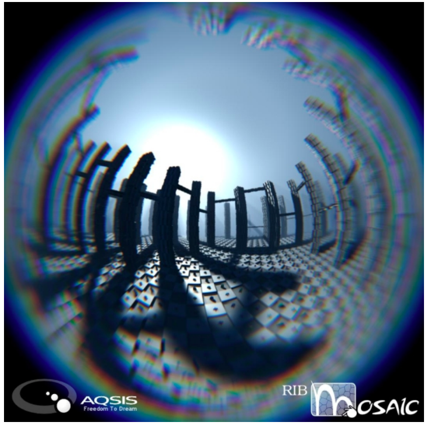
Introduction to Gravitational lensing I

Anupreeta More
(IUCAA)

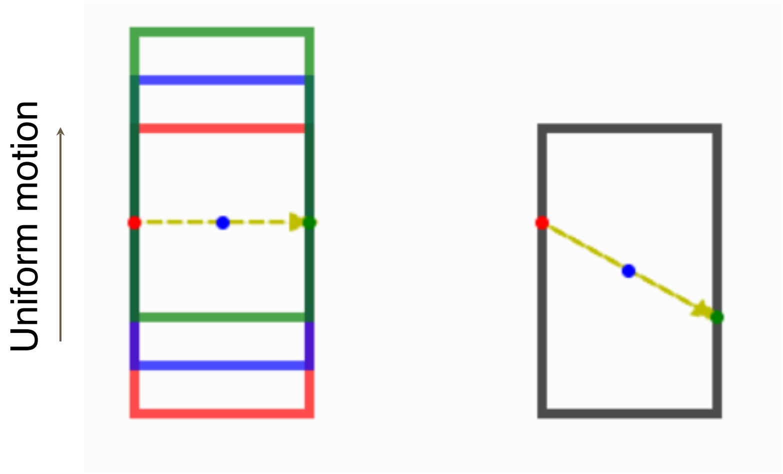
Sep 5, 2024

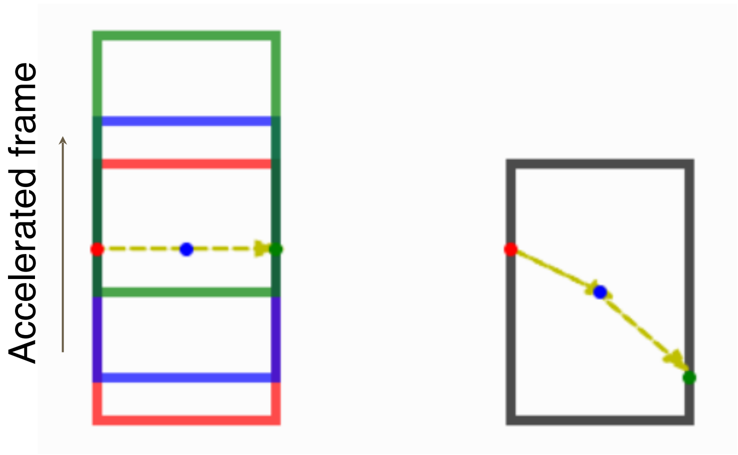
RESCEU Summer School 2024, Kanazawa


corbis


AQSIS
Freedom To Dream
RIB
MOSAIC

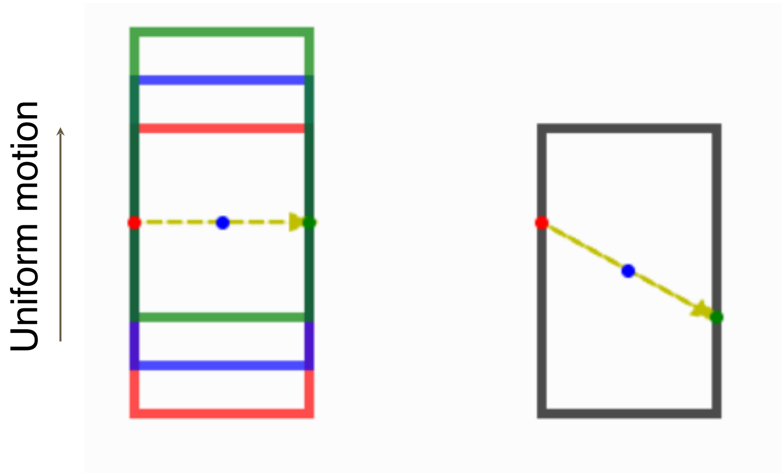
Equivalence principle: Deflection of light

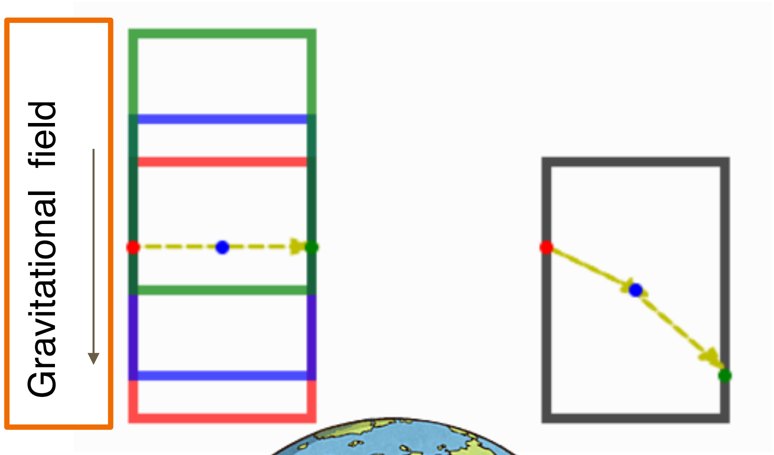




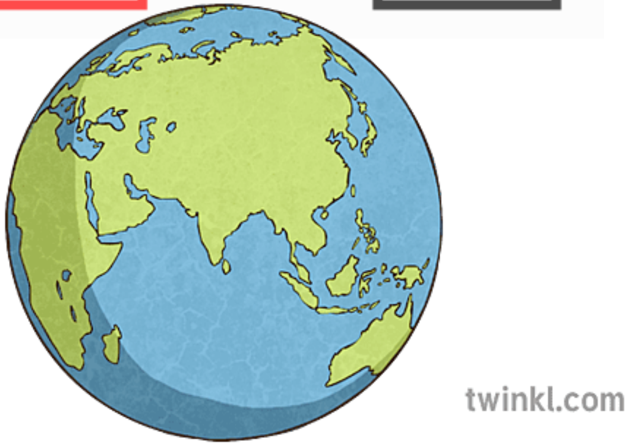
- Experiments conducted in accelerated frames vs in gravitational field produce results that are indistinguishable
- Light deflection in accelerated frames → light will show similar deflection in gravity

Equivalence principle: Deflection of light





- Experiments conducted in accelerated frames vs in gravitational field produce results that are indistinguishable
- Light deflection in accelerated frames → light will show similar deflection in gravity



Speed of light near massive objects

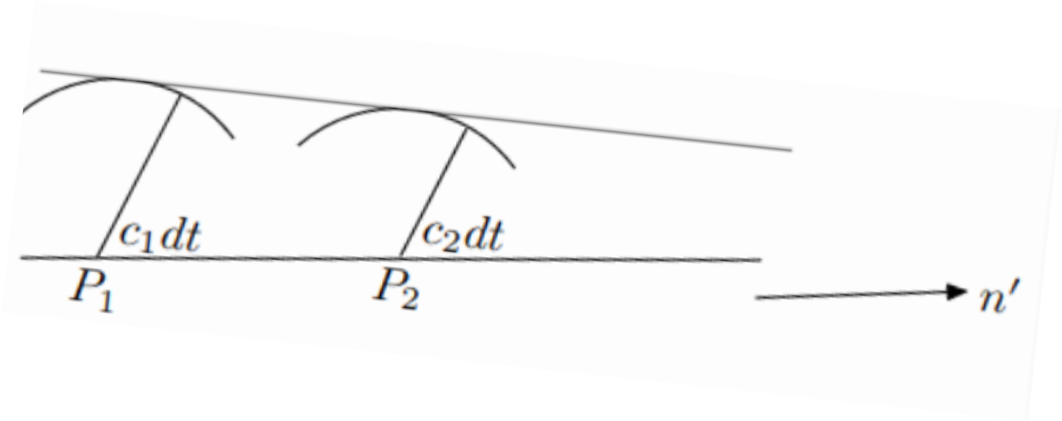
$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right)c^2dt^2 + \left(1 - \frac{2\Phi}{c^2}\right)dl^2$$

Metric in the weak field limit!

$$c' = \left[\frac{dl^2}{dt^2}\right]^{1/2} = c\left[\frac{\left(1 + \frac{2\Phi}{c^2}\right)}{\left(1 - \frac{2\Phi}{c^2}\right)}\right]^{1/2} \simeq c\left(1 + \frac{2\Phi}{c^2}\right)$$

Potential is negative, so light appears to slow down near gravitational potentials.

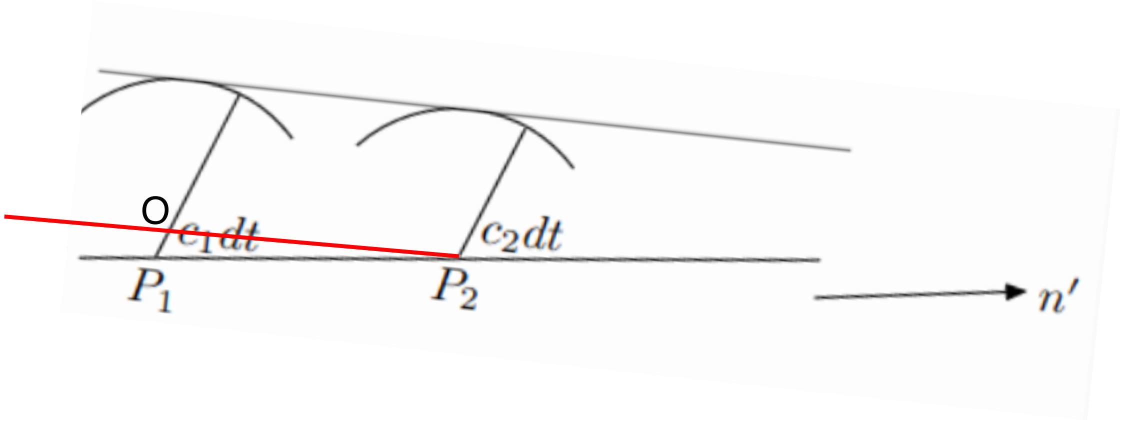
Light deflection due to differential speed



$$c' \simeq c\left(1 + \frac{2\Phi}{c^2}\right)$$

- In a paper in 1911, Einstein used Huygens principle to calculate the deflection of light due to the difference in speed of light

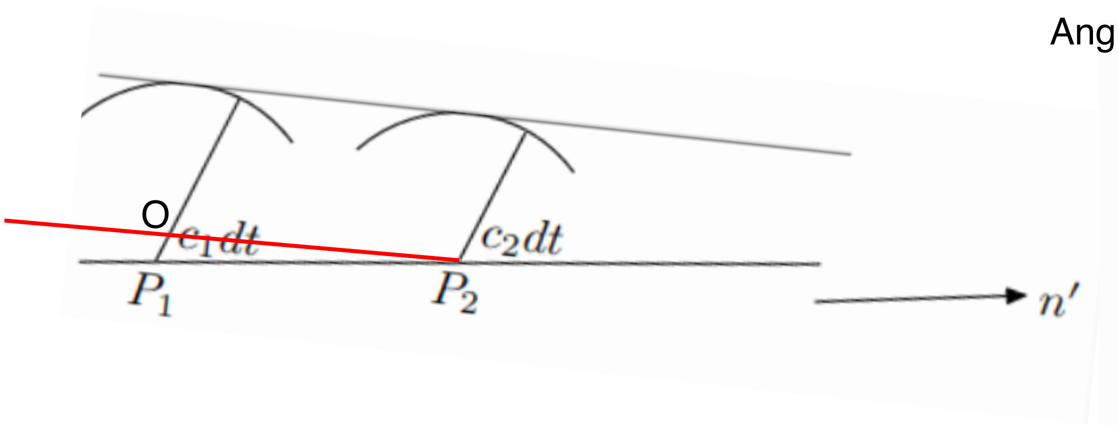
Light deflection due to differential speed



$$c' \simeq c\left(1 + \frac{2\Phi}{c^2}\right)$$

- In a paper in 1911, Einstein used Huygens principle to calculate the deflection of light due to the difference in speed of light

Light deflection due to differential speed



$$c' \simeq c\left(1 + \frac{2\Phi}{c^2}\right)$$

Angle OP_2P_1

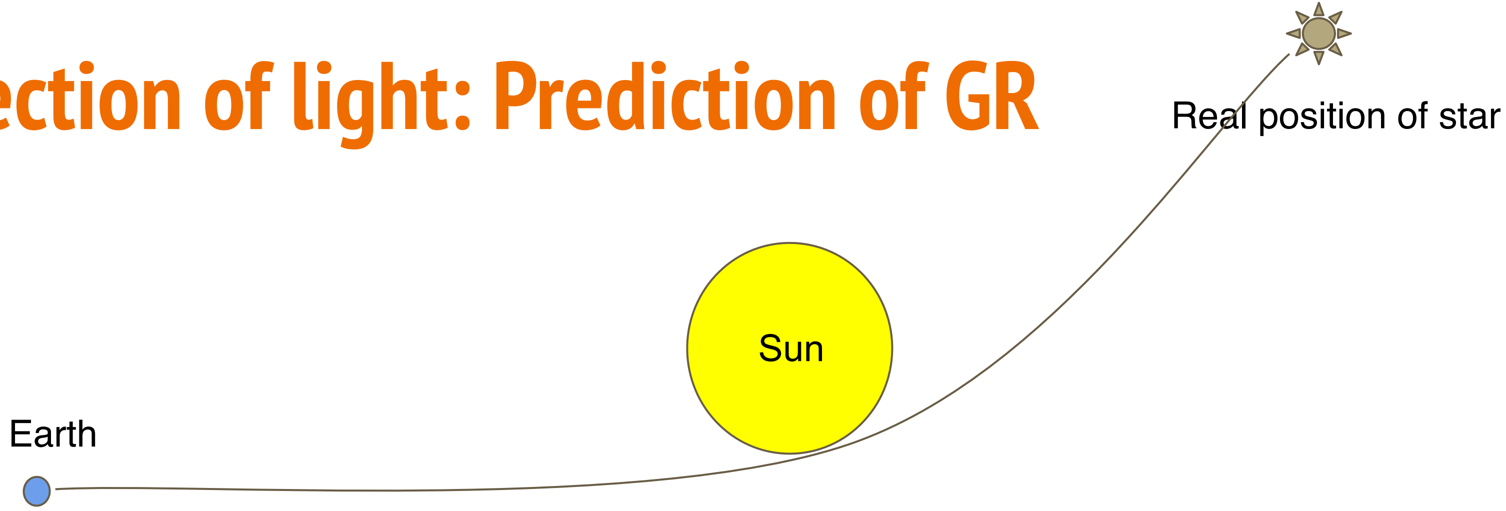
OP_1

P_1P_2

$$\begin{aligned}
\delta\alpha &= -\frac{\partial c'}{\partial x}\delta t \\
&= -\frac{2}{c}\frac{\partial \Phi}{\partial x}\delta t \\
&= -\frac{2}{c^2}\frac{\partial \Phi}{\partial x}c\delta t \\
&= -\frac{2}{c^2}\frac{\partial \Phi}{\partial x}\delta l
\end{aligned}$$

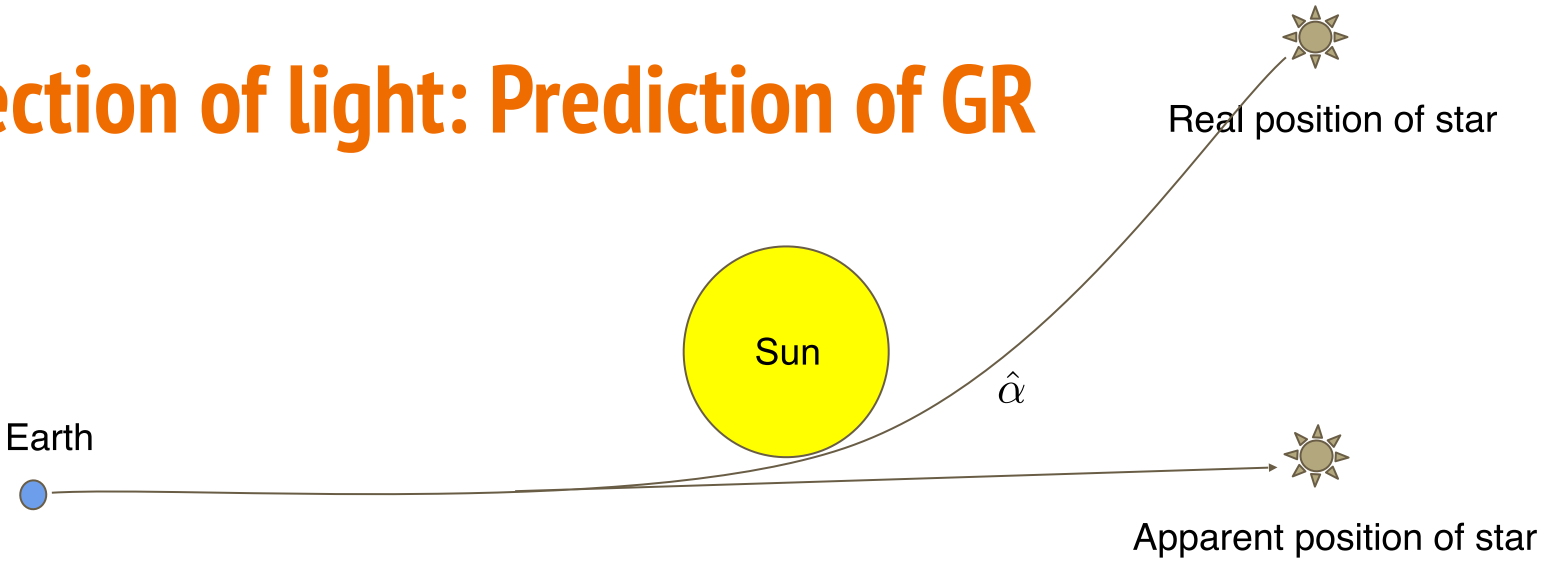
$$\alpha = \int \delta\alpha = -\int \frac{2}{c^2}\nabla_\perp \Phi dl$$

Deflection of light: Prediction of GR



- The deflection of light is a consequence of the equivalence principle

Deflection of light: Prediction of GR

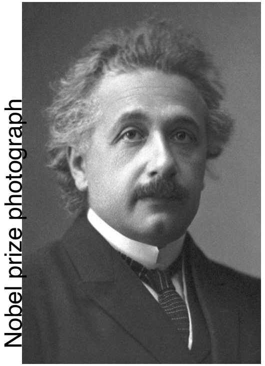


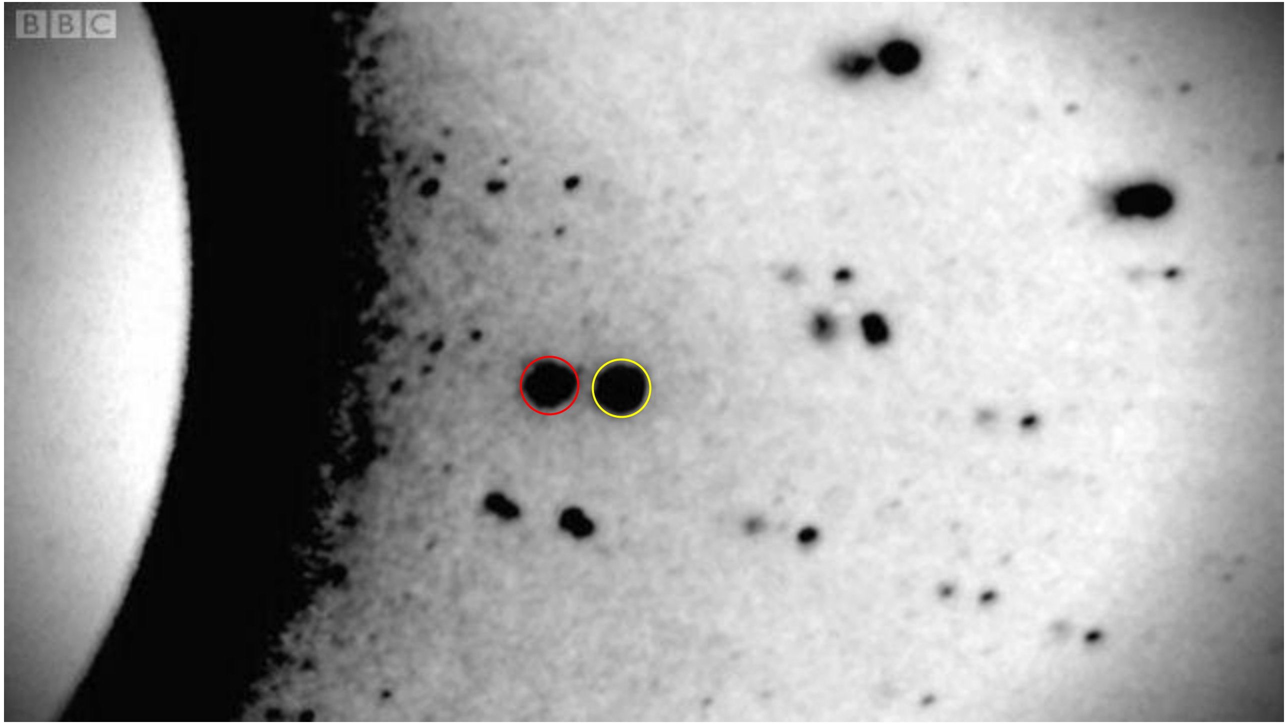
- The deflection of light is a consequence of the equivalence principle
- For a point mass: $$\hat{\alpha} = \frac{4GM}{c^2}\frac{\boldsymbol{\xi}}{|\boldsymbol{\xi}|^2} = 1.75'' \left(\frac{M}{M_\odot}\right)\left(\frac{\xi}{R_\odot}\right)^{-1}$$

ξ is the Impact Parameter

Deflection of light: Confirmation of GR


Library of Congress


Nobel prize photograph



Einstein and Eddington, BBC Documentary

Fritz Zwicky: an Oracle!

Caltech archives

Nebulae as Gravitational Lenses

Einstein recently published[1] some calculations concerning a suggestion made by R. W. Mandl, namely, that a star B may act as a "gravitational lens" for light coming from another star A which lies closely enough on the line of sight behind B. As Einstein remarks the chance to observe this effect for stars is extremely small.

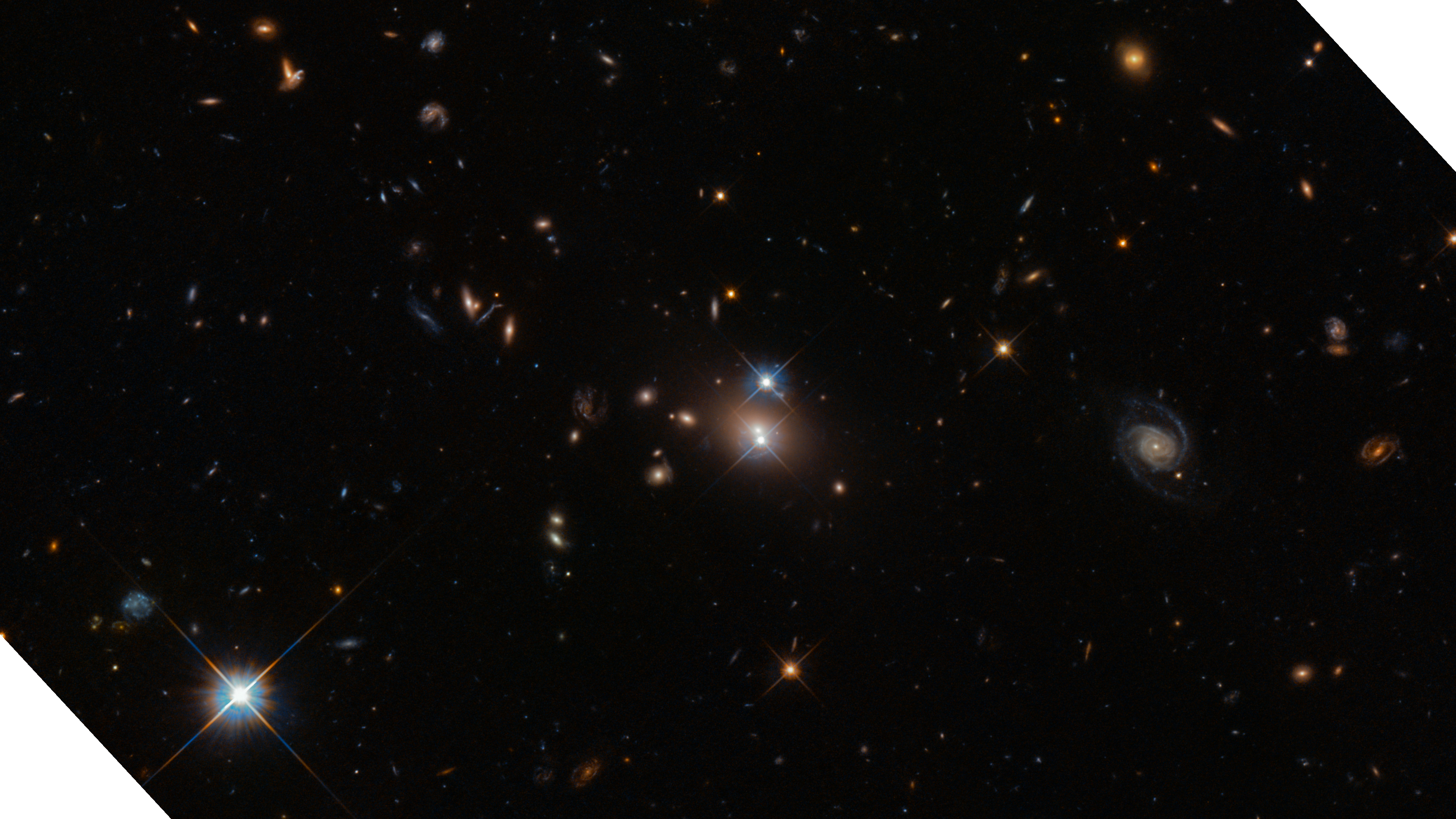
Zwicky F., Phys. Rev. 1937, 50, 291

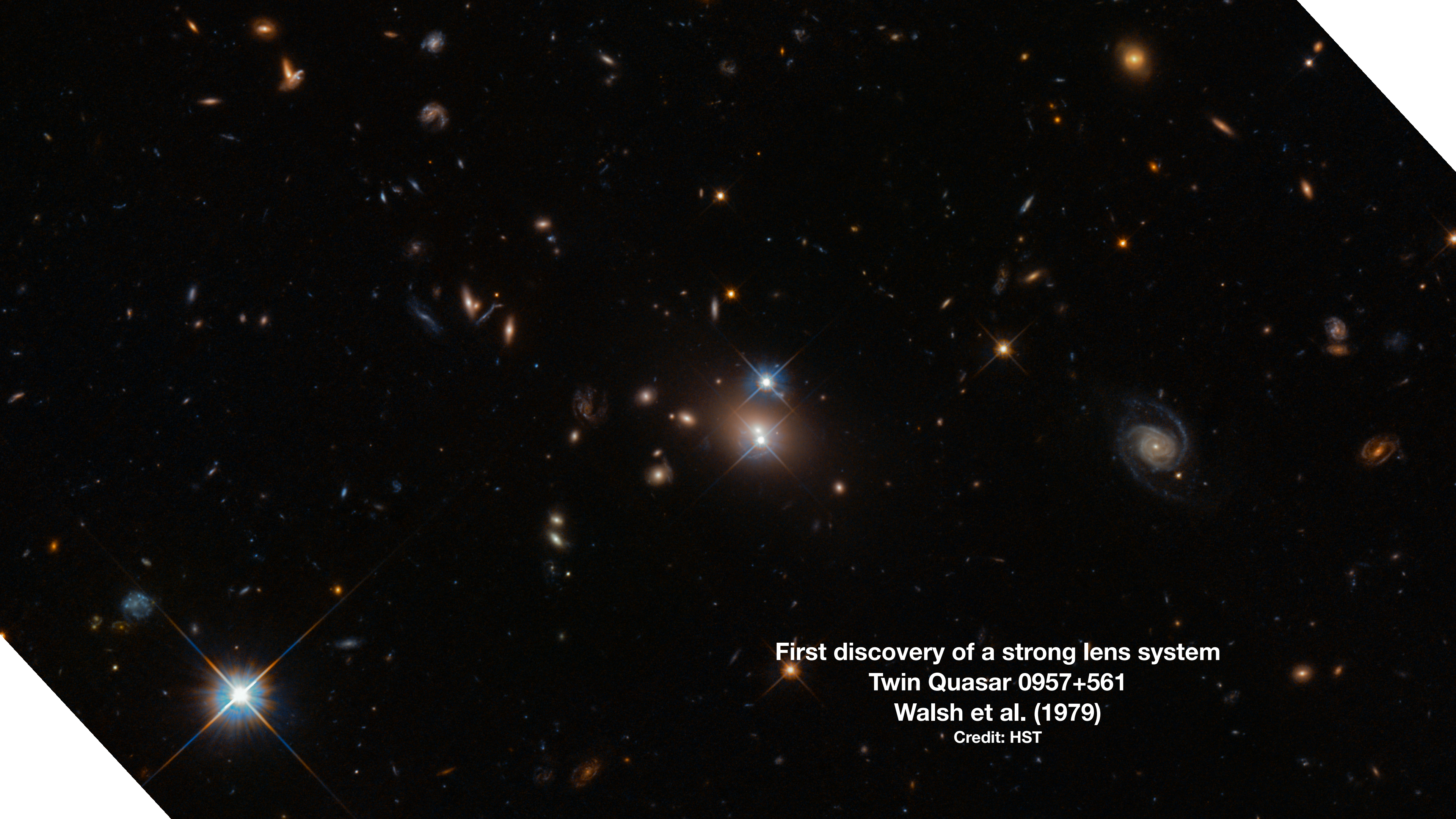
The discovery of images of nebulae which are formed through the gravitational fields of nearby nebulae would be of considerable interest for a number of reasons.

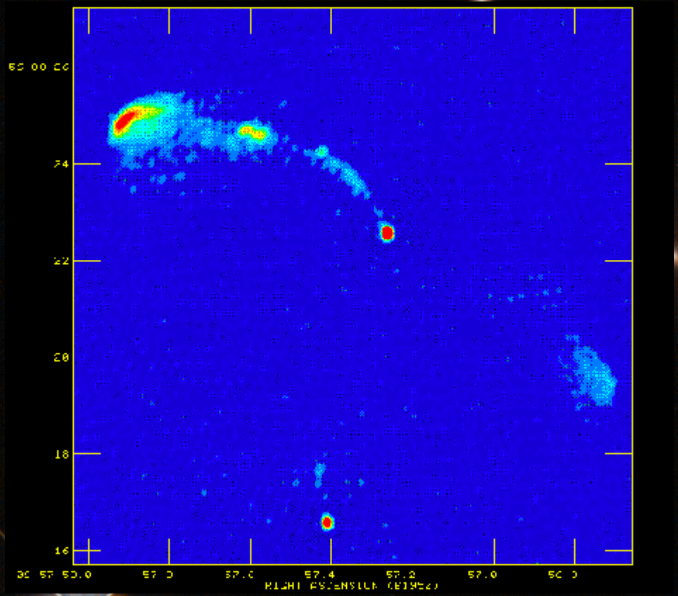
(1) It would furnish an additional test for the general theory of relativity.

(2) It would enable us to see nebulae at distances greater than those ordinarily reached by even the greatest telescopes. Any such *extension* of the known parts of the universe promises to throw very welcome new light on a number of cosmological problems.

(3) The problem of determining nebular masses at present has arrived at a stalemate. The mass of an average nebula until recently was thought to be of the order of $M_N = 10^9\, M_\odot$, where $M_\odot$ is the mass of the sun. This estimate is based on certain deductions drawn from data on the intrinsic brightness of nebulae as well as their spectrographic rotations. Some time ago, however, I showed[2] that a straightforward application of the virial theorem to the great cluster of nebulae in Coma leads to an average nebular mass four hundred times greater than the one mentioned, that is, $M_N' = 4 \times 10^{11} M_\odot$. This result has recently been verified by an investigation of the Virgo cluster.[3] Observations on the deflection of light around nebulae may provide the most direct determination of nebular masses and clear up the above-mentioned discrepancy.




First discovery of a strong lens system
Twin Quasar 0957+561
Walsh et al. (1979)
Credit: HST



First discovery of a strong lens system

Twin Quasar 0957+561

Walsh et al. (1979)

Credit: HST

First lensed arcs galaxies

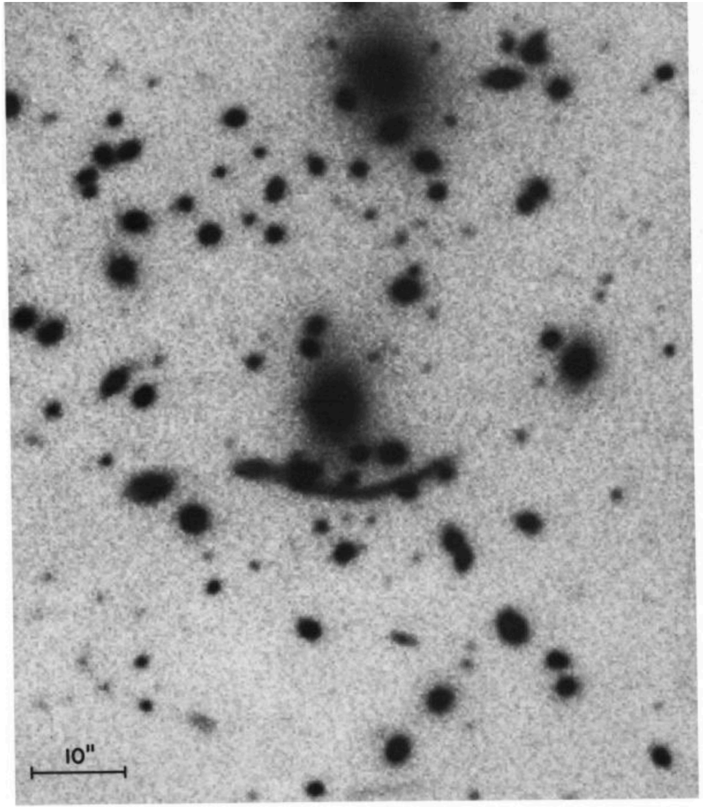


FIG. 1.—Central region of Abell 370 showing the arc. North is at the top, and east is to the left.

Lynds and Petrosian (1986)

First lensed arcs galaxies

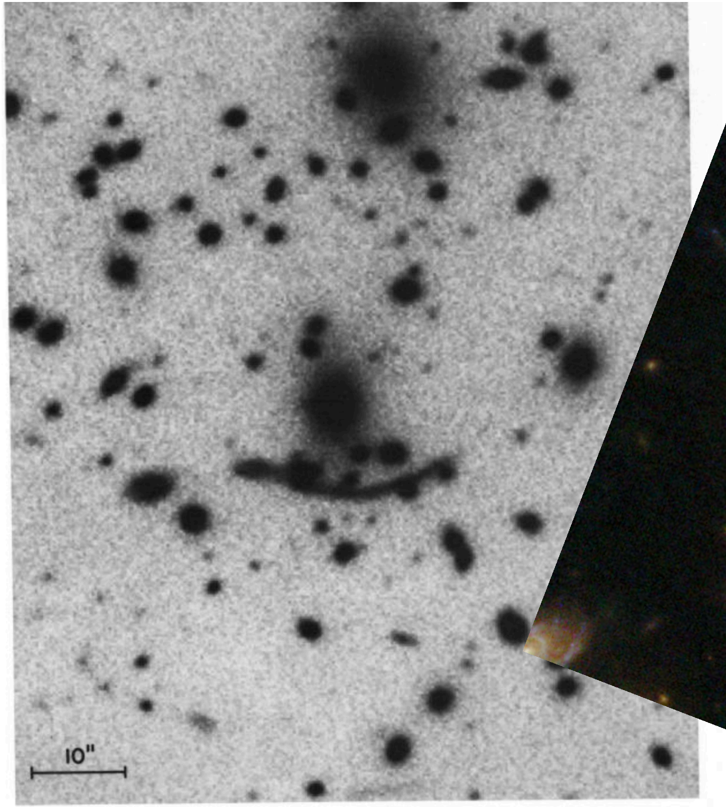
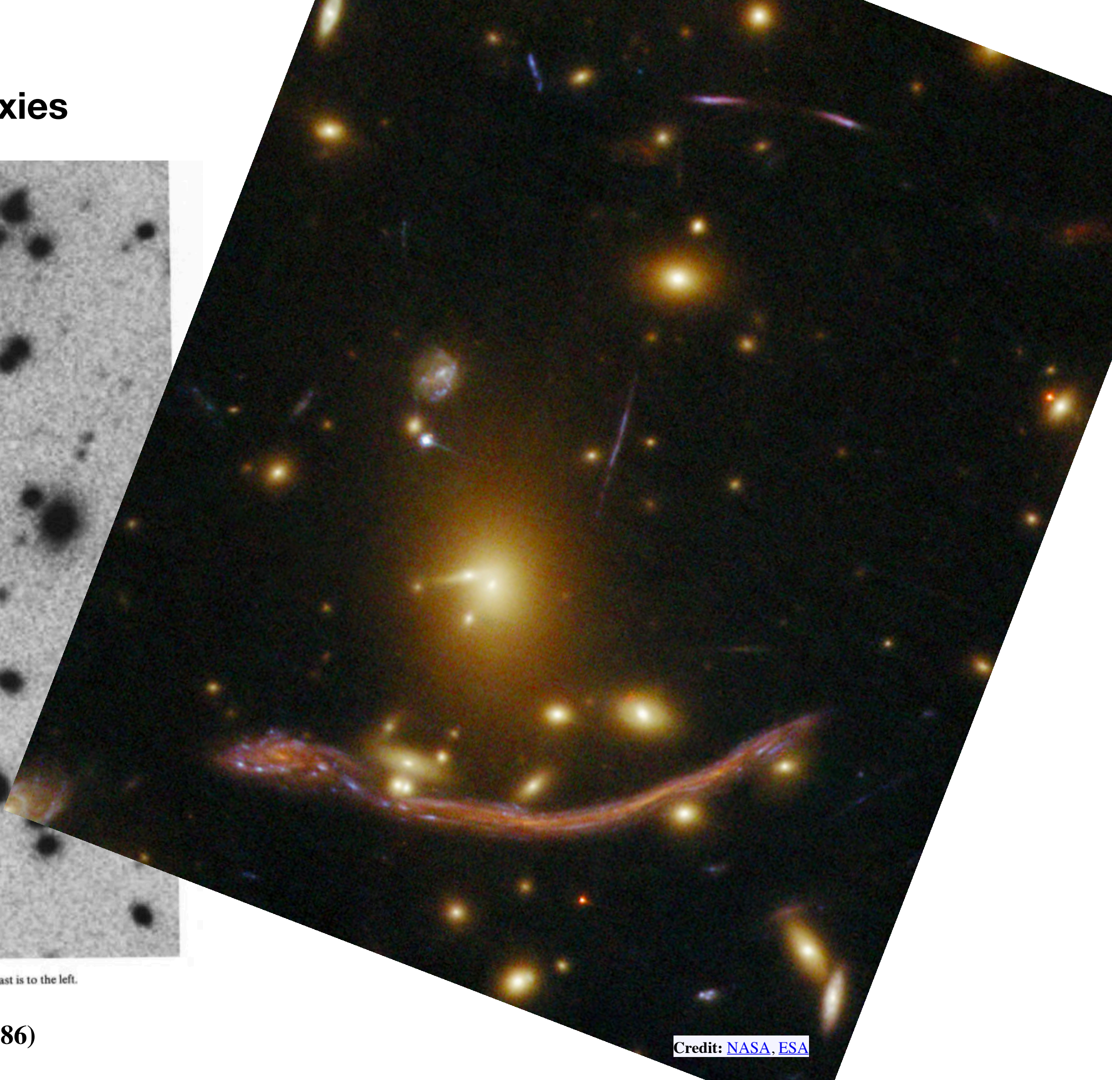
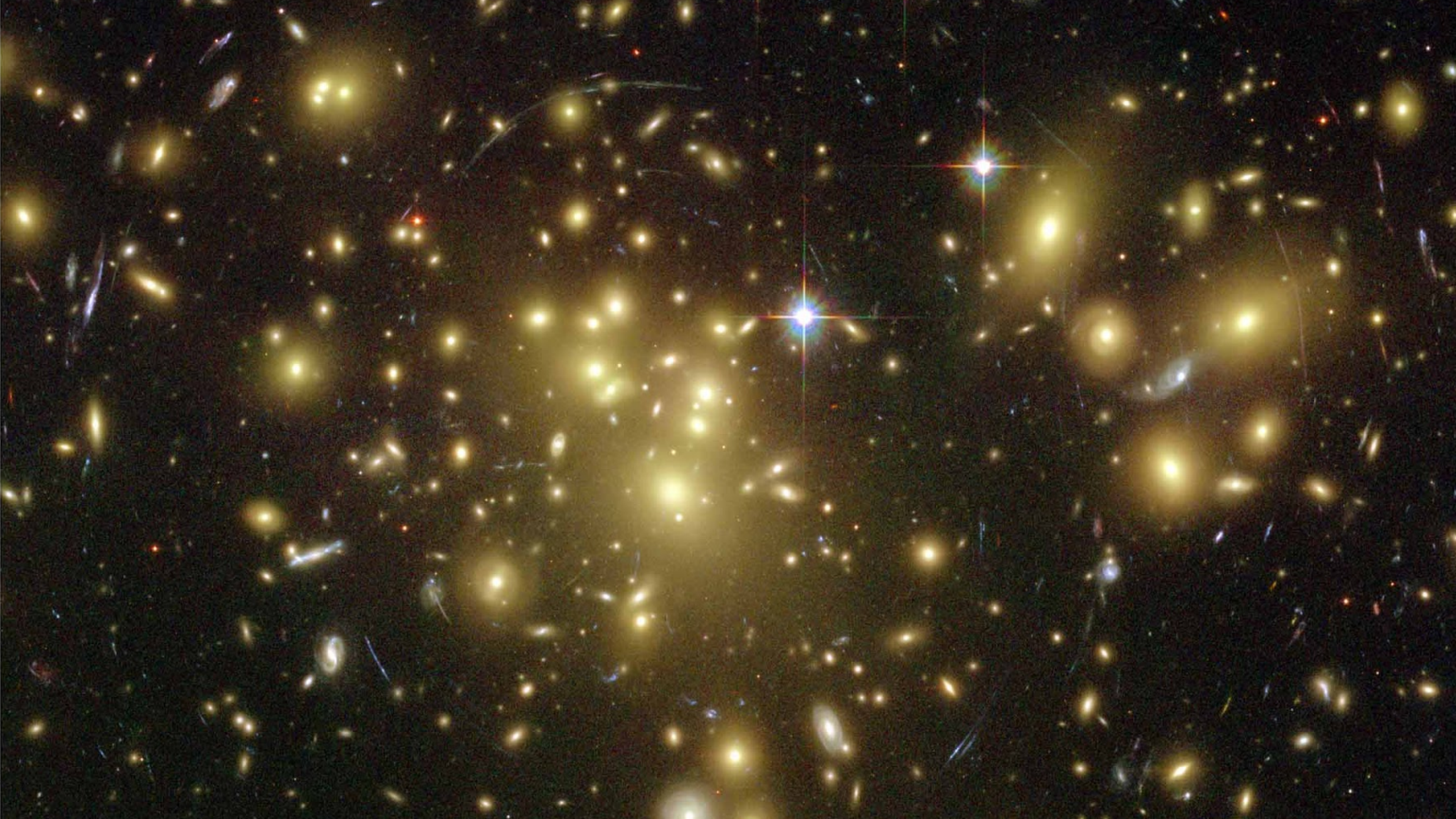


FIG. 1.—Central region of Abell 370 showing the arc. North is at the top, and east is to the left.

Lynds and Petrosian (1986)



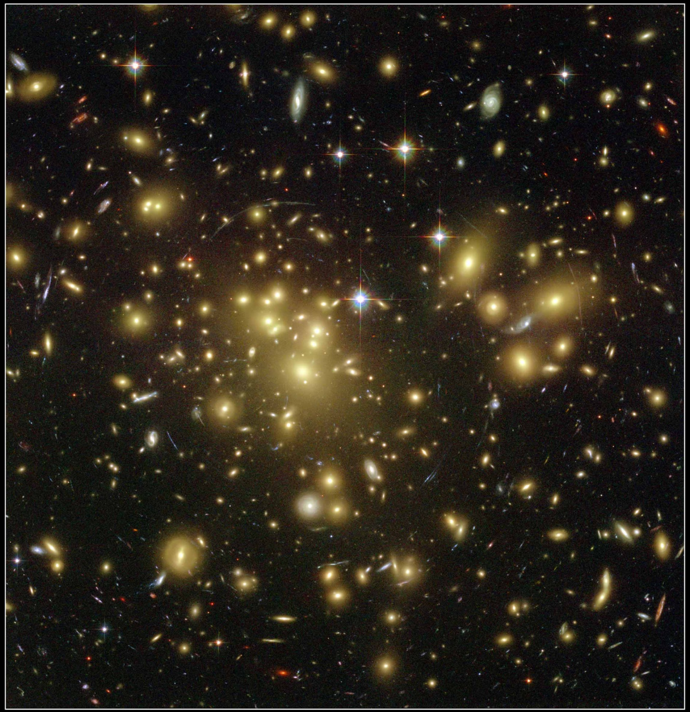
Credit: NASA, ESA




All galaxy images that you see are gravitational mirages

Gravitational lensing

- Bending of light due to the gravitational field of massive objects.
- Different forms
  - **Strong gravitational lensing**
  - **Weak gravitational lensing**
  - **Microlensing**



Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys

NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA
STScI-PRC03-01a

Gravitational lensing: one tool to probe them all

Gravitational lensing: one tool to probe them all

- Measure cosmological parameters of the Universe
- Initial mass function of stars
- Understand the galaxy dark matter halo connection
- Search for exo planets
- Tests of general relativity
- Abundances of compact halo objects
- Understand nature of dark matter
- Galaxies in the early Universe
- Study of black hole disks

Mirror mirror on the wall

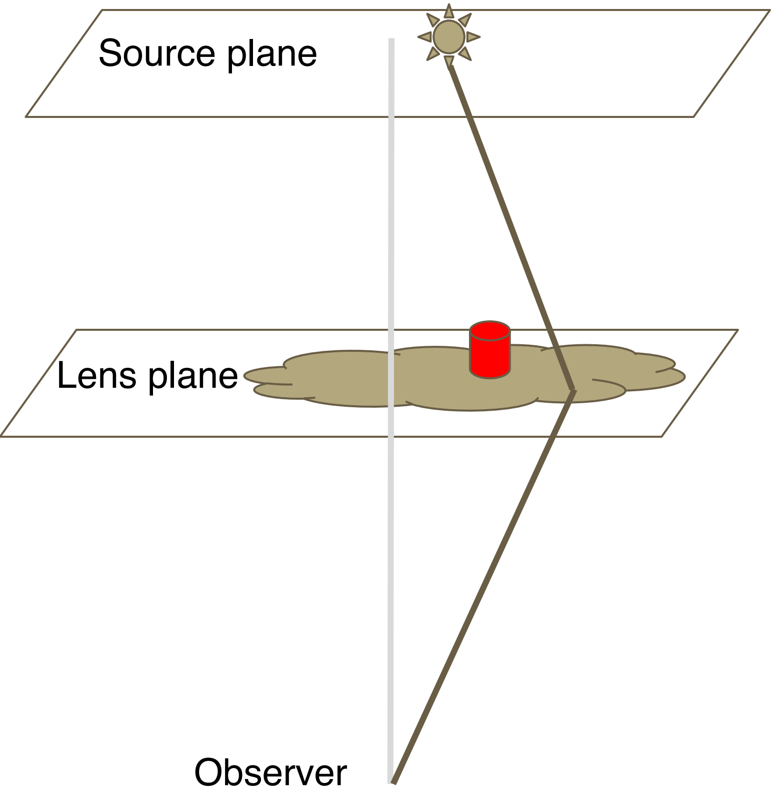
Funplanners.com

- Formation of multiple images of the same object.
- Magnification (or demagnification) of certain locations.
- Time of arrival of the different images.

Funplanners.com

The observed image properties can inform us about the curvature of the mirror and the features of the “lensed” object

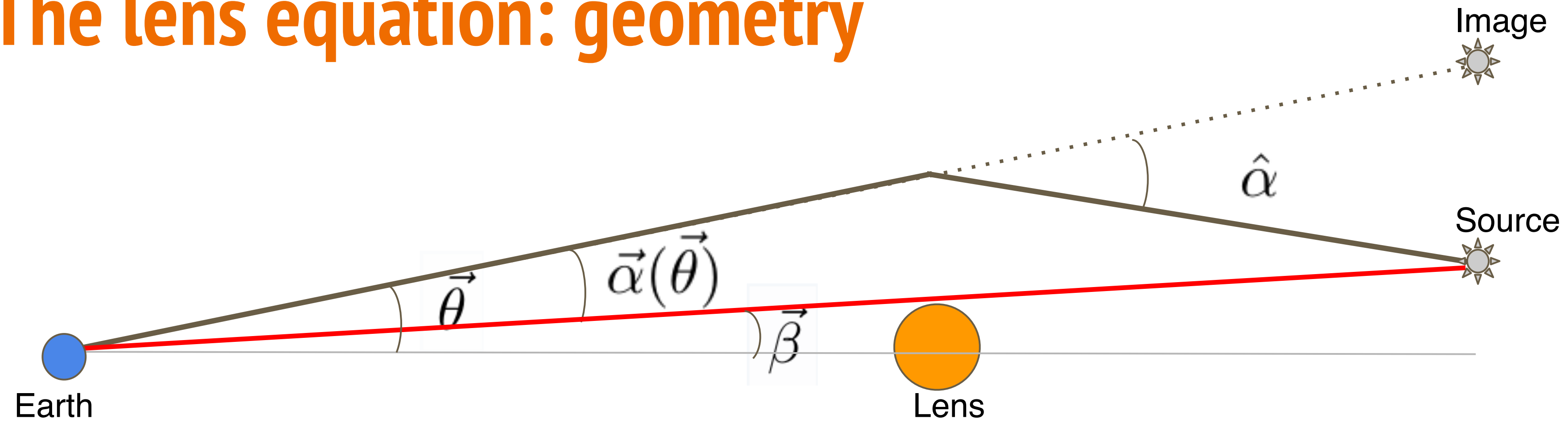
The deflection angle: general lens model



- Virtually all astrophysically relevant cases of gravitational lensing correspond to the thin lens approximation.
- Discretization gives the total deflection angle for a general lens model.

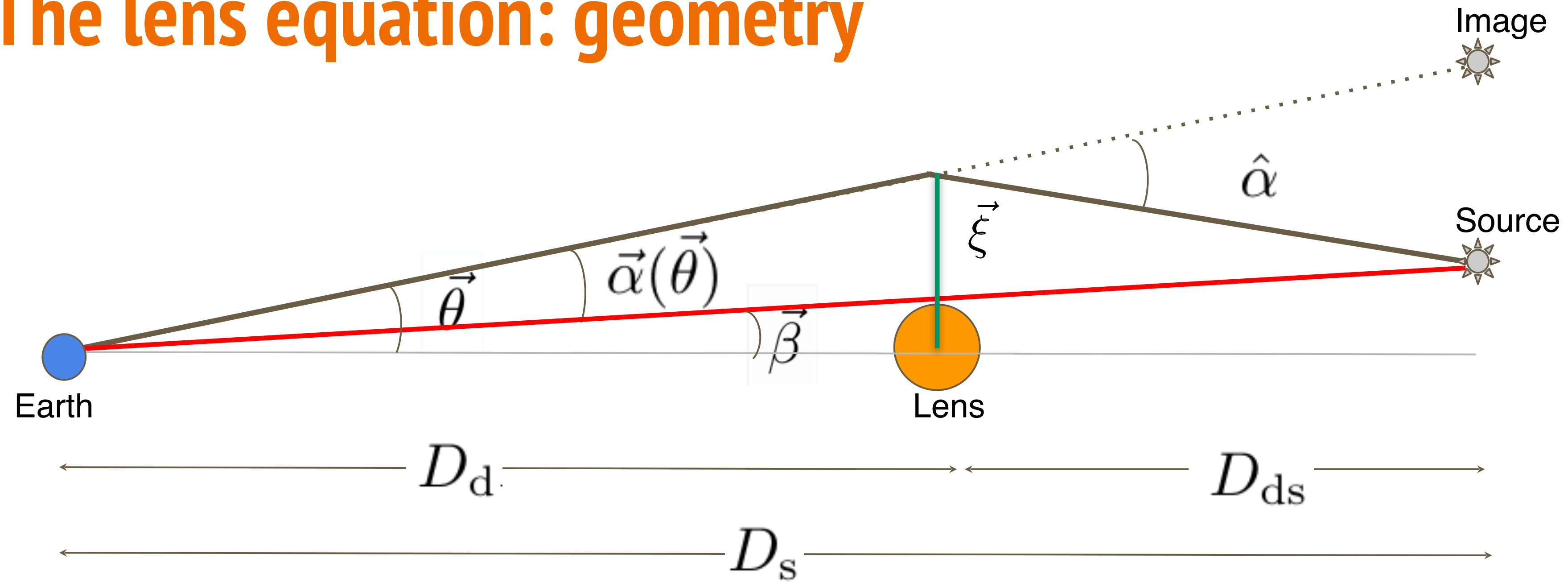
$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2} \int \mathrm{d}^2\xi' \, \Sigma(\boldsymbol{\xi}') \, \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2} \, ,$$

The lens equation: geometry



$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

The lens equation: geometry

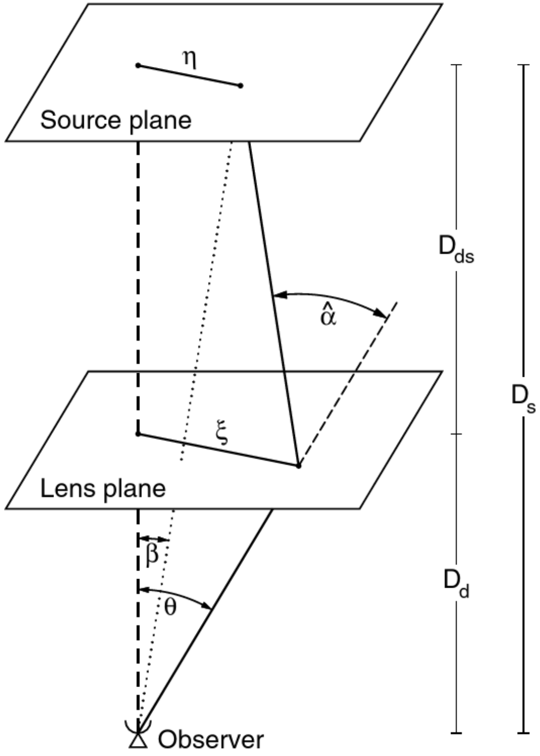


$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\boldsymbol{\xi} = D_d \boldsymbol{\theta}$$

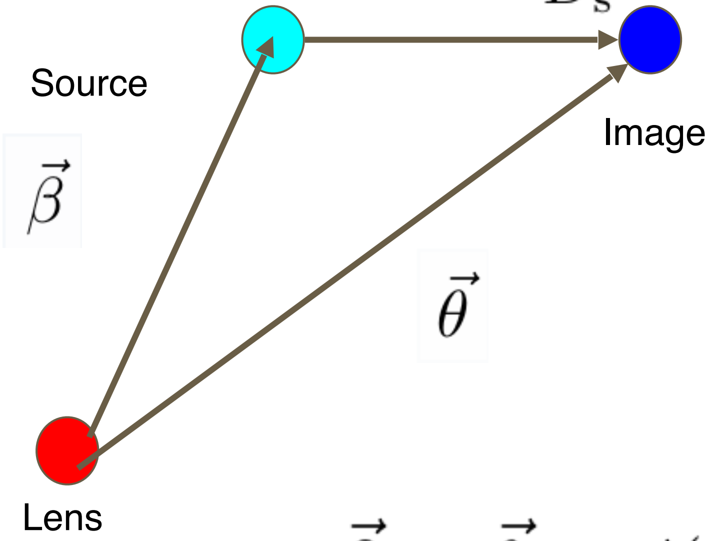
$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\rm ds}}{D_{\rm s}} \hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

The lens equation: geometry



SAAS Fee Courses Vol 33

As seen on sky



$$\vec{\alpha} = \hat{\alpha} \frac{D_{\rm ds}}{D_{\rm s}}$$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Scaled deflection angle

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\rm ds}}{D_{\rm s}} \hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

$$\boldsymbol{\xi} = D_d \boldsymbol{\theta}$$

$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2} \int {\rm d}^2\xi' \, \Sigma(\boldsymbol{\xi}') \, \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2} \, ,$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\rm ds}}{D_{\rm s}} \left[\frac{4G}{c^2} \int D_{\rm d}^2 d^2\boldsymbol{\theta}' \, \Sigma(D_{\rm d}\boldsymbol{\theta}') \frac{D_{\rm d}(\boldsymbol{\theta} - \boldsymbol{\theta}')}{D_{\rm d}^2 |\boldsymbol{\theta} - \boldsymbol{\theta}'|^2} \right]$$

Scaled deflection angle

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\rm ds}}{D_{\rm s}}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

$$\boldsymbol{\xi} = D_d\boldsymbol{\theta}$$

$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2}\int \mathrm{d}^2\xi' \, \Sigma(\boldsymbol{\xi}') \frac{\boldsymbol{\xi}-\boldsymbol{\xi}'}{|\boldsymbol{\xi}-\boldsymbol{\xi}'|^2} ,$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi}\left[\int d^2\boldsymbol{\theta}' \frac{\Sigma(D_{\rm d}\boldsymbol{\theta}')}{\frac{c^2 D_{\rm s}}{4\pi G D_{\rm ds} D_{\rm d}}} \frac{(\boldsymbol{\theta}-\boldsymbol{\theta}')}{|\boldsymbol{\theta}-\boldsymbol{\theta}'|^2}\right.$$

Scaled deflection angle

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

$$\boldsymbol{\xi} = D_d \boldsymbol{\theta}$$

$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2} \int \mathrm{d}^2\xi' \, \Sigma(\boldsymbol{\xi}') \, \frac{\boldsymbol{\xi} - \boldsymbol{\xi}'}{|\boldsymbol{\xi} - \boldsymbol{\xi}'|^2} \,,$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi} \left[\int d^2\boldsymbol{\theta}' \, \kappa(\boldsymbol{\theta}) \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}')}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2} \right]$$

$$\Sigma_{\mathrm{crit}} = \frac{c^2 D_{\mathrm{s}}}{4\pi G D_{\mathrm{ds}} D_{\mathrm{d}}}$$

$$\kappa = \frac{\Sigma}{\Sigma_{\mathrm{crit}}}$$

Lensing potential

Define lensing potential:

$$\psi = \frac{1}{\pi} \int \kappa(\boldsymbol{\theta}') d^2\boldsymbol{\theta}' \log(|\boldsymbol{\theta} - \boldsymbol{\theta}'|)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{1}{\pi}\left[\int d^2\boldsymbol{\theta}' \, \kappa(\boldsymbol{\theta}) \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}')}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2}\right]$$

$$\boldsymbol{\alpha} = \nabla_{\boldsymbol{\theta}} \psi(\boldsymbol{\theta})$$

$$\nabla^2_{\boldsymbol{\theta}} \psi(\boldsymbol{\theta}) = 2\kappa(\boldsymbol{\theta})$$

Magnification

- Lens equation relates positions in the source plane to positions in the image plane: a one (source) to many (images) mapping.
- Magnification gives the mapping between angular image area to the angular source area

$$M(\boldsymbol{\theta}) = |\mathcal{M}(\boldsymbol{\theta})| = \left|\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{\beta}}\right|_{\boldsymbol{\theta}}$$

Both angles are two dimensional quantities, thus the magnification is a 2x2 matrix.

- The magnification of each image is related to the Jacobian of the transformation equation.

Inverse of the Magnification matrix

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_j \partial \theta_i}$$

$$\mathcal{M}^{-1}(\boldsymbol{\theta}) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\kappa = \frac{1}{2}(\psi_{,11} + \psi_{,22}) \qquad \gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) \, , \quad \gamma_2 = \psi_{,12}$$

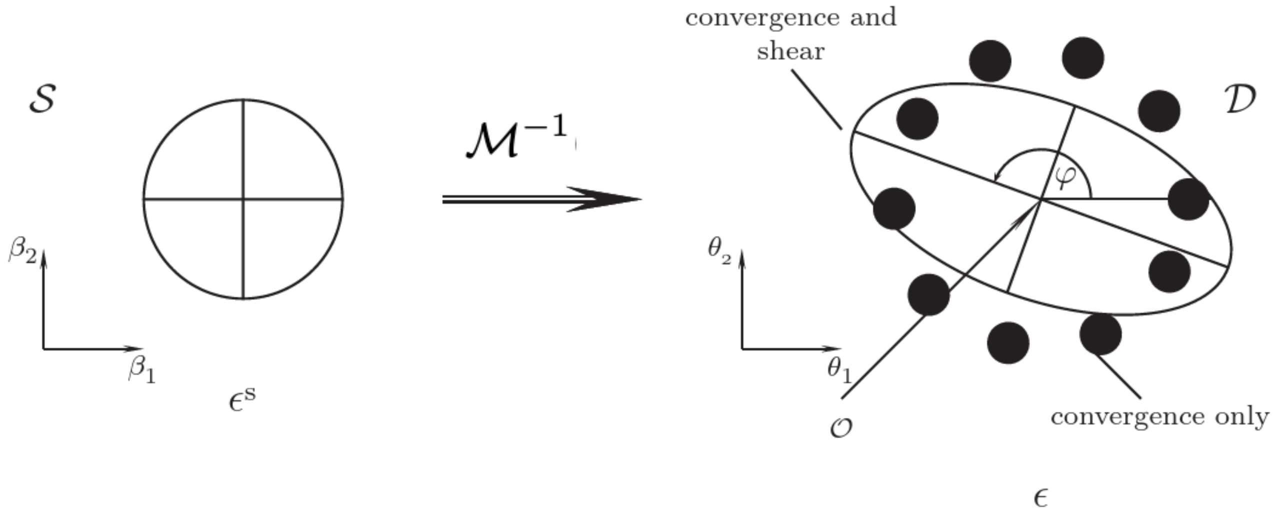
Inverse of the Magnification matrix

$$\mathcal{M}^{-1}(\boldsymbol{\theta}) = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

Inverse of the Magnification matrix

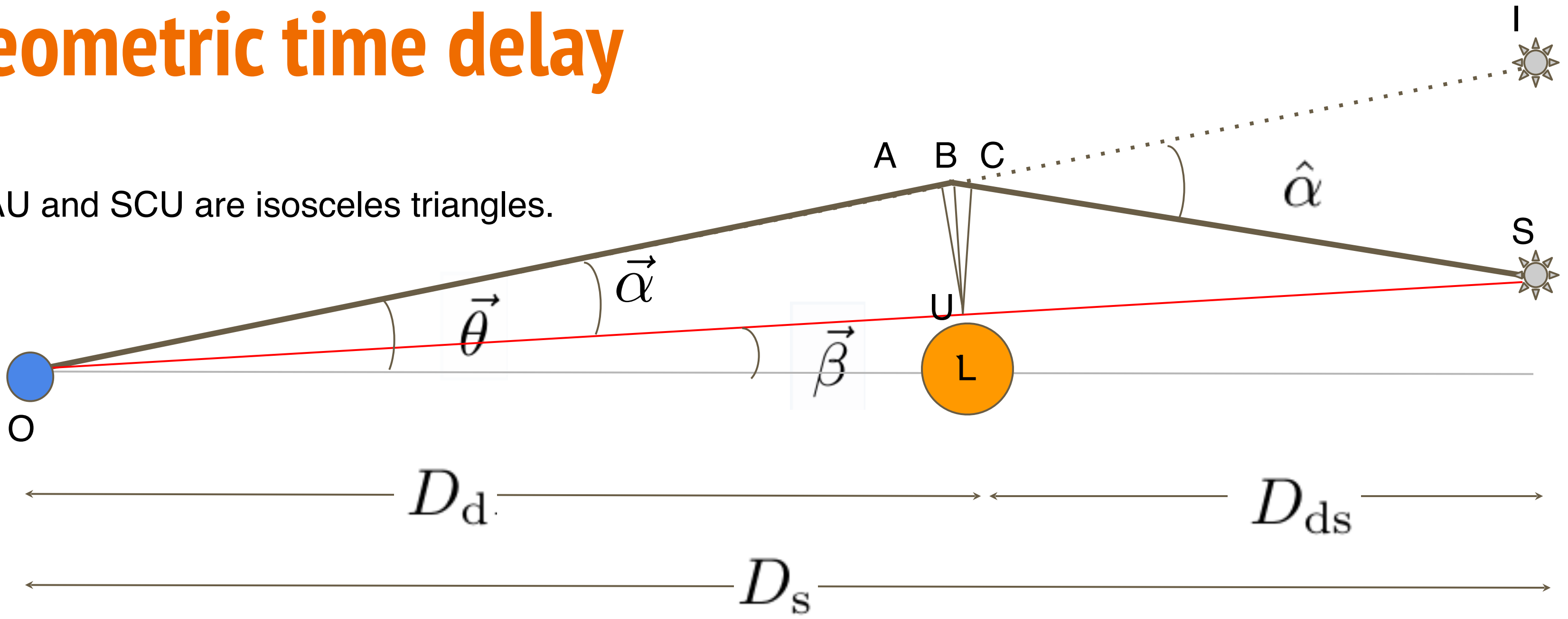
SAAS Fee Courses Vol 33



Convergence magnifies uniformly, shear distorts the shape

Geometric time delay

OAU and SCU are isosceles triangles.



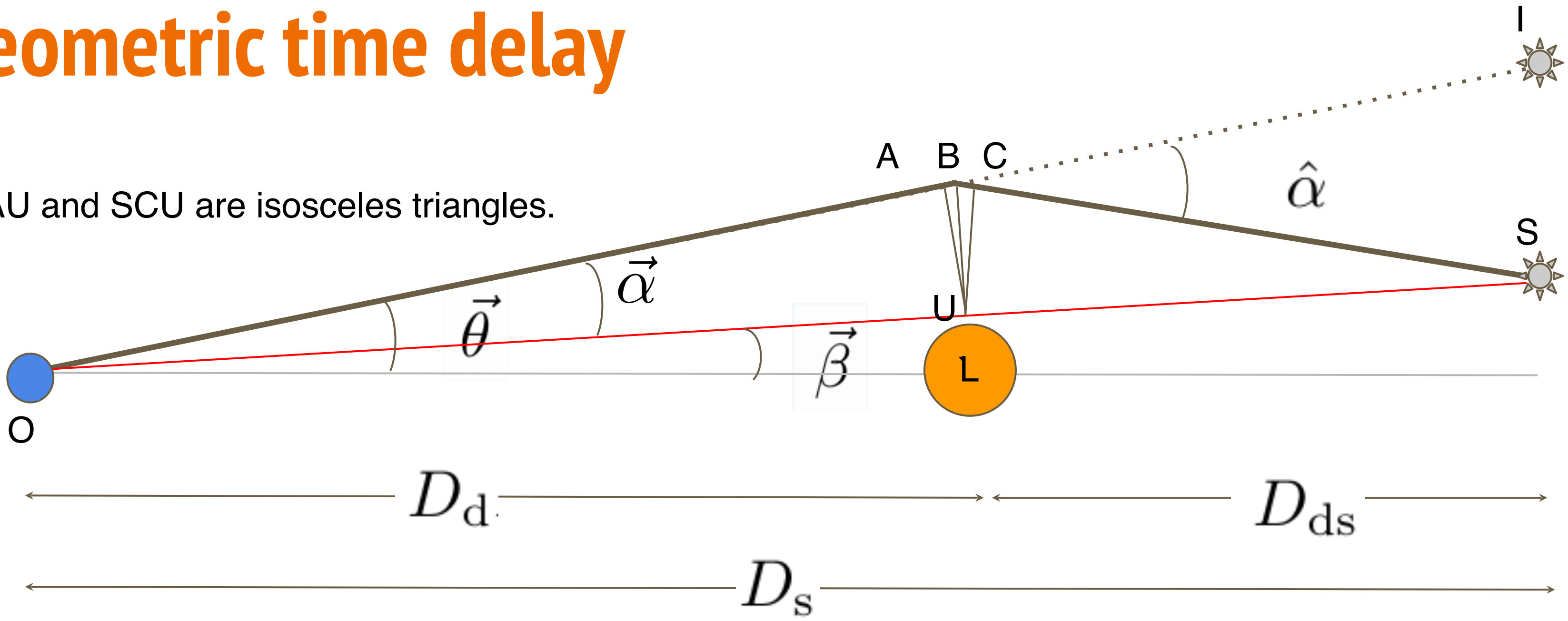
Extra path length = AB + BC = BU * ∠AUC = Dd α * ∠AUC

AUO + AUC + CUS = 180 => 90-AOU/2 + AUC + 90 - CSU/2 = 180

AUC = (AOU + CSU)/2 = $\hat{\alpha}/2$ = $(\alpha/2)\,(D_s/D_{ds})$

Geometric time delay

OAU and SCU are isosceles triangles.



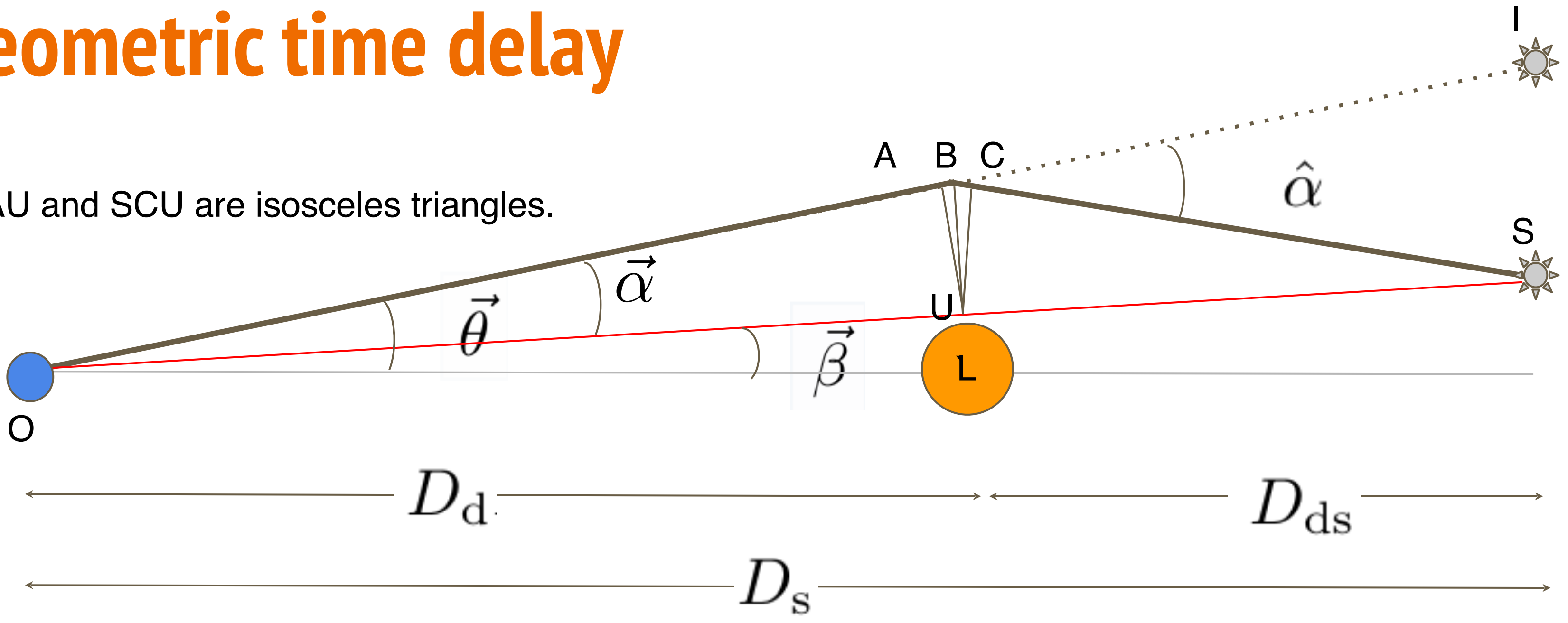
Extra path length = 1/2 α^2 (D_d D_s/D_{ds}) = 1/2 (θ - β)^2 (D_d D_s/D_{ds})

AUO + AUC + CUS = 180 => 90-AOU/2 + AUC + 90 - CSU/2 = 180

AUC = (AOU + CSU)/2 = $\hat{\alpha}$/2 = (α/2) (D_s/D_{ds})

Geometric time delay

OAU and SCU are isosceles triangles.



Geom. time delay = $(1+z)/[2c]\ (\theta - \beta)^2\ (D_d\ D_s/D_{ds})$

AUO + AUC + CUS = 180 => 90-AOU/2 + AUC + 90 - CSU/2 = 180

AUC = (AOU + CSU)/2 = $\hat{\alpha}/2 = (\alpha/2)\ (D_s/D_{ds})$

Gravitational time delay

$$\Delta T_{\rm grav} = \int \frac{dl}{c'} - \int \frac{dl}{c}$$

c' = c (1 + 2Φ/c²)

$$= -(1+z_{\rm d}) \int dl \frac{2\Phi}{c^3}$$

c'⁻¹ - c⁻¹ = -2Φ/c³ and cosmological expansion

$$= -(1+z_{\rm d}) \nabla_\perp^{-1} \nabla_\perp \int dl \frac{2\Phi}{c^3}$$

Introduce gradient and inverse gradient operator

$$= -\frac{(1+z_{\rm d})}{c} \nabla_\perp^{-1} \hat{\vec{\alpha}}$$

Definition of the deflection angle

$$= -\frac{(1+z_{\rm d})}{c} \frac{D_{\rm s}}{D_{\rm d}} \nabla_\perp^{-1} \vec{\alpha}$$

Transform to scaled deflection angle

$$= -\frac{(1+z_{\rm d})}{c} \frac{D_{\rm s} D_{\rm d}}{D_{\rm d}} \nabla_{\vec{\theta}}^{-1} \vec{\alpha}$$

Convert spatial derivative to angular derivative

$$= -\psi \frac{(1+z_{\rm d}) D_{\rm s} D_{\rm d}}{D_{\rm ds} c}$$

Use definition of the lensing potential

Grav. time delay = (1+z)/c (D_d D_s/D_{ds}) Ψ(θ)

Gravitational time delay

$$\begin{aligned}\Delta T_{\rm grav} &= \int \frac{dl}{c'} - \int \frac{dl}{c} \\ &= -(1+z_{\rm d}) \int dl \frac{2\Phi}{c^3} \\ &= -(1+z_{\rm d}) \nabla_\perp^{-1} \nabla_\perp \int dl \frac{2\Phi}{c^3} \\ &= -\frac{(1+z_{\rm d})}{c} \nabla_\perp^{-1} \hat{\vec{\alpha}} \\ &= -\frac{(1+z_{\rm d})}{c} \frac{D_{\rm s}}{D_{\rm d}} \nabla_\perp^{-1} \vec{\alpha} \\ &= -\frac{(1+z_{\rm d})}{c} \frac{D_{\rm s} D_{\rm d}}{D_{\rm d}} \nabla_{\vec{\theta}}^{-1} \vec{\alpha} \\ &= -\psi \frac{(1+z_{\rm d}) D_{\rm s} D_{\rm d}}{D_{\rm ds} c}\end{aligned}$$

$c' = c\,(1 + 2\Phi/c^2)$

$c'^{-1} - c^{-1} = -2\Phi/c^3$ and cosmological expansion

Introduce gradient and inverse gradient operator

Definition of the deflection angle

Transform to scaled deflection angle

Convert spatial derivative to angular derivative

Use definition of the lensing potential

Grav. time delay = $(1+z)/c\ (D_d\ D_s/D_{ds})\ \Psi(\theta)$

Arrival time of images: Fermat potential

$$\tau(\boldsymbol{\theta};\boldsymbol{\beta}) = \frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\beta})^2 - \psi(\boldsymbol{\theta})\,, \longrightarrow \quad t_{\mathrm{delay}} = (1+z)\frac{D_{\mathrm{s}}D_{\mathrm{d}}}{cD_{\mathrm{ds}}}\tau$$

$$\nabla\tau(\boldsymbol{\theta};\boldsymbol{\beta}) = \mathbf{0} \longrightarrow \quad \boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

- Images of a source at $\boldsymbol{\beta}$ form at the stationary points of the Fermat potential surface.
- Time difference between two images of the same source has two components: a geometric component and a gravitational time delay

Reference to follow

- Gravitational lensing, strong, weak and micro, SAAS Fee Advanced Courses, Vol 33 (here on Springer link)
    - Introduction to Gravitational lensing (Peter Schneider)
    - Strong gravitational lensing (Chris Kochanek)
    - Weak gravitational lensing (Peter Schneider)
    - Microlensing (Joachim Wambsganss)