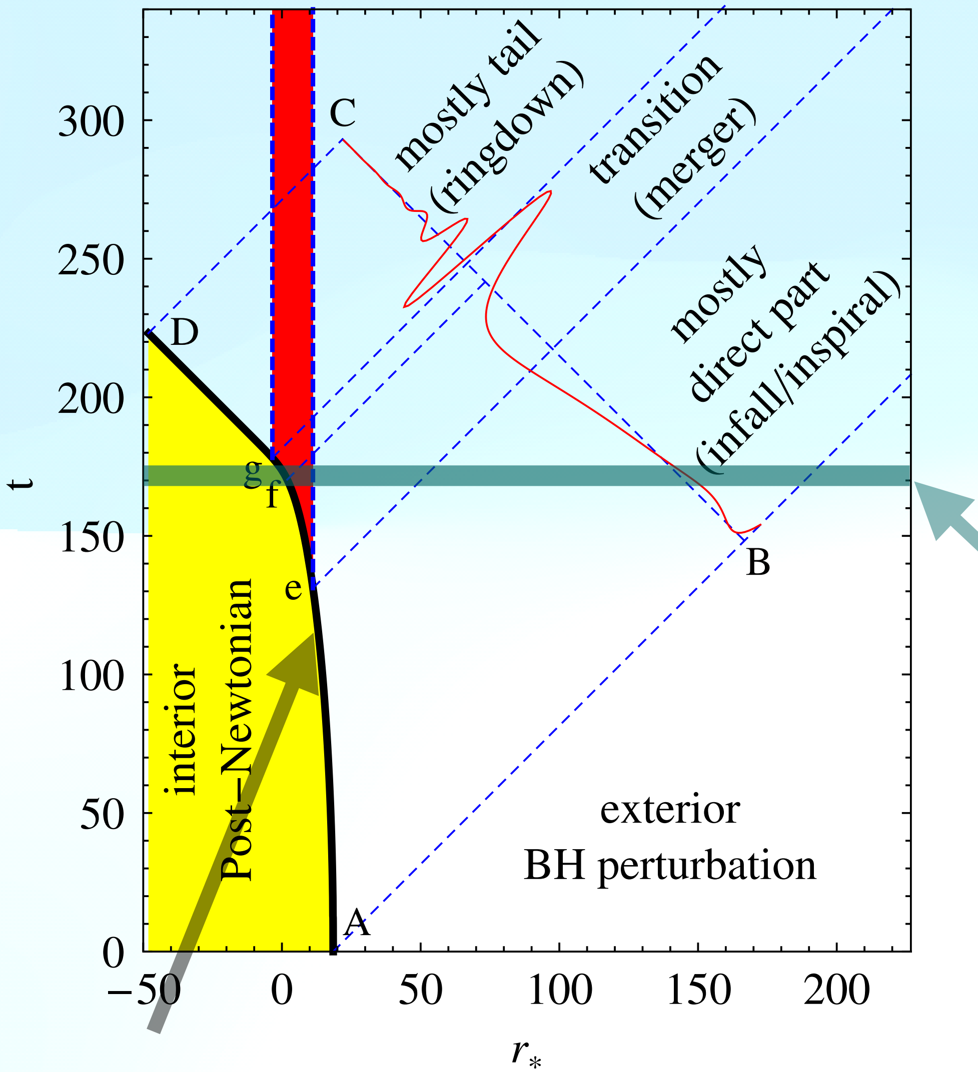
Name	$\|\vec{\Omega}^{\rm orb\ ini}\|$	$M^{\rm ini,1}/M^{\rm ini,2}$	$\chi_{\rm eff}$	$\chi_{\perp}^{\rm ref,1}$	$\chi_{\perp}^{\rm ref,2}$	$e^{\rm ref}$	$N^{\rm orbits}$	Files
	From: To:	From: To:	From: To:	From: To:	From: To:	From: To:	From: To:	
SXS:BBH:0001	1.220×10^{-2}	1.000	1.216×10^{-7}	9.733×10^{-10}	1.430×10^{-9}	2.569×10^{-4}	28.12	
SXS:BBH:0002	1.129×10^{-2}	1.000	9.399×10^{-8}	7.182×10^{-10}	1.473×10^{-9}	1.746×10^{-4}	32.42	
SXS:BBH:0003	1.128×10^{-2}	1.000	2.707×10^{-4}	0.4994	7.429×10^{-8}	2.869×10^{-4}	32.34	
SXS:BBH:0004	1.131×10^{-2}	1.000	-0.2498	5.018×10^{-11}	1.403×10^{-9}	3.802×10^{-4}	30.19	
SXS:BBH:0005	1.217×10^{-2}	1.000	0.2498	6.905×10^{-11}	2.745×10^{-9}	2.355×10^{-4}	30.19	
SXS:BBH:0006	1.446×10^{-2}	1.345	-0.1351	0.2770	0.1115	2.485×10^{-4}	20.08	
SXS:BBH:0007	1.220×10^{-2}	1.500	1.140×10^{-7}	7.405×10^{-10}	3.566×10^{-10}	4.338×10^{-4}	29.09	
SXS:BBH:0008	1.443×10^{-2}	1.500	1.831×10^{-7}	1.870×10^{-9}	8.263×10^{-9}	1.586×10^{-3}	21.28	
SXS:BBH:0009	1.737×10^{-2}	1.500	0.2998	1.206×10^{-10}	5.697×10^{-9}	$<9.500\times10^{-5}$	17.10	
SXS:BBH:0010	1.448×10^{-2}	1.500	-0.2596	0.2499	5.689×10^{-8}	4.385×10^{-4}	19.40	
SXS:BBH:0011	1.438×10^{-2}	1.500	0.2599	0.2489	2.302×10^{-8}	6.030×10^{-5}	23.42	
SXS:BBH:0012	1.449×10^{-2}	1.500	-0.2998	3.793×10^{-11}	1.406×10^{-9}	5.960×10^{-5}	19.08	

Pages: **1** 2 3 4 5 6 7 8 9 10 11 12 13 14 15 ... Next Last 1 of 68

Black-Hole: Ringdown

potential barrier (light ring)

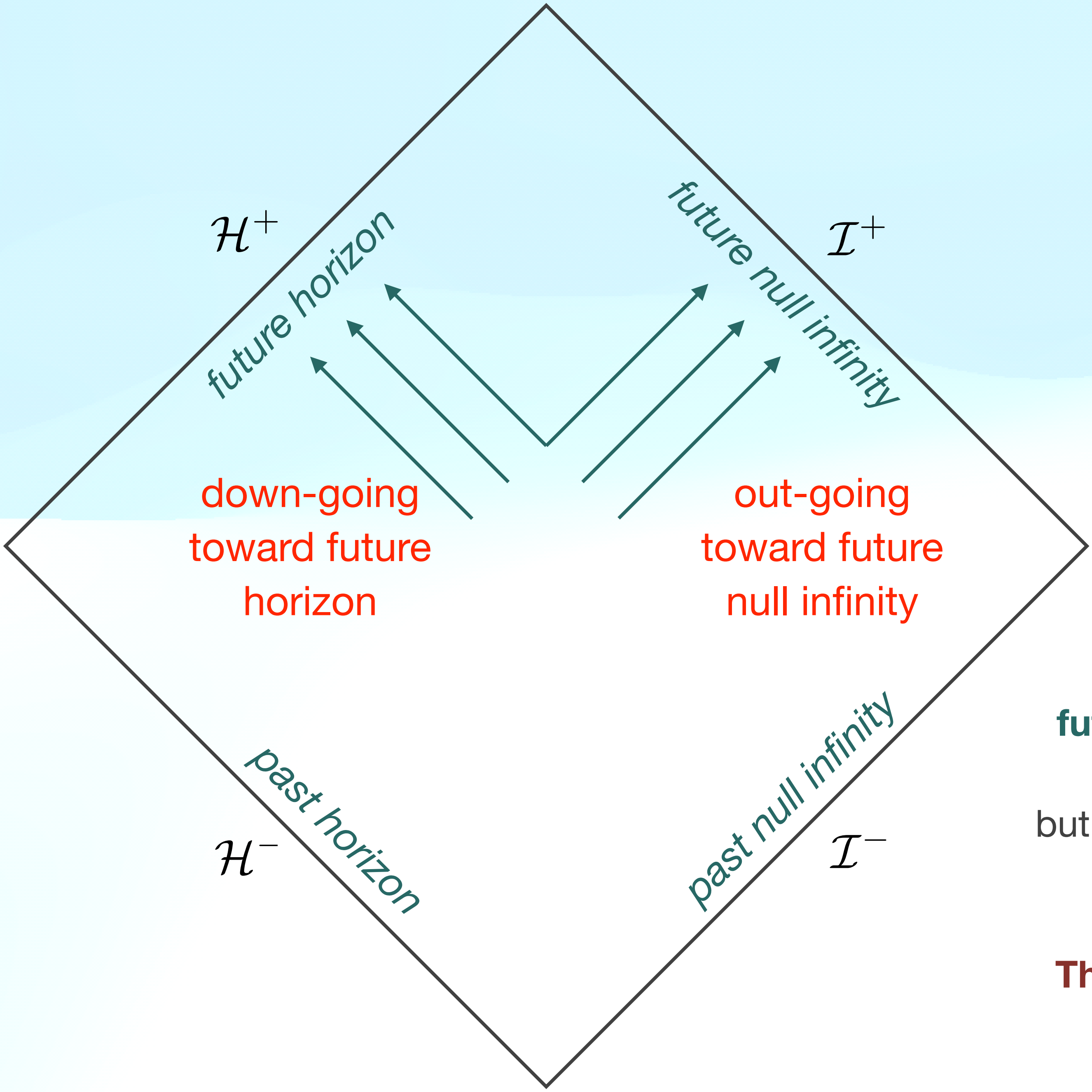


Posing Data on Inflating Shell: Hybrid Approach
[Nichols and Chen, 2010; Zimmerman and Chen, 2011]

Richard Price (1970s):
GW at the end of gravitational collapse is made up from QNMs, because the star no longer sources radiation that can escape the potential barrier

Close-Limit Approximation
[Campanelli, Lousto, Baker, Price, et al., 1990s]

Black-Hole: Quasi-Normal Modes



Waves will reach the **future horizon**, and distort it.

but the horizon simply passively receives the radiation.

The horizon will not radiate.

Teukolsky Equation

$$\nabla^{\alpha}\nabla_{\alpha}h_{\mu\nu} + 2\,{}^{B}R^{\alpha}{}_{\mu\nu}{}^{\beta}h_{\alpha\beta} = 0$$

too many components!

In 1973, Teukolsky found decoupled equation for Weyl scalars ψ_0 and ψ_4.

TABLE 1

FIELD QUANTITIES ψ, SPIN-WEIGHT s, AND SOURCE TERMS T FOR EQUATION (4.7)

ψ	s	T
Φ	0	$\Box\Phi = 4\pi T$
χ_0 $\rho^{-1}\chi_1$	$\frac{1}{2}$ $-\frac{1}{2}$	See references in Appendix B
ϕ_0 $\rho^{-2}\phi_2$	1 -1	J_0 (eq. [3.6]) $\rho^{-2}J_2$ (eq. [3.8])
${\psi_0}^{B}$ $\rho^{-4}{\psi_4}^{B}$	2 -2	$2T_0$ (eq. [2.13]) $2\rho^{-4}T_4$ (eq. [2.15])

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2}$$
$$- \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi}$$
$$- 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,. \quad (4.7)$$

Black-Hole: Quasi-Normal Modes

$$\psi = e^{-i\omega t} e^{im\varphi} S(\theta) R(r)$$

Separation of Variables

$$\Delta^{-s} \frac{d}{dr}\left(\Delta^{s+1} \frac{dR}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda\right) R = 0\,,$$

$$K \equiv (r^2 + a^2)\omega - am$$

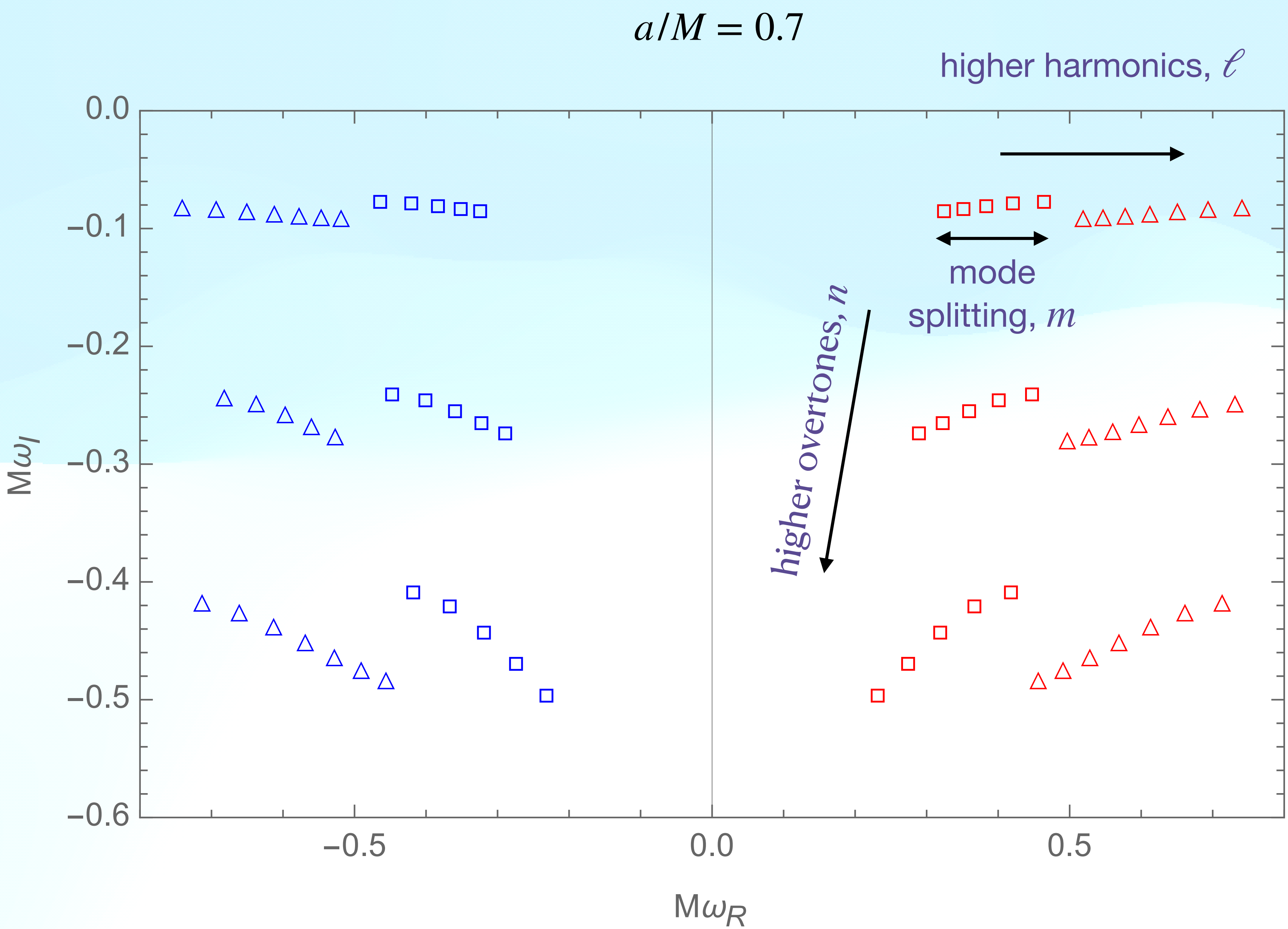
Radial Equation

$$\frac{1}{\sin\theta} \frac{d}{d\theta}\left(\sin\theta \frac{dS}{d\theta}\right) +$$

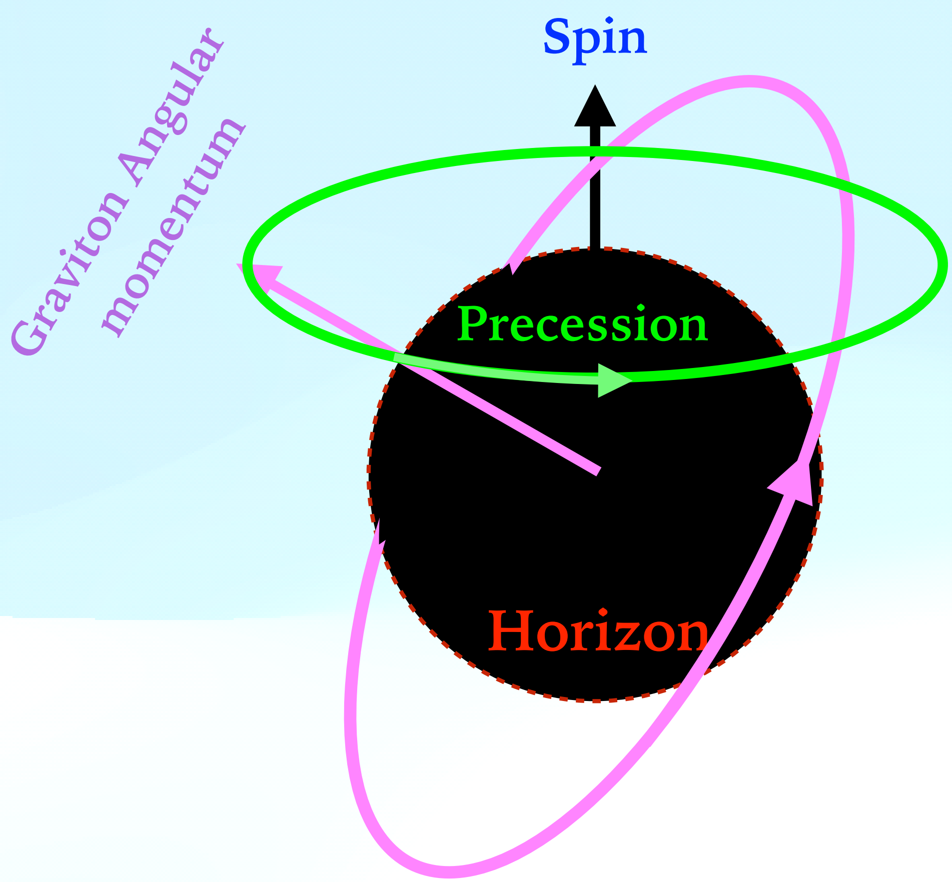
$$\left(a^2\omega^2 \cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s \cos\theta - \frac{2ms\cos\theta}{\sin^2\theta} - s^2 \cot^2\theta + s + A\right) S = 0\,,$$

Angular Equation (Spin-Weighted Spheroidal Harmonics)

Quasi-Normal Mode Spectrum



Black-Hole: Quasi-Normal Modes



$$\omega_R = L\left(\omega_{\rm orb} + \frac{m}{L}\omega_{\rm prec}\right) \quad \gamma = (n + \tfrac{1}{2})\gamma_L$$

QNMs are features of the photosphere

[Huan Yang *et al.*, PRD **86**, 104006 (2012)]

Photosphere: Graviton Circular Orbits

$$e^{iS} = e^{-iEt + im\phi + iR(r) + i\Theta(\theta)}$$

$$g^{\alpha\beta}\frac{\partial S}{\partial x^\alpha}\frac{\partial S}{\partial x^\beta} = 0$$

L and *m*: number of nodes in the angular directions

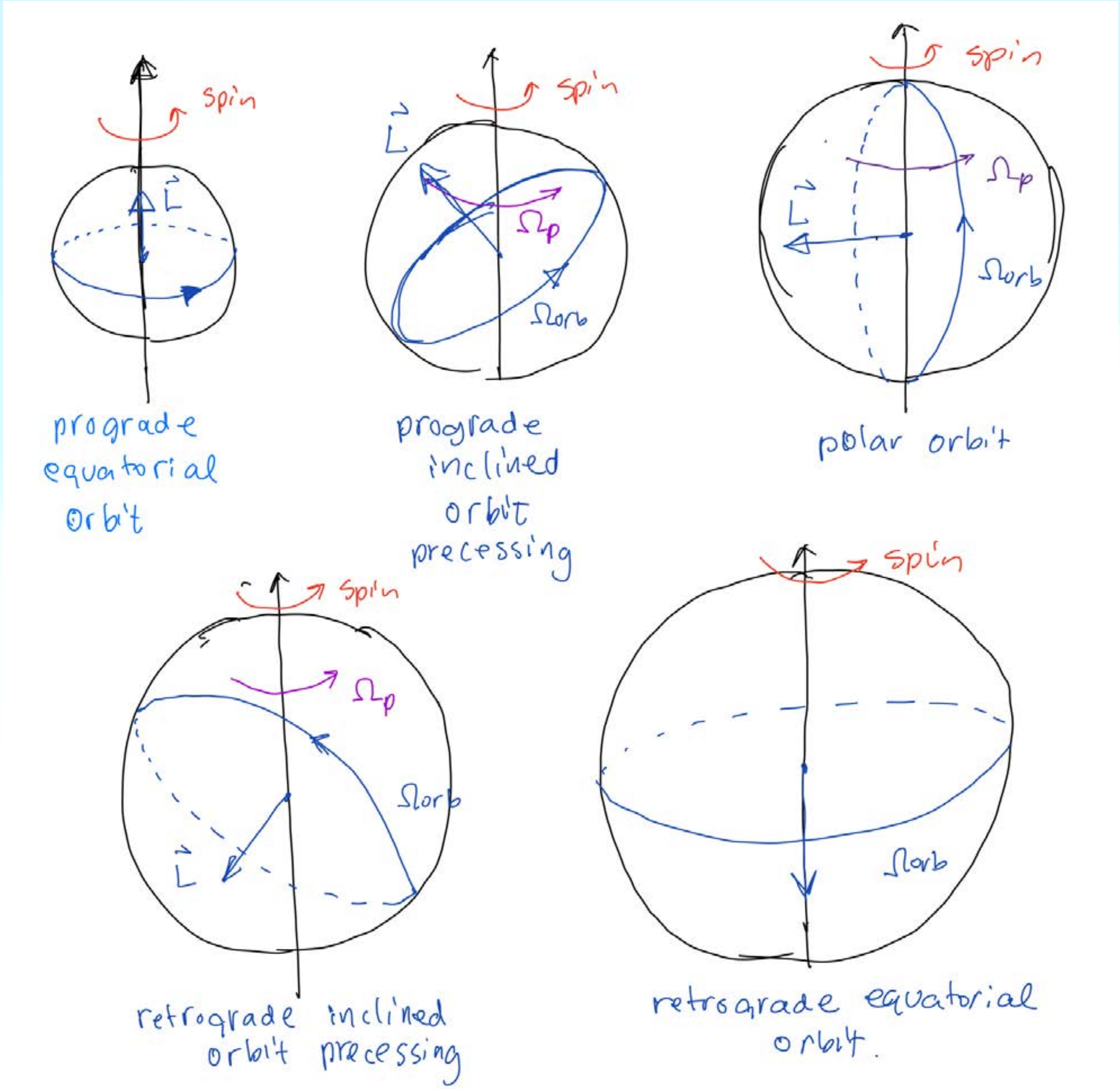
n: radial pattern

$\omega_{\rm orb}$: orbital frequency

$\omega_{\rm prec}$: precession frequency (splitting)

γ_L: orbital Lyapunov exponent

Black-Hole: Quasi-Normal Modes



Excitation of QNMs

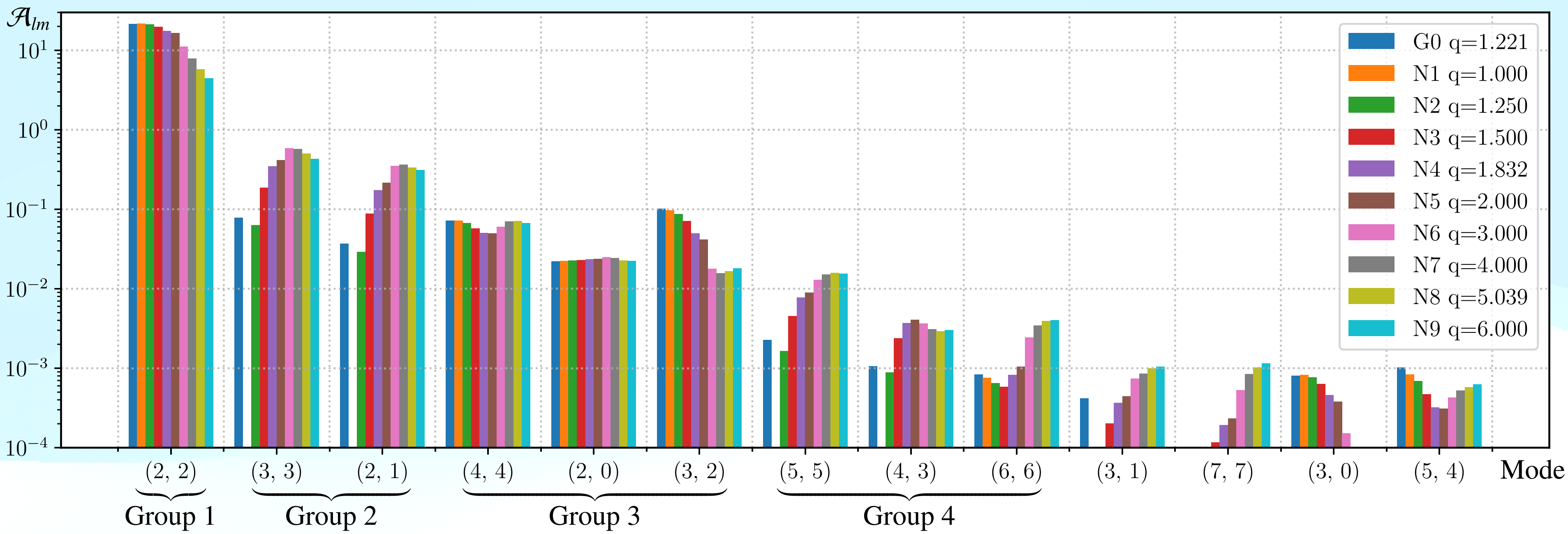
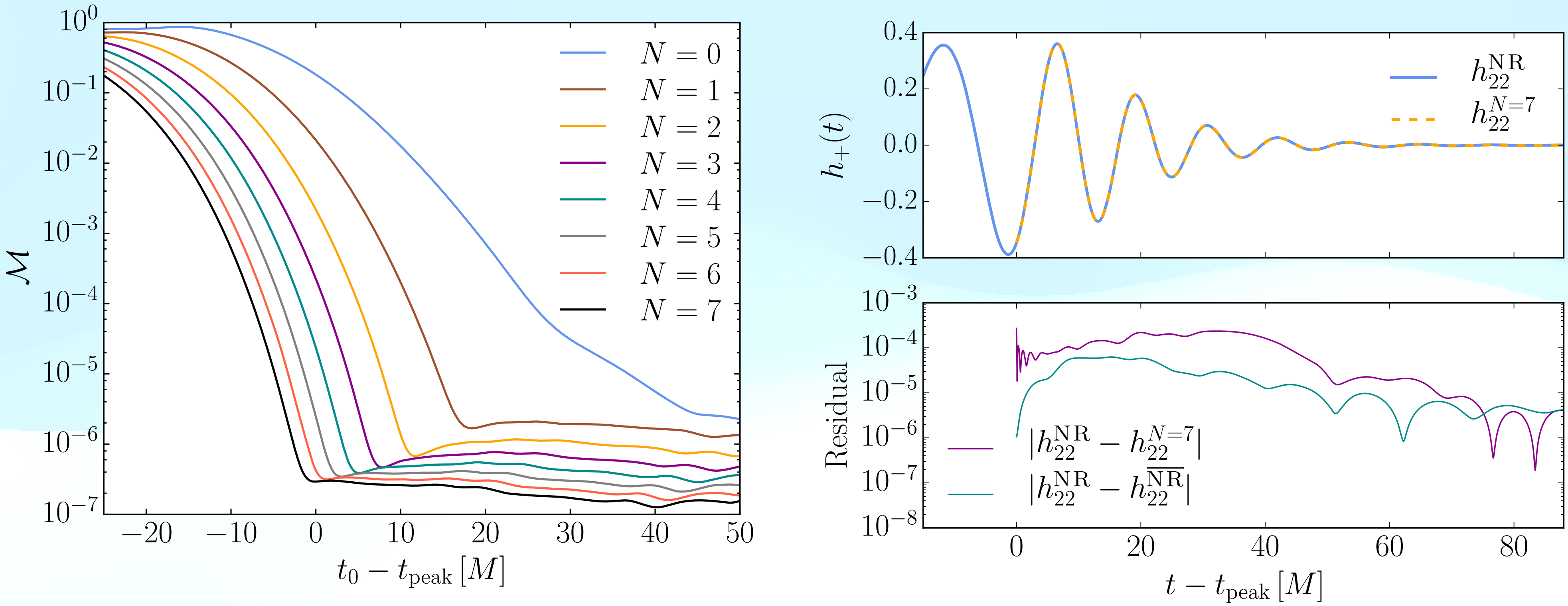


FIG. 4. The relative importance $\mathcal{A}_{lm}$, defined in Eq. (16) as the strain component in spin-weighted spherical mode (l, m) squared and integrated from t_{peak} to $t_{\text{peak}} + 100M$. Groups 1–4 of the (l, m) modes are defined according to their relative importance in the QNM expansion and are added to the fitting models in order. See details in Sec. III A.

[Xiang Li et al., 2021]

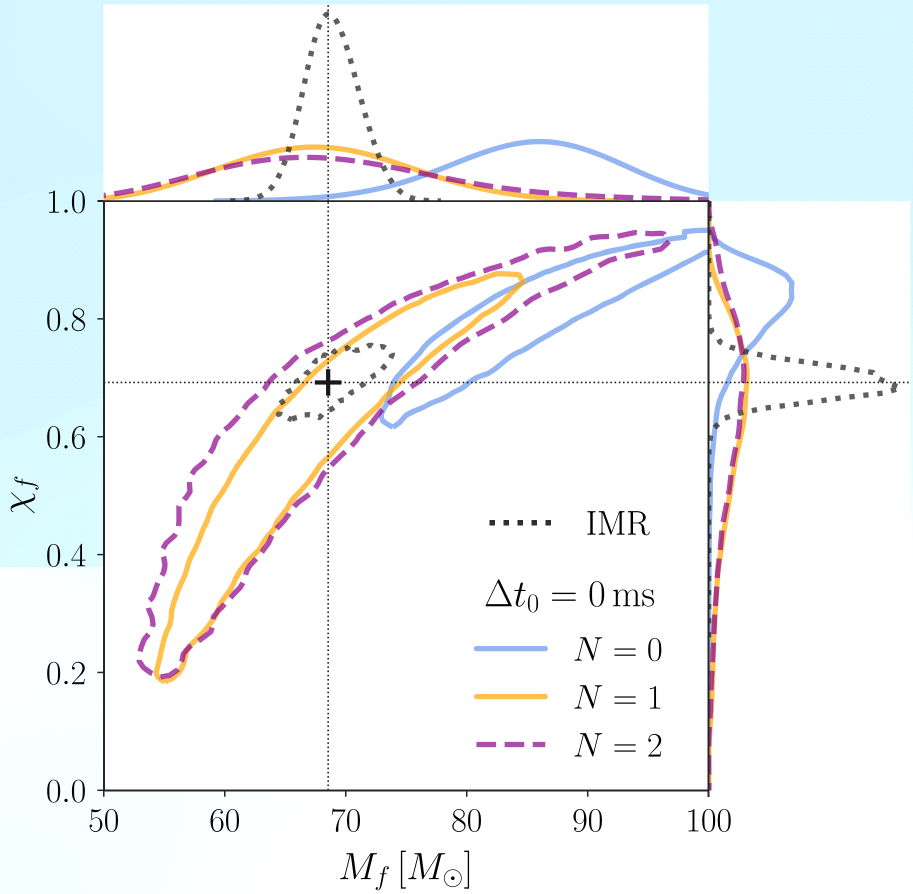
Overtones in Ringdown Waves: Theory

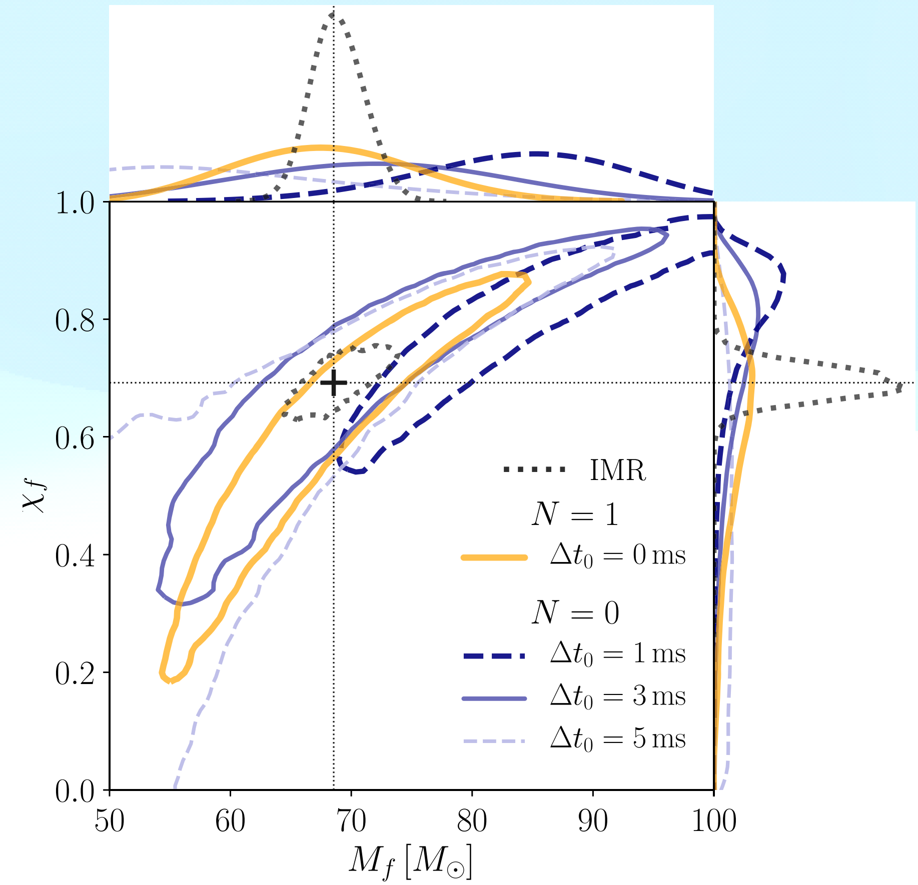


"Mismatch" between numerical waveform and sums of QNM overtones
for $(l, m) = (2,2)$

[Giesler, Isi, Scheel and Teukolsky, 2019]

Overtones in Ringdown Waves: GW150914



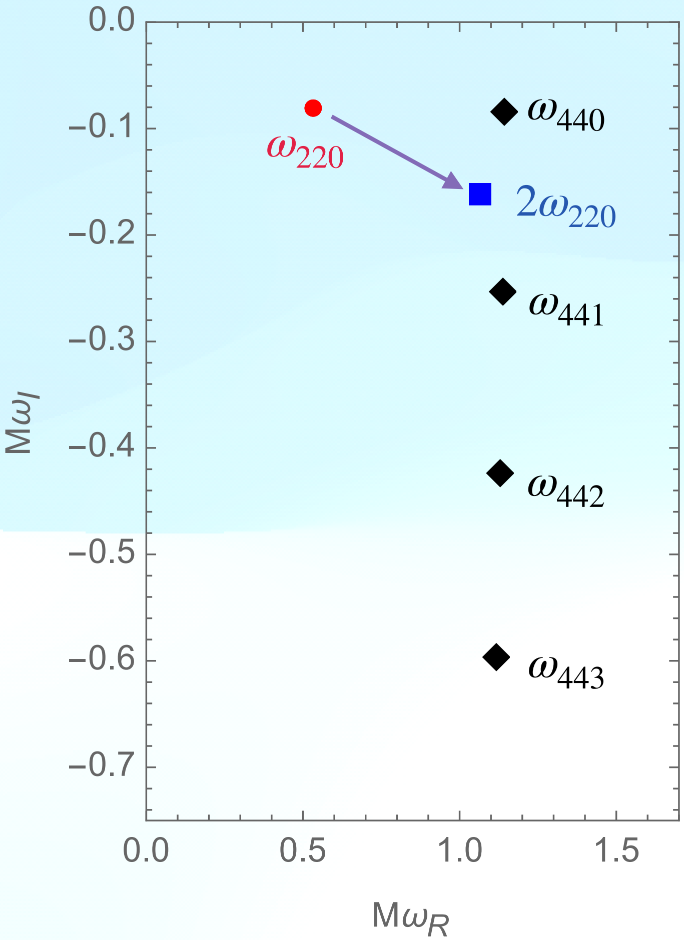


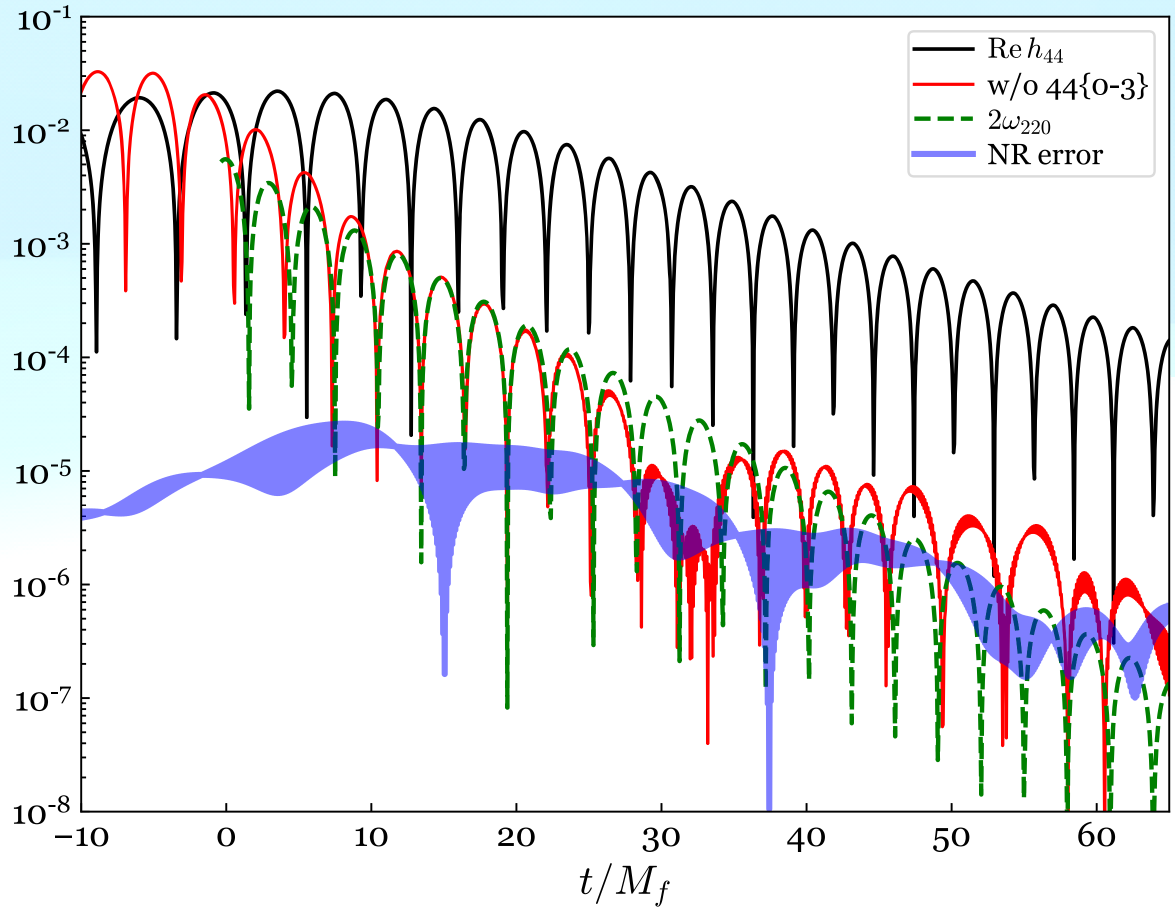
With early starting time, more overtones are better!

With additional overtone, better to start early!

Extracting overtone frequencies allow "no-hair" test!

Nonlinearity



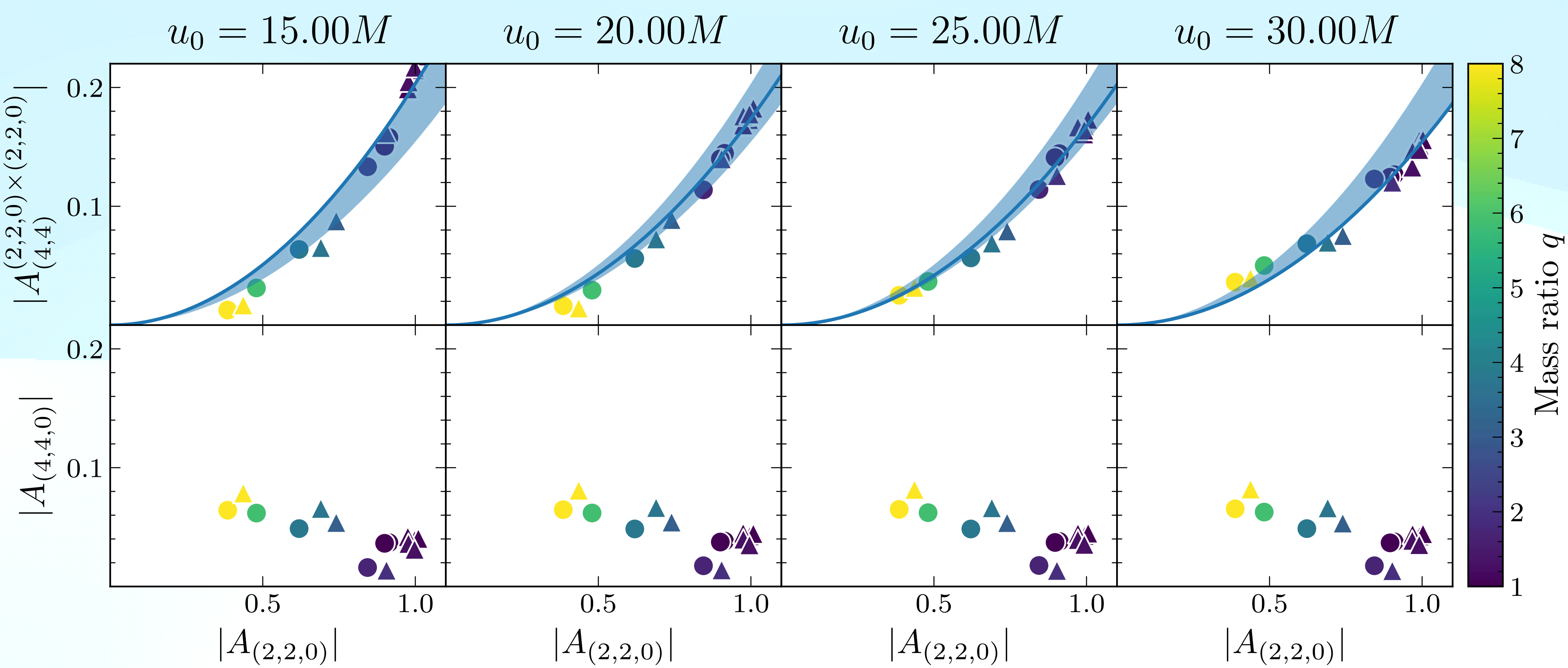


Effective Stress-Energy of the 220 mode will drive a (4,4) spatial mode with $2\omega_{220}$ frequency!

[Keefe Mitman et al., 2023; Sizheng Ma et al., 2022]

Nonlinearity

across different binaries: amplitude of $2\omega_{22}$ mode is quadratic in $\omega_{2,2}$ mode



[Mitman et al., 2023]

Other Tests of Relativity

Alternative Theories

- Scalar-Tensor Theory
- Higher-Derivative Terms
- Chern-Simons Gravity
- Stars phase-transition instead of forming BHs

Parametrized Deviations

- How many polarizations?
- Speed of GW?
- Waveform's Amplitude/ Phase deviates from GR?

Confirm Phenomena

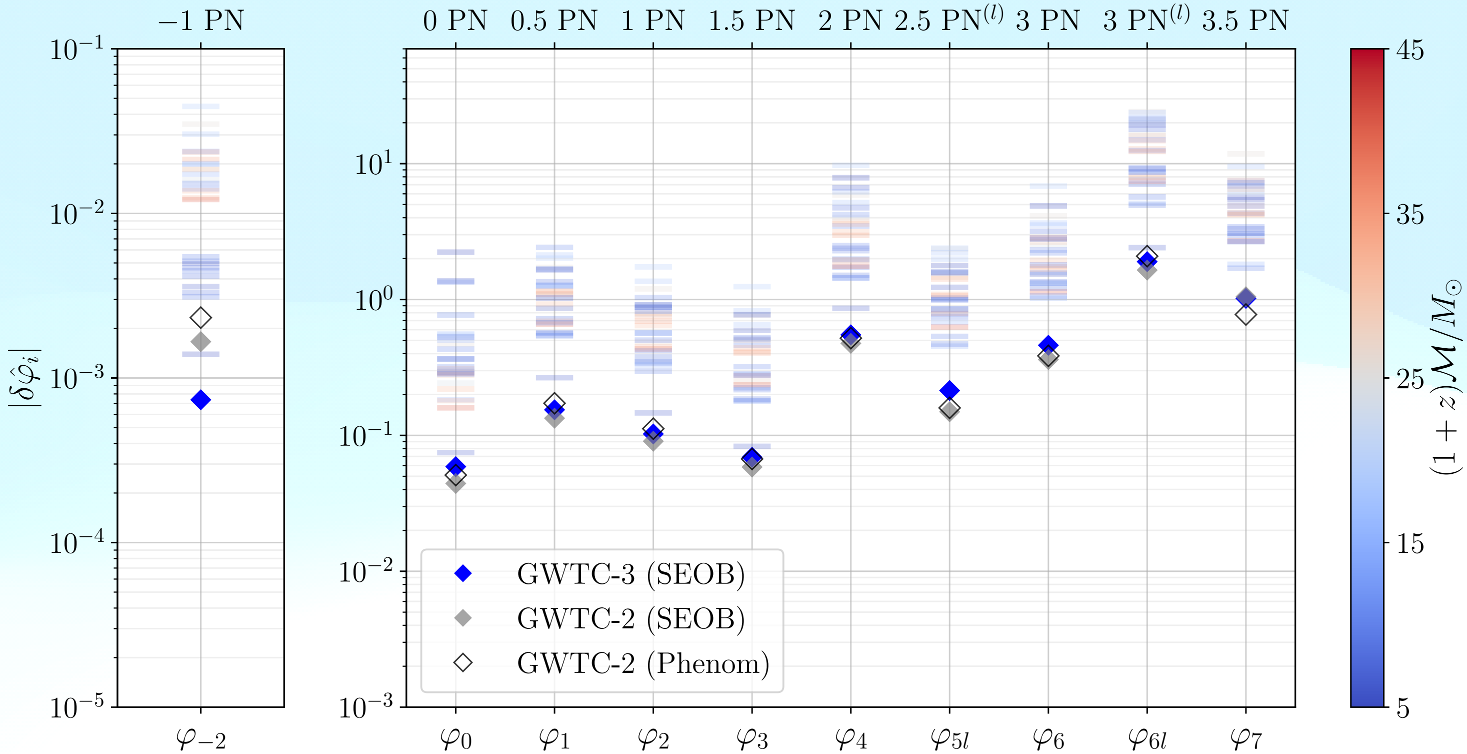
- Is Energy/Angular Momentum Lost during Merger?
- GW Memory?
- Is Horizon Black?
- Are there quantum fluctuations around BH?

Environment?

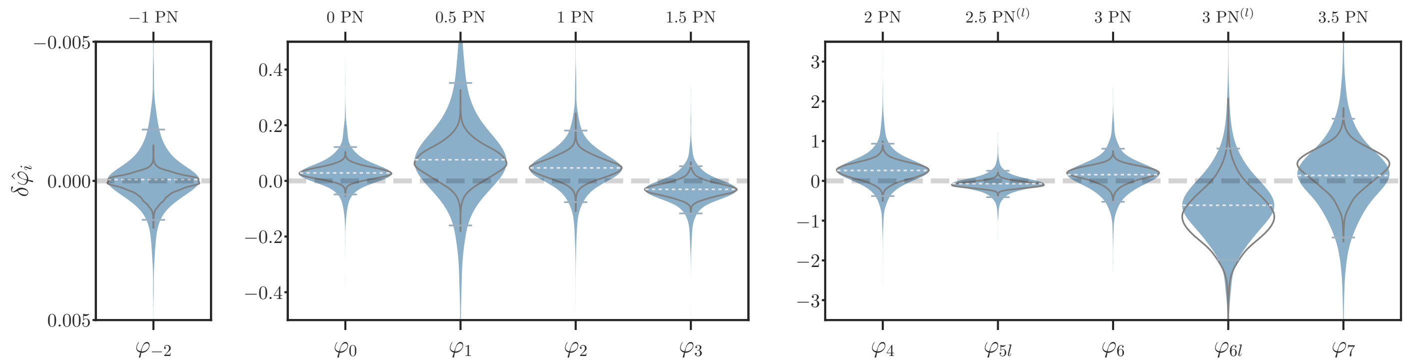
- Is there a third object around our binary?
- Are there other things? Like Dark-Matter Halo, Axion Cloud, etc?

Parametrized Test of GR

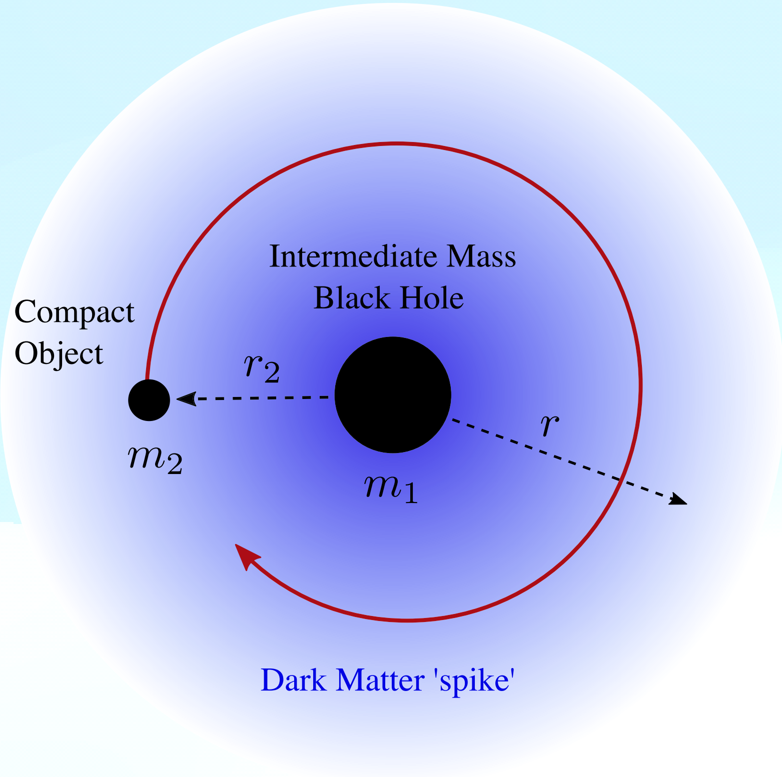
Individual Events



"Combined"

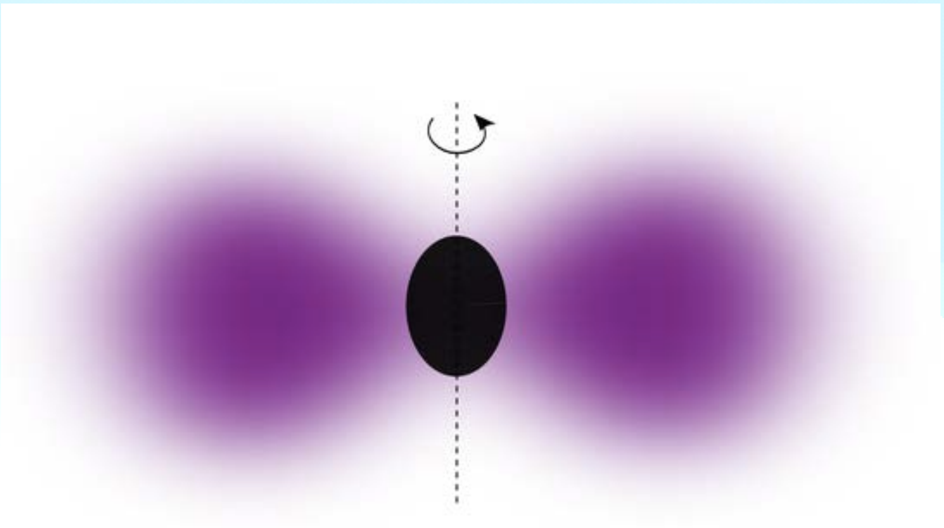


Superradiance of Black Holes and Dark Matter



Dark matter around BH causes change in waveform

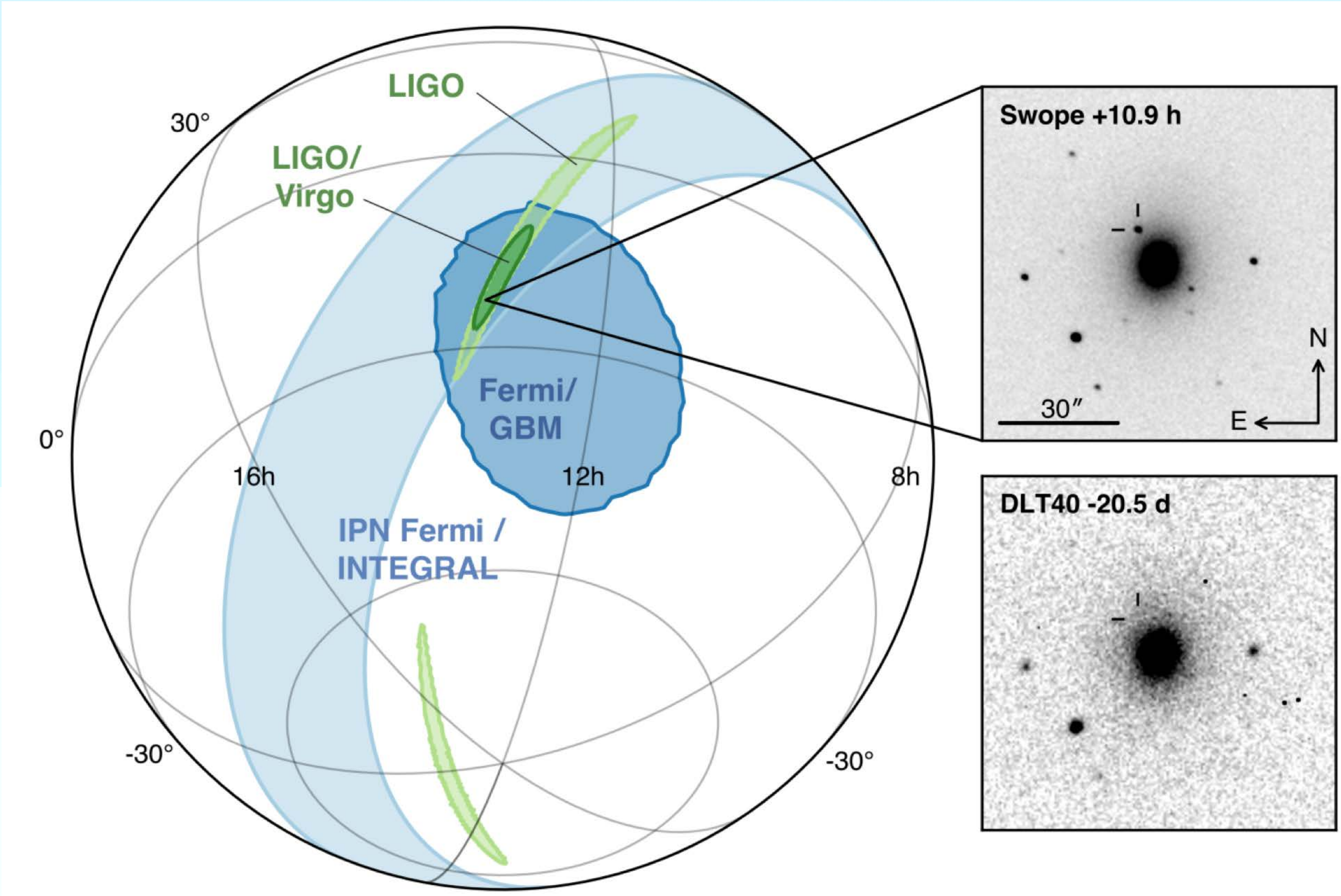
[Kavanagh et al., 2020]



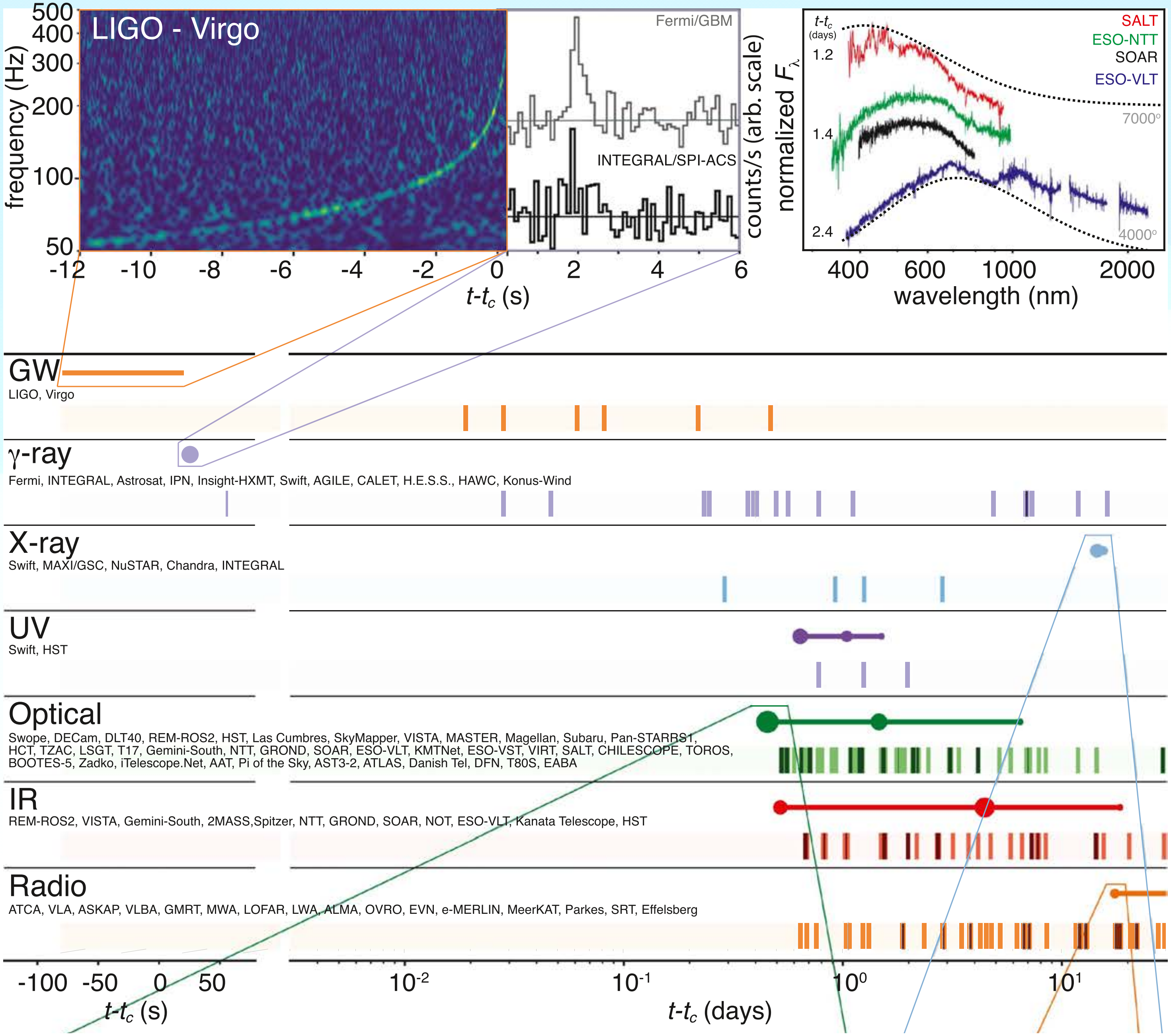
Cloud may spontaneously form around spinning black hole, extracting rotation energy if Compton wavelength comparable to size of black hole.

[Arvanitaki et al., 2010-2011]

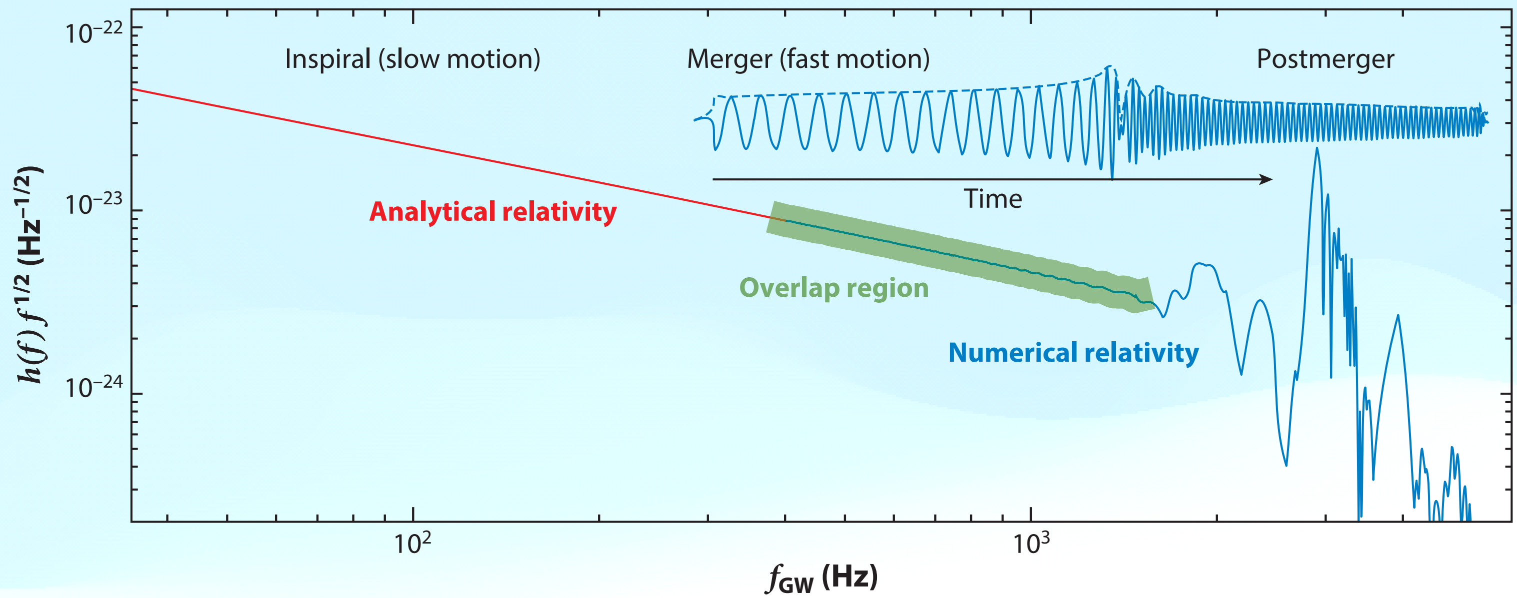
GW170817: Merger of Binary Neutron Stars

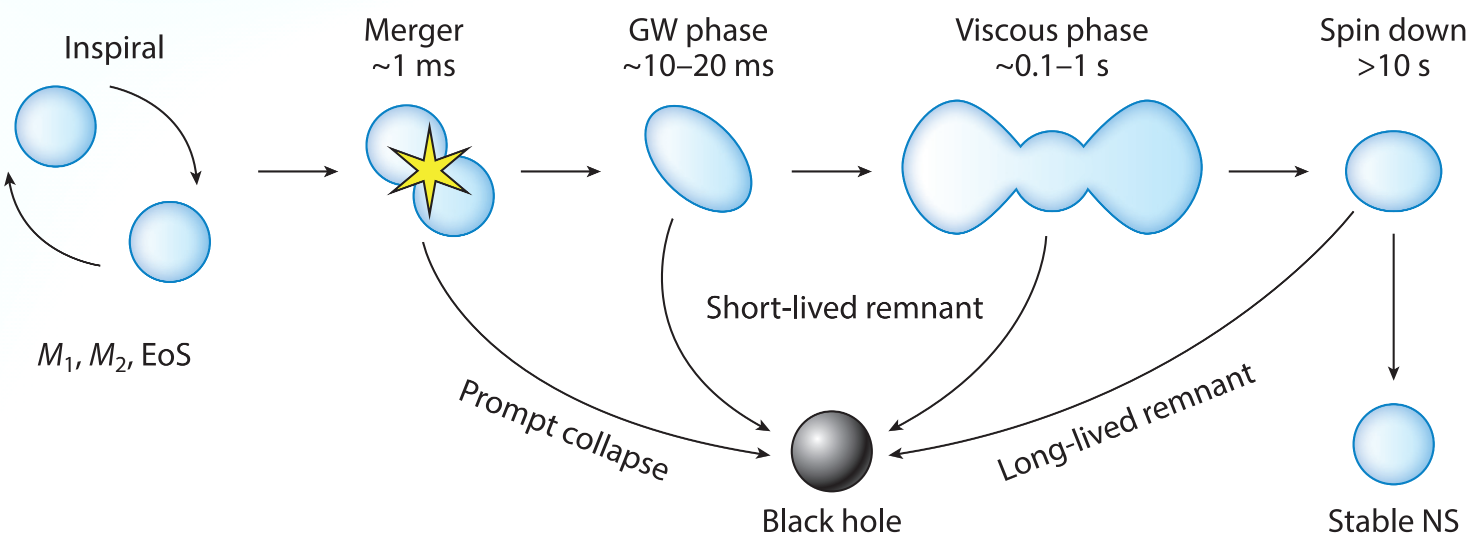


EM Counterparts



GW from Neutron Star Binaries





Neutron-Star Tidal Deformability

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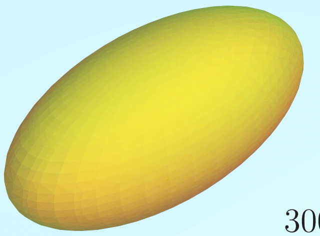
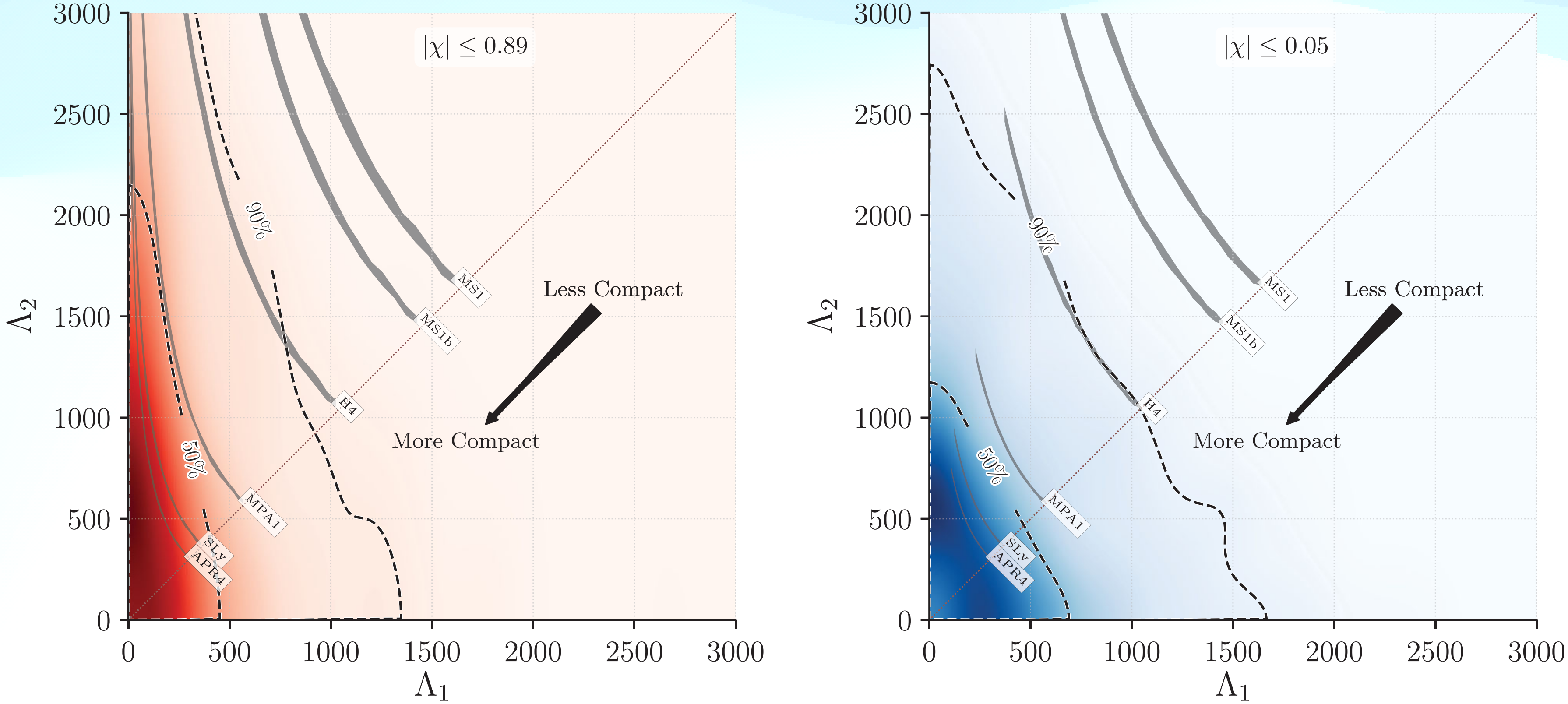


$$\phi = -\frac{M}{r} + \frac{1}{2}\mathcal{E}_{ij}x^i x^j - \frac{3}{2}\frac{Q^{ij}n_i n_j}{r^3} + \ldots \qquad Q^{ij} = -\lambda \mathcal{E}^{ij}$$



[Constraints from GW170817, LIGO-VIRGO Collaboration, 2017]

Summary of Lecture 1

- General relativity describes gravity using space-time geometry

- Confirmed by waveforms from binary black holes
  - Weak-field effects: spin/orbital precession.
  - Strong-field effects: black-hole quasi-normal modes.
  - Precision tests require improved sensitivity.

- Neutron star binary merger
  - probes state of matter at high densities