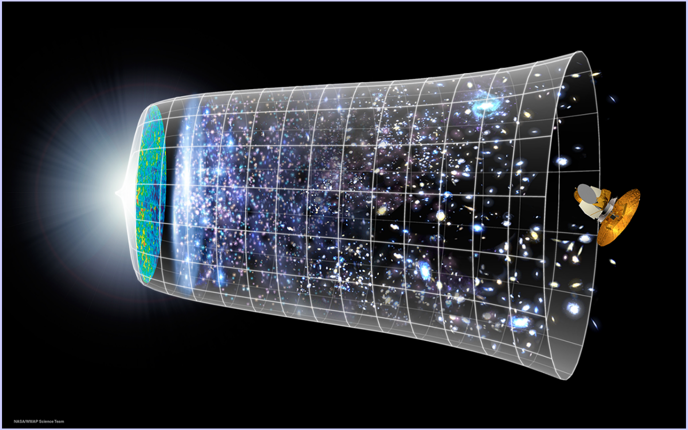
String Gas Cosmology

Robert Brandenberger
McGill University

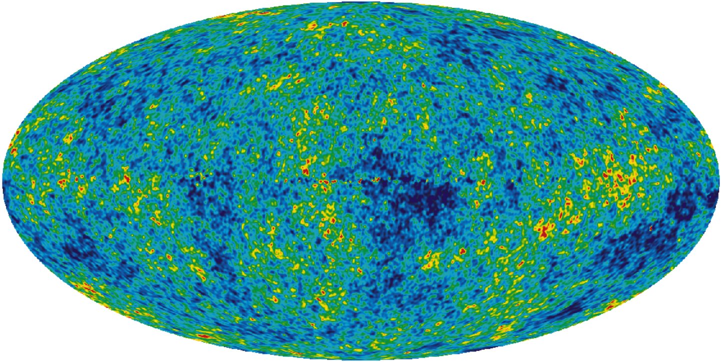
Takayama Lectures, August 2016

Outline

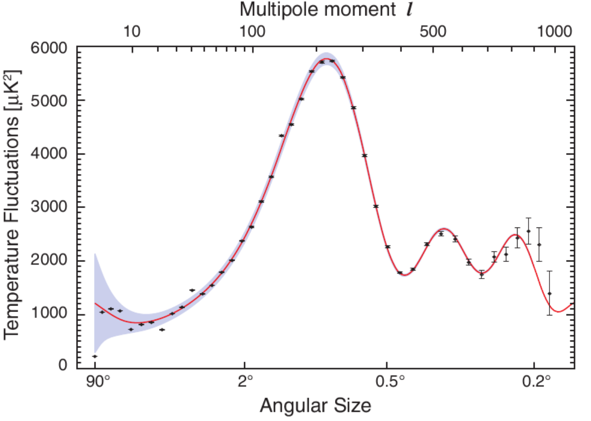
Plan



Credit: NASA/WMAP Science Team



Credit: NASA/WMAP Science Team



Credit: NASA/WMAP Science Team

Predicting the Data

1970Ap&SS...7....3S

1970

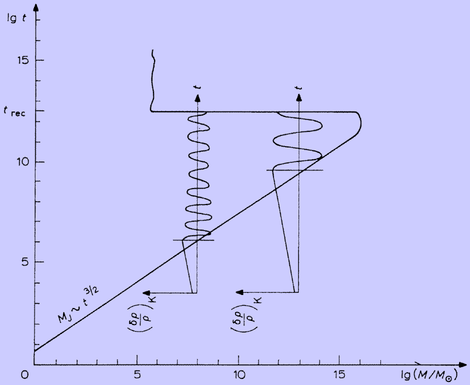


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.

Predictions of Sunyaev & Zeldovich, and Peebles & Yu

1970Ap&SS...7....3S

1970 paper

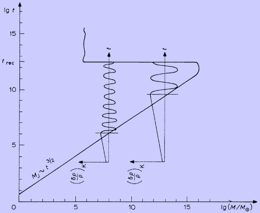


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

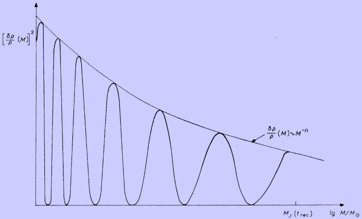


Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.
- **Inflation** is the first model to yield such a primordial spectrum from causal physics.
- But it is NOT the only one.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.
- **Inflation** is the first model to yield such a primordial spectrum from causal physics.
- But it is NOT the only one.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.
- **Inflation** is the first model to yield such a primordial spectrum from causal physics.
- But it is NOT the only one.

Horizon vs. Hubble radius

$$\text{Metric}: \; ds^2 \,=\, dt^2 - a(t)^2 d\mathbf{x}^2$$

Horizon: forward light cone, carries causality information

$$l_f(t) \,=\, a(t) \int_0^t dt' a(t')^{-1} \,.$$

Hubble radius: relevant to dynamics of cosmological fluctuations

$$l_H(t) \,=\, H^{-1}(t)$$

Horizon vs. Hubble radius

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

Horizon: forward light cone, carries causality information

$$l_f(t) = a(t) \int_0^t dt' a(t')^{-1} .$$

Hubble radius: relevant to dynamics of cosmological fluctuations

$$l_H(t) = H^{-1}(t)$$

Horizon vs. Hubble radius

$$\text{Metric}: \ ds^2 \ = \ dt^2 - a(t)^2 d\mathbf{x}^2$$

Horizon: forward light cone, carries causality information

$$l_f(t) \ = \ a(t) \int_0^t dt' a(t')^{-1} \, .$$

Hubble radius: relevant to dynamics of cosmological fluctuations

$$l_H(t) \ = \ H^{-1}(t)$$

Requirements I

Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius (Criterium 2)
- Long propagation on super-Hubble scales (Criterium 3)
- Scale-invariant spectrum of adiabatic cosmological perturbations (Criterium 4).

Requirements I

Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius (Criterium 2)
- Long propagation on super-Hubble scales (Criterium 3)
- Scale-invariant spectrum of adiabatic cosmological perturbations (Criterium 4).

Requirements I

Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius (Criterium 2)
- Long propagation on super-Hubble scales (Criterium 3)
- Scale-invariant spectrum of adiabatic cosmological perturbations (Criterium 4).

Requirements I

Model must yield a successful structure formation scenario:

- Scales of cosmological interest today must originate inside the Hubble radius (Criterium 2)
- Long propagation on super-Hubble scales (Criterium 3)
- Scale-invariant spectrum of adiabatic cosmological perturbations (Criterium 4).

Requirements II

Model must refer to the problems of Standard Cosmology which the inflationary scenario addresses.

- Solution of the horizon problem: horizon $\gg$ Hubble radius (Criterium 1).
- Solution of the flatness problem.
- Solution of the size and entropy problems.

Plan

Inflationary Cosmology

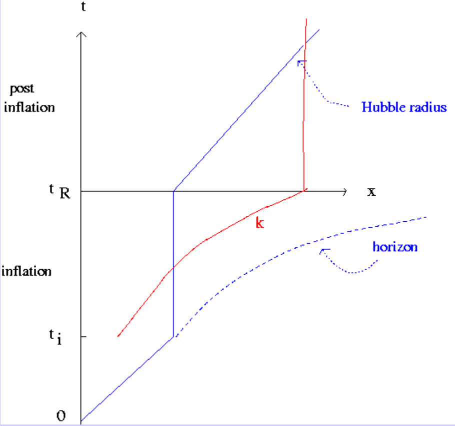
R. Brout, F. Englert and E. Gunzig (1978), A. Starobinsky (1978), K. Sato (1981), A. Guth (1981)

Idea: phase of almost exponential expansion of space $t \in [t_i, t_R]$

Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Space-time sketch of Inflationary Cosmology



Note:

- $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length at the beginning of inflation $\rightarrow$ Trans-Planckian Problem for cosmological fluctuations (J. Martin and R.B., 2000).

Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length at the beginning of inflation $\rightarrow$ Trans-Planckian Problem for cosmological fluctuations (J. Martin and R.B., 2000).

Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length at the beginning of inflation $\rightarrow$ Trans-Planckian Problem for cosmological fluctuations (J. Martin and R.B., 2000).

Addressing the Criteria

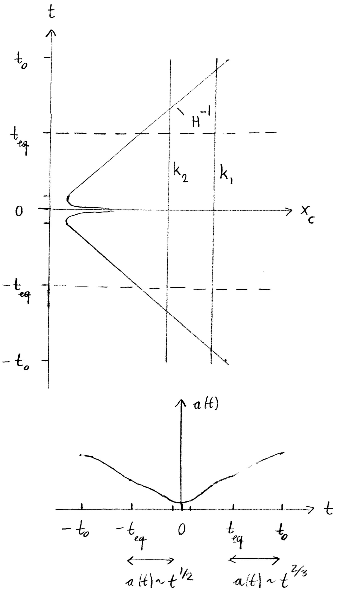
- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length at the beginning of inflation $\rightarrow$ Trans-Planckian Problem for cosmological fluctuations (J. Martin and R.B., 2000).

Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length at the beginning of inflation $\rightarrow$ Trans-Planckian Problem for cosmological fluctuations (J. Martin and R.B., 2000).

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Overview of the Matter Bounce

- Begin with a matter phase of contraction during which fluctuations of current cosmological interest exit the Hubble radius.
- Later in the contraction phase the equation of state of matter may be different (e.g. radiation).
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Overview of the Matter Bounce

- Begin with a matter phase of contraction during which fluctuations of current cosmological interest exit the Hubble radius.
- Later in the contraction phase the equation of state of matter may be different (e.g. radiation).
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Overview of the Matter Bounce

- Begin with a matter phase of contraction during which fluctuations of current cosmological interest exit the Hubble radius.
- Later in the contraction phase the equation of state of matter may be different (e.g. radiation).
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Overview of the Matter Bounce

- Begin with a matter phase of contraction during which fluctuations of current cosmological interest exit the Hubble radius.
- Later in the contraction phase the equation of state of matter may be different (e.g. radiation).
- New physics provides a nonsingular (or singular) cosmological bounce.
- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

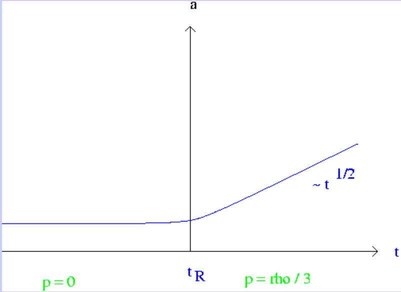
Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

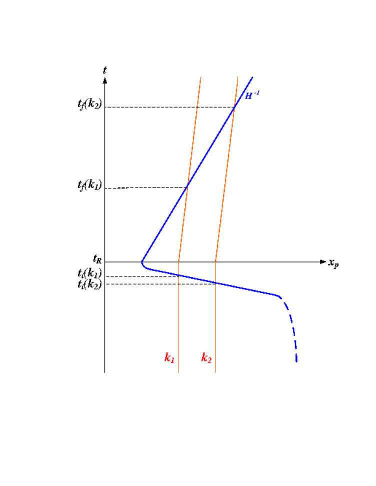
- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Emergent Universe Scenario
R.B. and C. Vafa, 1989



Space-time sketch of an Emergent Universe

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal fluctuations.

Overview of the Emergent Universe Scenario

- The Universe begins in a quasi-static phase.
- After a phase transition there is a transition to the Hot Big Bang phase of Standard Cosmology.
- Fluctuations originate as thermal perturbations on sub-Hubble scales in the emergent phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales if the thermal fluctuations have holographic scaling.

Overview of the Emergent Universe Scenario

- The Universe begins in a quasi-static phase.
- After a phase transition there is a transition to the Hot Big Bang phase of Standard Cosmology.
- Fluctuations originate as thermal perturbations on sub-Hubble scales in the emergent phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales if the thermal fluctuations have holographic scaling.

Overview of the Emergent Universe Scenario

- The Universe begins in a quasi-static phase.
- After a phase transition there is a transition to the Hot Big Bang phase of Standard Cosmology.
- Fluctuations originate as thermal perturbations on sub-Hubble scales in the emergent phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales if the thermal fluctuations have holographic scaling.

Overview of the Emergent Universe Scenario

- The Universe begins in a quasi-static phase.
- After a phase transition there is a transition to the Hot Big Bang phase of Standard Cosmology.
- Fluctuations originate as thermal perturbations on sub-Hubble scales in the emergent phase.
- Adiabatic fluctuation mode acquires a scale-invariant spectrum of curvature perturbations on super-Hubble scales if the thermal fluctuations have holographic scaling.

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length in the initial state $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations .

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length in the initial state $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations .

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length in the initial state $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations .

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length in the initial state $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations .

Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.
- Note: Wavelengths of interesting fluctuation modes $\ll$ Planck length in the initial state $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations .

Plan

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$
\begin{aligned}
ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\
\varphi &= \varphi_0 + \delta\varphi
\end{aligned}
$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$
\begin{aligned}
S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\
v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\
z &= a\frac{\varphi_0'}{\mathcal{H}}
\end{aligned}
$$

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$\begin{aligned} ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\ \varphi &= \varphi_0 + \delta\varphi \end{aligned}$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\ v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\ z &= a\frac{\varphi_0'}{\mathcal{H}} \end{aligned}$$

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$
\begin{aligned}
ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\
\varphi &= \varphi_0 + \delta\varphi
\end{aligned}
$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations
Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$
\begin{aligned}
S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\
v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\
z &= a\frac{\varphi_0'}{\mathcal{H}}
\end{aligned}
$$

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$
\begin{aligned}
ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\
\varphi &= \varphi_0 + \delta\varphi
\end{aligned}
$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations
Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$
\begin{aligned}
S^{(2)} &= \frac{1}{2}\int d^4x\big((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\big) \\
v &= a\big(\delta\varphi + \frac{z}{a}\Phi\big) \\
z &= a\frac{\varphi_0'}{\mathcal{H}}
\end{aligned}
$$

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p+\rho}\delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

Gravitational Waves

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right]$$

- $h_{ij}(\mathbf{x}, t)$ transverse and traceless
- Two polarization states

$$h_{ij}(\mathbf{x}, t) = \sum_{a=1}^{2} h_a(\mathbf{x}, t)\epsilon_{ij}^a$$

- At linear level each polarization mode evolves independently.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) = a(t)h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k'' + (k^2 - \frac{a''}{a})u_k = 0 .$$

Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) = a(t)h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0 .$$

Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) \;=\; a(t)h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k^{''} + \left(k^2 - \frac{a^{''}}{a}\right)u_k \;=\; 0\,.$$

Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

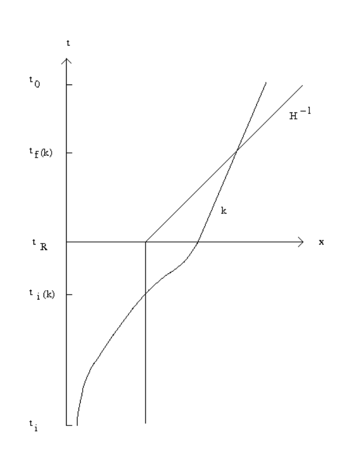
Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

Plan

Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3|\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k,t_i(k))\Big(\frac{a(t)}{a(t_i(k))}\Big)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3 |\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k,t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3 |\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \text{const}$

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3|\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k,t_i(k))\Big(\frac{a(t)}{a(t_i(k))}\Big)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3|\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k,t_i(k))\left(\frac{a(t)}{a(t_i(k))}\right)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0$ $\rightarrow$ red spectrum, $n_t < 0$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0 \rightarrow$ red spectrum, $n_t < 0$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))} \right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0 \rightarrow$ red spectrum, $n_t < 0$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0$ $\rightarrow$ red spectrum, $n_t < 0$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3|h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0$ $\rightarrow$ red spectrum, $n_t < 0$

Matter Bounce: Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) \,=\, k^3|\zeta(k)|^2 \,\sim\, k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \,=\, c_1\eta^2 + c_2\eta^{-1} \,,$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$P_\zeta(k,\eta) \quad \sim \quad k^3|v_k(\eta)|^2 a^{-2}(\eta)$$
$$\sim \quad k^3|v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2}$$

Matter Bounce: Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) = k^3|\zeta(k)|^2 \sim k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) = c_1\eta^2 + c_2\eta^{-1},$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$P_\zeta(k,\eta) \sim k^3|v_k(\eta)|^2 a^{-2}(\eta)$$
$$\sim k^3|v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2}$$

Matter Bounce: Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_{\zeta}(k) \, = \, k^3 |\zeta(k)|^2 \, \sim \, k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \, = \, c_1 \eta^2 + c_2 \eta^{-1} \, ,$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_{\zeta}(k, \eta) \quad &\sim \quad k^3 |v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim \quad k^3 |v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \end{aligned}$$

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gang et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Signature in the Bispectrum: formalism

$$
\begin{aligned}
&< \zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3) > \\
&\quad = i \int_{t_i}^{t} dt' < [\zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3), L_{int}(t')] > ,
\end{aligned}
$$

$$
\begin{aligned}
< \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) > \quad &= \quad (2\pi)^7 \delta(\sum \vec{k}_i) \frac{P_\zeta^2}{\prod k_i^3} \\
&\quad \times \mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \, ,
\end{aligned}
$$

$$
|\mathcal{B}|_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{10}{3} \frac{\mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sum_i k_i^3} \, .
$$

String Gas
Brandenberger
Introduction
Paradigms
Inflationary Expansion
Matter Dominated Contraction
Emergent
Perturbations
Applications
Inflation
Bounce
SGC
String Gas Cosmology
SGC Structure
Moduli
Other
Discussion
Conclusions

Signature in the Bispectrum: Results

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*

If we project the resulting shape function $\mathcal{A}$ onto some popular shape masks we get

$$|\mathcal{B}|_{NL}^{\mathrm{local}} = -\frac{35}{8},$$

for the local shape ($k_1 \ll k_2 = k_3$). This is negative and of order $O(1)$.
For the equilateral form ($k_1 = k_2 = k_3$) the result is

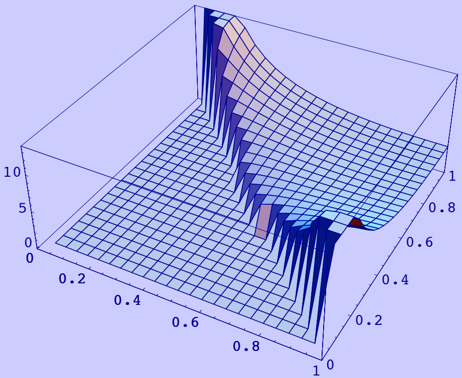
$$|\mathcal{B}|_{NL}^{\mathrm{equil}} = -\frac{255}{64},$$

For the folded form ($k_1 = 2k_2 = 2k_3$) one obtains the value

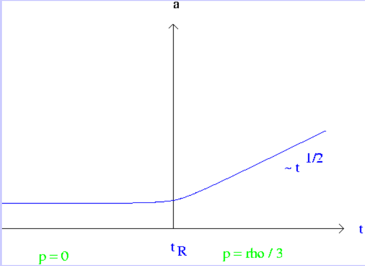
$$|\mathcal{B}|_{NL}^{\mathrm{folded}} = -\frac{9}{4}.$$

Bispectrum of the Matter Bounce Scenario

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*

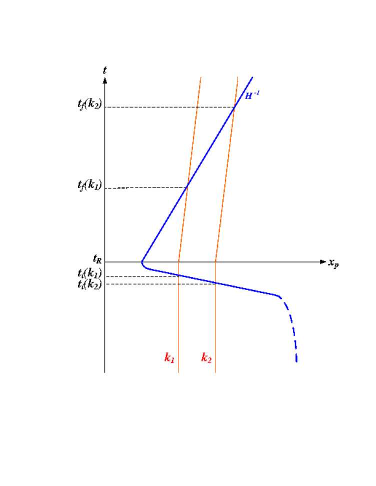


Background for Emergent Cosmology



Structure Formation in Emergent Cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal fluctuations.

Method

- Calculate matter correlation functions in the static phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, .$$

Key assumption: holographic scaling of thermodynamical quantities: $C_V \sim R^2$

Example: for string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \, .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key assumption: holographic scaling of thermodynamical quantities: $C_V \sim R^2$

Example: for string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_{\Phi}(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2 k^2 < (\delta M)^2 >_R \\
&= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\
&\sim 8G^2 T
\end{aligned}
$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

Power spectrum of cosmological fluctuations

$$\begin{aligned} P_\Phi(k) &= 8G^2k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2k^2 < (\delta M)^2 >_R \\ &= 8G^2k^{-4} < (\delta\rho)^2 >_R \\ &\sim 8G^2T \end{aligned}$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum

Requirements

- static phase $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$
- Cosmological fluctuations in the IR are described by Einstein gravity.

Signature in Non-Gaussianities

M. He, ..., RB, arXiv:1608.05079

- Non-Gaussianities order 1 on microscopic scales, but Poisson-suppressed on cosmological scales.
- **Exception**: if topological defects such as cosmic strings or superstrings are formed.
- **Scale-dependent** non-Gaussianities.

Signature in Non-Gaussianities

M. He, ..., RB, arXiv:1608.05079

- Non-Gaussianities order 1 on microscopic scales, but Poisson-suppressed on cosmological scales.
- **Exception**: if topological defects such as cosmic strings or superstrings are formed.
- **Scale-dependent** non-Gaussianities.

Signature in Non-Gaussianities

M. He, ..., RB, arXiv:1608.05079

- Non-Gaussianities order 1 on microscopic scales, but Poisson-suppressed on cosmological scales.
- **Exception**: if topological defects such as cosmic strings or superstrings are formed.
- **Scale-dependent** non-Gaussianities.

Plan

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

T-Duality

T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

T-Duality

T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

T-Duality

T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

T-Duality

T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

T-Duality

T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

T-Duality

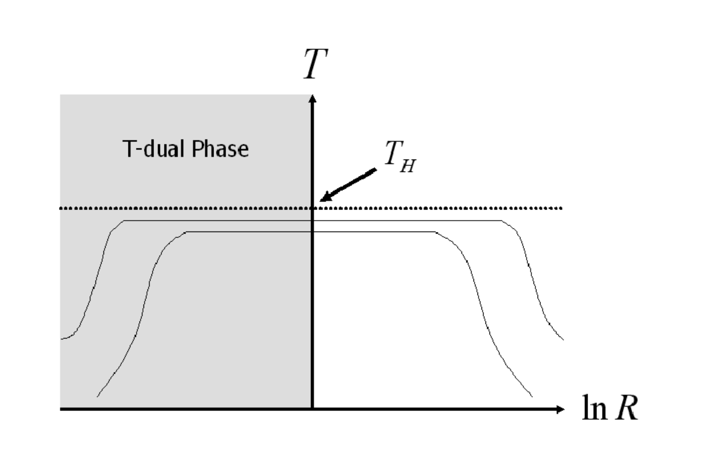
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Adiabatic Considerations

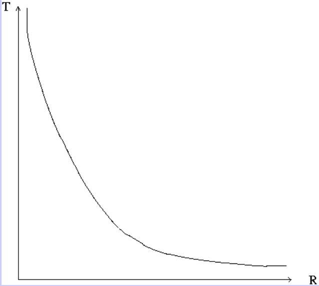
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



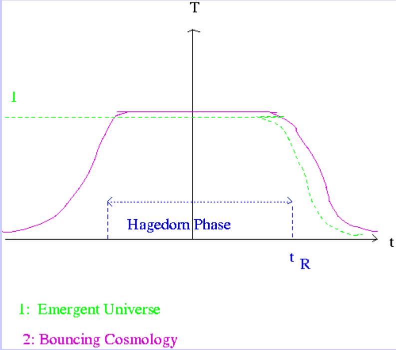
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



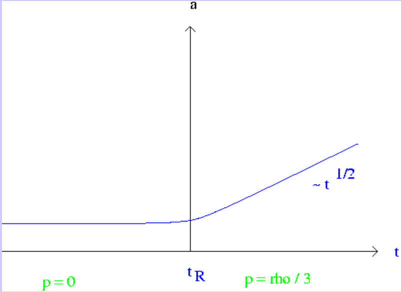
Dynamics

Assume some action gives us $R(t)$



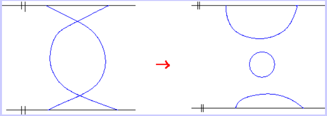
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



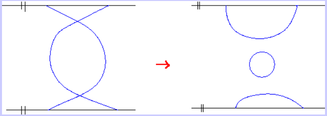
Dynamics III

The transition from the Hagedorn phase to the radiation phase of standard cosmology is given by the unwinding of winding modes:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.

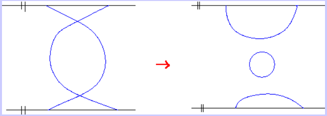


- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.

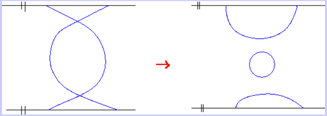


- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.

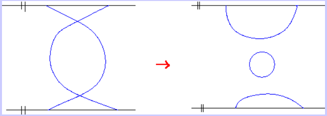


- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow \quad V_{eff}(R)$ has a minimum at a finite value of $R, \quad \rightarrow \quad R_{min}$
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow \; V_{eff}(R)$ has a minimum at a finite value of $R, \; \rightarrow \; R_{min}$
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., arXiv:0802.1557]
- Gaugino condensation induces high scale supersymmetry breaking [S. Mishra, W. Xue, R. B, and U. Yajnik, arXiv:1103.1389].

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., arXiv:0802.1557]
- Gaugino condensation induces high scale supersymmetry breaking [S. Mishra, W. Xue, R. B, and U. Yajnik, arXiv:1103.1389].

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., arXiv:0802.1557]
- Gaugino condensation induces high scale supersymmetry breaking [S. Mishra, W. Xue, R. B, and U. Yajnik, arXiv:1103.1389].

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., arXiv:0802.1557]
- Gaugino condensation induces high scale supersymmetry breaking [S. Mishra, W. Xue, R. B, and U. Yajnik, arXiv:1103.1389].

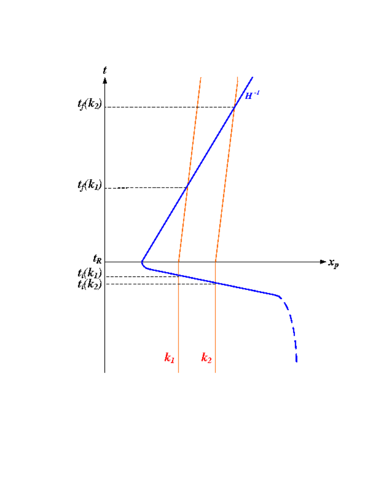
Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., arXiv:0802.1557]
- Gaugino condensation induces high scale supersymmetry breaking [S. Mishra, W. Xue, R. B, and U. Yajnik, arXiv:1103.1389].

Plan

Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 \,=\, a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big)\,.$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle \,=\, 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle\,,$$

$$\langle|h(k)|^2\rangle \,=\, 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle\,.$$

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

Power Spectrum of Cosmological Perturbations

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2 k^2 < (\delta M)^2 >_R \\
&= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
\end{aligned}
$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_\Phi(k) &= 8G^2k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2k^2 < (\delta M)^2 >_R \\
&= 8G^2k^{-4} < (\delta\rho)^2 >_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
\end{aligned}
$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

Sketch of the Derivation

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^2 > = \frac{1}{V^2}\frac{d^2 \log Z}{d\beta^2} = T\left[L^4 l_s^3 (1 - \frac{T}{T_H})\right]^{-1}$$

Using constraint equation:

$$\Phi_k \sim 4\pi G \delta\rho_k \left(\frac{a}{k}\right)^2$$

we obtain

$$< \Phi_k^2 > \simeq (4\pi G)^2 \frac{T}{l_s^3 (1 - T/T_H)} k^{-3}$$

$$\mathcal{P}_\Phi(k) \simeq 8\left(\frac{l_{pl}}{l_s}\right)^4 \frac{1}{1 - T/T_H}$$

Sketch of the Derivation

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^2 > = \frac{1}{V^2}\frac{d^2 \log Z}{d\beta^2} = T\left[L^4 l_s^3 (1 - \frac{T}{T_H})\right]^{-1}$$

Using constraint equation:

$$\Phi_k \sim 4\pi G \delta\rho_k \left(\frac{a}{k}\right)^2$$

we obtain

$$< \Phi_k^2 > \simeq (4\pi G)^2 \frac{T}{l_s^3 (1 - T/T_H)} k^{-3}$$

$$\mathcal{P}_\Phi(k) \simeq 8\left(\frac{l_{pl}}{l_s}\right)^4 \frac{1}{1 - T/T_H}$$

Sketch of the Derivation

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^2 > = \frac{1}{V^2}\frac{d^2 \log Z}{d\beta^2} = T\left[L^4 l_s^3 (1 - \frac{T}{T_H})\right]^{-1}$$

Using constraint equation:

$$\Phi_k \sim 4\pi G \delta\rho_k \left(\frac{a}{k}\right)^2$$

we obtain

$$< \Phi_k^2 > \simeq (4\pi G)^2 \frac{T}{l_s^3 (1 - T/T_H)} k^{-3}$$

$$\mathcal{P}_\Phi(k) \simeq 8 \left(\frac{l_{pl}}{l_s}\right)^4 \frac{1}{1 - T/T_H}$$

Sketch of the Derivation

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^2 > = \frac{1}{V^2}\frac{d^2 \log Z}{d\beta^2} = T\left[L^4 l_s^3 (1 - \frac{T}{T_H})\right]^{-1}$$

Using constraint equation:

$$\Phi_k \sim 4\pi G \delta\rho_k \left(\frac{a}{k}\right)^2$$

we obtain

$$< \Phi_k^2 > \simeq (4\pi G)^2 \frac{T}{l_s^3 (1 - T/T_H)} k^{-3}$$

$$\mathcal{P}_\Phi(k) \simeq 8\left(\frac{l_{pl}}{l_s}\right)^4 \frac{1}{1 - T/T_H}$$

Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$
\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
&= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\
&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H)
\end{aligned}
$$

Key ingredient for string thermodynamics

$$
< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)
$$

Key features:

- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$
\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
&= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\
&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H)
\end{aligned}
$$

Key ingredient for string thermodynamics

$$
< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)
$$

Key features:

- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)

Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

Non-Gaussianities
B. Chen, Y. Wang, W. Xue and RB, arXiv:0712.2477

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^3 > = \frac{1}{V^3}\frac{d^3 \log Z}{d\beta^3} = T\left[L^7 l_s^3 (1 - \frac{T}{T_H})^2\right]^{-1}$$

$$< \Phi_k^3 > \simeq (4\pi G)^3 \frac{T^2 H(t_H(k))}{l_s^3 (1 - T/T_H)^2} k^{-9/2}$$

$$f_{NL}(k) \sim k^{-3/2} \frac{< \Phi_k^3 >}{< \Phi_k^2 >< \Phi_k^2 >} \sim \frac{l_s^3 H(t_H(k))}{4\pi l_p^2}$$

Using Hubble radius crossing condition $k = aH$ we get

$$f_{NL}(k) \sim (\frac{l_s}{l_{pl}})^2 (\frac{k}{k_H})$$

Non-Gaussianities

B. Chen, Y. Wang, W. Xue and RB, arXiv:0712.2477

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^3 > = \frac{1}{V^3}\frac{d^3 \log Z}{d\beta^3} = T\left[L^7 l_s^3 (1 - \frac{T}{T_H})^2\right]^{-1}$$

$$< \Phi_k^3 > \simeq (4\pi G)^3 \frac{T^2 H(t_H(k))}{l_s^3 (1 - T/T_H)^2} k^{-9/2}$$

$$f_{NL}(k) \sim k^{-3/2} \frac{< \Phi_k^3 >}{< \Phi_k^2 >< \Phi_k^2 >} \sim \frac{l_s^3 H(t_H(k))}{4\pi l_p^2}$$

Using Hubble radius crossing condition $k = aH$ we get

$$f_{NL}(k) \sim (\frac{l_s}{l_{pl}})^2 (\frac{k}{k_H})$$

Non-Gaussianities

B. Chen, Y. Wang, W. Xue and RB, arXiv:0712.2477

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^3 > = \frac{1}{V^3}\frac{d^3 \log Z}{d\beta^3} = T\left[L^7 l_s^3 (1 - \frac{T}{T_H})^2\right]^{-1}$$

$$< \Phi_k^3 > \simeq (4\pi G)^3 \frac{T^2 H(t_H(k))}{l_s^3 (1 - T/T_H)^2} k^{-9/2}$$

$$f_{NL}(k) \sim k^{-3/2} \frac{< \Phi_k^3 >}{< \Phi_k^2 >< \Phi_k^2 >} \sim \frac{l_s^3 H(t_H(k))}{4\pi l_p^2}$$

Using Hubble radius crossing condition $k = aH$ we get

$$f_{NL}(k) \sim (\frac{l_s}{l_{pl}})^2 (\frac{k}{k_H})$$

Non-Gaussianities

B. Chen, Y. Wang, W. Xue and RB, arXiv:0712.2477

Thermal equilibrium relations: Given a box of length L

$$< \delta\rho^3 > = \frac{1}{V^3}\frac{d^3 \log Z}{d\beta^3} = T\left[L^7 l_s^3 (1 - \frac{T}{T_H})^2\right]^{-1}$$

$$< \Phi_k^3 > \simeq (4\pi G)^3 \frac{T^2 H(t_H(k))}{l_s^3 (1 - T/T_H)^2} k^{-9/2}$$

$$f_{NL}(k) \sim k^{-3/2} \frac{< \Phi_k^3 >}{< \Phi_k^2 >< \Phi_k^2 >} \sim \frac{l_s^3 H(t_H(k))}{4\pi l_p^2}$$

Using Hubble radius crossing condition $k = aH$ we get

$$f_{NL}(k) \sim (\frac{l_s}{l_{pl}})^2 (\frac{k}{k_H})$$

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].

To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order.

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].
To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].

To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].
To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].

To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order.

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].
To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].
To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order

Comparison with Galileon Inflation

By violating the *Null Energy Condition* one can construct inflationary models with $\dot{H} > 0$ which lead to gravitational waves with a blue spectrum [T. Kobayashi, M. Yamaguchi and J. Yokoyama, arXiv:1008.0603].

To distinguish between String Gas Cosmology and Galileon Inflation note that [M. He, ..., RB, arXiv:1608.05079]:

- Consistency relation between spectral indices
  $n_s = 1 - n_t$ for String Gas Cosmology
  $n_s - 1 = -2\epsilon - \eta + f_1(\epsilon, \eta)$ and $n_t = -2\epsilon$ for inflation.
- Amplitude of the non-Gaussianities
  String Gas Cosmology: Poisson-suppressed on cosmological scales
  Galileon inflation: large amplitude unless fine tuning.
- Scale-dependence of non-Gaussianities
  String Gas Cosmology: large blue tilt
  Galileon Inflation: scale-independent at leading order.

Plan

Action

S. Patil and R.B., 2004, 2005

Action: Dilaton gravity plus string gas matter

$$S = \frac{1}{\kappa}\left(S_g + S_\phi\right) + S_{SG}\,,$$

$$S_{SG} = -\int d^{10}x\sqrt{-g}\sum_\alpha \mu_\alpha \epsilon_\alpha\,,$$

where

- μ_α: number density of strings in the state α
- ϵ_α: energy of the state α.

Introduce comoving number density:

$$\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}\,,$$

Action

S. Patil and R.B., 2004, 2005

Action: Dilaton gravity plus string gas matter

$$S = \frac{1}{\kappa}\left(S_g + S_\phi\right) + S_{SG} \, ,$$

$$S_{SG} = -\int d^{10}x\sqrt{-g}\sum_\alpha \mu_\alpha \epsilon_\alpha \, ,$$

where

- μ_α: number density of strings in the state α
- ϵ_α: energy of the state α.

Introduce comoving number density:

$$\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}} \, ,$$

Action

S. Patil and R.B., 2004, 2005

Action: Dilaton gravity plus string gas matter

$$S = \frac{1}{\kappa}\left(S_g + S_\phi\right) + S_{SG}\,,$$

$$S_{SG} = -\int d^{10}x\sqrt{-g}\sum_\alpha \mu_\alpha \epsilon_\alpha\,,$$

where

- μ_α: number density of strings in the state α
- ϵ_α: energy of the state α.

Introduce comoving number density:

$$\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}\,,$$

Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \,,$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \,,$$

$$p_\alpha^i = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3} \,,$$

$$p_\alpha^a = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g} \alpha'} \left(\frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right) .$$

Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \,,$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \,,$$

$$p_\alpha^i = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3} \,,$$

$$p_\alpha^a = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}\alpha'} \left(\frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right) \,.$$

Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \,,$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \,,$$

$$p_\alpha^i = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3} \,,$$

$$p_\alpha^a = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}\alpha'} \left(\frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right) \,.$$

Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \,,$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \,,$$

$$p_\alpha^i = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3} \,,$$

$$p_\alpha^a = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}\alpha'} \left(\frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right) \,.$$

Single string energy

ϵ_α is the energy of the string state α:

$$\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}} \Big[\alpha' p_d^2 + b^{-2}(n, n) + b^2(w, w) + 2(n, w) + 4(N - 1) \Big]^{1/2} ,$$

where

- $\vec{n}$ and $\vec{w}$: momentum and winding number vectors in the internal space
- $\vec{p}_d$: momentum in the large space

Single string energy

ϵ_α is the energy of the string state α:

$$\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}} \Big[\alpha' p_d^2 + b^{-2}(n,n) + b^2(w,w) + 2(n,w) + 4(N-1)\Big]^{1/2} ,$$

where

- $\vec{n}$ and $\vec{w}$: momentum and winding number vectors in the internal space
- $\vec{p}_d$: momentum in the large space

Background equations of motion

Radion equation:

$$\ddot{b} \; + \; \dot{b}(3\frac{\dot{a}}{a} + 5\frac{\dot{b}}{b}) \; = \; \frac{8\pi G\mu_{0,\alpha}}{\alpha' \sqrt{\hat{G}_a \epsilon_\alpha}}$$

$$\times \quad \left[\frac{n_a^2}{b^2} - w_a^2 b^2 + \frac{2}{(D-1)}[b^2(w,w) + (n,w) + 2(N-1)]\right]$$

Scale factor equation:

$$\ddot{a} \; + \; \dot{a}(2\frac{\dot{a}}{a} + 6\frac{\dot{b}}{b}) \; = \; \frac{8\pi G\mu_{0,\alpha}}{\sqrt{\hat{G}_i \epsilon_\alpha}}$$

$$\times \quad \left[\frac{p_d^2}{3} + \frac{2}{\alpha'(D-1)}[b^2(w,w) + (n,w) + 2(N-1)]\right] \; ,$$

Special states

Enhanced symmetry states

$$b^2(w,w) + (n,w) + 2(N-1) \,=\, 0\,.$$

Stable radion fixed point:

$$\frac{n_a^2}{b^2} - w_a^2 b^2 \,=\, 0\,.$$

Special states

Enhanced symmetry states

$$b^2(w, w) + (n, w) + 2(N - 1) = 0 .$$

Stable radion fixed point:

$$\frac{n_a^2}{b^2} - w_a^2 b^2 = 0 .$$

Gaugino condensation

Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton

Kähler potential: (standard)

$$\mathcal{K}(S) = -\ln(S+\bar{S}), \quad S = e^{-\Phi} + ia .$$

$$\Phi = 2\phi - 6\ln b \tag{1}$$

Φ :4-d dilaton, b: radion, a: axion.

Non-perturbative superpotential (from gaugino condensation):

$$W = M_P^3 \left(C - Ae^{-a_0 S} \right)$$

Gaugino condensation

Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton

Kähler potential: (standard)

$$\mathcal{K}(S) = -\ln(S + \bar{S}) \, , \quad S = e^{-\Phi} + ia \, .$$

$$\Phi = 2\phi - 6 \ln b \tag{1}$$

Φ :4-d dilaton, b: radion, a: axion.

Non-perturbative superpotential (from gaugino condensation):

$$W = M_P^3 \left(C - Ae^{-a_0 S} \right)$$

Gaugino condensation

Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton

Kähler potential: (standard)

$$\mathcal{K}(S) = -\ln(S + \bar{S}), \quad S = e^{-\Phi} + ia .$$

$$\Phi = 2\phi - 6\ln b \tag{1}$$

Φ :4-d dilaton, b: radion, a: axion.

Non-perturbative superpotential (from gaugino condensation):

$$W = M_P^3 \left(C - Ae^{-a_0 S} \right)$$

Dilaton potential I

Yields a potential for the dilaton (and radion)

$$
\begin{aligned}
V &= \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[\frac{C^2}{4} e^{2\Phi} + ACe^{\Phi} \left(a_0 + \frac{1}{2} e^{\Phi} \right) e^{-a_0 e^{-\Phi}} \right. \\
&\left. + A^2 \left(a_0 + \frac{1}{2} e^{\Phi} \right)^2 e^{-2a_0 e^{-\Phi}} \right] .
\end{aligned}
$$

Expand the potential about its minimum:

$$
\begin{aligned}
V &= \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \\
&\times \left(e^{-\Phi} - e^{-\Phi_0} \right)^2 .
\end{aligned}
$$

Dilaton potential I

Yields a potential for the dilaton (and radion)

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[\frac{C^2}{4} e^{2\Phi} + ACe^{\Phi} \left(a_0 + \frac{1}{2} e^{\Phi} \right) e^{-a_0 e^{-\Phi}} + A^2 \left(a_0 + \frac{1}{2} e^{\Phi} \right)^2 e^{-2a_0 e^{-\Phi}} \right] .$$

Expand the potential about its minimum:

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \times \left(e^{-\Phi} - e^{-\Phi_0} \right)^2 .$$

Dilaton potential II

Lift the potential to 10-d, redefining b to be in the Einstein frame.

$$
\begin{aligned}
V(b,\phi) &= \frac{M_{10}^{16}\hat{V}}{4} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0}\right)^2 e^{-2a_0 e^{-\Phi_0}} \\
&\quad \times e^{-3\phi/2} \left(b^6 e^{-\phi/2} - e^{-\Phi_0}\right)^2 .
\end{aligned}
$$

Dilaton potential in 10d Einstein frame

$$
V \simeq n_1 e^{-3\phi/2} \left(b^6 e^{-\phi/2} - n_2\right)^2
$$

Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization

Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

$$\begin{aligned} g^E_{\mu\nu} &= e^{-\phi/2} g^s_{\mu\nu} \\ b_s &= e^{\phi/4} b_E \\ T^E_{\mu\nu} &= e^{2\phi} T^s_{\mu\nu} \,. \end{aligned}$$

Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization
Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

$$\begin{aligned} g^E_{\mu\nu} &= e^{-\phi/2} g^s_{\mu\nu} \\ b_s &= e^{\phi/4} b_E \\ T^E_{\mu\nu} &= e^{2\phi} T^s_{\mu\nu} \,. \end{aligned}$$

Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization
Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.
Step 1: convert the string gas matter contributions to the 10-d Einstein frame

$$
\begin{aligned}
g^E_{\mu\nu} &= e^{-\phi/2} g^s_{\mu\nu} \\
b_s &= e^{\phi/4} b_E \\
T^E_{\mu\nu} &= e^{2\phi} T^s_{\mu\nu} \,.
\end{aligned}
$$

Joint analysis II

Step 2: Consider both dilaton and radion equations:

$$
\begin{aligned}
- \quad & \frac{M_{10}^8}{2}\Big(3a^2\dot{a}b^6\dot{\phi} + 6a^3b^5\dot{b}\dot{\phi} + a^3b^6\ddot{\phi}\Big) \\
+ \quad & \frac{3}{2}n_1 a^3 b^6 e^{-3\phi/2}\Big(b^6 e^{-\phi/2} - n_2\Big)^2 \\
+ \quad & a^3 b^{12} n_1 e^{-2\phi}\Big(b^6 e^{-\phi/2} - n_2\Big) \\
+ \quad & \frac{1}{2\epsilon}e^{\phi/4}\Big(-\mu_0\epsilon^2 + \mu_0|p_d|^2 \\
+ \quad & 6\mu_0\left[\frac{n_a^2}{\alpha'}e^{-\phi/2}b^{-2} - \frac{w^2}{\alpha'}e^{\phi/2}b^2\right]\Big) \\
= \quad & 0\,,
\end{aligned}
$$

Joint analysis III

$$
\begin{aligned}
\ddot{b} \quad + \quad & 3\frac{\dot{a}}{a}\dot{b} + 5\frac{\dot{b}^2}{b} = -\frac{n_1 b}{M_{10}^8} e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \\
& -\frac{2n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right) \\
& +\frac{1}{2-D}\left[-\frac{10b}{M_{10}^8} n_1 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2\right. \\
& \left. -\frac{12n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right)\right] \\
& +\frac{8\pi G_D \mu_0}{\alpha' \sqrt{\hat{G}_{a\epsilon}}} e^{2\phi}\left[n_a^2 b^{-2} e^{-\phi/2} - w_a^2 b^2 e^{\phi/2}\right. \\
& \left. +\frac{2}{D-1}\left(e^{\phi/2} b^2 w^2 + n \cdot w + 2(N-1)\right)\right]
\end{aligned}
$$

Joint analysis IV

Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.

Joint analysis IV

Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.

Joint analysis IV

Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.

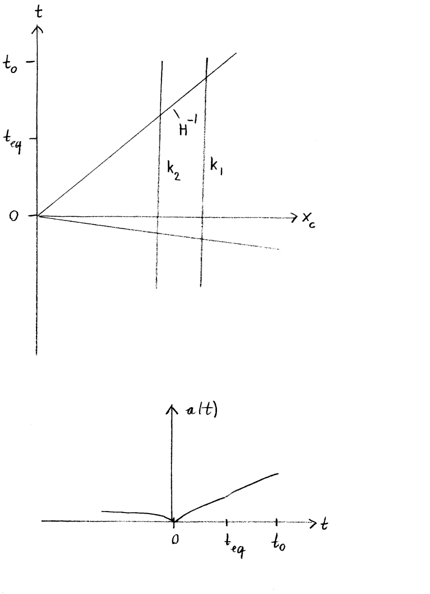
Plan

String Inflation
D. Baumann and L. McAllister, arXiv:1404.2601

- Many effective field theory models **motivated** by string theory exist.
- No model has been proven to be consistent from the point of view of superstring theory.
- Most promising approach: axion monodromy inflation [L. McAllister, E. Silverstein, A. Westphal, arXiv:0808.0706]

Ekpyrotic Bounce

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok *Phys. Rev. D64, 123522 (2001)*



Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Entropy fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Entropy fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Entropy fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Entropy fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Entropy fluctuations starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination.
- Note: Wavelengths of interesting fluctuation modes $\gg$ Planck length throughout the evolution $\rightarrow$ No Trans-Planckian Problem for cosmological fluctuations.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1 / Z_2 \,, \tag{2}$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential**.
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1/Z_2 \,, \qquad (2)$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential**.
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1/Z_2 , \quad (2)$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential**.
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1/Z_2 \,, \tag{2}$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential.**
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1/Z_2 , \tag{2}$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential.**
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

String Theory Context

- Consider heterotic M-theory [P. Horava and E. Witten]

$$\mathcal{M} = \mathcal{M}_4 x \mathrm{CY}_6 x S_1 / Z_2 \,, \tag{2}$$

- Orbifold S_1/Z_2 bounded by **orbifold fixed planes.**
- Our matter fields confined to one of the orbifold fixed planes.
- Radius of **orbifold** larger than that of CY_6.
- **Assumption**: radius r of orbifold is time-dependent due to the effects of a **potential.**
- Effective field theory: four-dimensions with an extra scalar field $\varphi \sim \ln(r/r_{pl})$
- **Assumption**: negative exponential potential.

Obtaining a Phase of Ekpyrotic Contraction

Introduce a scalar field with negative exponential potential and AdS minimum:

$$V(\phi) = -V_0 exp(-(\frac{2}{p})^{1/2}\frac{\phi}{m_{pl}}) \quad 0 < p \ll 1 \tag{3}$$

Motivated by potential between branes in heterotic M-theory
In the homogeneous and isotropic limit, the cosmology is given by

$$a(t) \sim a(t)^p \tag{4}$$

and the equation of state is

$$w \equiv \frac{p}{\rho} = \frac{2}{3p} - 1 \gg 1\,. \tag{5}$$

Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
- Horizon problem: absent.
- Flatness problem: addressed - see later.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
- Horizon problem: absent.
- Flatness problem: addressed - see later.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
- Horizon problem: absent.
- Flatness problem: addressed - see later.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
- Horizon problem: absent.
- Flatness problem: addressed - see later.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

Overview of the Ekpyrotic Bounce

- Fluctuations originate as quantum vacuum perturbations on sub-Hubble scales in the contracting phase.
- Adiabatic fluctuation mode not scale invariant.
- Entropic fluctuation modes acquire a scale-invariant spectrum of curvature perturbations on super-Hubble scales.
- Transfer of to adiabatic fluctuations on super-Hubble scales (similar to curvaton scenario).
- Horizon problem: absent.
- Flatness problem: addressed - see later.
- Size and entropy problems: not present if we assume that the universe begins cold and large.

Solution to the flatness problem

The energy density in the Ekpyrotic field scales as

$$\rho(a) = \rho_0 a^{-3(1+w)} \qquad (6)$$

and thus dominates all other forms of energy density (including anisotropic stress) as the universe shrinks $\rightarrow$ quasi-homogeneous bounce, no chaotic mixmaster behavior.

Spectrum of Adiabatic Fluctuations

If $a(t) \sim t^p$ then conformal time scales as $\eta \sim t^{1-p}$.

The solution of the mode equation for v is

$$v_k(\eta) = c_1 \eta^{-\alpha} + c_2 \eta \,, \quad (7)$$

where c_1 and c_2 are constant coefficients and $\alpha \simeq p$ for $p \ll 1$.

Hence, the power spectrum in not scale invariant:

$$P_\zeta(k,t) = \left(\frac{z(t)}{v(t_H(k))}\right)^2 k^3 |v_k(t_H(k))|^2 \sim k^3 k^{-1} k^{-2p} \sim k^{2(1-p)} \,. \quad (8)$$

Spectrum of Entropy Fluctuations I

Consider a second scalar field χ with the same negative exponential potential

$$\delta\ddot{\chi}_k + \left(k^2 + V''\right)\delta\chi_k = 0\,. \tag{9}$$

$$\delta\ddot{\chi}_k + \left(k^2 - \frac{2}{t^2}\right)\delta\chi_k = 0\,. \tag{10}$$

Vacuum initial conditions

$$\delta\chi_k \rightarrow \frac{1}{\sqrt{2k}} e^{ikt} \text{ as } k(-t) \rightarrow \infty \tag{11}$$

Spectrum of Entropy Fluctuations II

Solution:

$$\delta \chi_k \sim H^{(1)}_{3/2}(-kt) \sim k^{-3/2} \tag{12}$$

in the super-Hubble limit.

Hence

$$P_\chi(k) \sim k^3 k^{-3} \sim k^0 \,, \tag{13}$$

i.e. a scale-invariant power spectrum.

Origin of the Entropy Mode

- New Ekpyrotic Scenario (Buchbinder, Khoury and Ovrut (2007); Creminelli and Senatore (2007); Lehners et al (2007)) Assume a second scalar field χ with the same Ekpyrotic potential.
- Extra metric degrees of freedom which arise when the Ekpyrotic scenario is considered in terms of its 5-d M-theoretic origin (T. Battefeld, RB and S. Patil (2005)).

Challenges for the Ekpyrotic Scenario

- Description of the bounce.
- Initial conditions for fluctuations.

Plan

Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

Review of Inflationary Cosmology

Context:

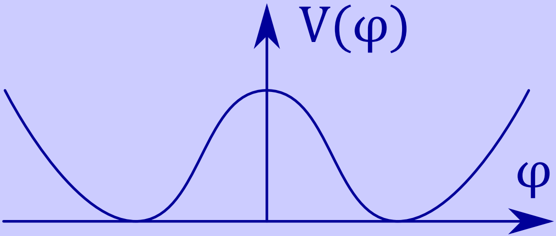
- General Relativity
- Scalar Field Matter

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long
- $\rightarrow$ field values $|\varphi| \gg m_{pl}$ or fine tuning.

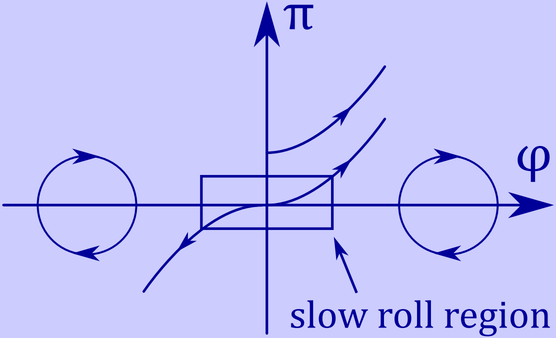
Initial Condition Problem for Small Field Inflation

D. Goldwirth and T. Piran, Phys. Rev. Lett., 1990



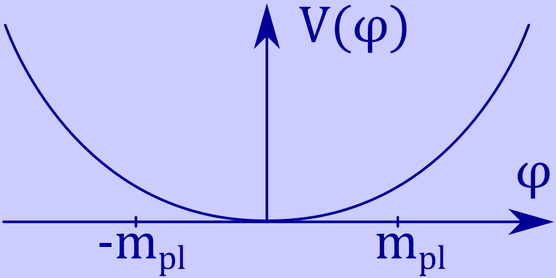
Phase Space Diagram for Small Field Inflation

Recent review: RB, arXiv:1601.01918

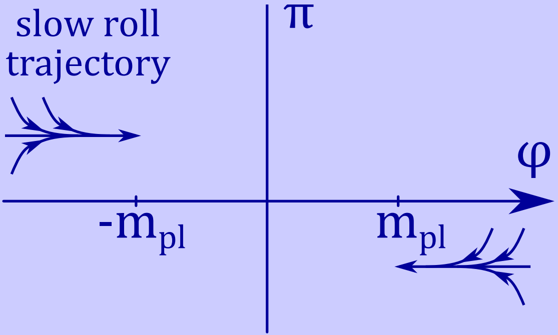


No Initial Condition Problem for Large Field Inflation

A. Starobinsky and H-J. Schmidt, 1987, J. Kung and RB, Phys. Rev. D42, 2008 (1990)



Phase Space Diagram for Large Field Inflation



Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

Successes of Inflation

- Solves horizon problem
- Solves flatness problem
- Solves size and entropy problems
- Causal generation mechanism for cosmological fluctuations
- **Predicted** slight red tilt of the power spectrum of cosmological perturbations.
- **Predicted** nearly Gaussian fluctuations.
- **Little sensitivity** on initial conditions.
- **Self consistent** effective field theory formulation.

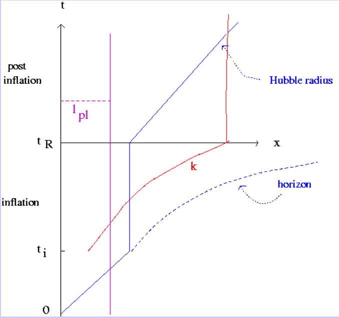
Conceptual Problems of Inflationary Cosmology

RB, hep-ph/9910410

- Singularity problem
- Trans-Planckian problem for cosmological fluctuations
- Cosmological constant problem
- Nature of the scalar field φ (the “inflaton")
- Applicability of General Relativity?
- Consistency with String Theory?

Trans-Planckian Problem

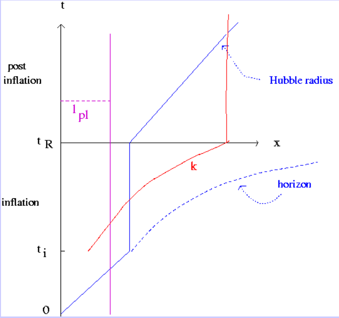
J. Martin and RB, hep-th/0005209



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

Trans-Planckian Problem

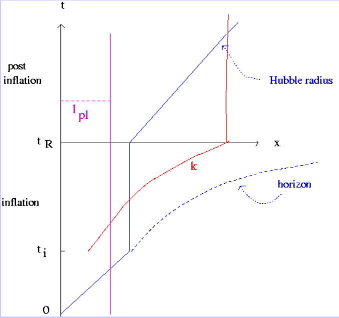
J. Martin and RB, hep-th/0005209



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

Trans-Planckian Problem

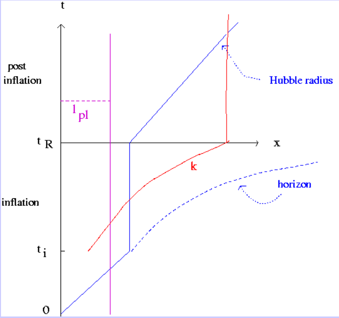
J. Martin and RB, hep-th/0005209



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

Trans-Planckian Problem

J. Martin and RB, hep-th/0005209



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

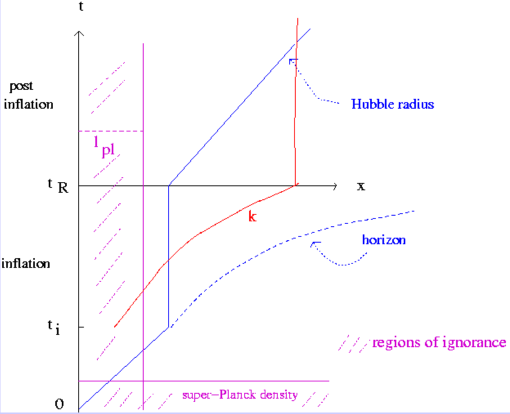
Recent Reference: A. Linde, V. Mukhanov and A. Vikman, arXiv:0912.0944

- It is not sufficient to show that the Hubble constant is smaller than the Planck scale.
- The frequencies involved in the analysis of the cosmological fluctuations are many orders of magnitude larger than the Planck mass. Thus, “the methods used in [1] are inapplicable for the description of the .. process of generation of perturbations in this scenario."

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Inflation and Fundamental Physics?

- In **effective field theory models** motivated by superstring theory there are many scalar fields, potential candidates for the inflaton.
- The quantum gravity / string theory corrections to the scalar field potentials are not under controle in most models.
- The key principles of superstring theory are not reflected in string inflation models.

Successes of String Gas Cosmology

- Solves horizon problem
- nonsingular
- No trans-Planckian problem for cosmological fluctuations.
- Causal generation mechanism for cosmological fluctuations
- **Explains** slight red tilt of the power spectrum of cosmological perturbations.
- **Explains** nearly Gaussian fluctuations.
- Natural initial state.
- Follows from basic principles of superstring theory.

Conceptual Problems of String Gas Cosmology

- Does **not** solve flatness problem.
- Does **not** solve size and entropy problems.
- **No Self consistent** effective field theory formulation.

Plan

Message

String Gas Cosmology is an alternative to cosmological inflation as a theory of the very early universe.

- Based on fundamental principles of superstring theory.
- Nonsingular.
- Fluctuations are thermal in origin.

String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Poisson-suppressed nin-Gaussianities.
- Scale-dependent non-Gaussianities.

Dynamical understanding of the Hagedorn phase is missing.

Message

String Gas Cosmology is an alternative to cosmological inflation as a theory of the very early universe.

- Based on fundamental principles of superstring theory.
- Nonsingular.
- Fluctuations are thermal in origin.

String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Poisson-suppressed nin-Gaussianities.
- Scale-dependent non-Gaussianities.

Dynamical understanding of the Hagedorn phase is missing.

Message

String Gas Cosmology is an alternative to cosmological inflation as a theory of the very early universe.

- Based on fundamental principles of superstring theory.
- Nonsingular.
- Fluctuations are thermal in origin.

String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Poisson-suppressed nin-Gaussianities.
- Scale-dependent non-Gaussianities.

Dynamical understanding of the Hagedorn phase is missing.

Message

String Gas Cosmology is an alternative to cosmological inflation as a theory of the very early universe.

- Based on fundamental principles of superstring theory.
- Nonsingular.
- Fluctuations are thermal in origin.

String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Poisson-suppressed nin-Gaussianities.
- Scale-dependent non-Gaussianities.

Dynamical understanding of the Hagedorn phase is missing.

Message

String Gas Cosmology is an alternative to cosmological inflation as a theory of the very early universe.

- Based on fundamental principles of superstring theory.
- Nonsingular.
- Fluctuations are thermal in origin.

String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Poisson-suppressed nin-Gaussianities.
- Scale-dependent non-Gaussianities.

Dynamical understanding of the Hagedorn phase is missing.