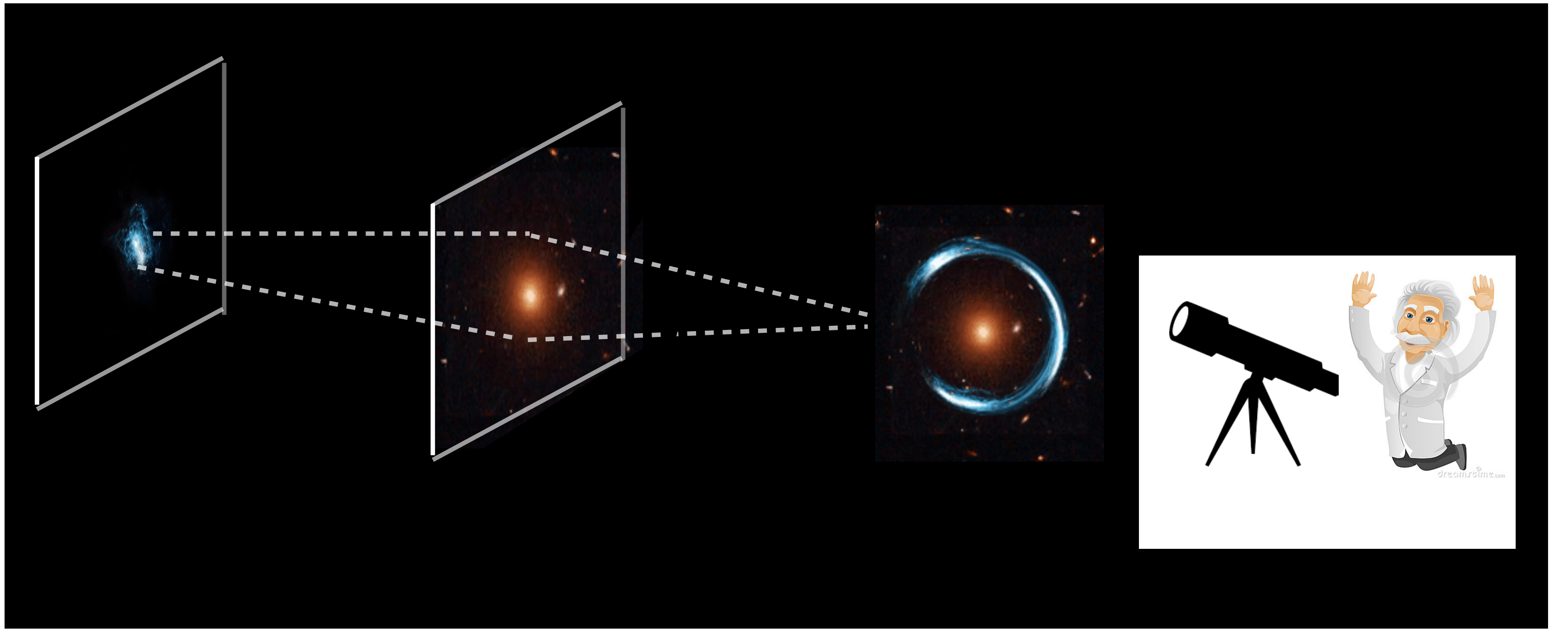
Strong Gravitational lensing

Anupreeta More
IUCAA

RESCEU Summer School, Kanazawa **Sep 6, 2024**

Gravitational lensing



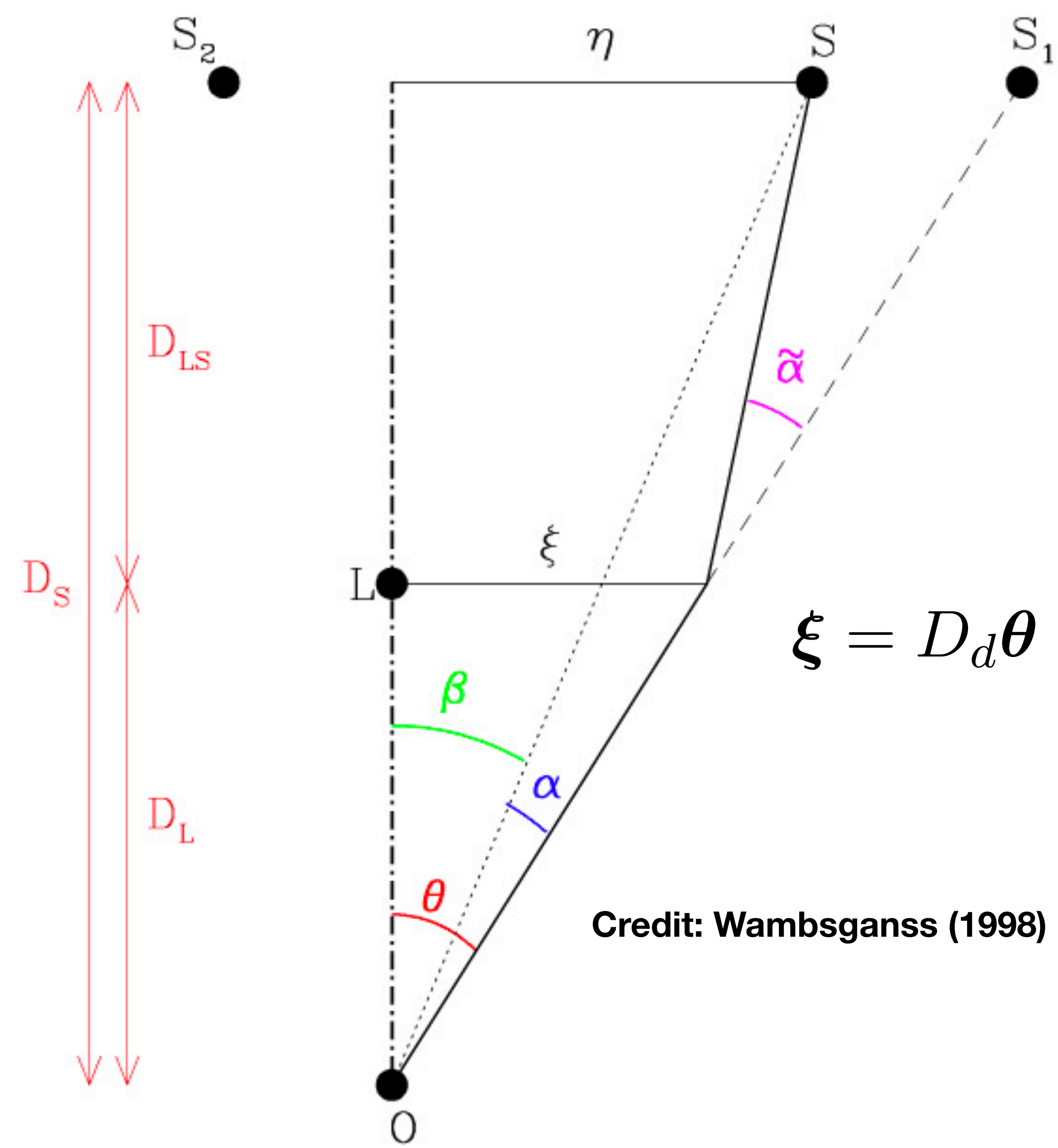
Deflection of light in the vicinity of a massive object due to gravity

Main observables are: image positions, magnifications and time delays

Image Positions

Lens Equation: $$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

Scaled deflection angle: $$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta})$$



$$\boldsymbol{\xi} = D_d\boldsymbol{\theta}$$

Credit: Wambsganss (1998)

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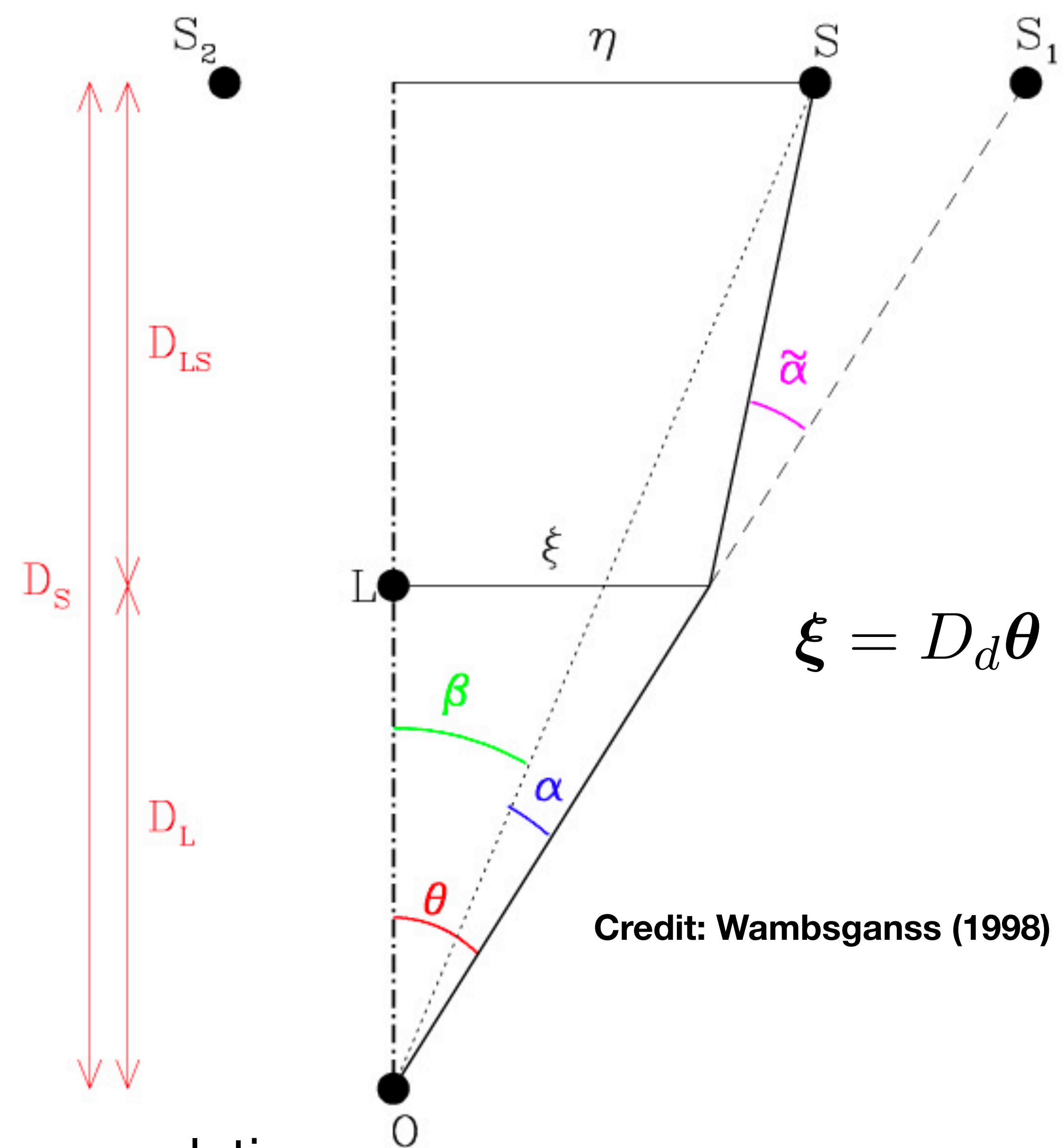

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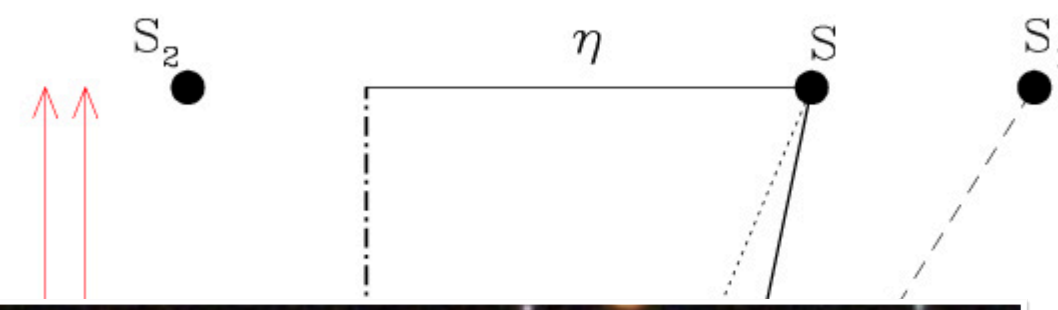
- Angles are 2D vectors.
- Distances are angular diameter distances
- True source position is unknown. Positions of the images relative to an origin (usually, the lens center) comprise the observables.

Image Positions



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Magnification

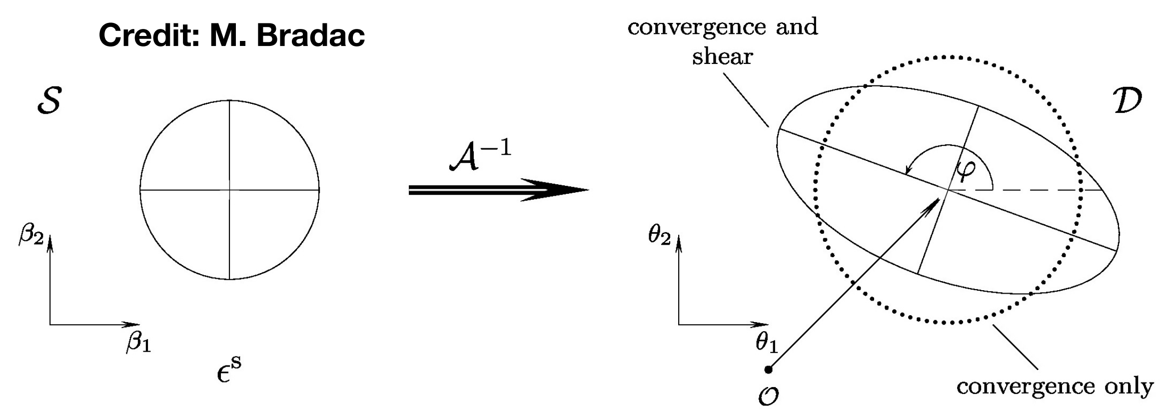
Define Jacobian matrix: $\mathcal{A}(\boldsymbol{\theta}) = \dfrac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}}$

where $\mathcal{A}_{ij} = \dfrac{\partial \beta_i}{\partial \theta_j}$

source plane

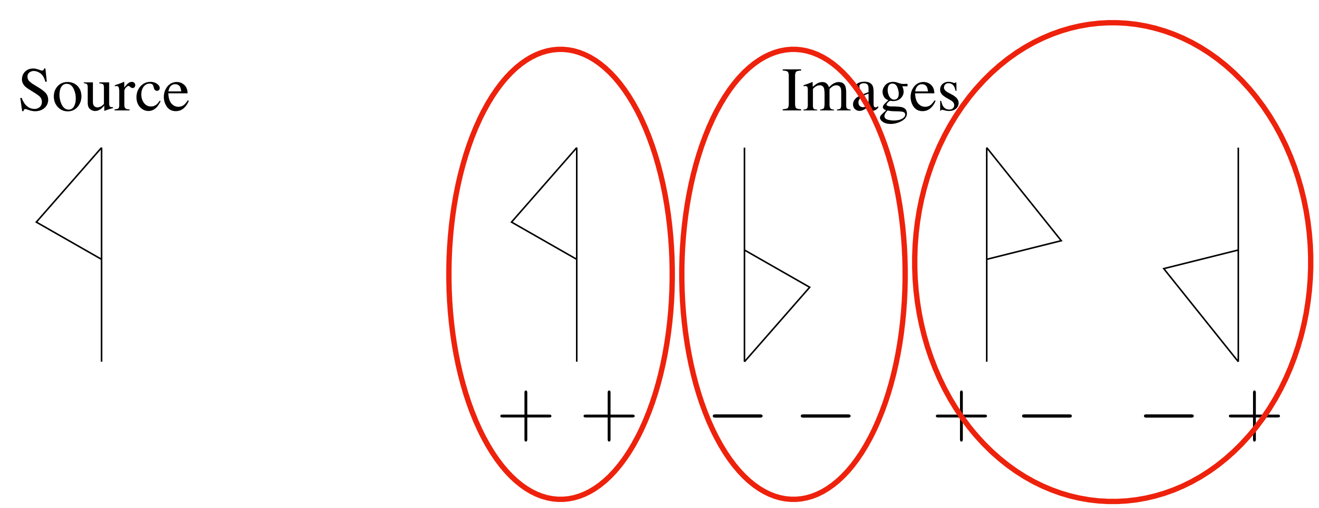
lens plane

Credit: M. Bradac



Magnification : $\mu(\boldsymbol{\theta}) = \dfrac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$

Types: **I** **III** **II**



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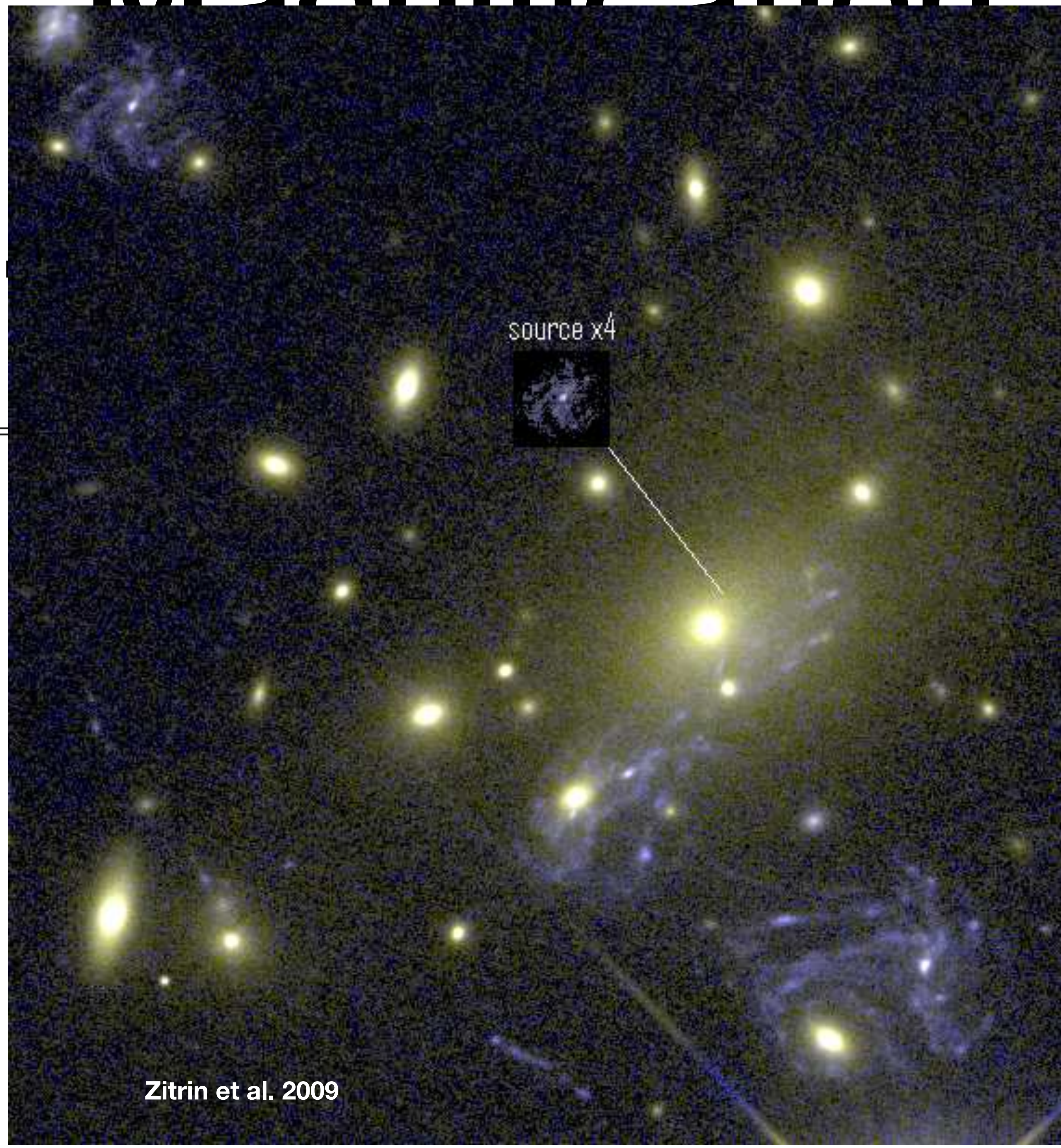

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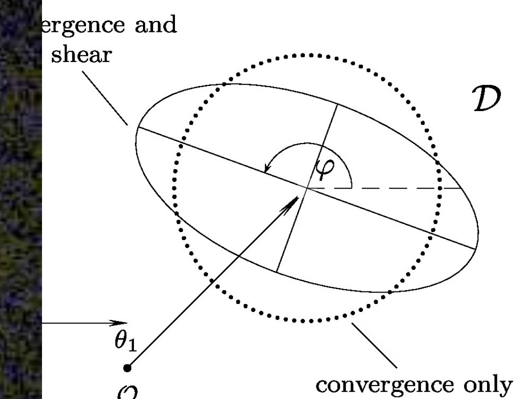
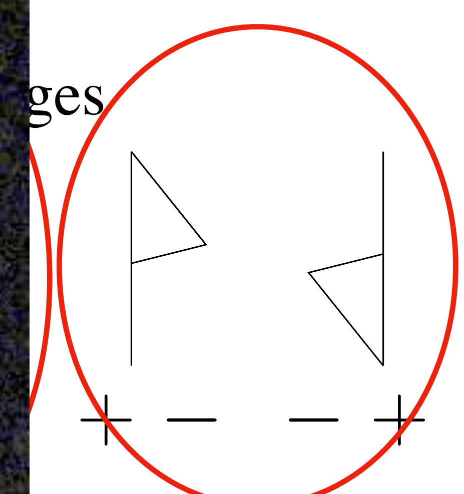


where $\mathcal{A}_{ij} =$

Magnification :

II

ges



Time delay

(Shapiro 1964)

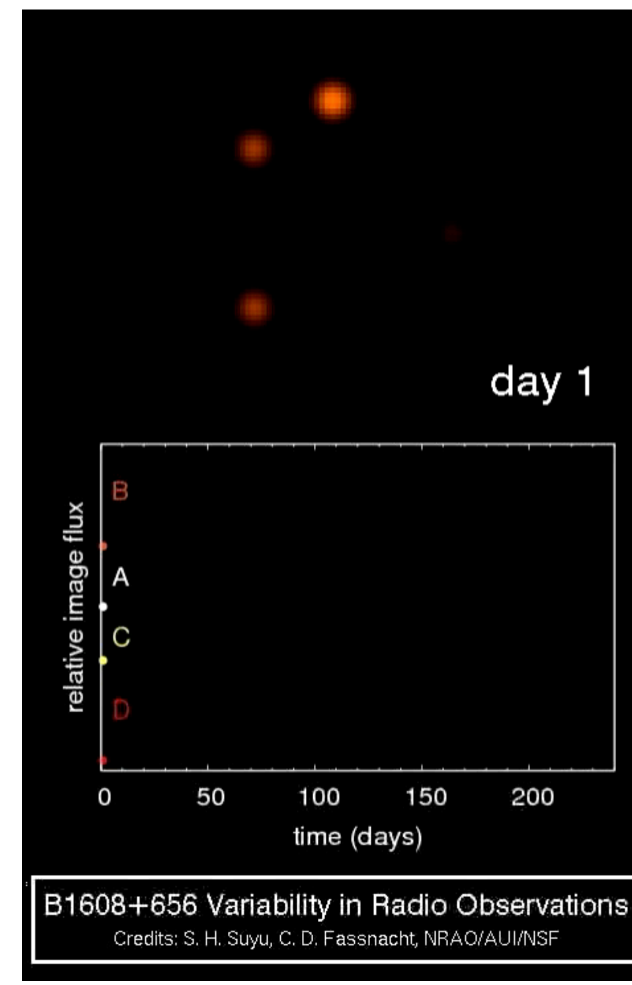
Time delay function or Fermat potential

$$\tau(\boldsymbol{\theta}) = \frac{(1+z_d)}{c}\frac{D_d D_s}{D_{ds}}\left[\overbrace{\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\beta})^2}^{t_{\rm geom}} - \overbrace{\psi(\boldsymbol{\theta})}^{t_{\rm grav}}\right]$$

Relative time delays measure Hubble constant

$$\Delta\tau_{ij} = \frac{(1+z_d)}{c}\frac{D_d D_s}{D_{ds}}\left[\phi\left(\boldsymbol{\theta}_i,\boldsymbol{\beta}\right) - \phi\left(\boldsymbol{\theta}_j,\boldsymbol{\beta}\right)\right]$$

$$\frac{D_d D_s}{D_{ds}} \propto \frac{1}{H_0}$$



B1608+656 Variability in Radio Observations
Credits: S. H. Suyu, C. D. Fassnacht, NRAO/AUI/NSF

Effect observable for variable sources only.

Unperturbed path length to the source is unknown. Hence, only relative time delays of the lensed images comprise the observables.

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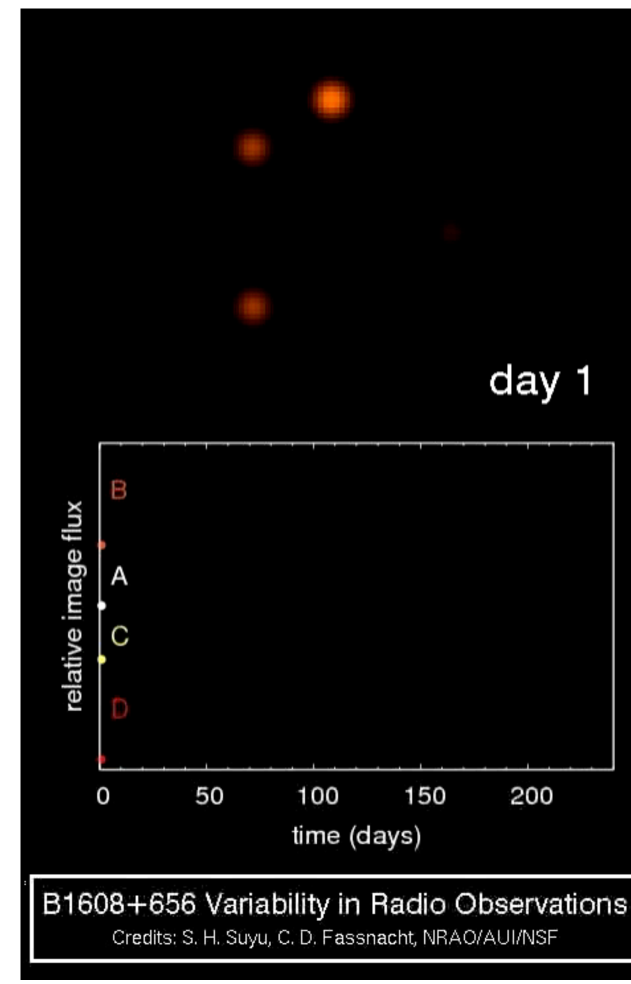

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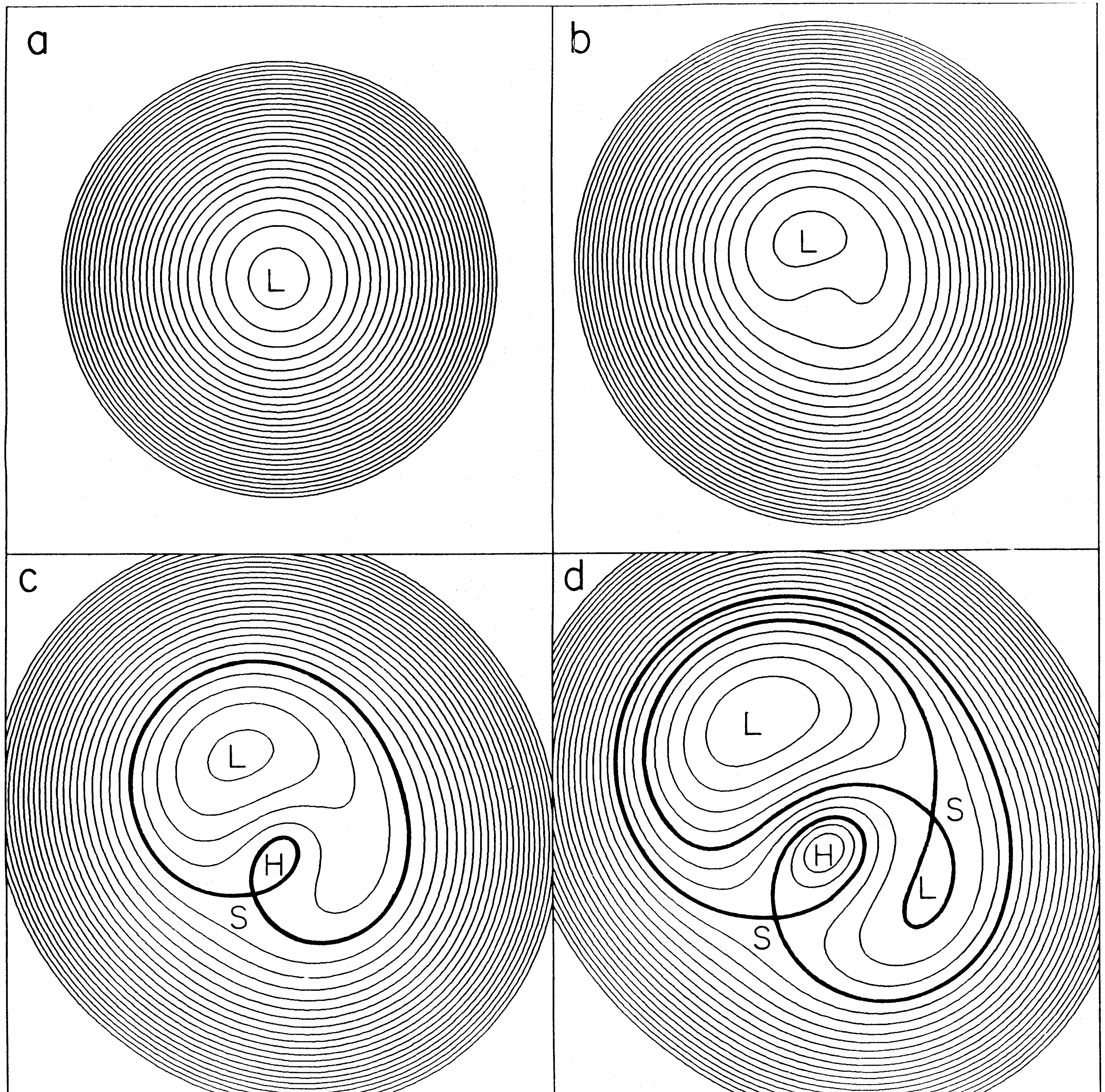
Time delay surface



Images form at

$$\nabla\tau(\boldsymbol{\theta};\boldsymbol{\beta}) = 0$$

Blandford & Narayan (1986)

$$\hat{\alpha} = \frac{4GM}{c^2}\frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|^2}$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta})$$

Point mass lens

Scaled Deflection angle

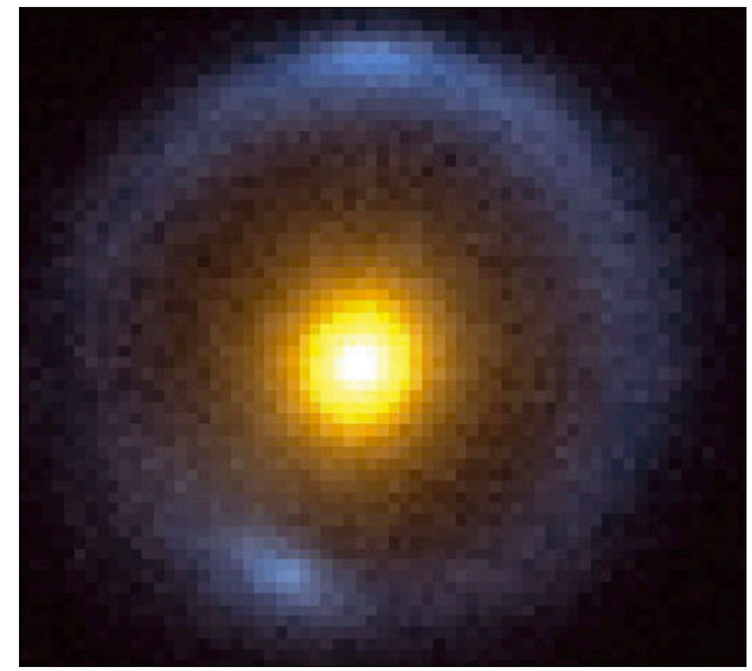
$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}\frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|^2}$$

Lens equation

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \theta_E^2 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|^2}$$

Einstein Radius

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$



Credit: C. Fassnacht / UCDavis

Narayan and Bartelmann 1996

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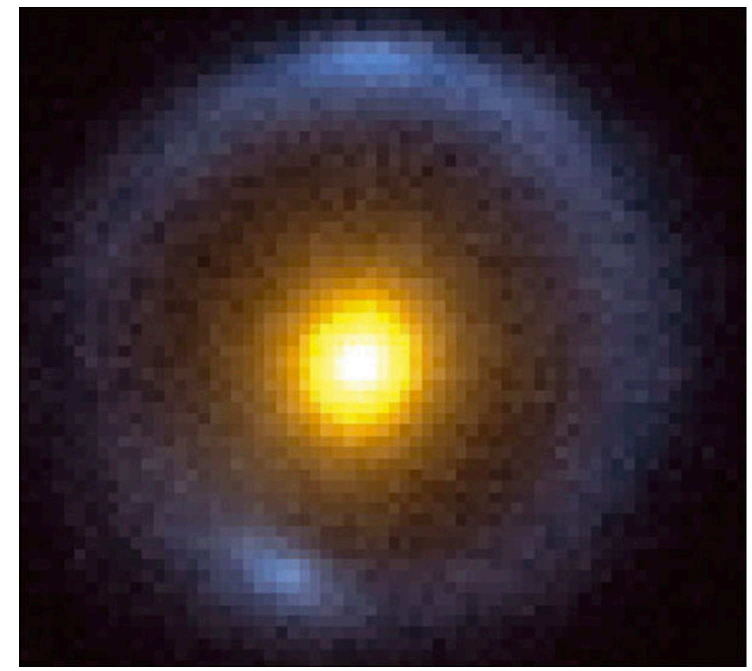

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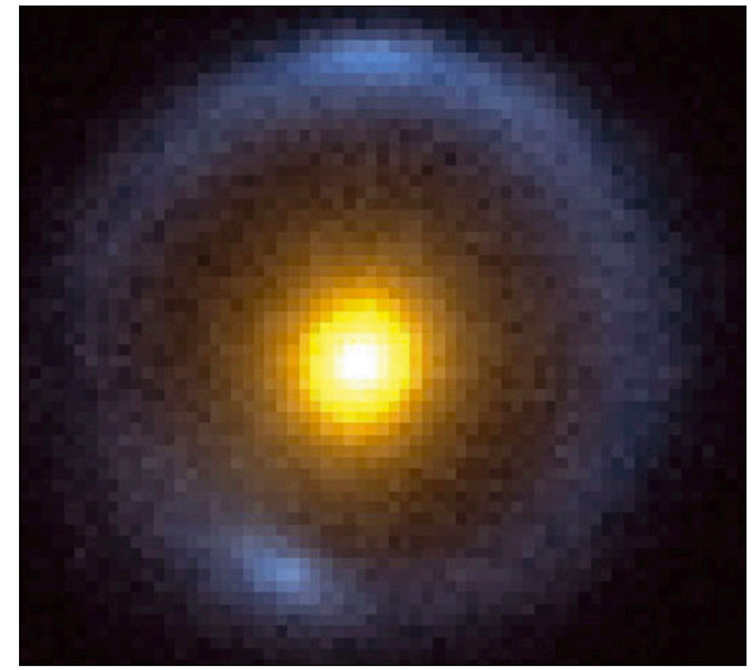

Credit: C. Fassnacht / UCDavis

let $D \equiv (D_{\rm l} D_{\rm s}/D_{\rm ls})$, then $\theta_{\rm E}$

$$\approx (10^{-3})'' \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{\rm kpc}\right)^{-1/2}$$

$$\approx 1'' \left(\frac{M}{10^{12} M_\odot}\right)^{1/2} \left(\frac{D}{\rm Gpc}\right)^{-1/2}$$

Narayan and Bartelmann 1996

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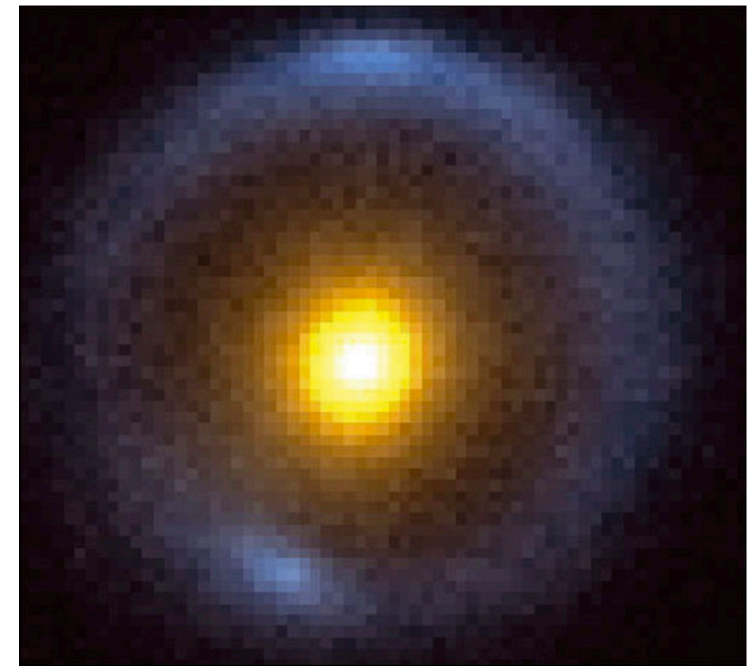

Credit: C. Fassnacht / UCDavis

Images

$$\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_{\rm E}^2}\right)$$

magnification

$$\mu_{\pm} = \left[1 - \left(\frac{\theta_{\rm E}}{\theta_{\pm}}\right)^4\right]^{-1}$$

let $D \equiv (D_{\rm l}D_{\rm s}/D_{\rm ls})$, then $\theta_{\rm E}$

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Narayan and Bartelmann 1996

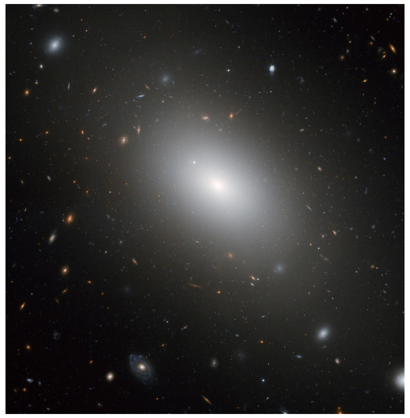
Extended mass lens

$$\hat{\alpha} = \frac{4GM(<\xi)}{c^2\xi}$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\rm ds}}{D_{\rm s}}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

Singular Isothermal sphere

Most frequently seen lenses are early-type galaxies



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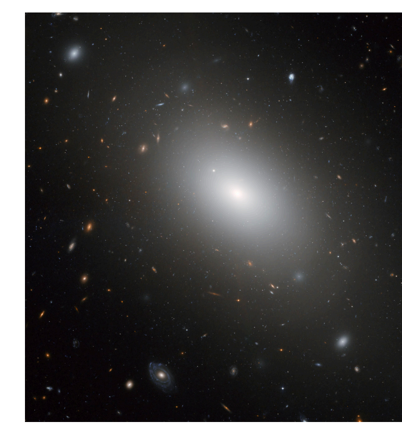

Density profile $$\rho(r) = \frac{\sigma_v^2}{2\pi G}\,\frac{1}{r^2}$$

Flat rotation curve $$v_{\rm rot}^2(r) = \frac{G\,M(r)}{r} = 2\,\sigma_v^2 = \text{constant}$$

Surface density $$\Sigma(\xi) = \frac{\sigma_v^2}{2G}\,\frac{1}{\xi}$$

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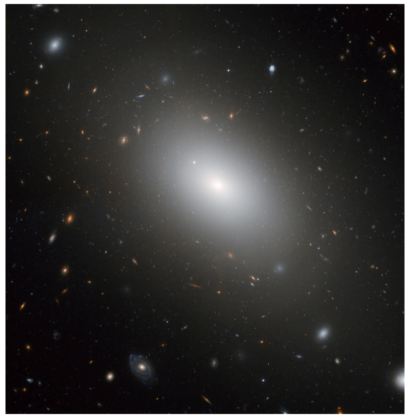

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$$M(<\xi) = \int 2\pi\xi' d\xi' \Sigma(\xi')$$

$$\hat{\alpha} = \frac{4G}{c^2}\,\frac{\sigma_v^2}{2G}\,\frac{2\pi\xi}{\xi}$$

Narayan and Bartelmann 1996

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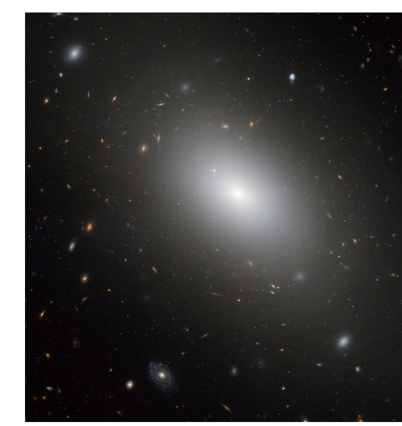

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Constant deflection angle
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Narayan and Bartelmann 1996

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Singular Isothermal sphere

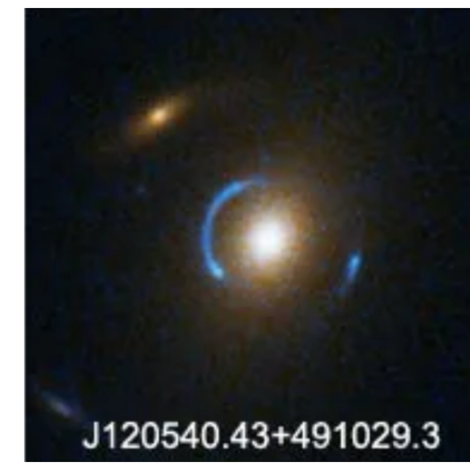
Constant deflection angle $\boxed{\hat{\alpha} = 4\pi \frac{\sigma_v^2}{c^2}}$

Scaled Deflection angle $\boldsymbol{\alpha}(\boldsymbol{\theta}) = 4\pi \frac{\sigma_v^2}{c^2} \frac{D_{\rm ds}}{D_{\rm s}}$

Lens Equation $\boldsymbol{\beta} = \boldsymbol{\theta} - \theta_E$

Einstein Radius $\theta_{\rm E} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{\rm ds}}{D_{\rm s}}$

Example of an early-type lens



Credit: HST/ACS, SLACS

Narayan and Bartelmann 1996

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Images $\theta_{\pm} = \beta \pm \theta_E$ for $|\beta| < \theta_{\rm E}$

Magnification $$\mu_{\pm} = \frac{\theta_{\pm}}{\beta} = 1 \pm \frac{\theta_{\rm E}}{\beta} = \left(1 \mp \frac{\theta_{\rm E}}{\theta_{\pm}}\right)^{-1}$$

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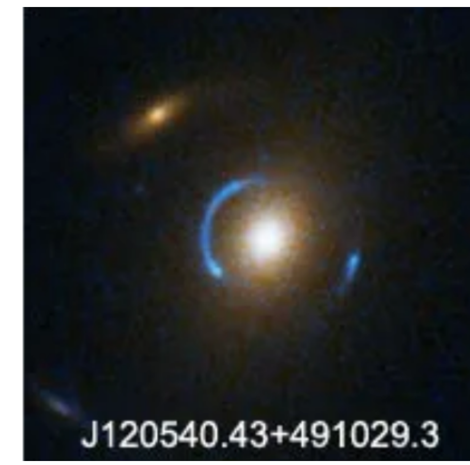

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Narayan and Bartelmann 1996

Critical curves & Caustics

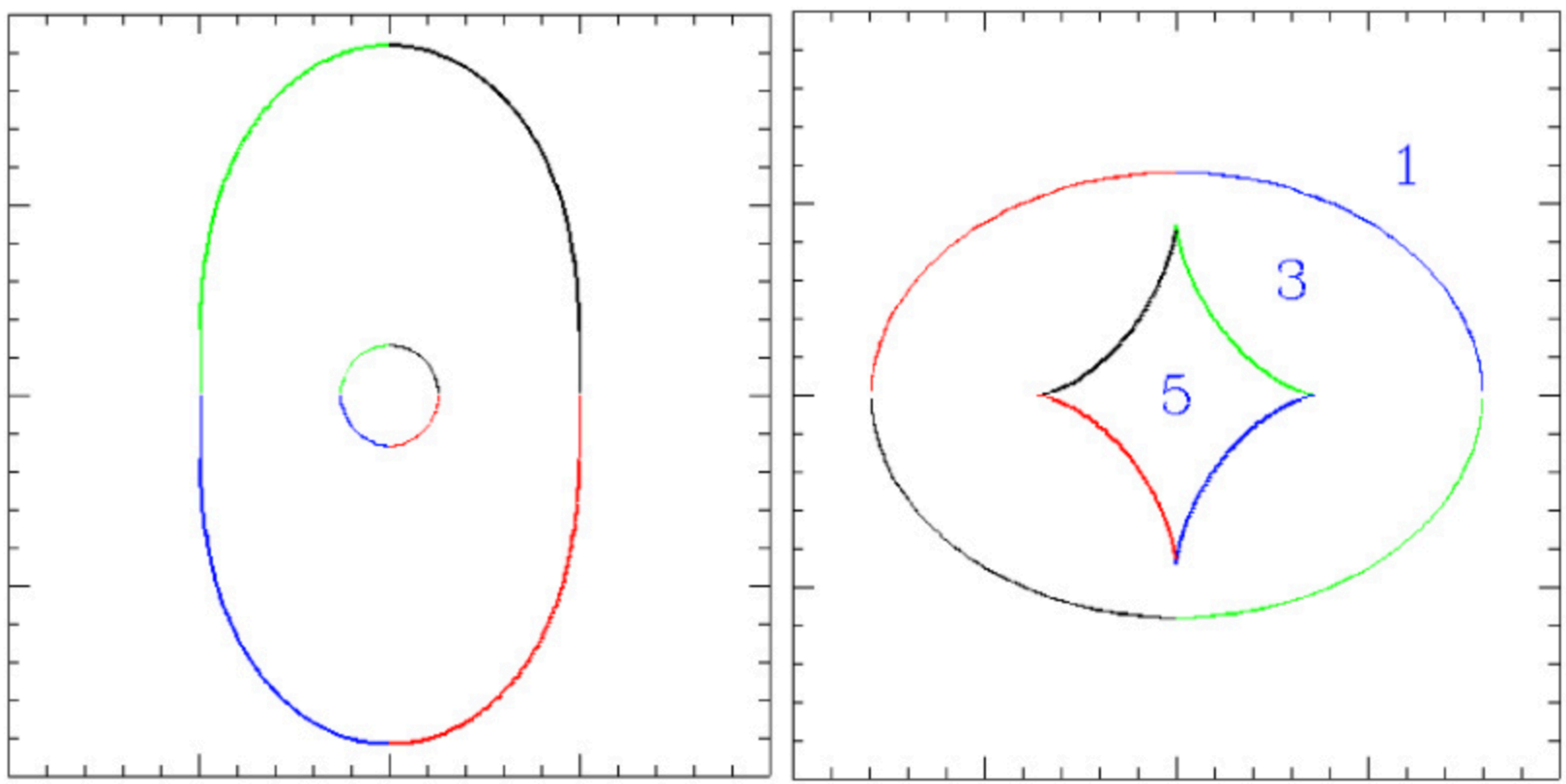
$\det \mathcal{A}(\boldsymbol{\theta}) = 0 \quad \mu = 1/\det \mathcal{A}$

correspond to critical curves (image plane) and caustics (source plane)

Critical curves

Caustics

Extended, elliptically symmetric mass distribution



Wambsganss 1998

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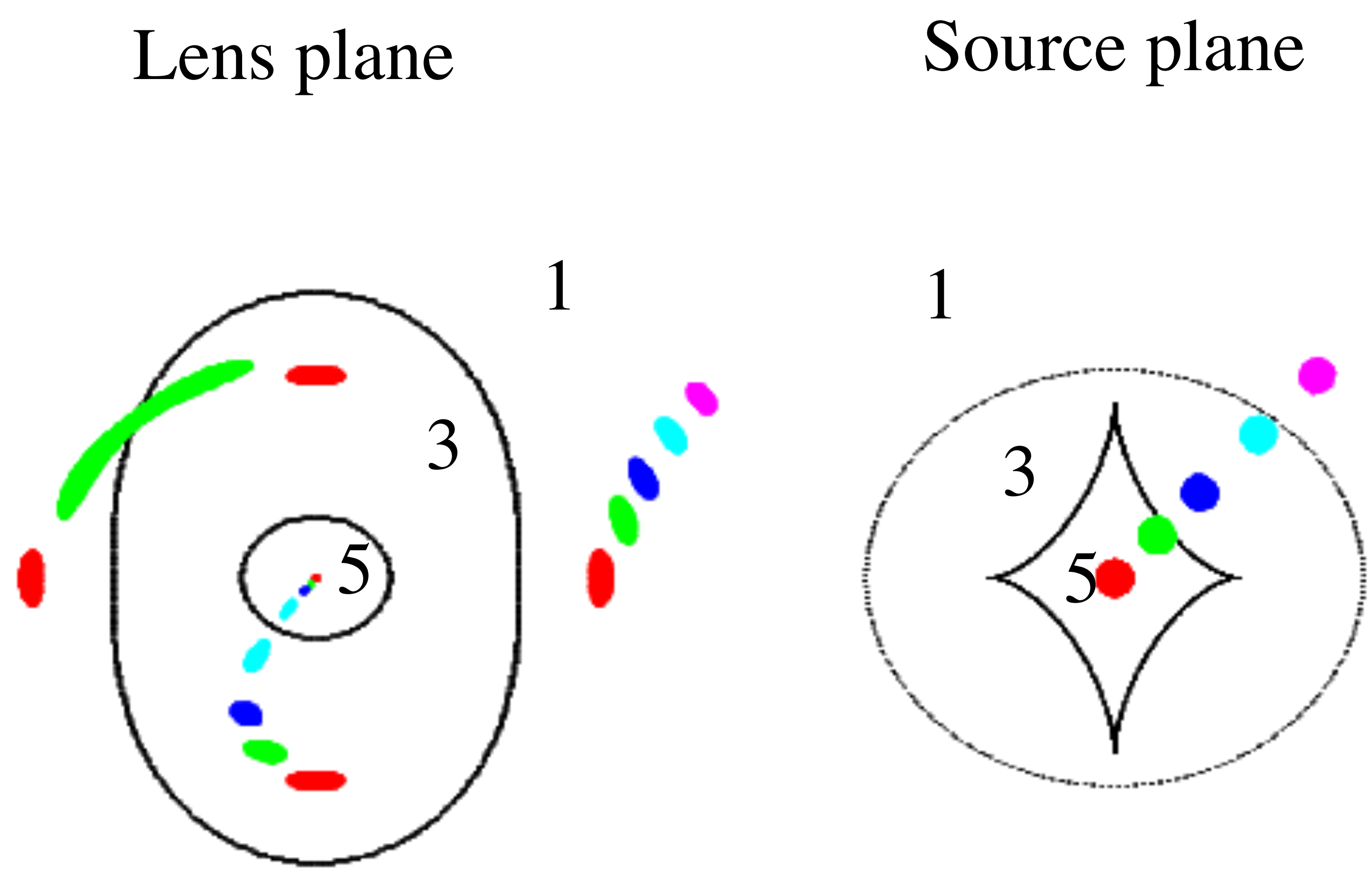

Narayan & Bartelmann (1996)

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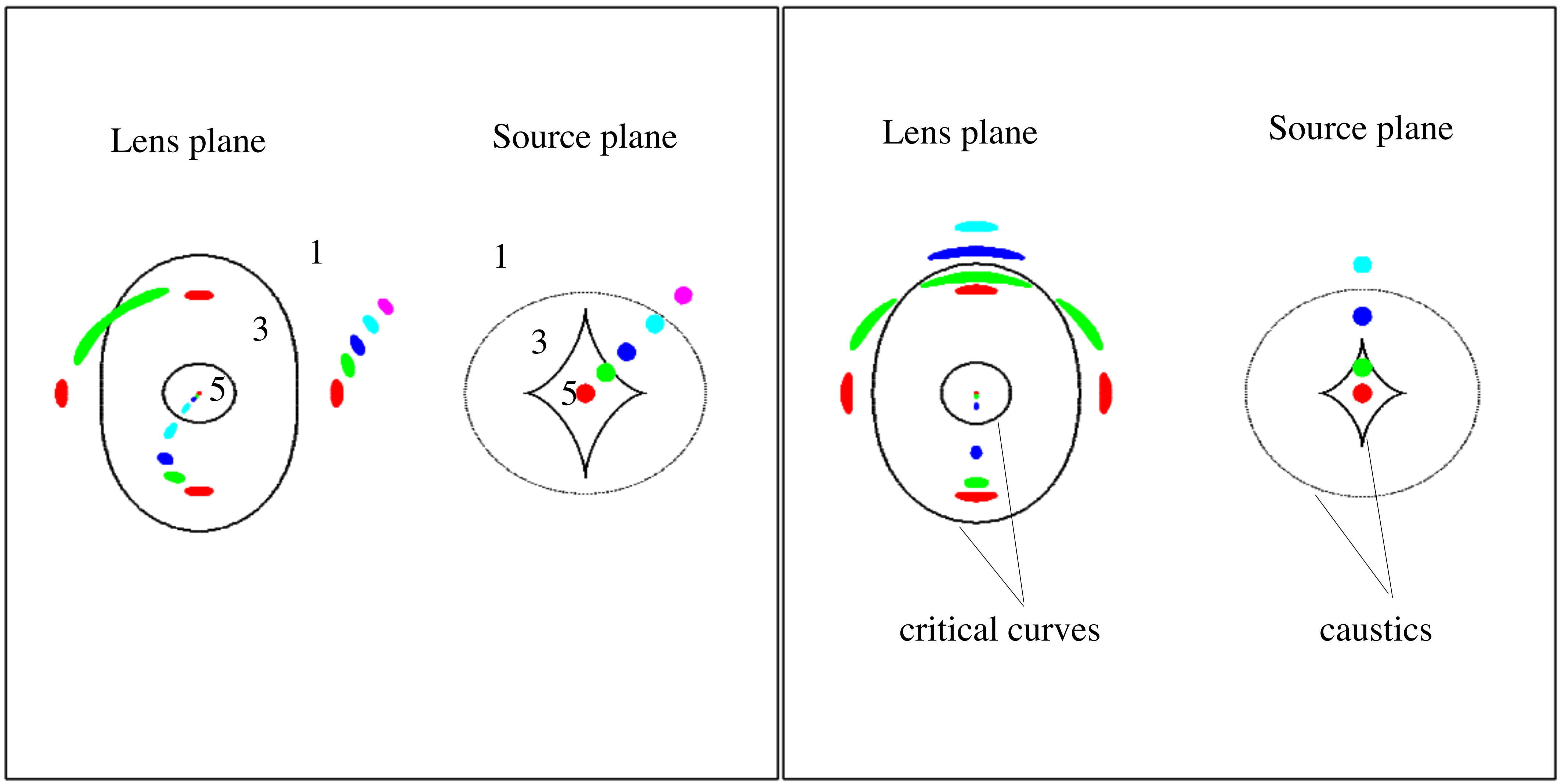

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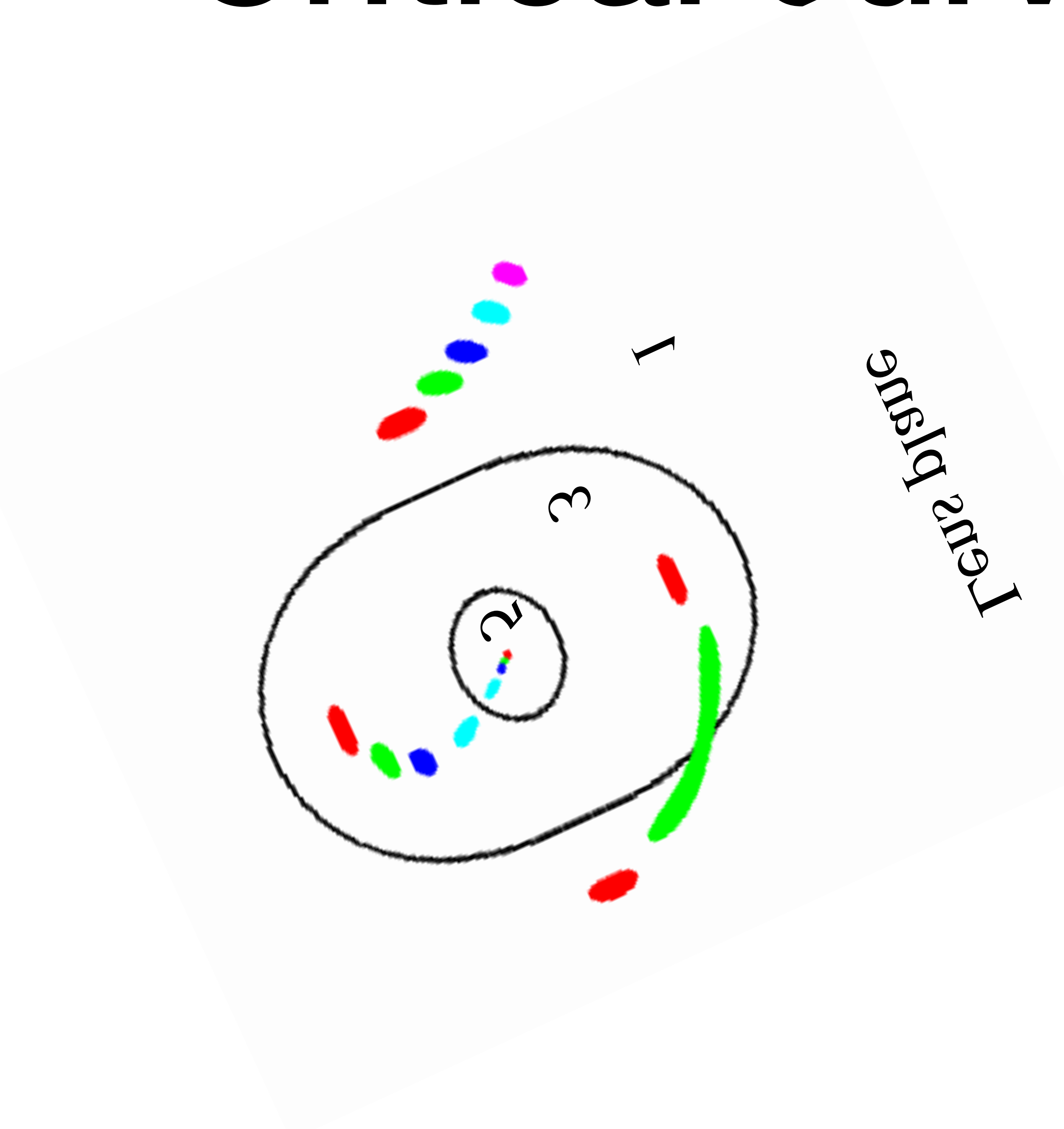


SDSS J1226+2149 Credit: ESA/Webb, NASA & CSA, J Rigby

Lens Search Methods

Lensing Search Algorithms

Source properties

- Galaxies
  - Extended, bluish ring/arc-like morphology e.g. SL2S (CFHTLS), SuGOHI (HSC)
  - Spectroscopic selection e.g. SLACS (SDSS), BELLS (BOSS)
- Quasars/AGNs
  - radio multi-band compact sources e.g. CLASS (NVSS,GB6)
  - color-color + point-like morphology e.g. STRIDES (DES)
  - spectroscopic survey e.g. SLQS (SDSS), BLQS (BOSS)
- Supernovae
  - Transient surveys - Difference imaging, color-mag, photo-z
  - Follow-up monitoring of known lenses (gals/clusters)

Lemon, C., Courbin, F., AM et a. (2024), Space Sci Review

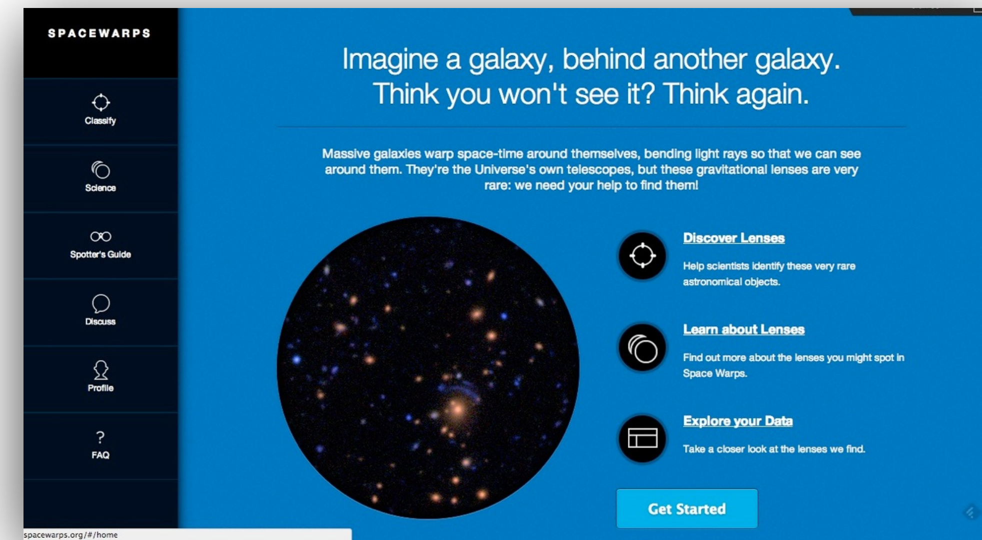
Lensing Search Algorithms

Lens properties

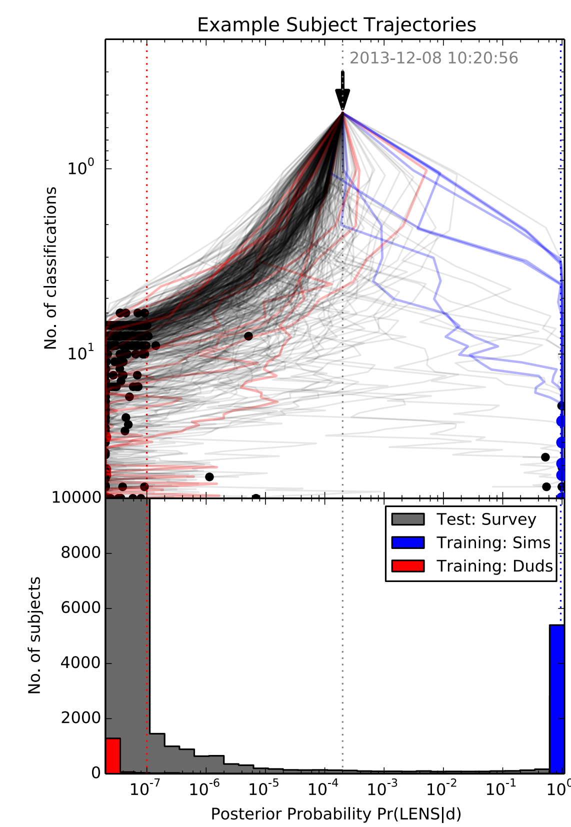
- Galaxies
  - Massive early type galaxies (photometric selection)
  - Spiral galaxies (spectroscopic selection) e.g. SWELLS
- Clusters
  - Serendipity/Visual inspection
  - Cluster-finding algorithm
    - optical richness e.g. SuGOHI-c (HSC), SGAS (SDSS)
    - X-ray e.g. MACS
    - Sunyaev Zeldovich e.g. SPT
  - Arc-finding algorithm e.g. SL2S (CFHTLS)

Lemon, C., Courbin, F., AM et a. (2024), Space Sci Review

Lensing Search Algorithms



- Space Warps (spacewarps.org) Citizen Science (*Supervised learning*) since 2013
    - pure visual inspection - CFHTLS
    - targeted searches - CS82, HSC and DES
- Machine learning algorithms (since 2017)
    - CNNs on Multi-band (color) images e.g. KiDS, CFHTLS, SuGOHI (HSC), DES
    - 1D CNN on Light curves e.g. lensed Supernovae
    - others?

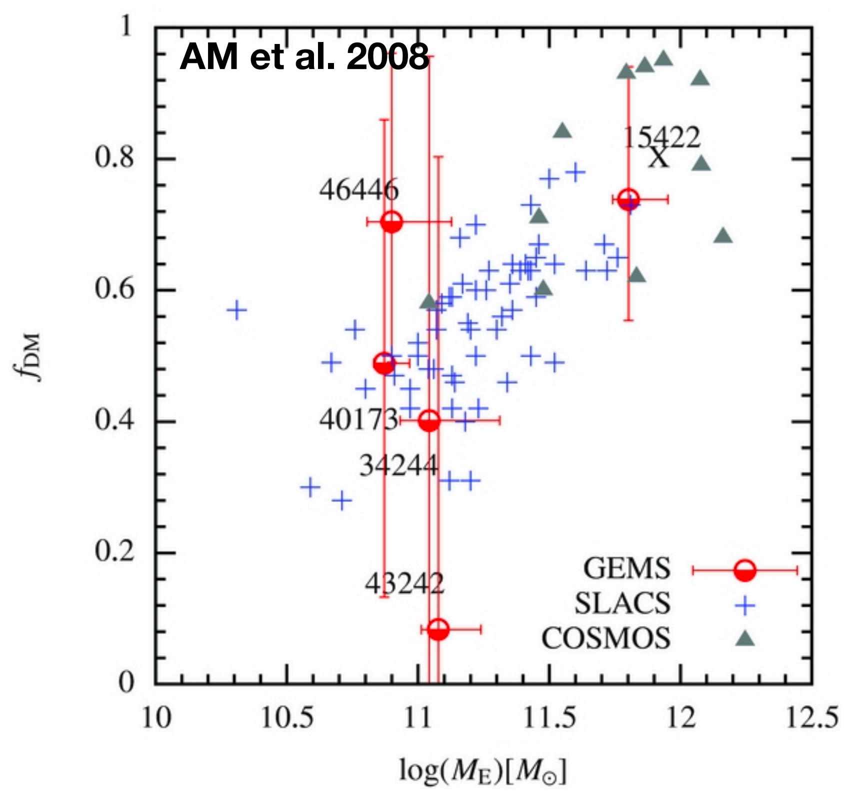


PI: Phil Marshall, Anupreeta More and Aprajita Verma

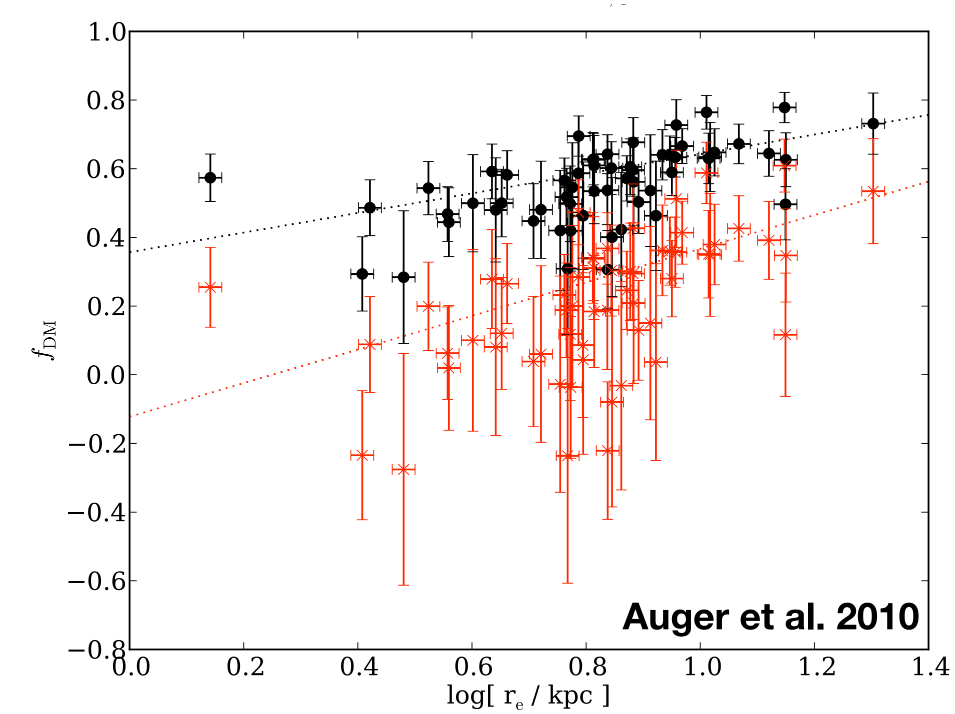
Applications

Lens: Dark Matter

- Mass (or mass fraction)



Fraction of mass in DM in galaxy-scale lensing halos



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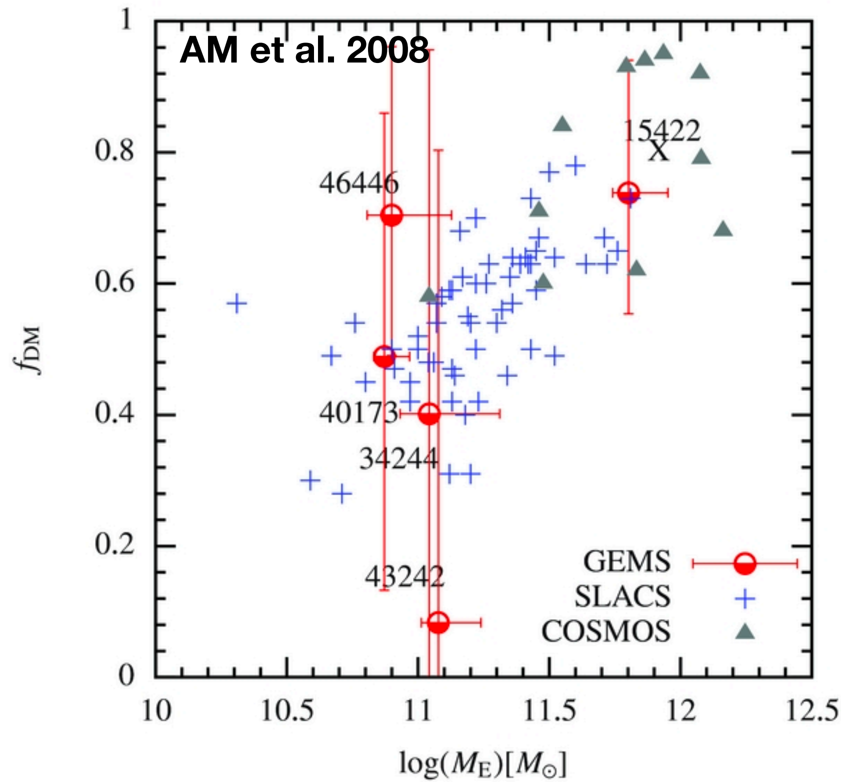

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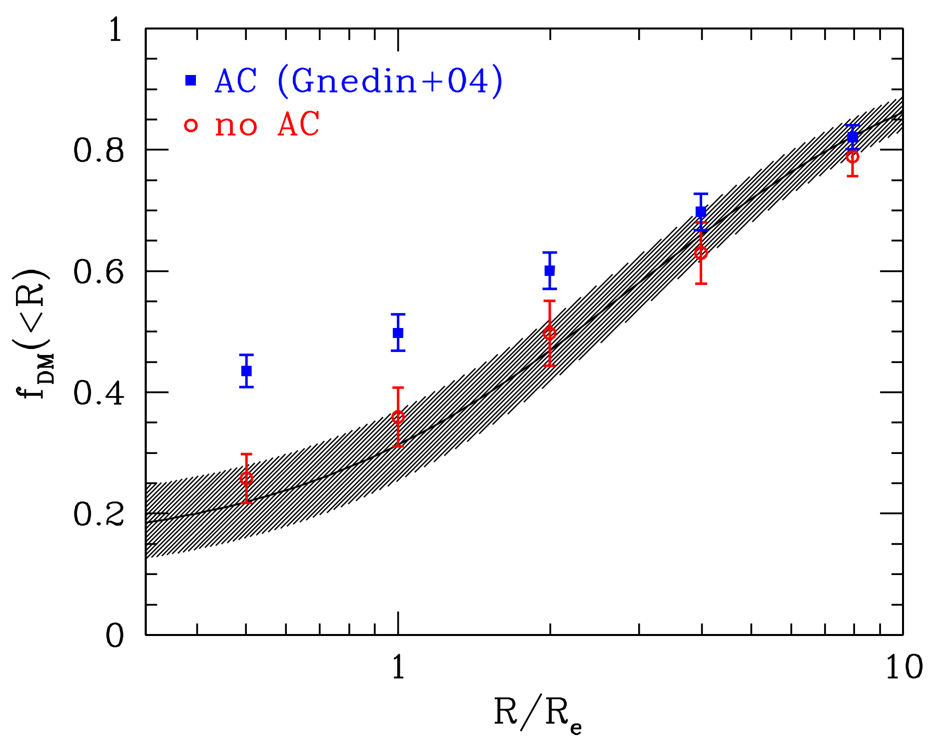
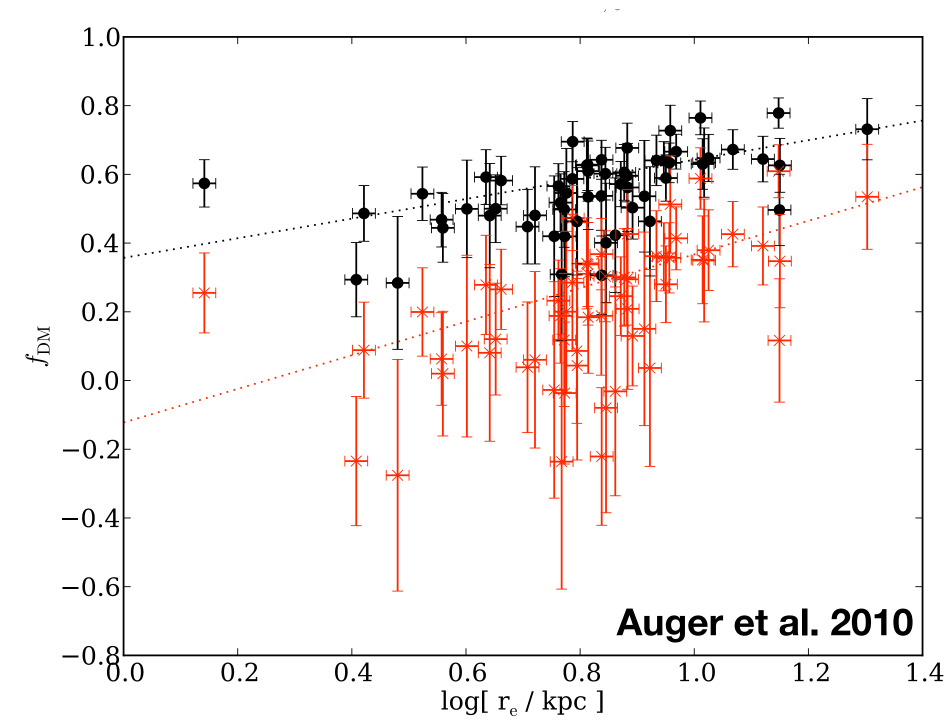


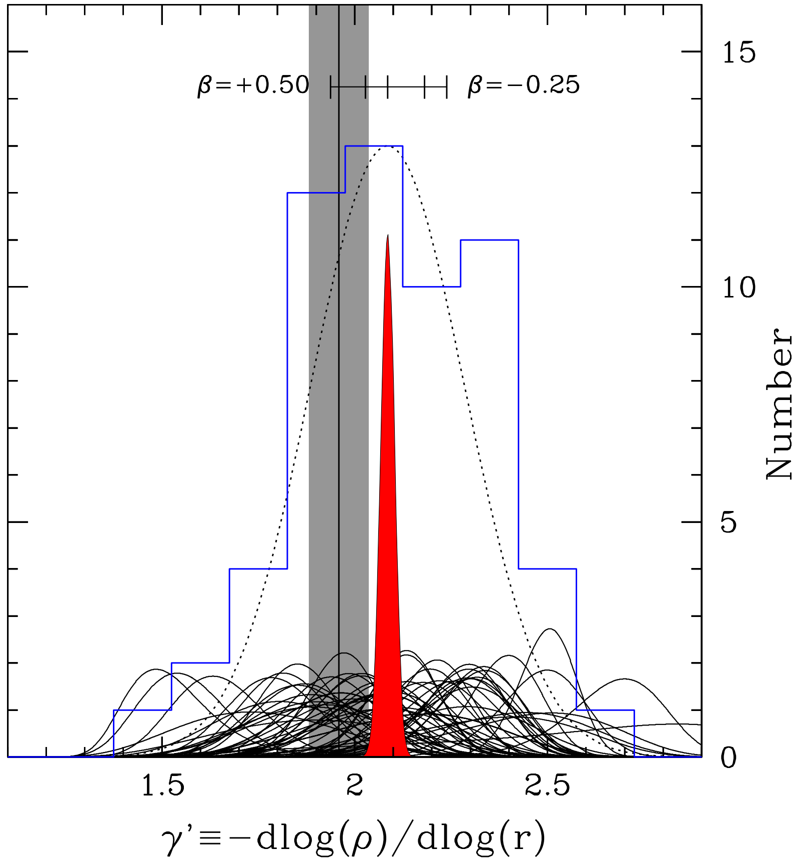
Statistical estimates of fraction of mass in DM combining systems with lensed galaxies and quasars (Oguri et al. 2014)



Lens: Dark Matter

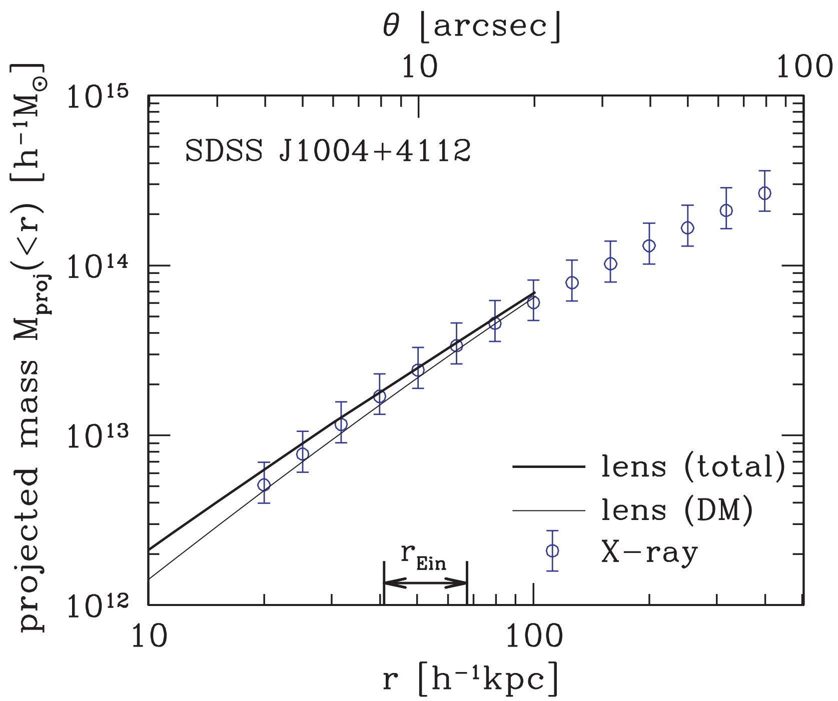
- Mass distribution

Constraining density profiles of a population of galaxies
Koopmans et al. 2009



Lens: Dark Matter

- Mass distribution



Mass distribution of a galaxy cluster using presence of the oddth image of a lensed quasar

Oguri 2010

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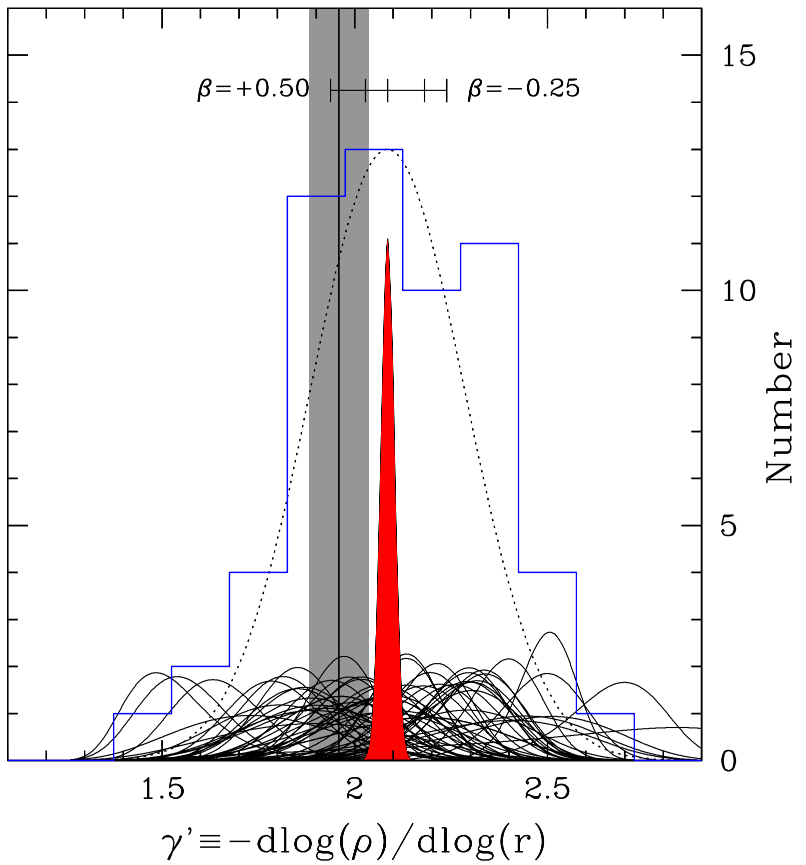

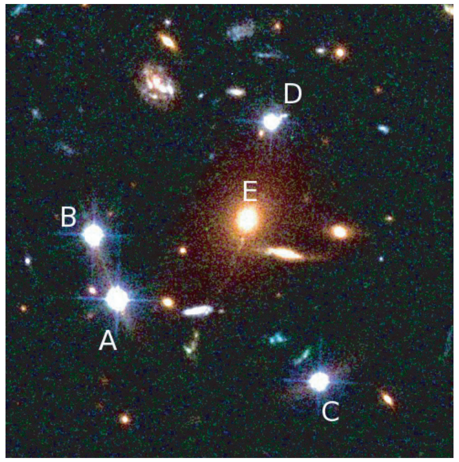

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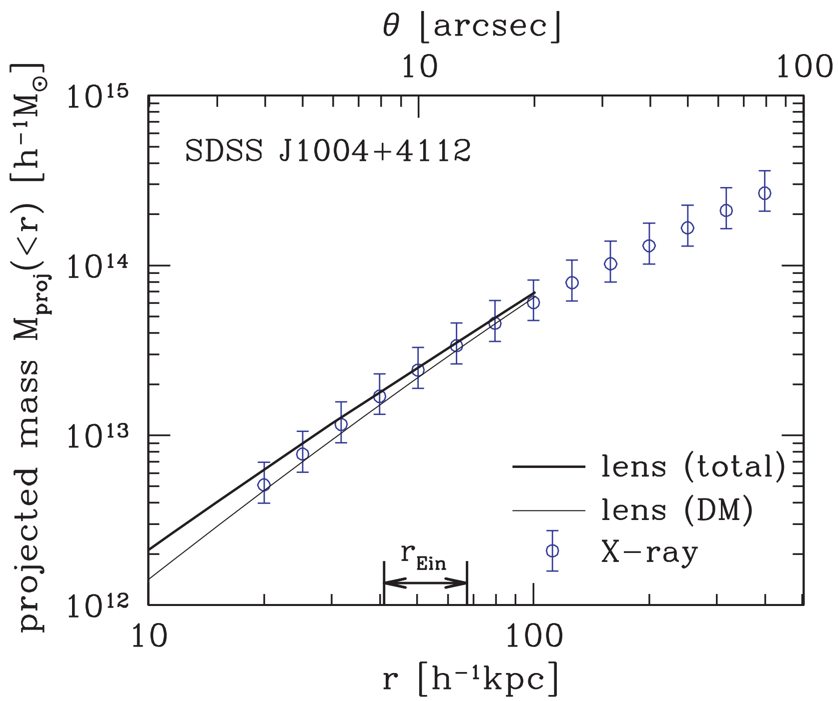

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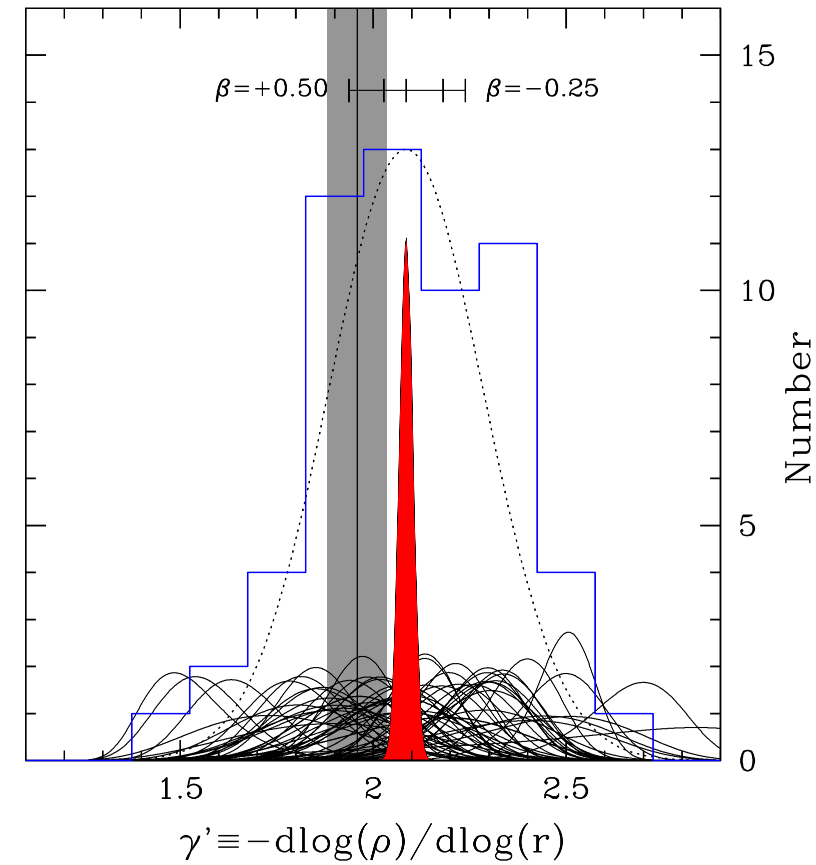

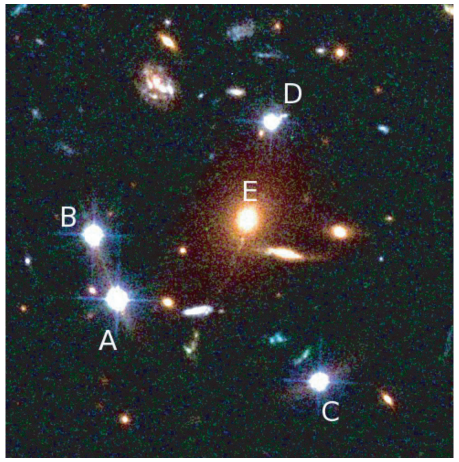

Lens: Dark Matter

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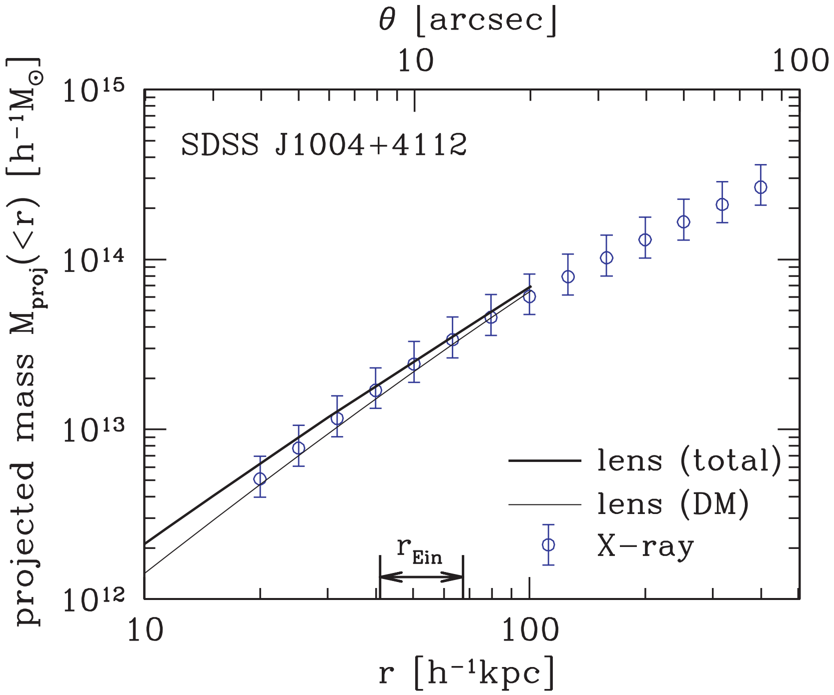

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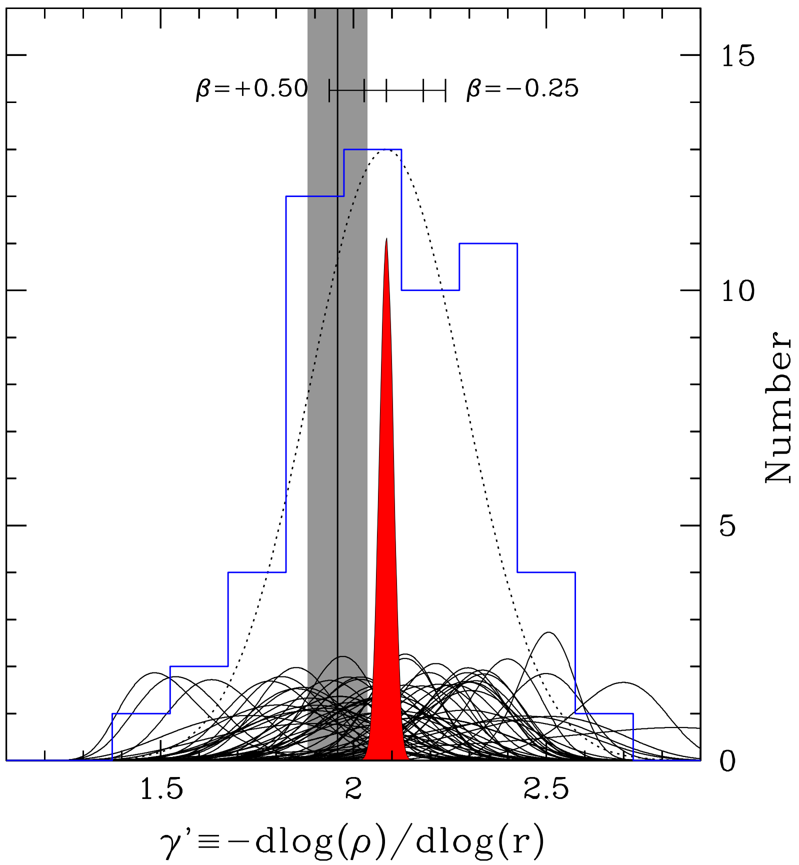
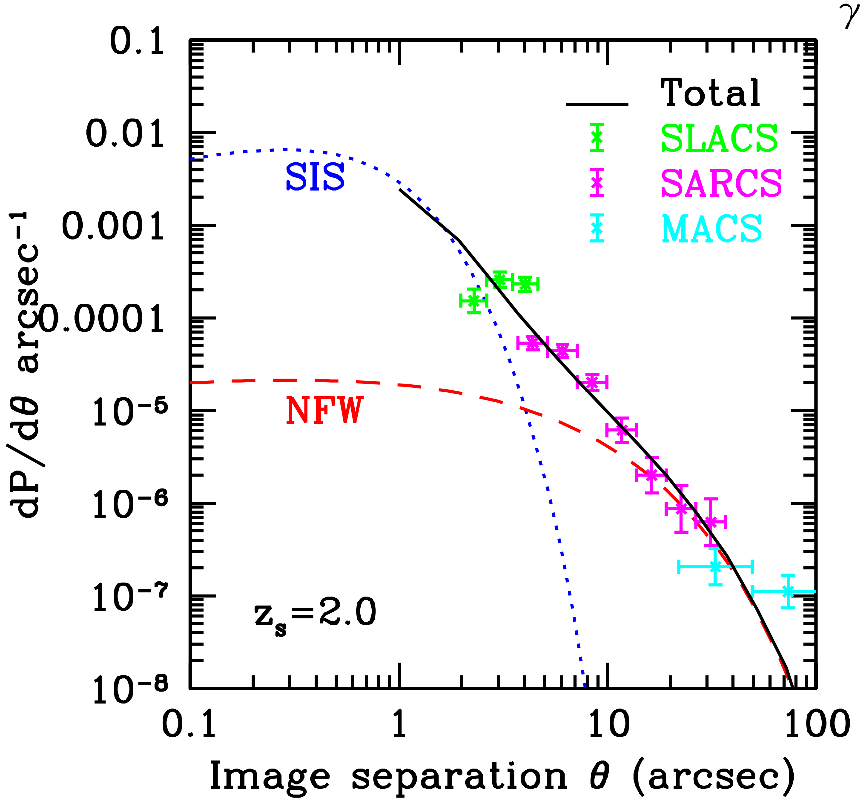
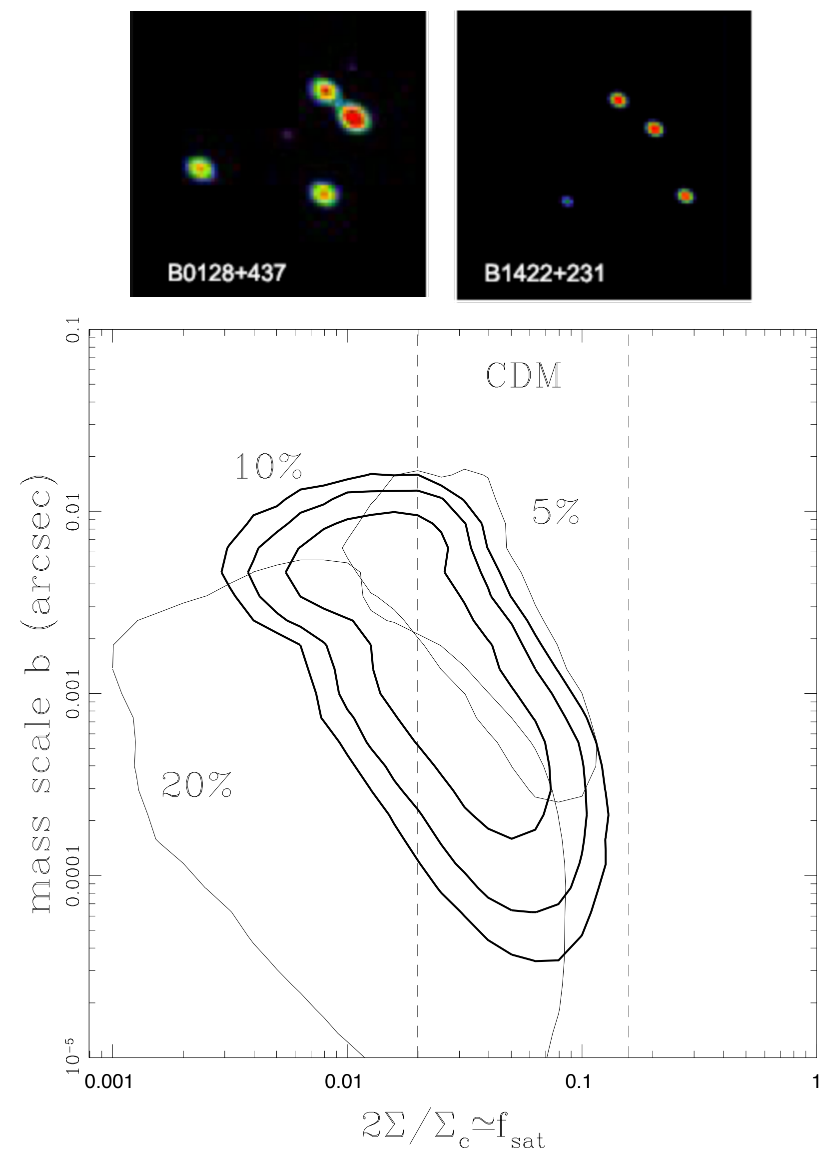




Statistical constraint on density profiles of galaxy groups
AM et al. 2012

Lens: Dark Matter

- Test of CDM (cosmological) model using subhalo/ substructure mass function



Anomalies in the flux ratios of lensed quasars to test LCDM predictions about subhalos (Dalal & Kochanek 2002)

Lens: Dark Matter

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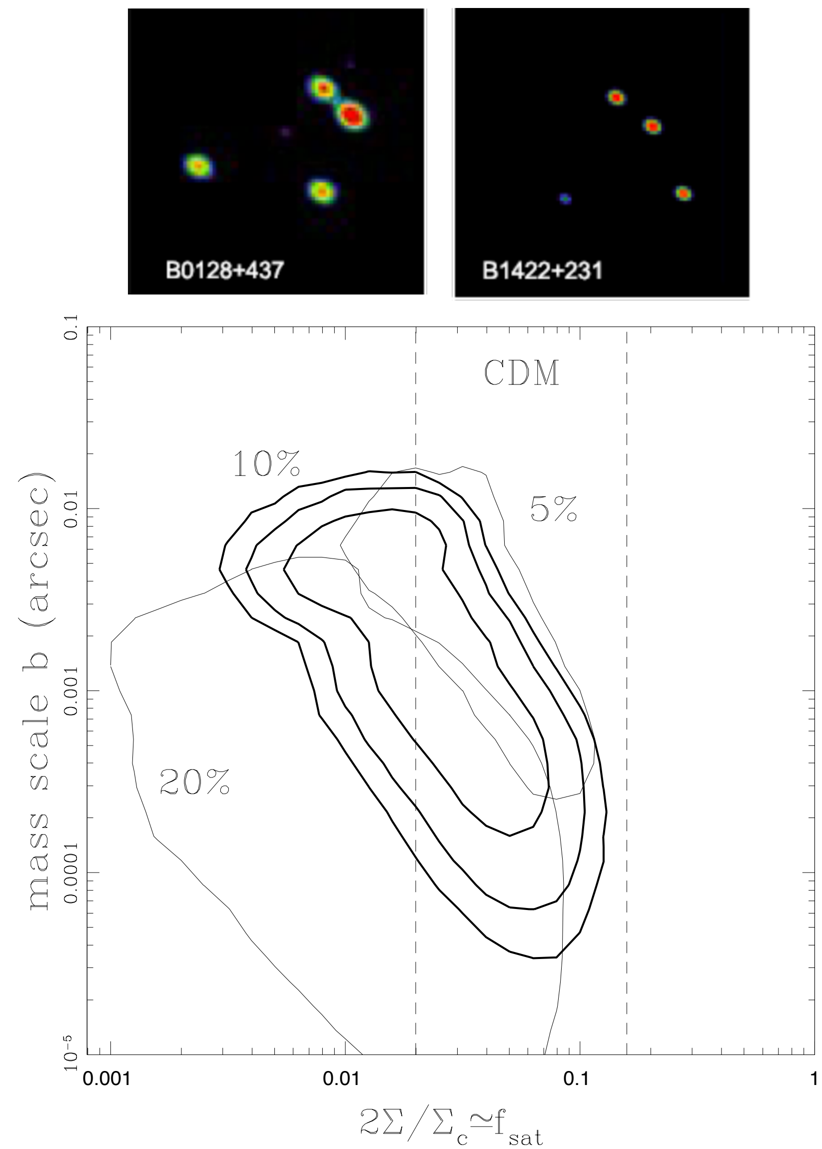
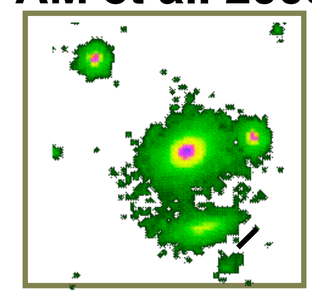
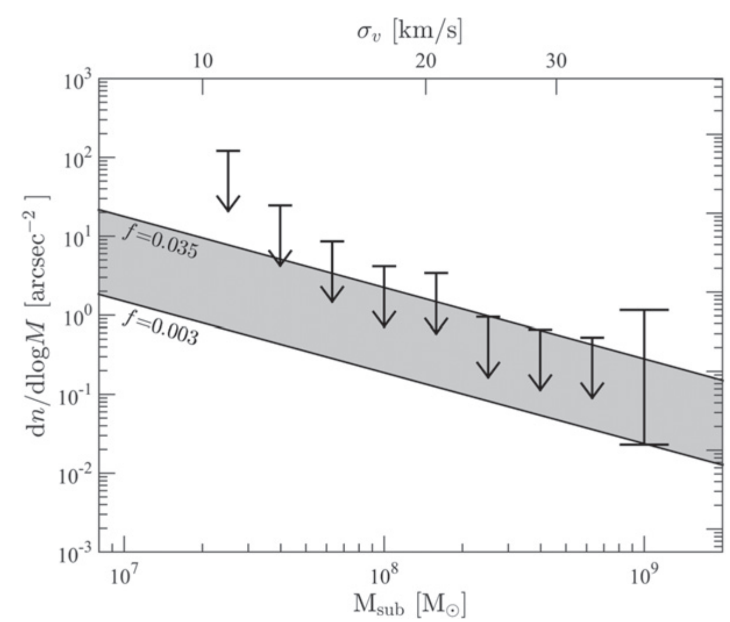


Anomalies in the flux ratios of lensed quasars to test LCDM predictions about subhalos (Dalal & Kochanek 2002)

AM et al. 2009





High resolution imaging can constrain small scale perturbations from subhalos AM et al. 2009, Hezaveh et al. 2016

Lens: Dark Matter

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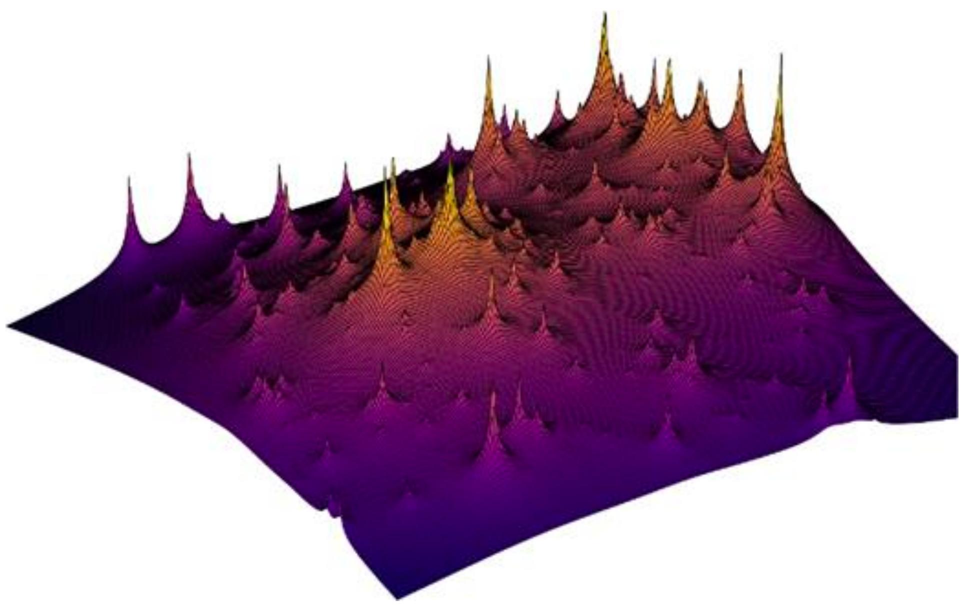
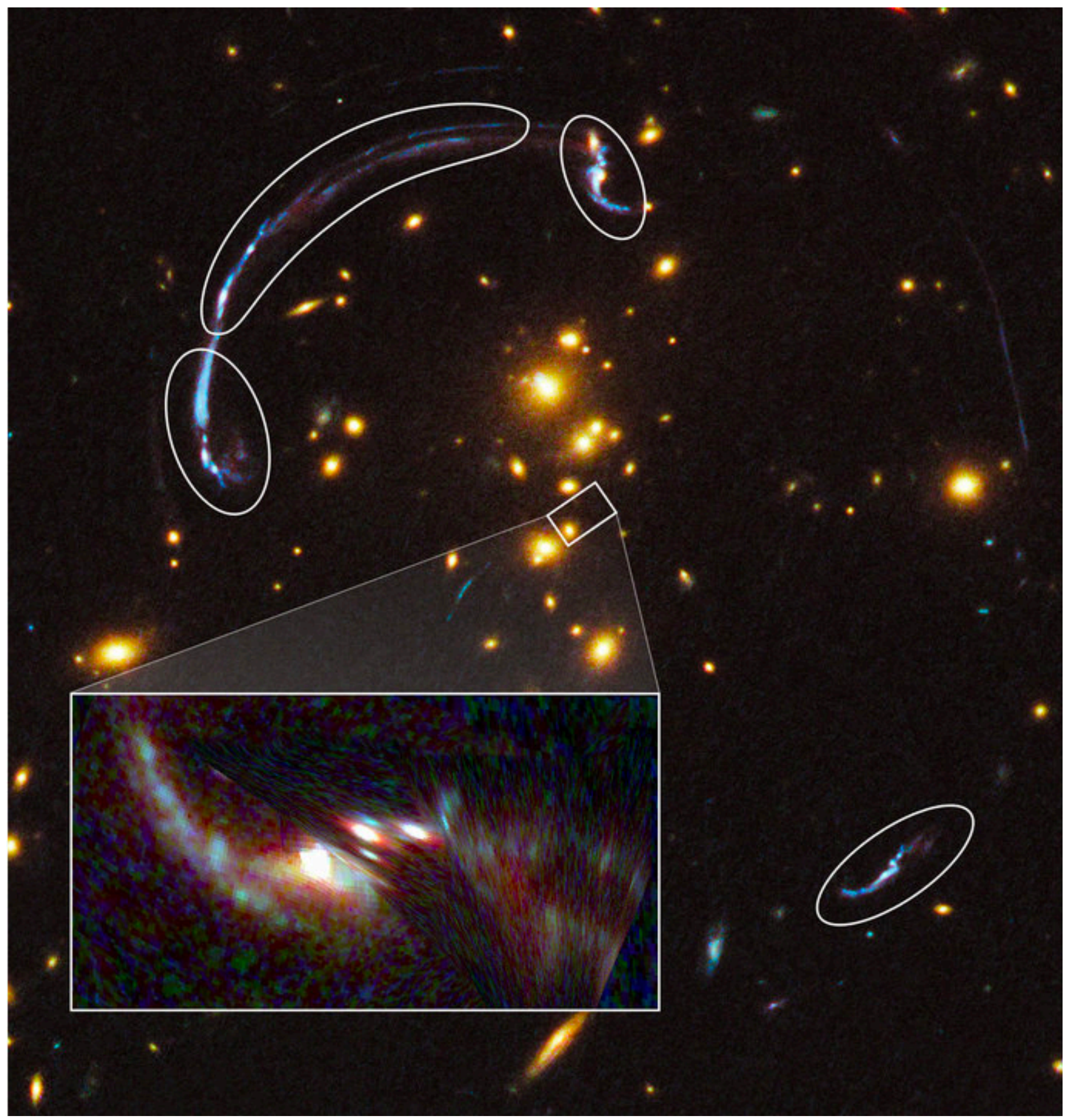


Mapping the DM subhalo mass function with strong lensing to test LCDM predictions
Natarajan et al. 2017

Source: Galaxies

- Stellar population (bulk) properties: luminosity, stellar mass, velocity dispersion, star-formation rate, clump sizes, metallicity



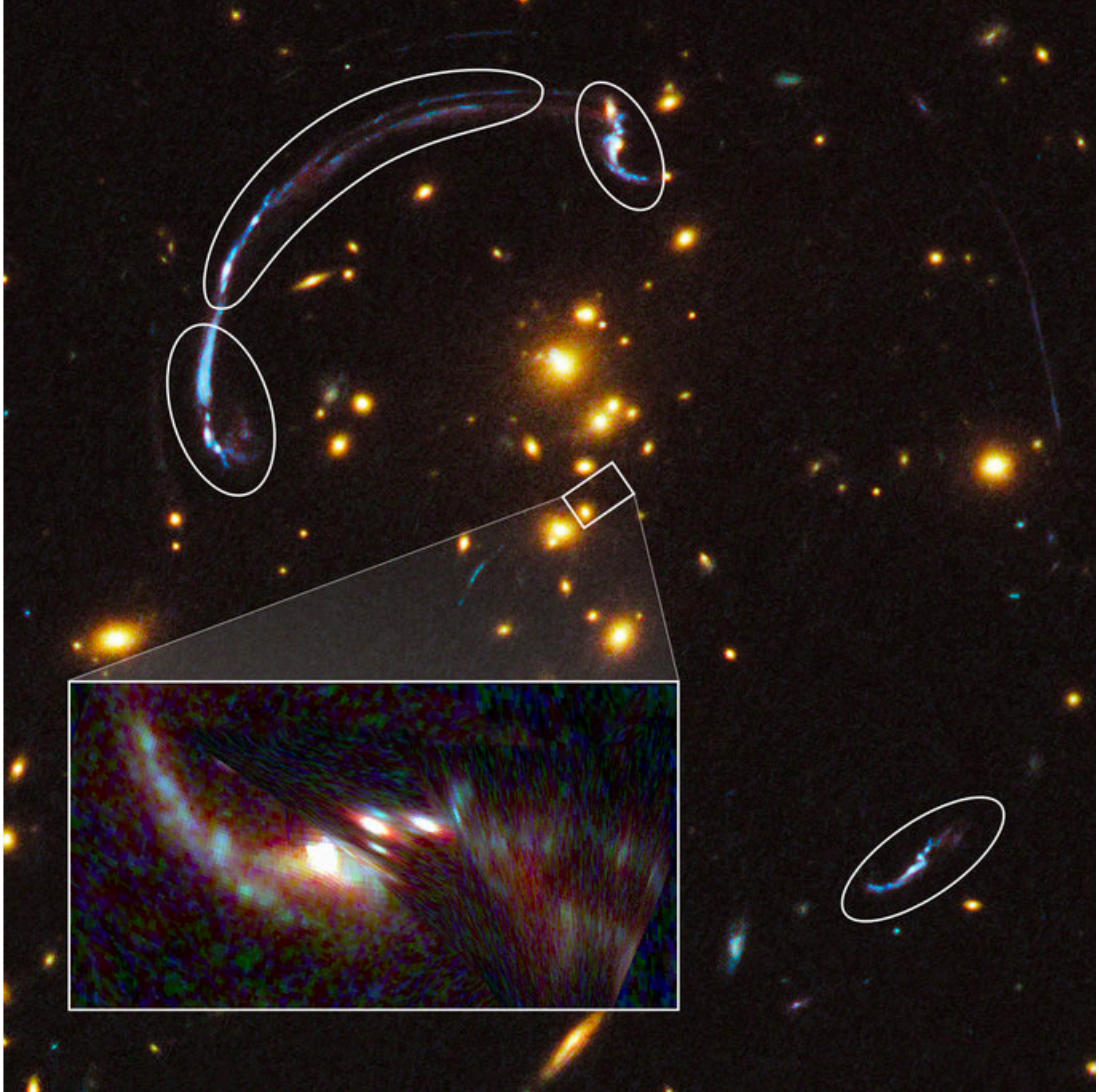
RCSGA 032727-132609: Reconstructed source (inset)
Sharon et al. 2012

Source redshift
$z = 1.70$

magnification
42.2 ± 5.5

Giant arc
$\sim 38''$ long

Source is part triply imaged and part quintuply imaged



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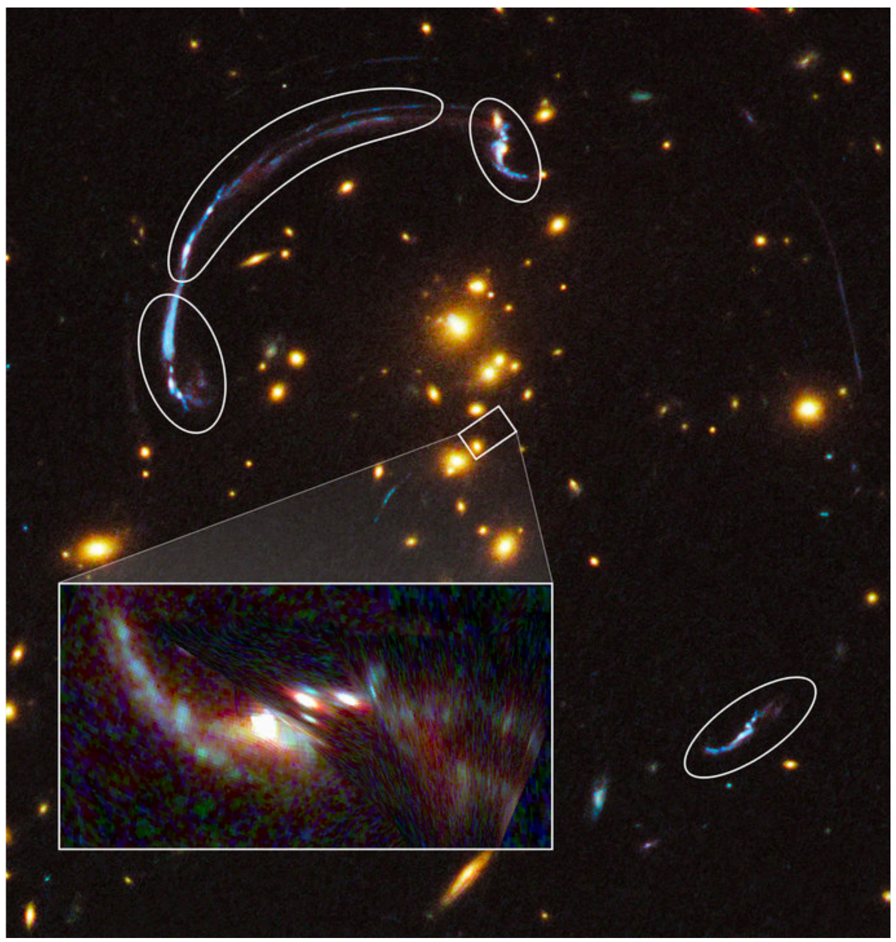
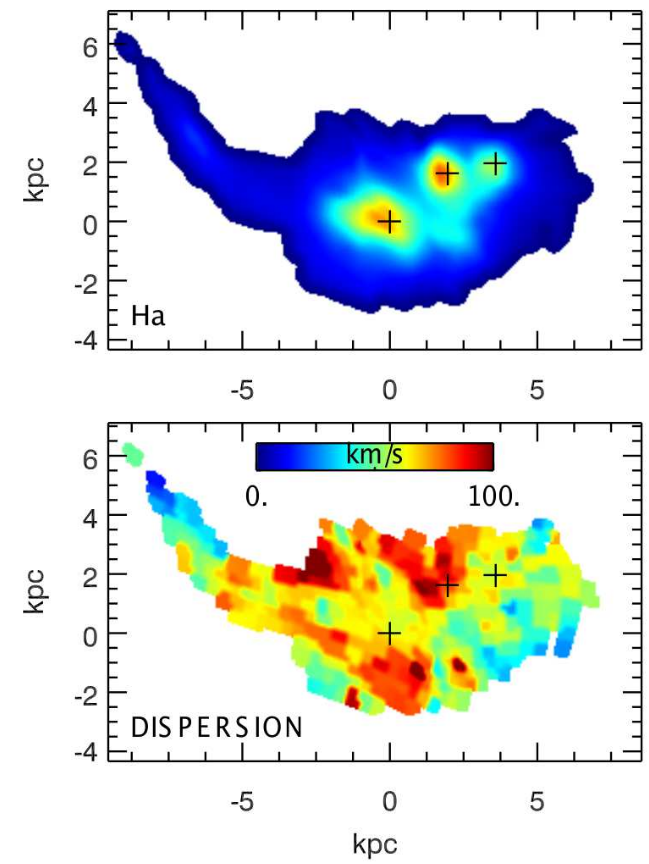
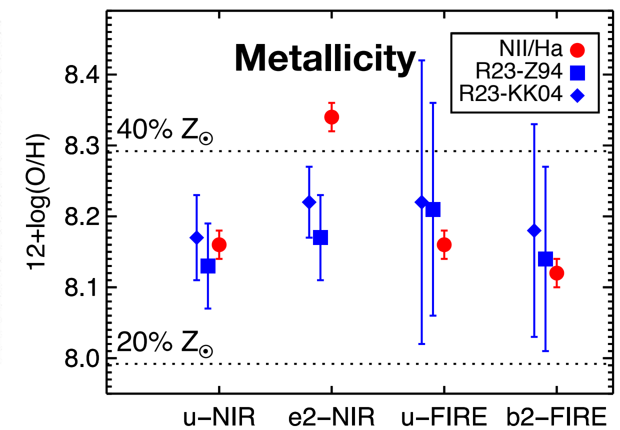
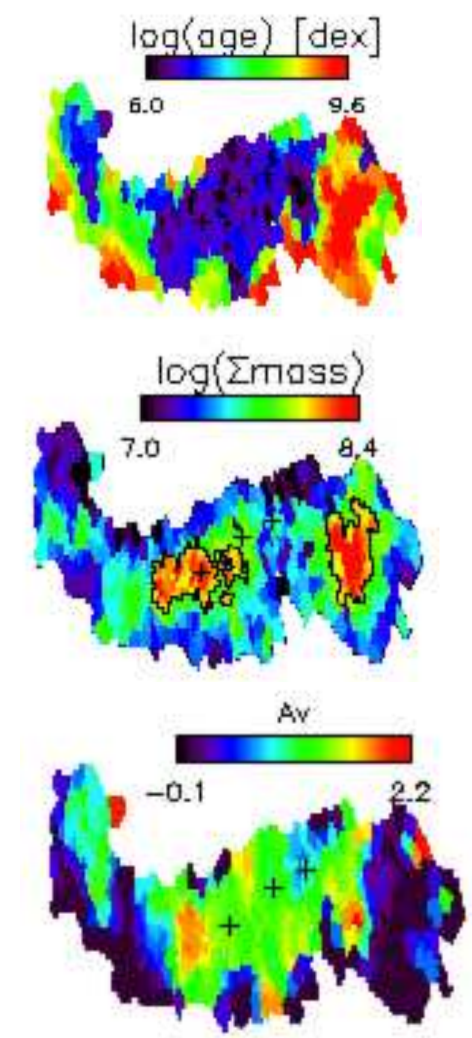
Wuyts et al. 2013



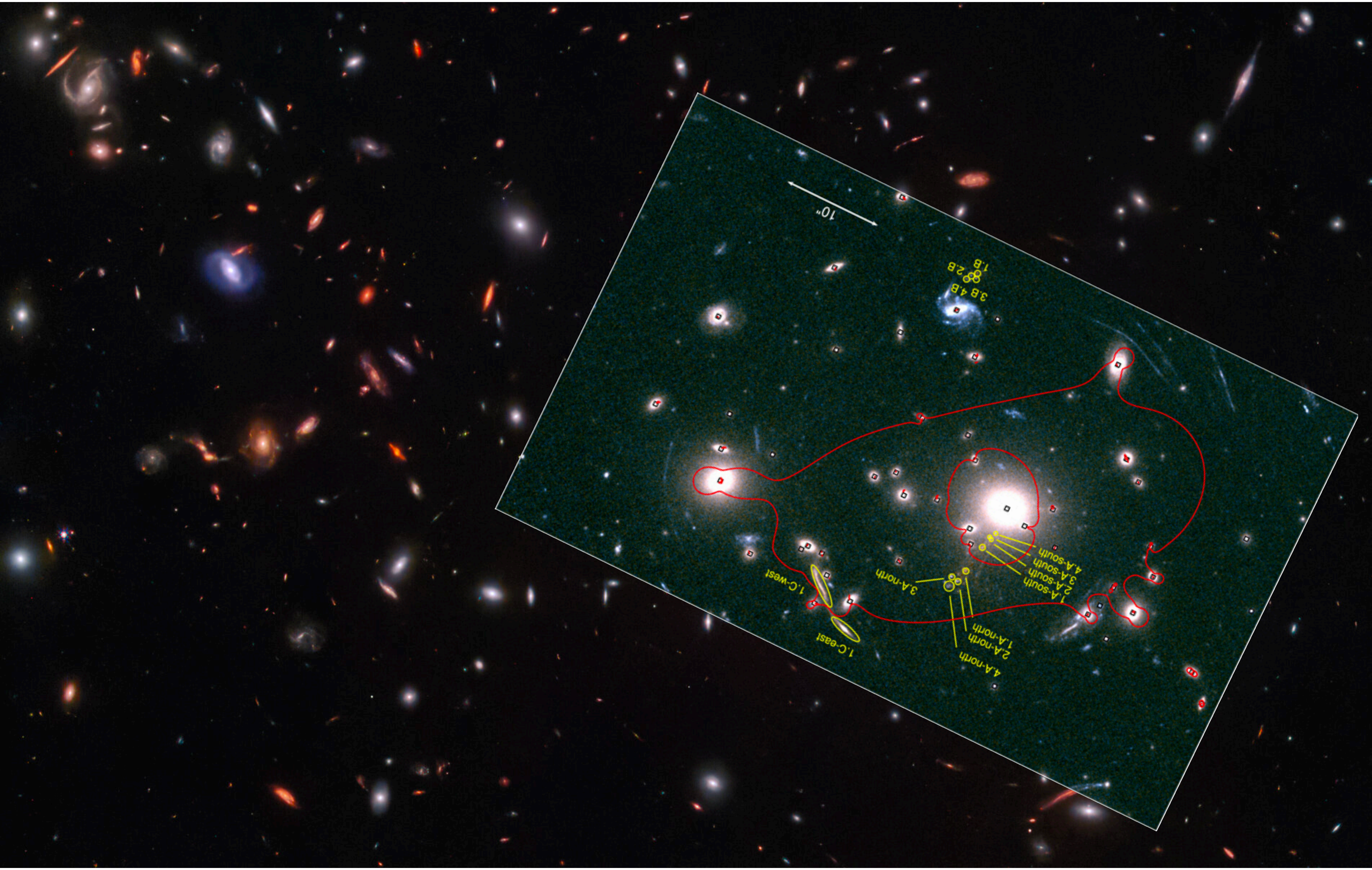
Reconstructed source (inset)
Sharon et al. 2012



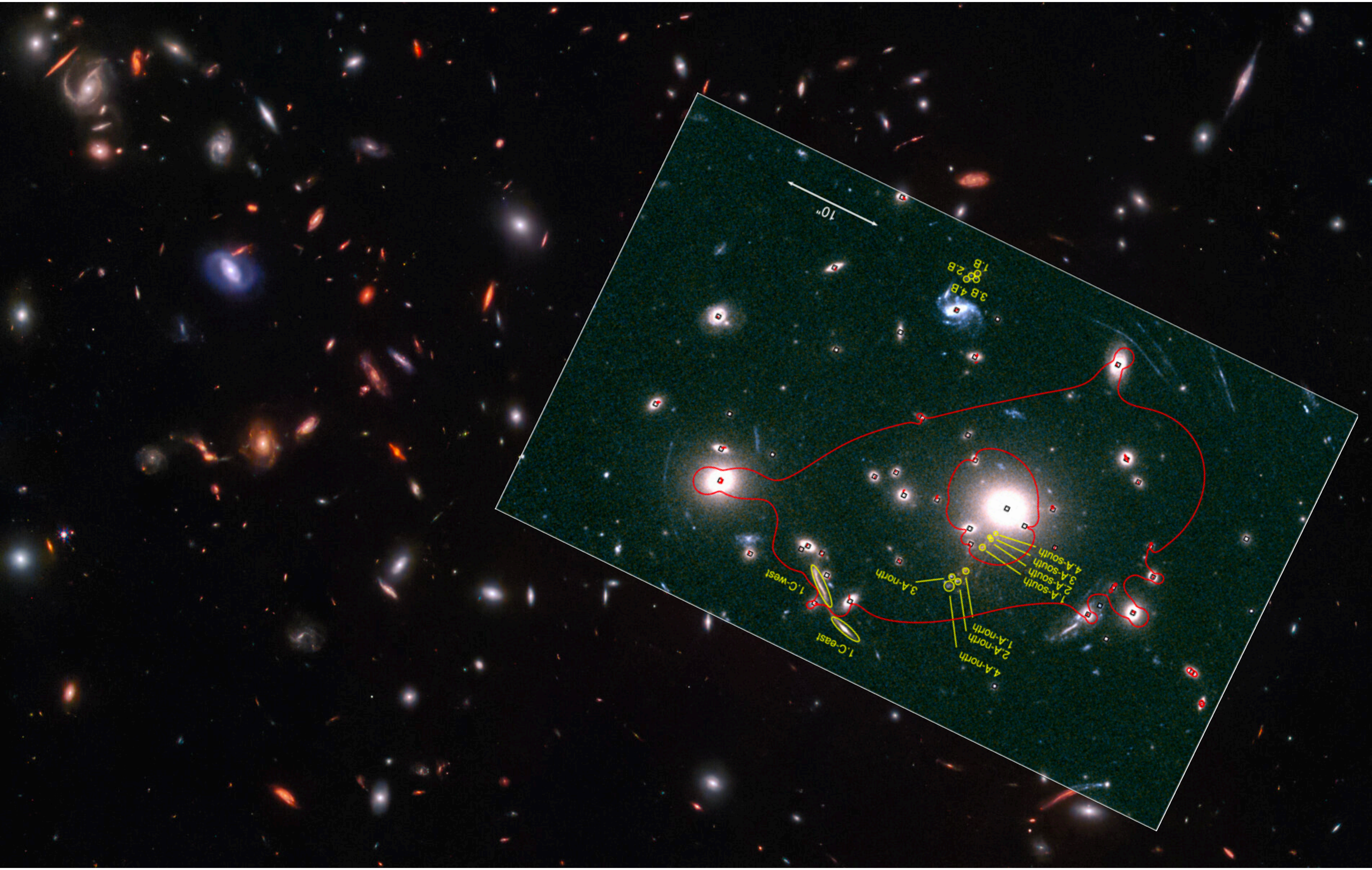




- **Ongoing interaction of 1 gas-rich and 1 gas-poor component**
- **large tidal tail, 7 SF regions size~500pc**
- **clumps show consistent VD-Lum relation with other high-z clumps (Wuyts et al. 2013)**

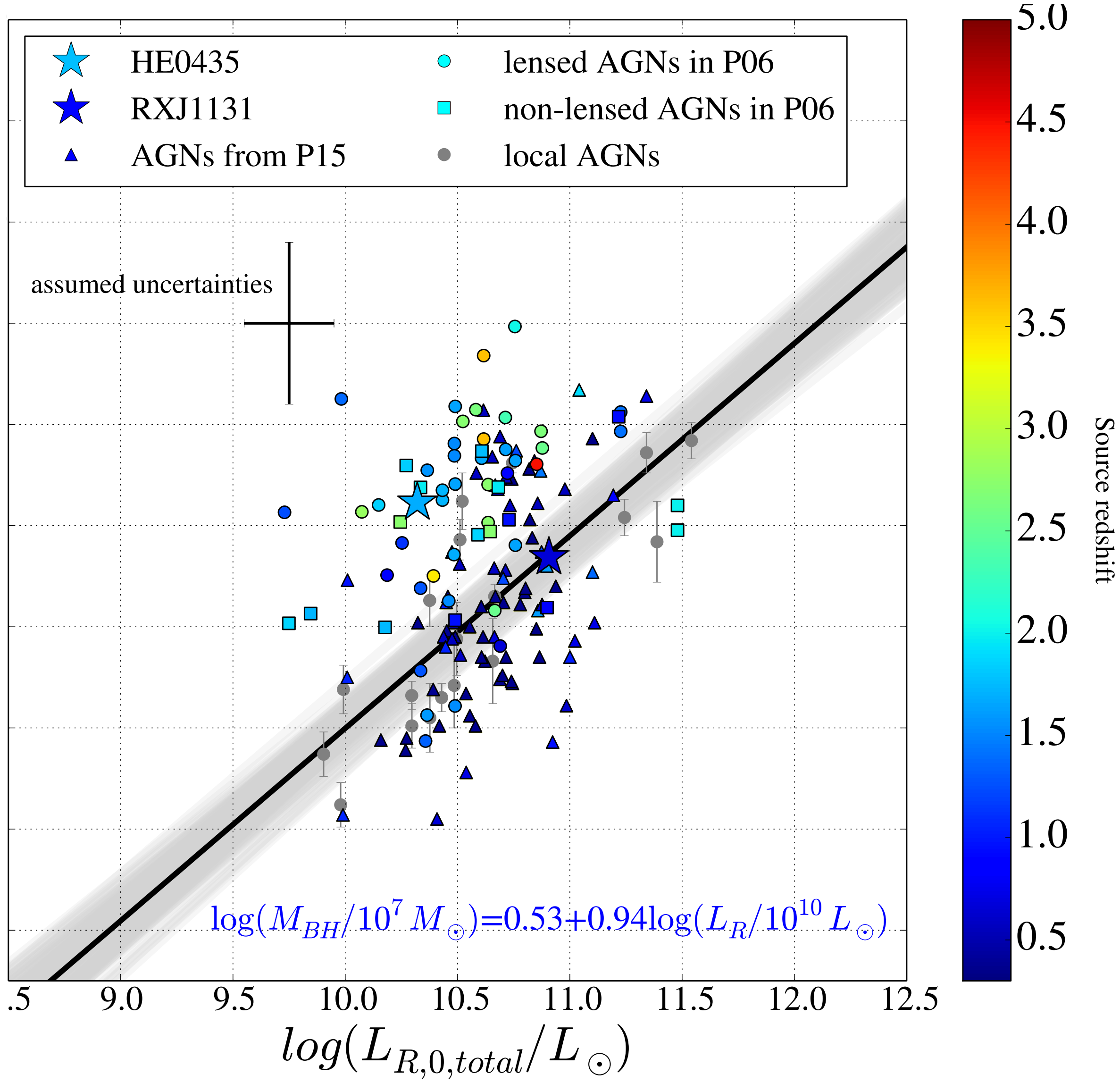


10"
1.B
2.B
3.B
4.B
1.A-south
2.A-south
3.A-south
4.A-south
3.A-north
1.A-north
2.A-north
4.A-north
1.C-west
1.C-east



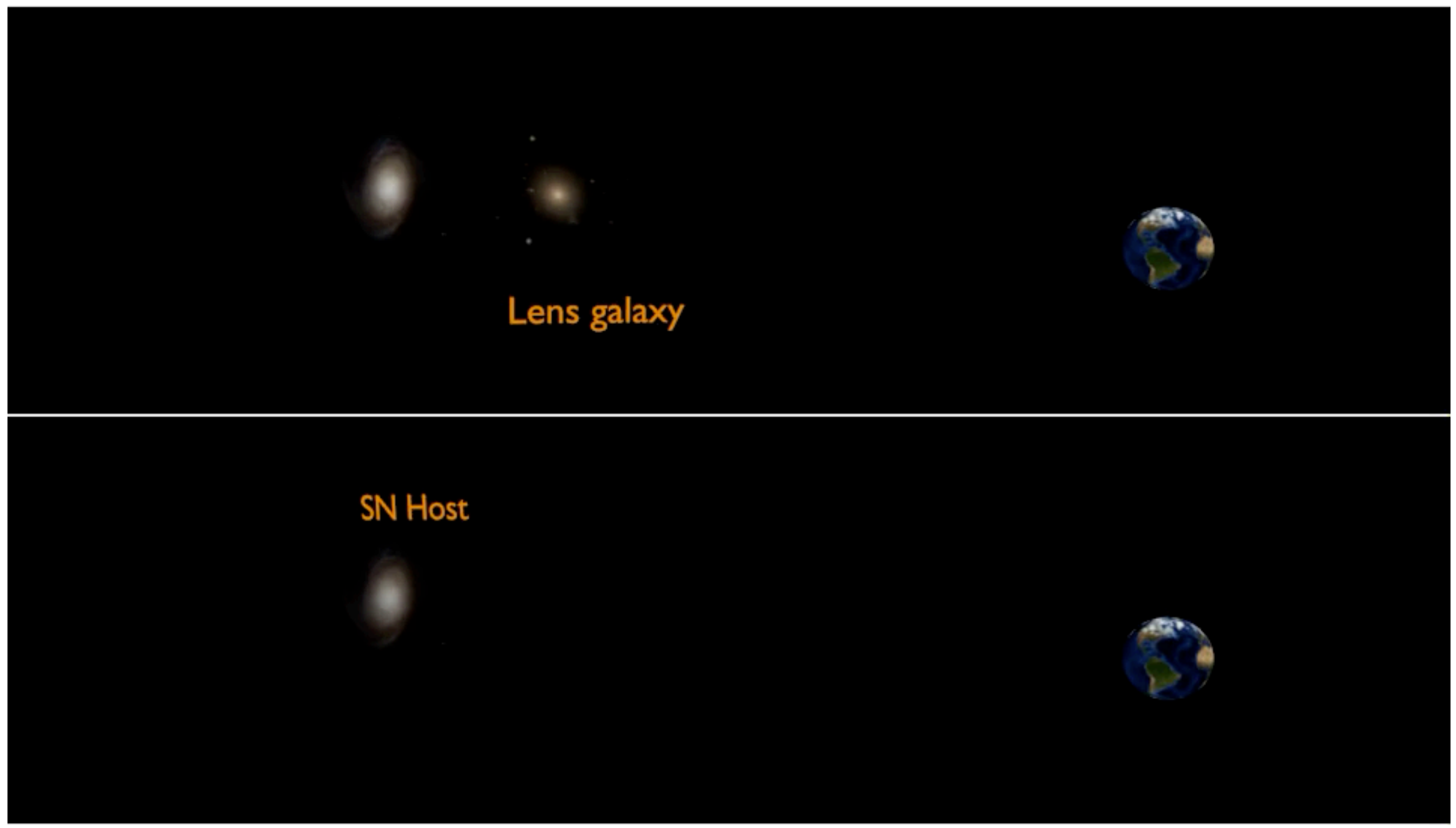
SDSS J1226+2149, galaxy cluster at 6.3 billion light-years
Credit: ESA/Webb, NASA & CSA, J. Rigby and HST

Source: Quasars



Black hole mass - host luminosity relation (Ding et al. 2017)

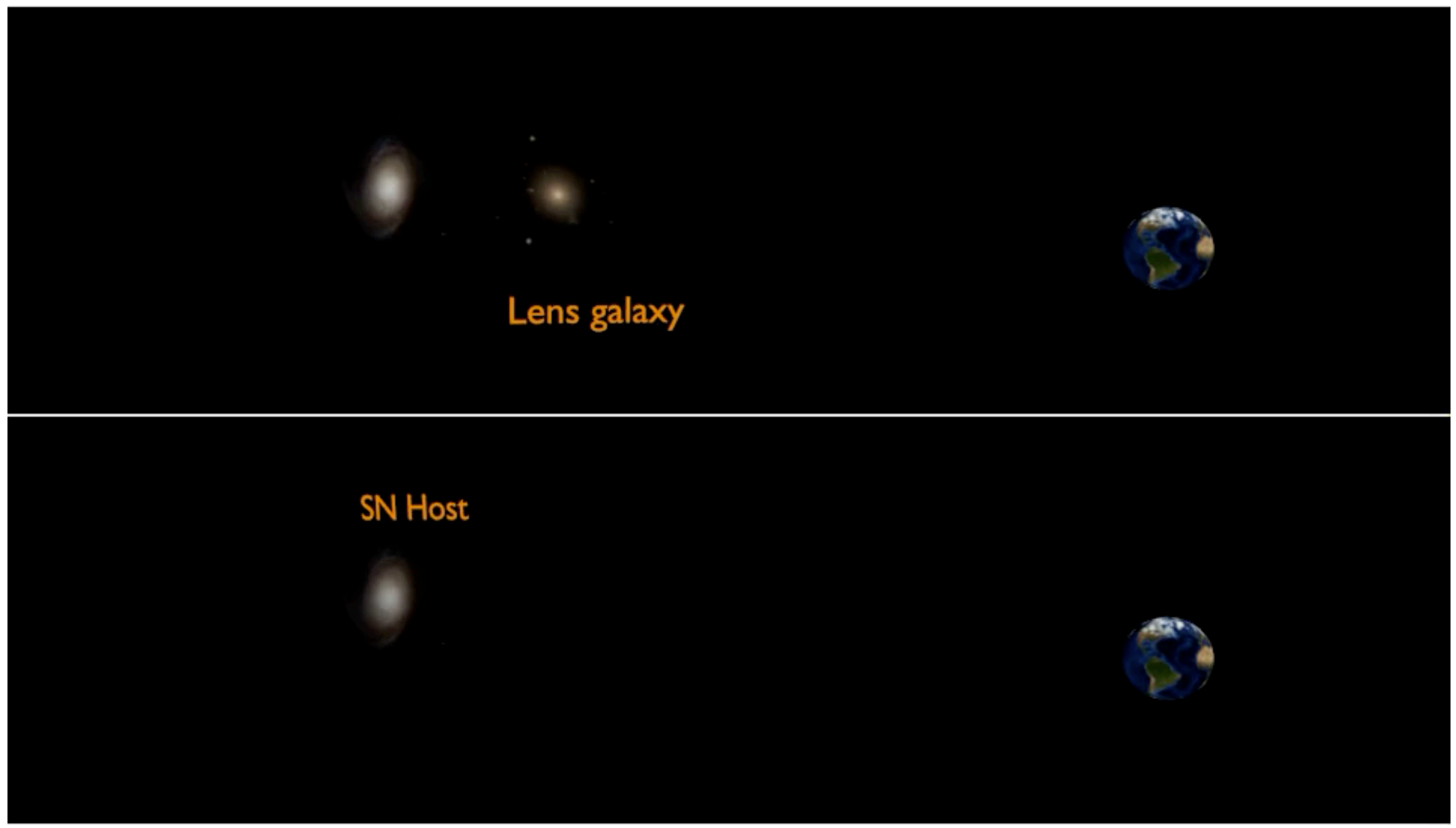
Cosmology



credit: Surhud More

- Hubble Constant using time delays between multiply lensed SNe (Refsdal 1964)

Cosmology



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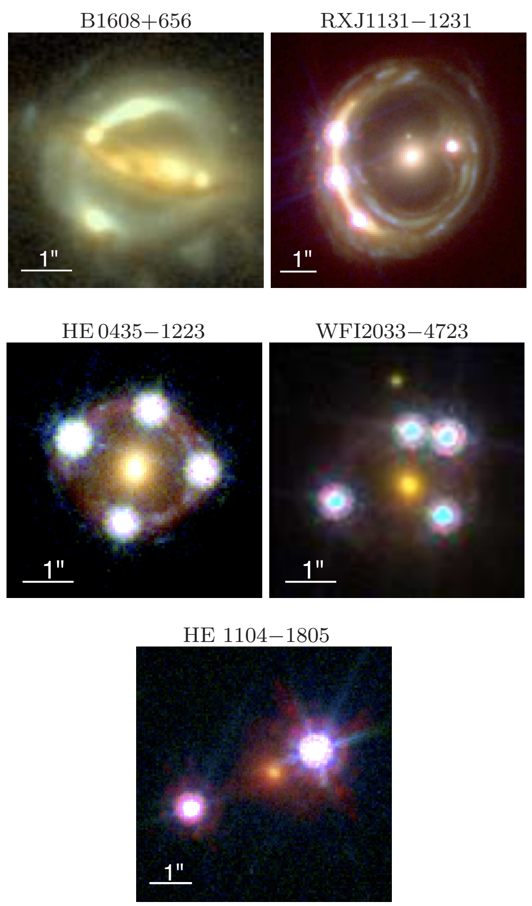
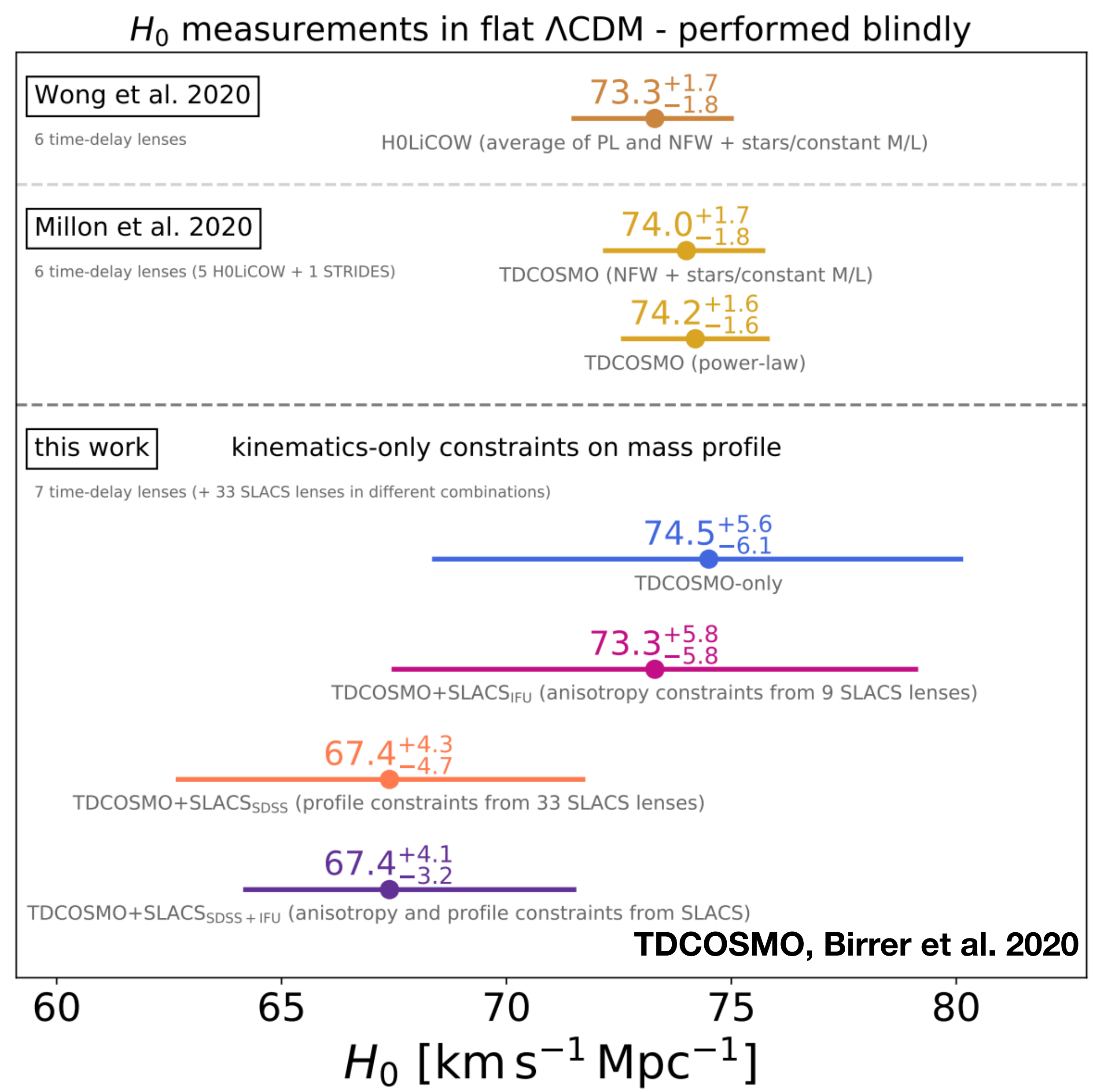
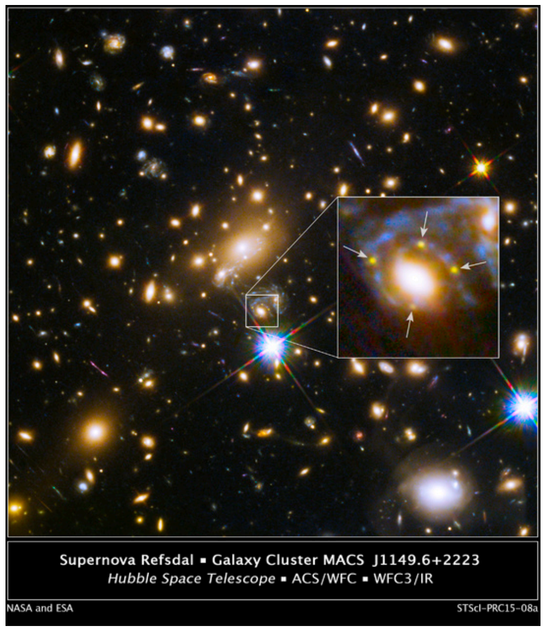
Cosmology



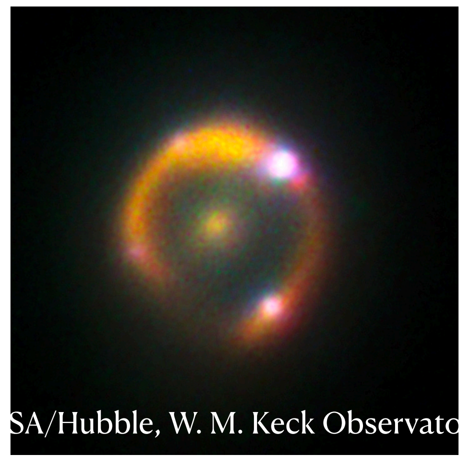


- Lensed Quasars have been used hitherto for H0 (Suyu 2012, Bonvin et al. 2017, Birrer et al. 2020 COSMOGRAIL, H0LiCoW and TDCOSMO collaborations)
- Pros: Much more abundant than lensed SNe; Cons: Painstakingly long monitoring observations spanning decades

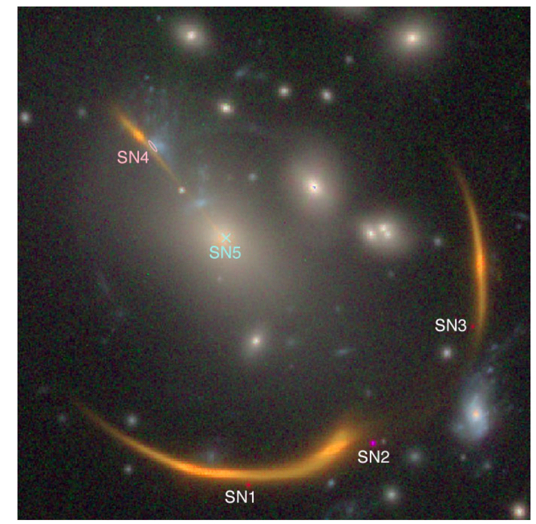
Cosmology



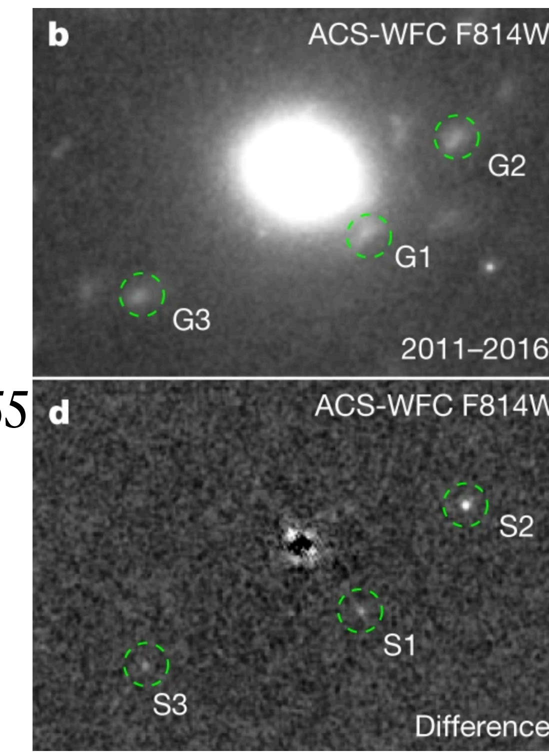
SN Refsdal (Kelly et al. 2015)



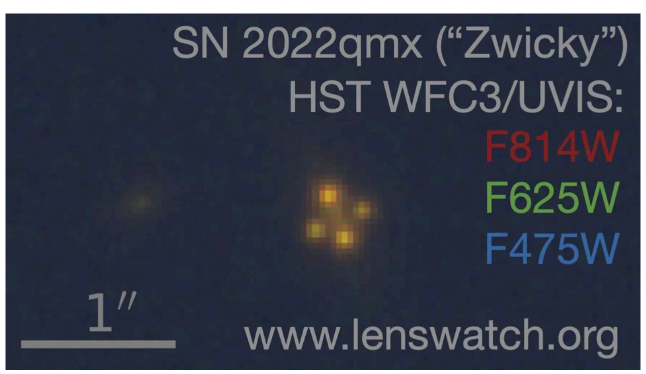
Discovery of iPTF16geu (Goobar et al. 2017), Microlensing (AM et al. 2017)



SN Requiem in MACS J0138.0-2155 (Rodney et al. 2021)



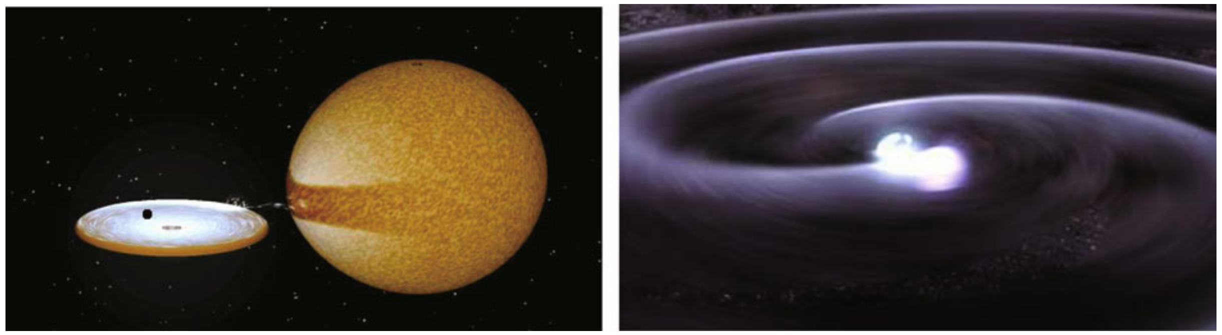
Lensed SN in Abell 370 (Chen et al. 2022)



SN Zwicky (Goobar et al. 2022)

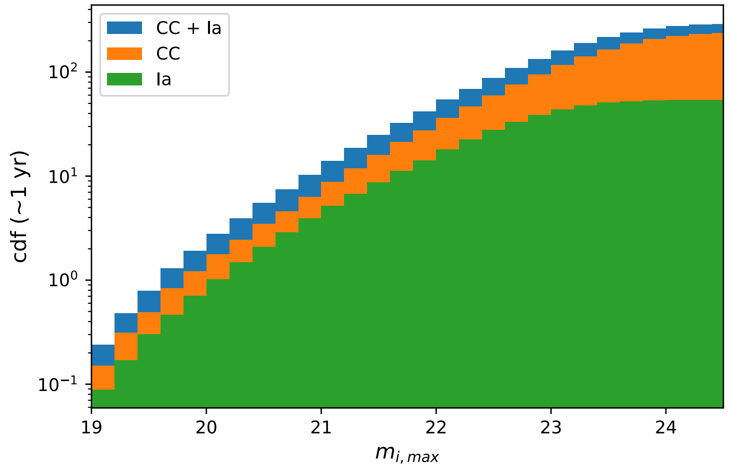
- Upcoming Surveys like LSST, Euclid will find several dozens of lensed SNe
- Pros: Standard candles - absolute magnification; transient - host only emission, monitoring easier
- Cons: Microlensing or mass distribution complex ; follow-up obs. essential

Source: Supernovae



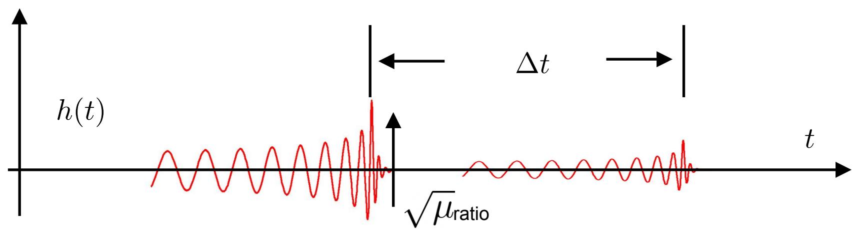
Different progenitor models

- Different progenitor models —> differences in light curves
- Lensed SNe
  - Higher redshifts
  - Long time delays —> accurate early predictions
  - Lower lensing systematics (e.g. microlensing) if in clusters



Expected detections in LSST survey per year
Goldstein et al. 2019

Cosmology

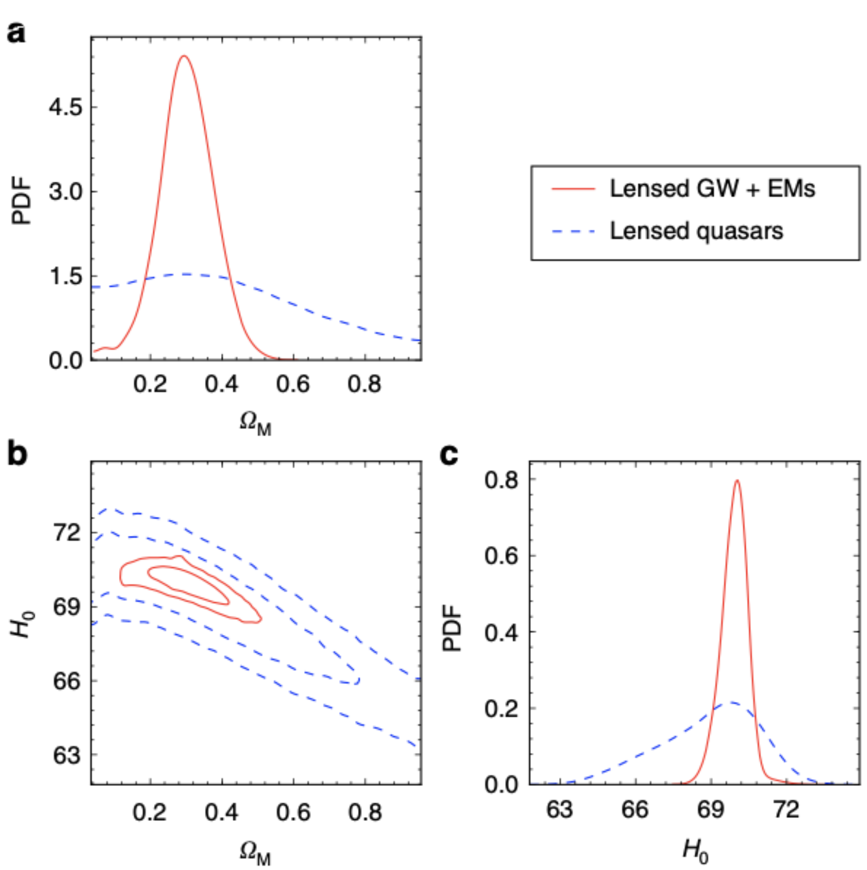


Lensed Gravitational Waves

Table 1 Relative uncertainties of three factors contributing to the accuracy of time-delay distance measurement

	$\delta\Delta t$	$\delta\Delta\psi$	δLOS
Lensed GW + EM	0%	0.6%	1%
Lensed quasar	3%	3%	1%

$\delta\Delta t$, $\delta\Delta\psi$, δLOS correspond to time delay, Fermat potential difference, and light-of-sight environment, respectively. We show the case for lensed gravitational wave (GW) + electromagnetic (EM) signals compared with standard technique in the EM domain using lensed quasars



Liao et al. Nature 2017

- Uncertainty on the lensing potential - lower for lensed transients
- Uncertainty on the time delay - negligible SNe results
- EM identification necessary

Cosmology

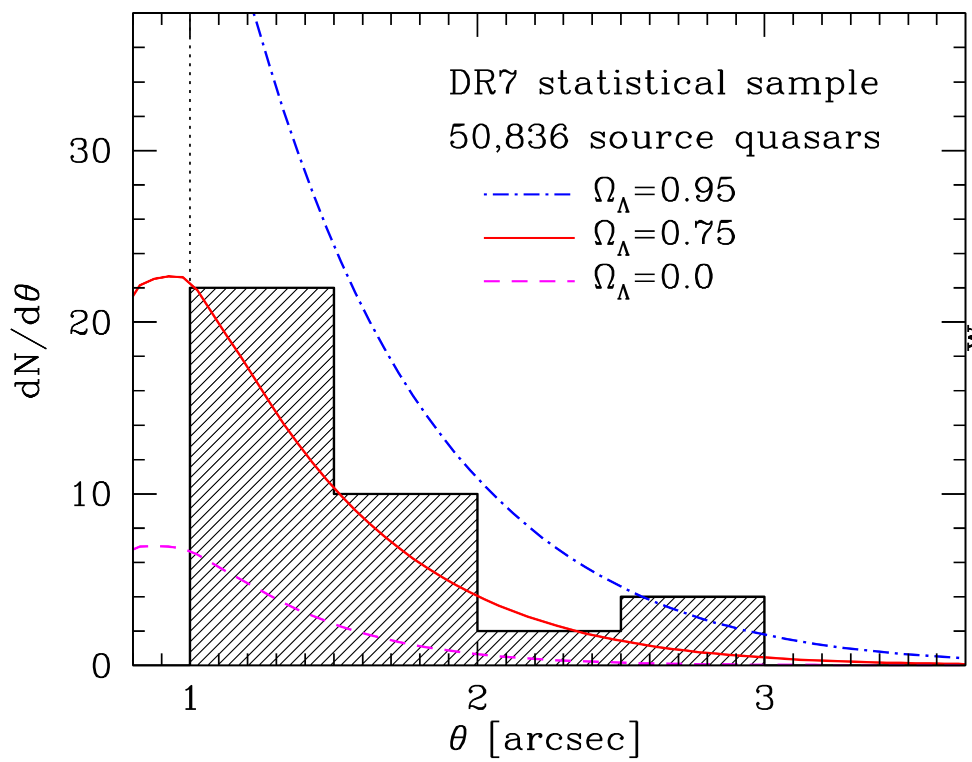
Image: NASA/ESA

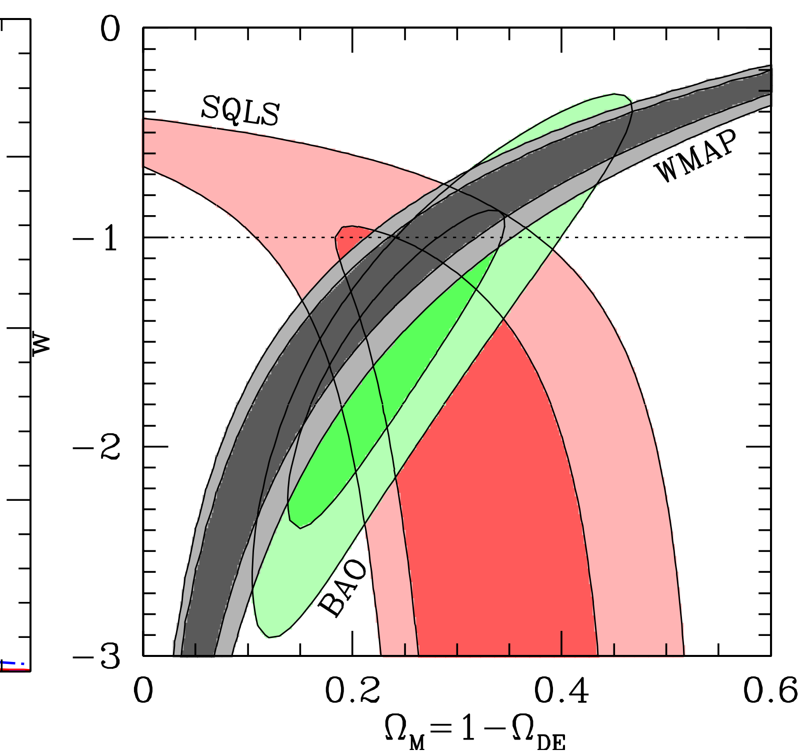
$$N_{\rm arcs} \sim \begin{cases} 2400 & \text{(OCDM)} \\ 280 & (\Lambda\text{CDM}) \\ 36 & (\text{S}/\tau\text{CDM}) \end{cases} .$$

Abundance of giant arcs in clusters to constrain cosmological parameters such as Dark Energy (Bartelmann 1998)

Cosmology

Image: NASA/ESA



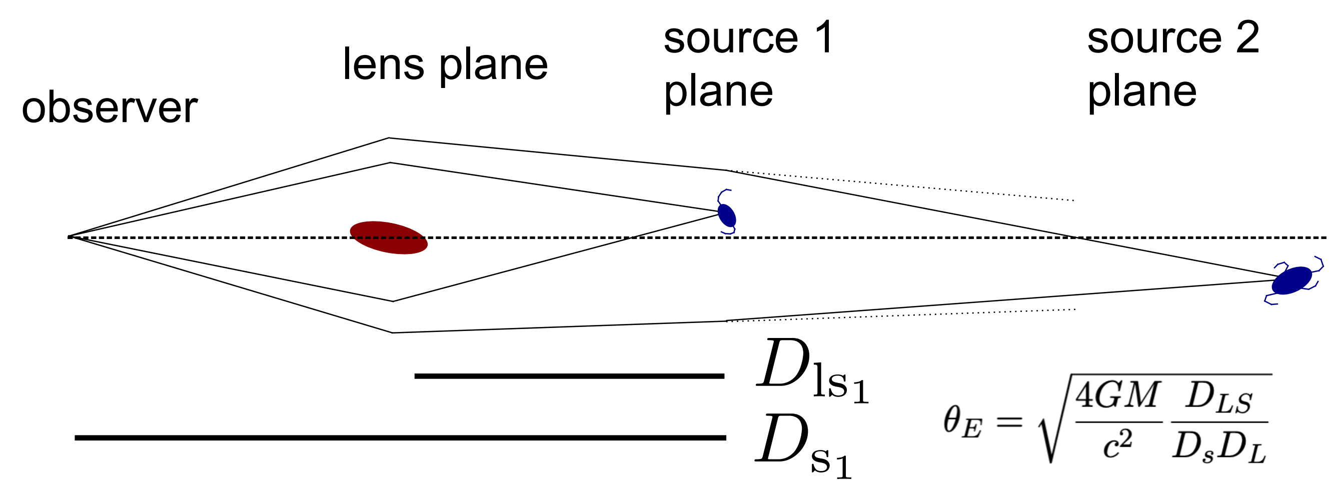


$$N_{\rm arcs} \sim \begin{cases} 2400 & (\text{OCDM}) \\ 280 & (\Lambda\text{CDM}) \\ 36 & (\text{S}/\tau\text{CDM}) \end{cases} .$$

Statistical samples of lensed quasars to constrain Dark Energy
Oguri et al. 2012

Abundance of giant arcs in clusters to constrain cosmological parameters such as Dark Energy
(Bartelmann 1998)

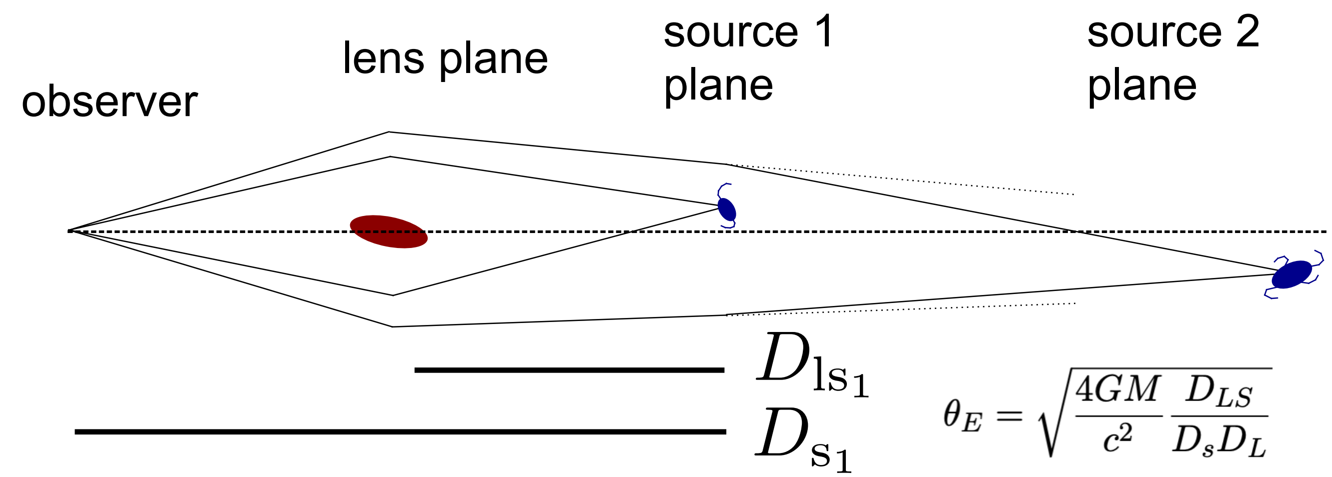
Cosmology



Collet et al. 2012

Cosmology

$$\frac{\theta_1}{\theta_2} = \frac{(D_{\mathrm{ls}_1}/D_{s_1})}{(D_{\mathrm{ls}_2}/D_{s_2})}$$



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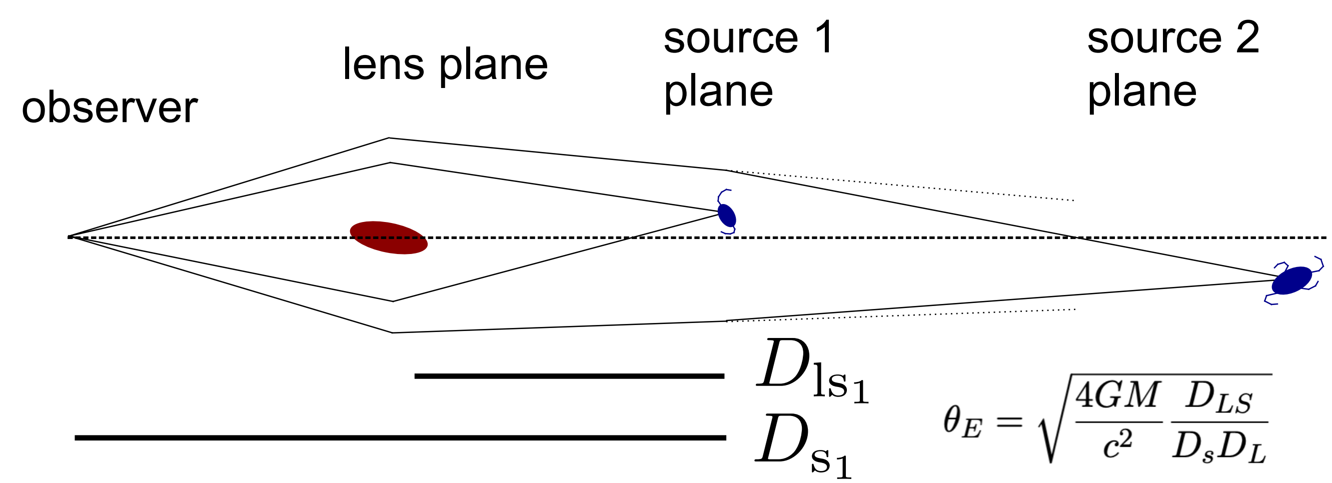

Collet et al. 2012

No dependence on Hubble constant

Complementarity with other cosmological probes e.g. Cosmic Microwave Background

Cosmology

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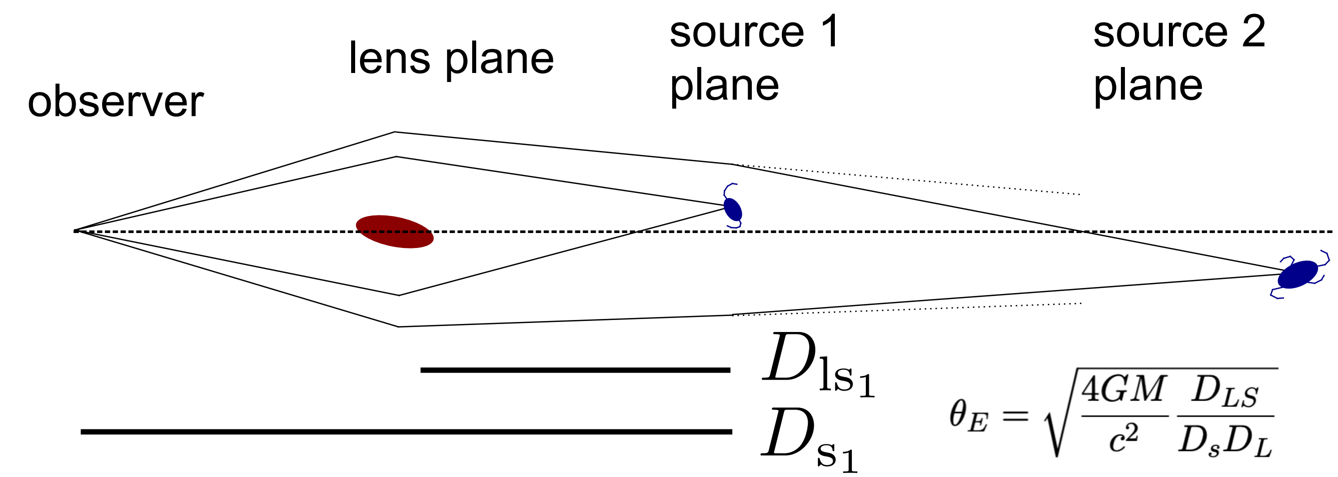
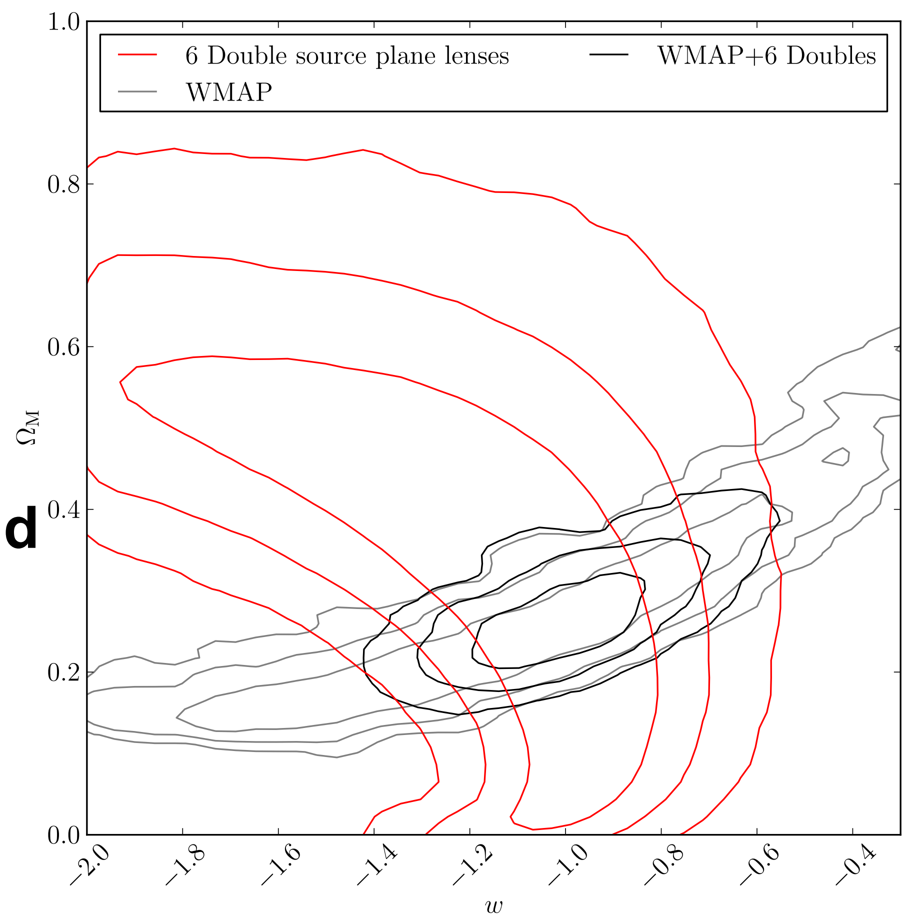


Collet et al. 2012



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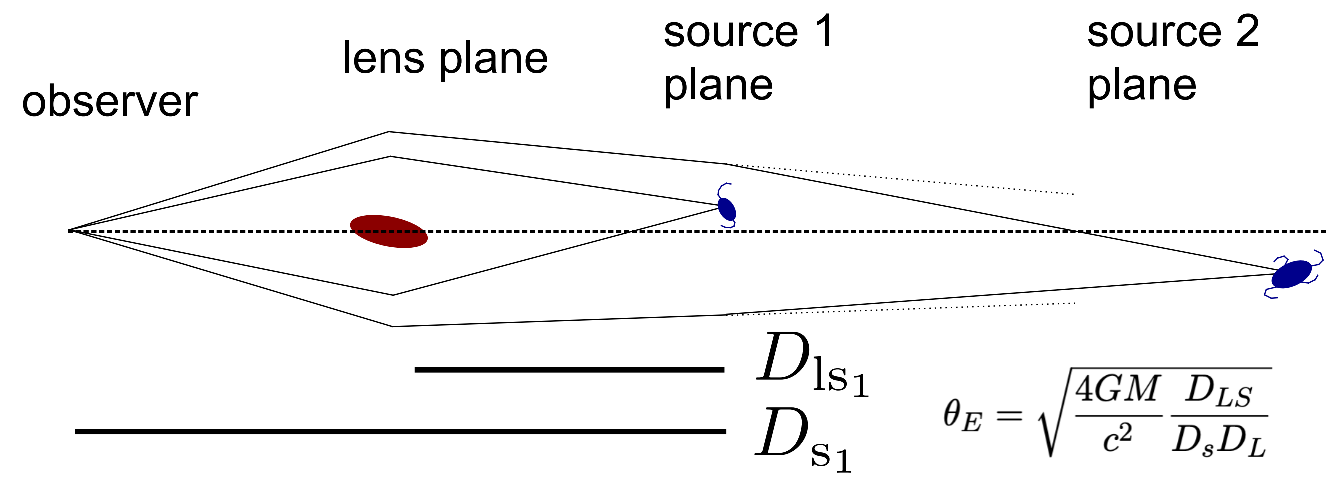
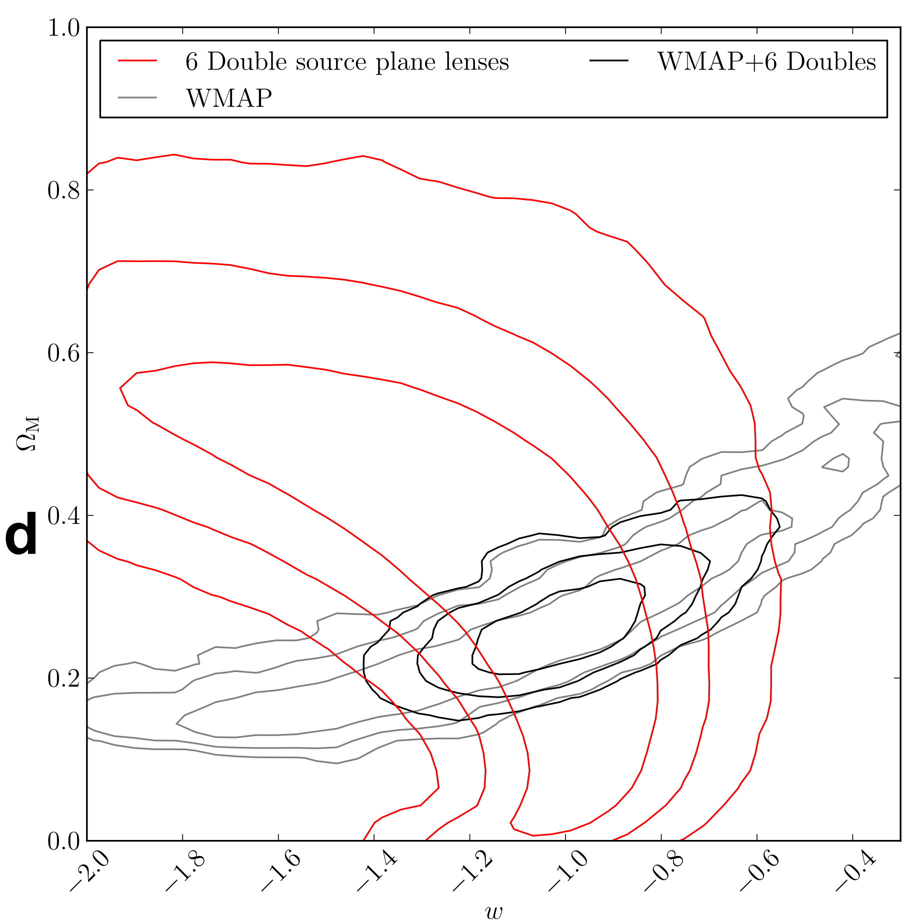


$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$

Collet et al. 2012



No dependence on Hubble constant

Complementarity with other cosmological probes e.g. Cosmic Microwave Background

References

- Material on lensing basics taken from multiple sources (Lectures from Pritchard, Suyu, Wambsganss, Narayan, Bartelmann, Blandford)
- Saas Fee Lectures by Schneider, Kochanek and Wambsganss
- Lens modelling software: Lensmodel/gravlens by Keeton and Glafic by Oguri