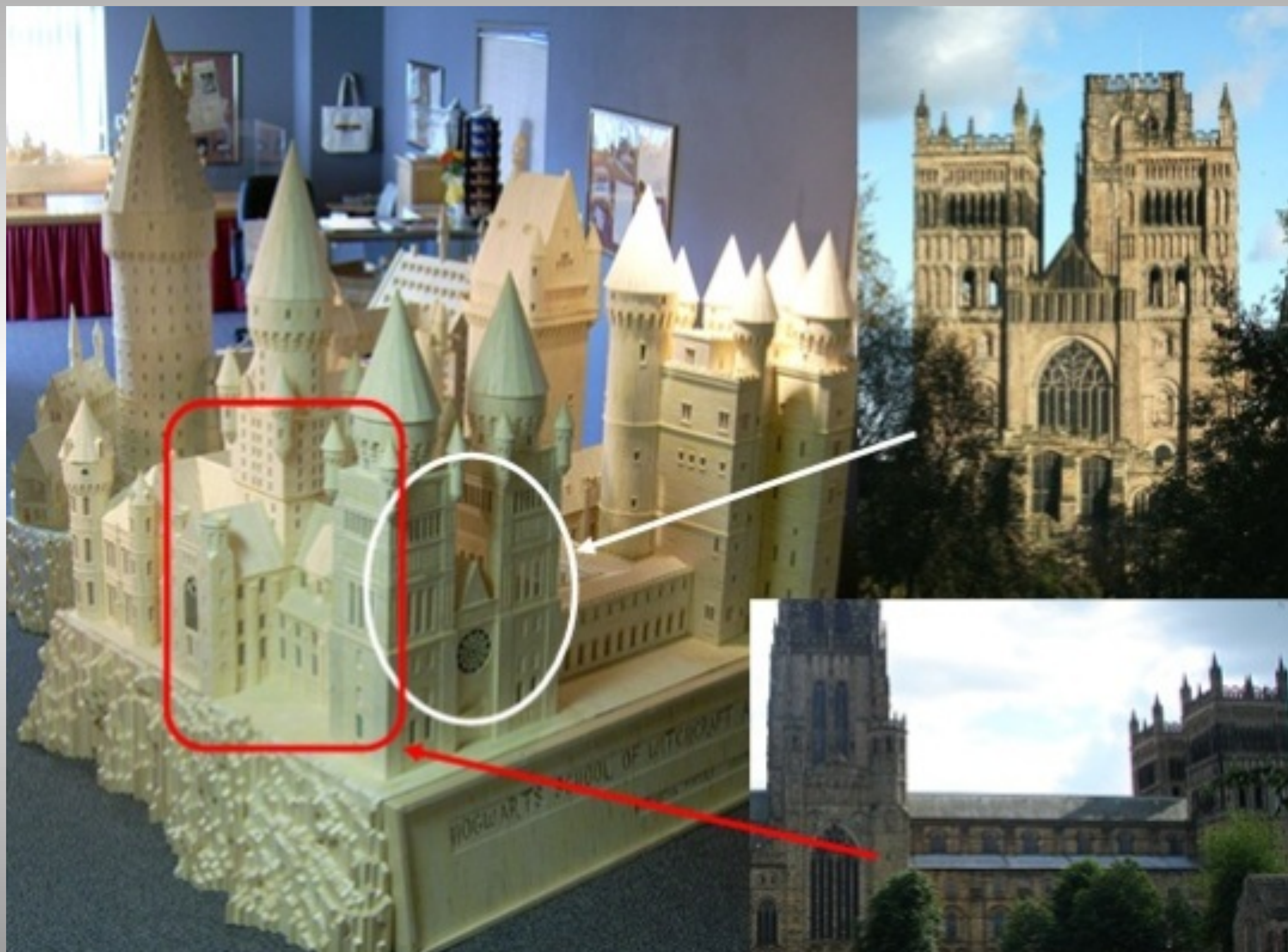
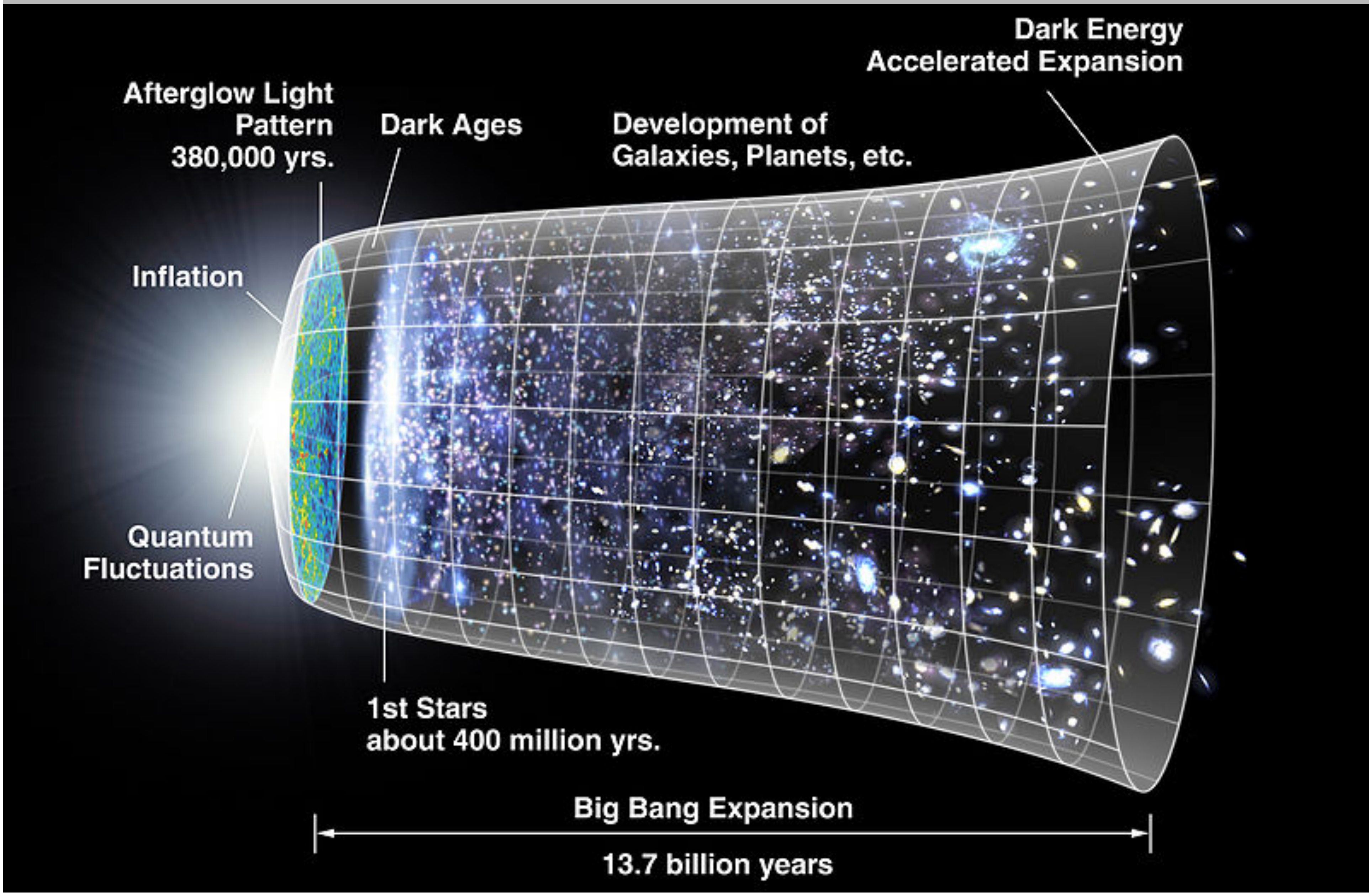
# Non-Perturbative Gravity in Cosmology

*Ruth Gregory*
Centre for Particle Theory

Durham University



Big picture well described by simplish physics and perturbation theory, but detail needs a closer look.



# For Further Detail

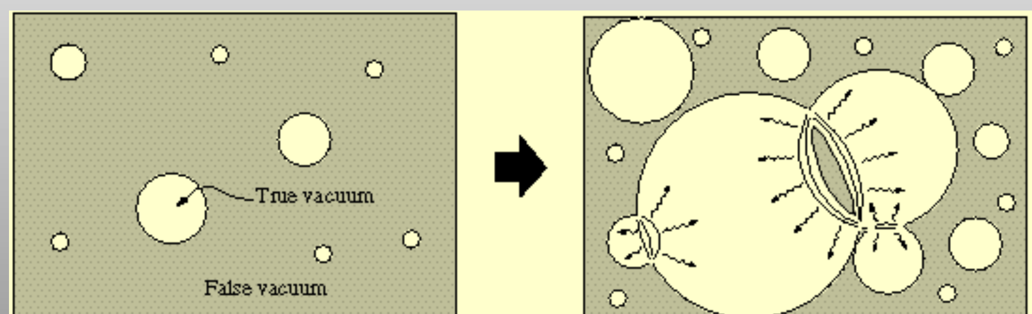
"Review" section of:
http://www.perimeterinstitute.ca/training/perimeter-scholars-international/lectures/2014/2015-psi-lectures

"Gravitational Physics"

# Non-Perturbative Gravity

- Strongly gravitating systems – black holes, cosmic structures

- First order phase transitions – bubbles, defects



# Non-Perturbative Gravity

- Phase transitions, relics, working with gravity
- Tunneling and Euclidean techniques in gravity
- Black holes, tunneling, the fate of our universe!

# Phase transitions and their relics

Working with Gravity : The Domain Wall

Israel's Equations

Walls in First Order Phase Transitions

# Gravity and Field Theory

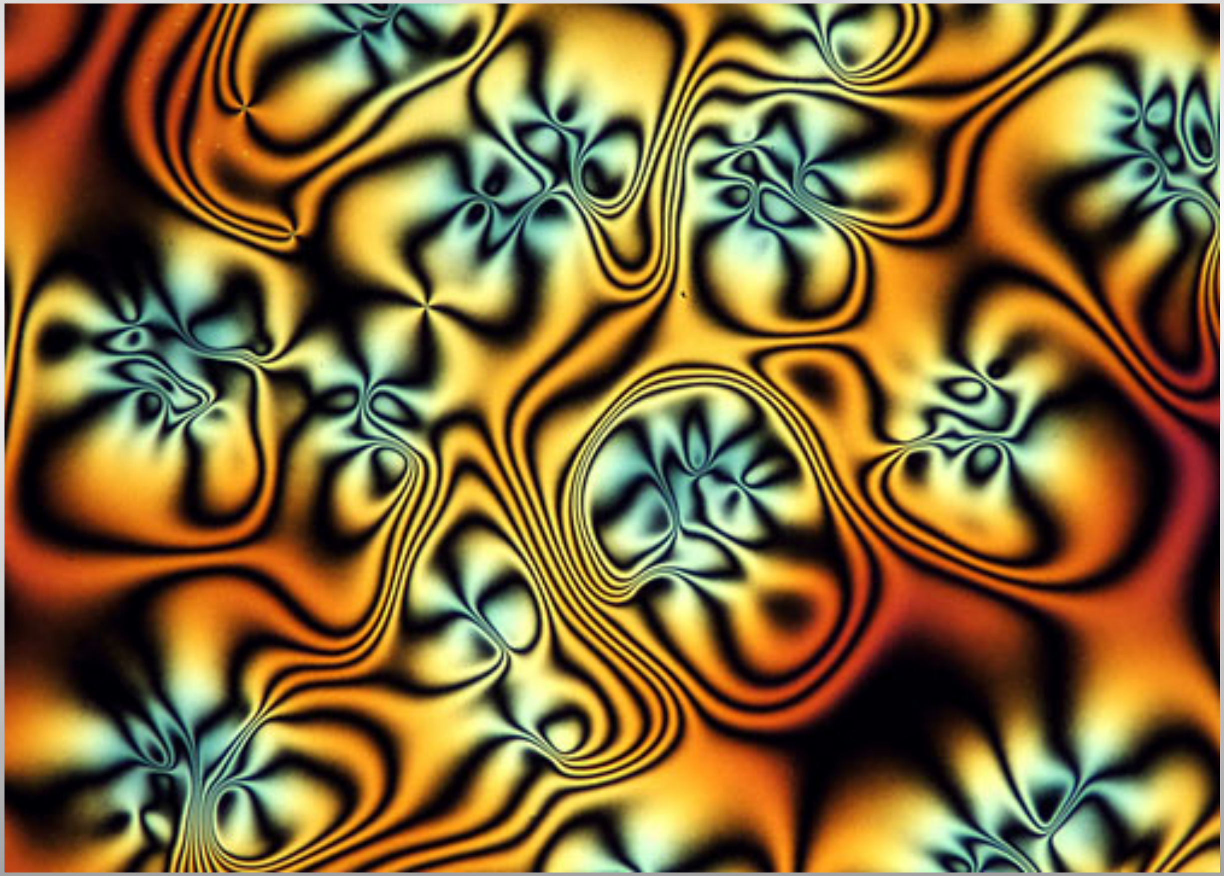
Lagrangian formalism natural to couple gravity to Particle Physics.

$$\mathcal{S} \sim \int d^4x\sqrt{g}\left[-\frac{R}{16\pi G}+\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

But at finite temperature, the effective potential is modified – at high T symmetry restored, but as T drops, the Universe undergoes phase transitions.

$$V_{eff} \sim V_0 + \phi^2 T^2 + ...$$

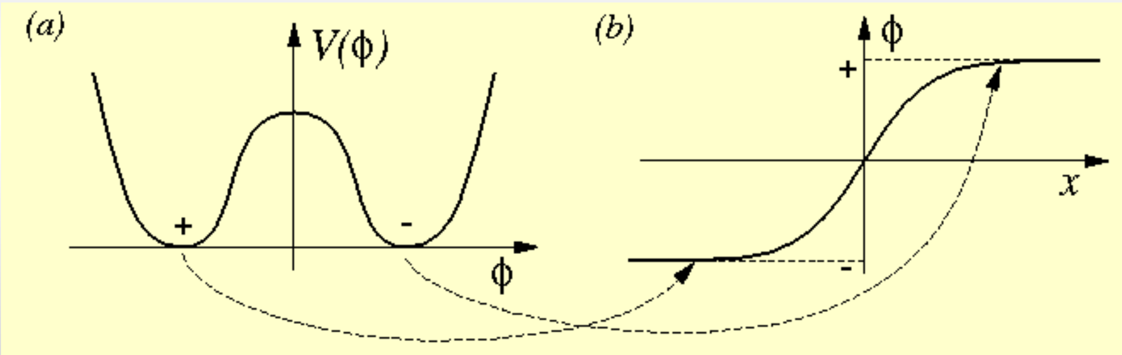
During a phase transition, the correlation length is typically somewhat less than the Hubble radius, hence there can be regions in different vacuum states. The boundaries between these regions become defects.



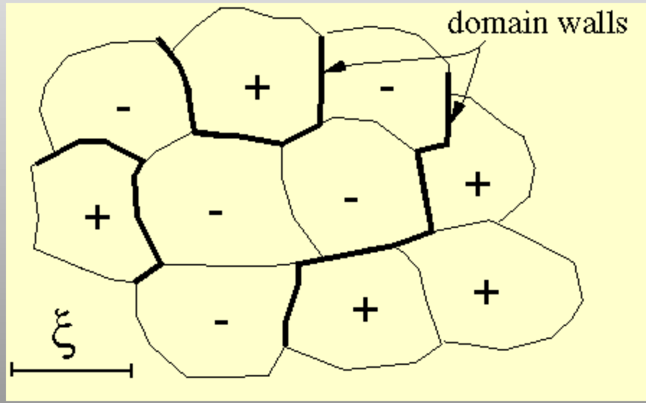
NEMATIC LIQUID CRYSTAL

# Simple example: Domain Wall

A simple example of a non-perturbative effect is the Domain Wall – forms when there is a transition between two distinct vacua.




Different regions exit into different vacua – the boundaries between are the “walls”.



# "The Kink"

The kink is an interpolation between two vacua:

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$

The equation of motion has an analytic solution (w/o gravity)

$$\nabla^2\phi + 2\lambda\phi(\phi^2 - \eta^2) = 0 \qquad\qquad \phi = \eta \tanh \sqrt{\lambda}\eta(z - z_0)$$

Characteristic width : $1/\sqrt{\lambda}\eta$

Energy momentum sharply localized.

$$\begin{aligned} T_{ab} &= \partial_a\phi\partial_b\phi - g_{ab}\left(\frac{1}{2}(\partial\phi)^2 - V\right) \\ &= \lambda\eta^4 \operatorname{sech}^2\sqrt{\lambda}\eta(z - z_0)\ \mathrm{diag}(1, -1, -1, 0) \end{aligned}$$

# Add Gravity

The wall is strongly localized in the z-direction, with finite energy per unit area

$$\sigma = \frac{4}{3}\sqrt{\lambda}\eta^3$$

.....but infinite area!


$T^0{}_0 = T^x{}_x = T^y{}_y$ suggests a constant curvature space parallel to wall:

$$ds^2 = A^2(z)\left[dt^2 - C_\kappa^2(t)(dx^2 + dy^2)\right] - dz^2$$

Get Friedmann-like equations:

$$\frac{\kappa}{\ell^2 A^2} - \frac{A'^2}{A^2} = \frac{8\pi G}{3}\left(V - \frac{1}{2}\phi'^2\right)$$
$$\frac{\kappa}{\ell^2 A^2} - \frac{A'^2}{A^2} - 2\frac{A''}{A} = 8\pi G\left(V + \frac{1}{2}\phi'^2\right)$$

With solution

$$\ell = \frac{3}{16\pi G\eta^2}$$

$$A = 1 - \frac{4\pi G\eta^2}{3}\left(4\log\left[2\cosh\sqrt{\lambda}\eta(z-z_0)\right] - \mathrm{sech}^2\sqrt{\lambda}\eta(z-z_0)\right)$$

Note, A -> 0 for z ~ $\ell$ – a coordinate singularity

Look near z ~ $\ell$

$$A'' \simeq 0 \quad , \qquad A \sim 1 \mp z/\ell$$

$$ds^2 \sim (1 - z/\ell)^2 \left[ dt^2 - \ell^2 \cosh^2(\frac{t}{\ell}) d\Omega_{II}^2 \right] - dz^2 \qquad \text{z>0}$$

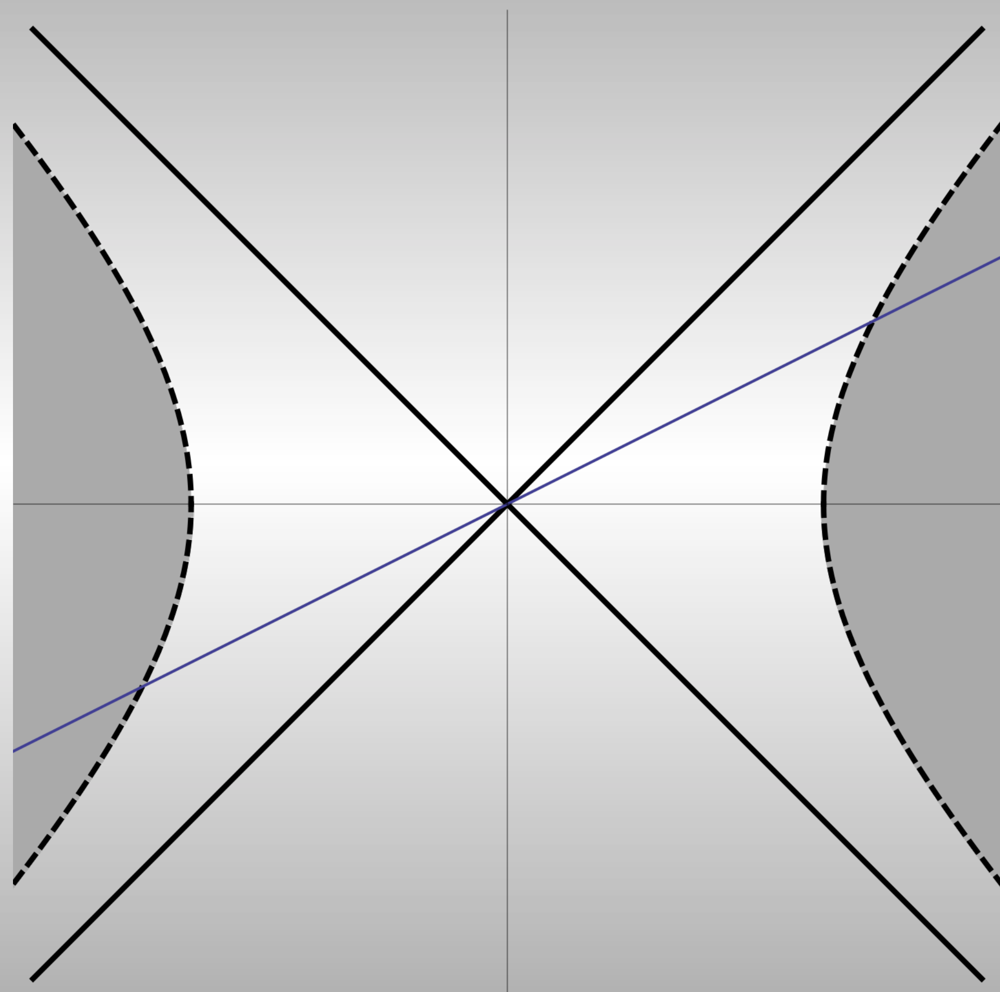
Change coords:

$$\left.\begin{aligned} \rho &= (\ell - z)\cosh(\frac{t}{\ell}) \\ \tau &= (\ell - z)\sinh(\frac{t}{\ell}) \end{aligned}\right\} \text{ flat spacetime}$$

$$z = \text{const.} \qquad\qquad \rho^2 - \tau^2 = (\ell - z)^2$$

Constant z lines are hyperbolae.

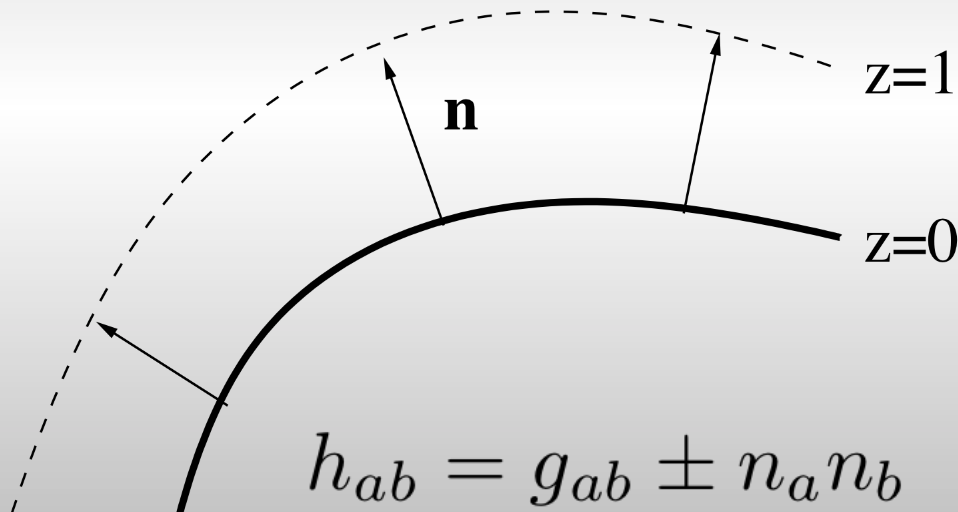
As A goes to zero, there is a horizon for an observer on the wall.




The picture for z<0 is a mirror image.

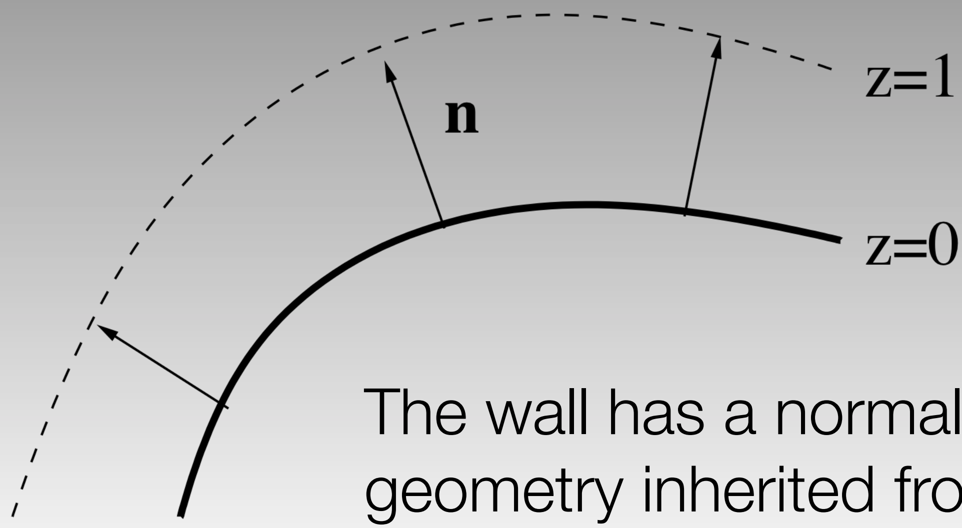
# Simpler Solution?

Since the wall is so “thin”, we can approximate with a delta-function – this is the ISRAEL approach.




$$h_{ab} = g_{ab} \pm n_a n_b$$

The wall inherits its metric from the space-time




The wall has a normal, n=(-)dz, and an intrinsic geometry inherited from our spacetime. The wall can curve in spacetime, this is measured by Extrinsic Curvature:

$$K_{ab} = \nabla_a n_b$$

Israel's equations then relate the jump in extrinsic curvature across the wall to its energy-momentum

$$\Delta K_{ab} - \Delta K h_{ab} = 8\pi G S_{ab}$$

For our wall, the hyperboloid in flat space has normal

$$\mathbf{n} = -dz = (\rho d\rho - \tau d\tau)/\sqrt{\rho^2 - \tau^2}$$

Extrinsic curvature:

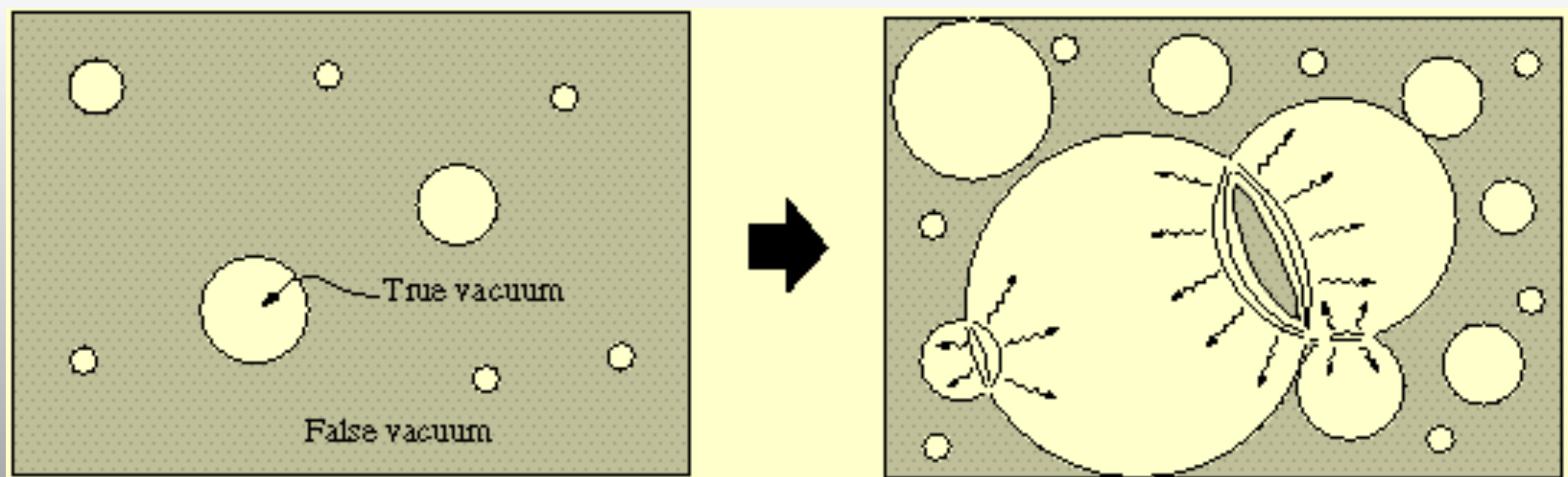
$$K_{\rho\rho} = \frac{-\tau^2}{(\rho^2 - \tau^2)^{5/2}} = -\frac{h_{\rho\rho}}{\ell} \quad ; \quad K_{\rho\tau} = -\frac{h_{\rho\tau}}{\ell} \quad ; \quad K_{\tau\tau} = -\frac{h_{\tau\tau}}{\ell}$$

So Israel gives:

$$\Delta K_{ab} - \Delta K h_{ab} = \frac{2}{\ell} h_{ab}$$

# Why Walls?

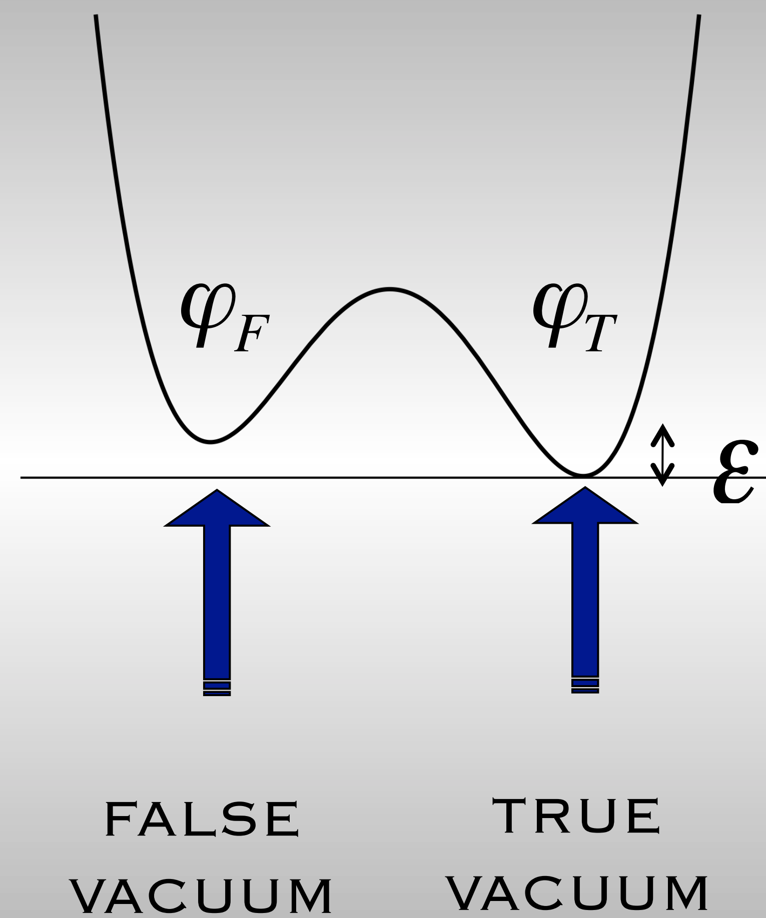
During a phase transition, if there is any nontrivial vacuum structure, defects will form. The wall comes from distinct vacua, so occurs with a first order phase transition



# Tunneling from the False Vacuum

A false vacuum is a local minimum, so at low energies we see a ‘normal’ particle spectrum (vacuum) and do not see it is not a global minimum.

The global minimum is reached at high energy, or by quantum tunnelling.

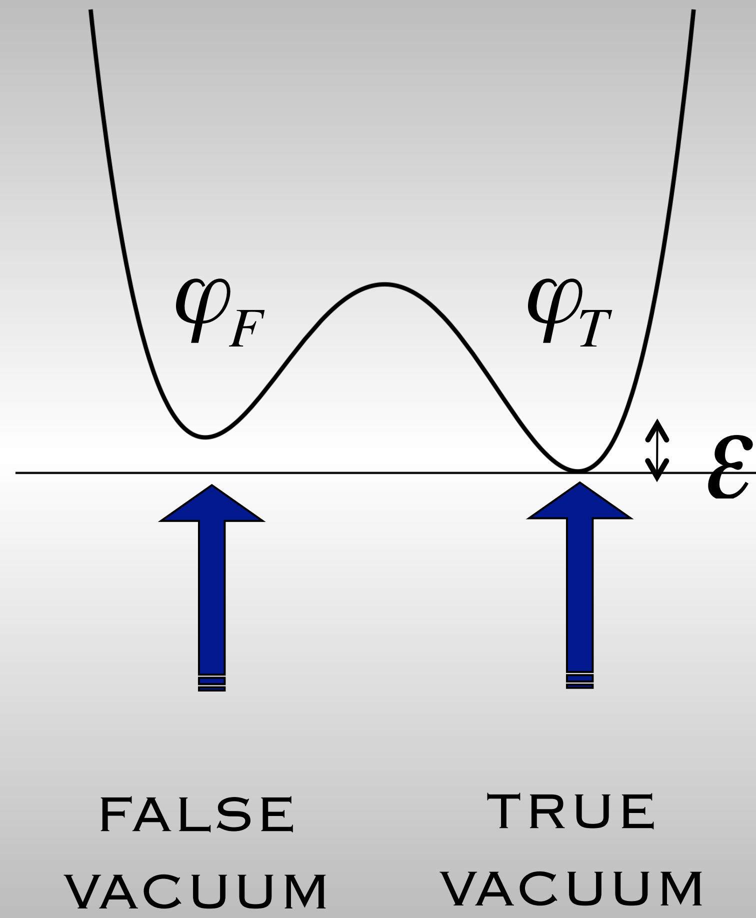


# Field Theory

This gives a first order phase transition, where we tunnel from one local energy minimum to another with lower overall energy.

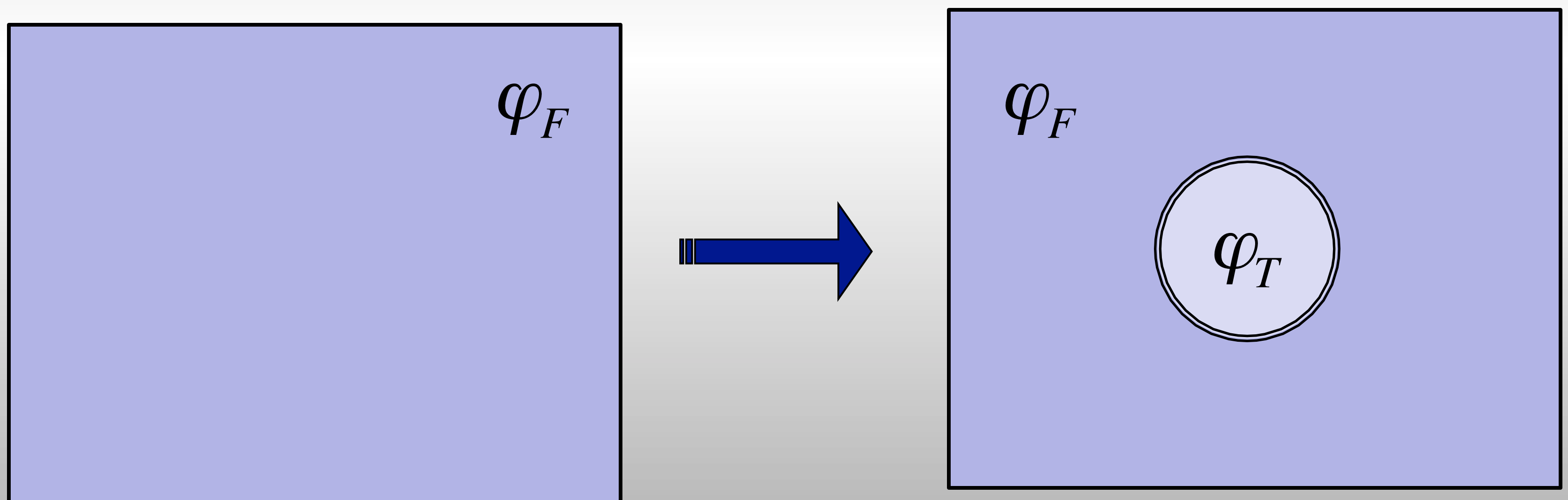
e.g. “old” inflation had a scalar field trapped in a false vacuum giving exponential expansion

Picture applies to any system with disconnected vacua.



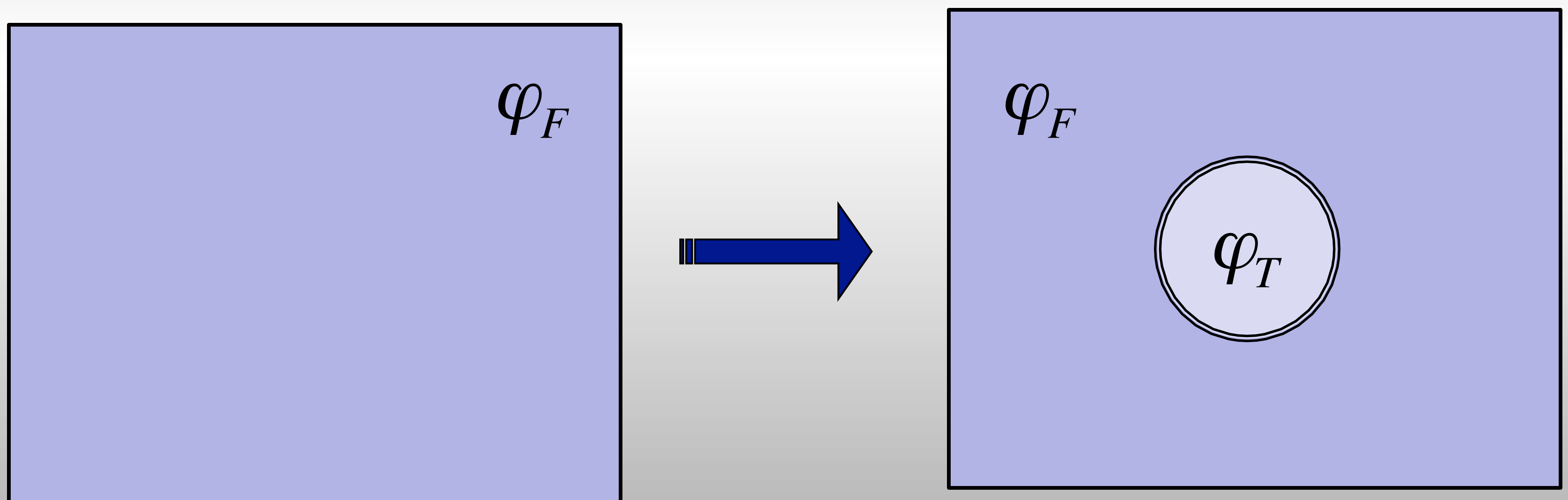
# Walls between vacua

A bubble of true vacuum forms inside the false vacuum:



# This is what we will calculate!

A bubble of true vacuum forms inside the false vacuum:



# Tunneling and Euclidean Gravity

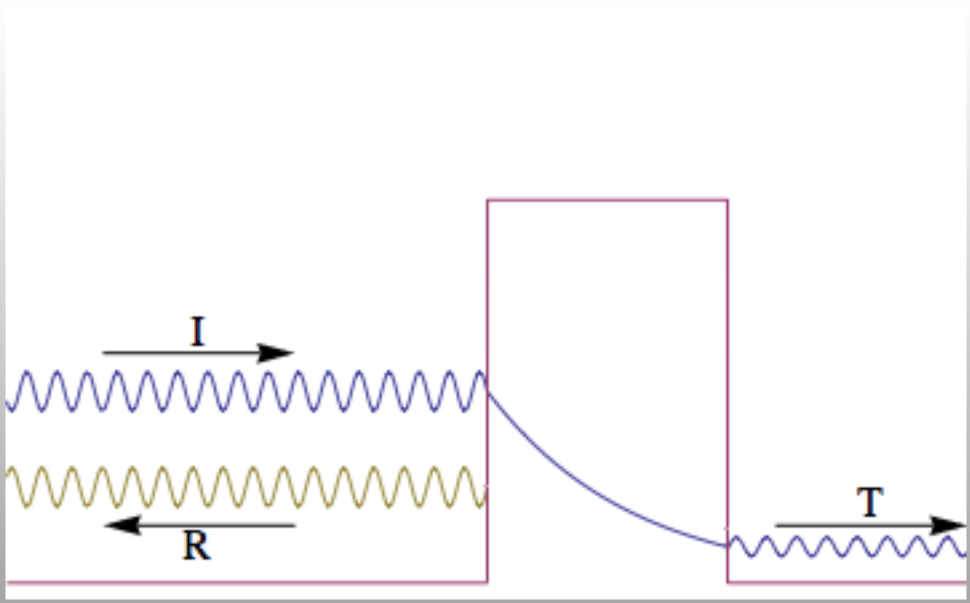
Tunneling, Euclideanisation & The Bounce

Coleman de Luccia

More on Euclidean Gravity

# Quantum Tunneling

Tunneling is an example of Quantum Mechanics in action – a classical particle with energy E less than a barrier of height V will rebound, but quantum mechanically the wave function never cuts off under a finite barrier, but decays – meaning that a little emerges through the other side:




$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

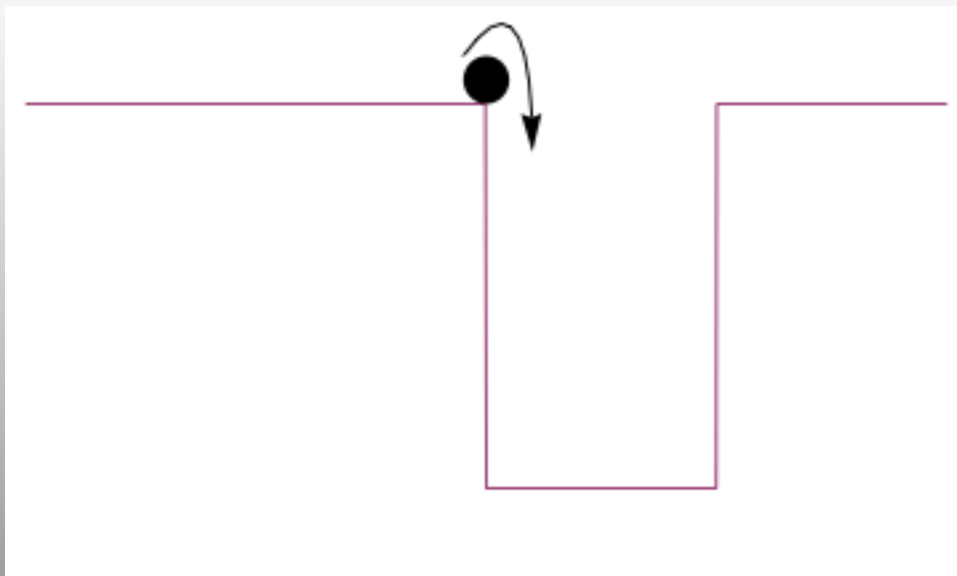
# Euclidean Perspective

Now rotate to imaginary time:

$$t \to i\tau$$

A classical particle moving in imaginary time has kinetic energy equal to the potential drop, so the amplitude $|T|^2$ now looks like the action integral for this classical motion.

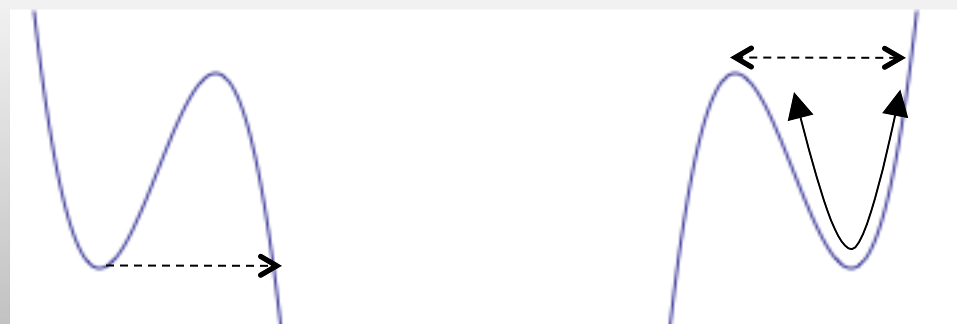
$$\frac{1}{2}\dot{x}^2 = \Delta V$$



$$\int \sqrt{2\Delta V}\,dx = \int 2\Delta V\,d\tau$$
$$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) d\tau$$

# Euclidean Trick

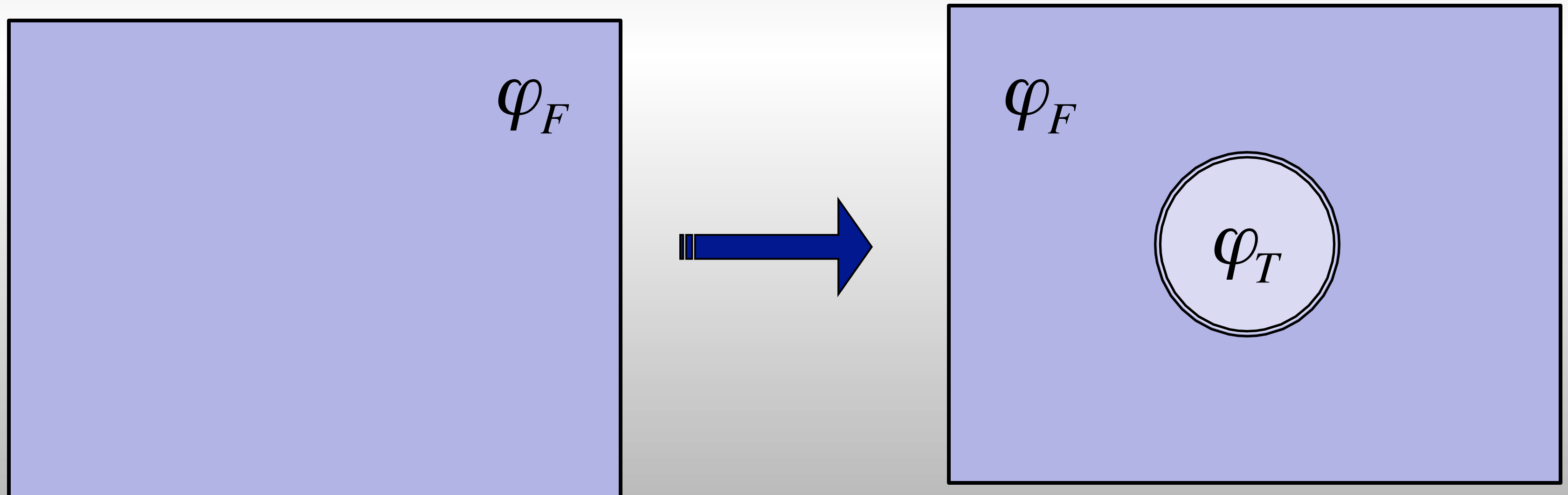
Generally, to compute leading behaviour of a tunneling amplitude take action of a classical particle moving in an inverted potential. The particle rolls from the (now) unstable point to the “exit” and back again – a “bounce”.



The action of this bounce gives the exponent in the amplitude of the wavefunction.

# Coleman Bounce

A bubble of true vacuum forms inside the false vacuum described by the Euclidean solution



# Coleman

Solving the Euclidean field equations should give the saddle point approximation for the tunneling solution.

$$\frac{d^2\phi}{d\tau^2} + \nabla^2\phi = -\frac{\partial V}{\partial\phi} = 2\lambda\phi(\phi^2 - \eta^2) + \mathcal{O}(\epsilon)$$

Original work of Coleman took a field theory with a “false” vacuum: in limit of small energy difference (relative to barrier) transition modeled by a “thin wall” bubble.

$$\phi'' + \frac{3}{\rho}\phi' = 2\lambda\phi(\phi^2 - \eta^2) \qquad [\rho^2 = \tau^2 + \mathbf{x}^2]$$

$$\phi \approx \eta \tanh[\sqrt{\lambda}\eta(\rho - \rho_0)]$$

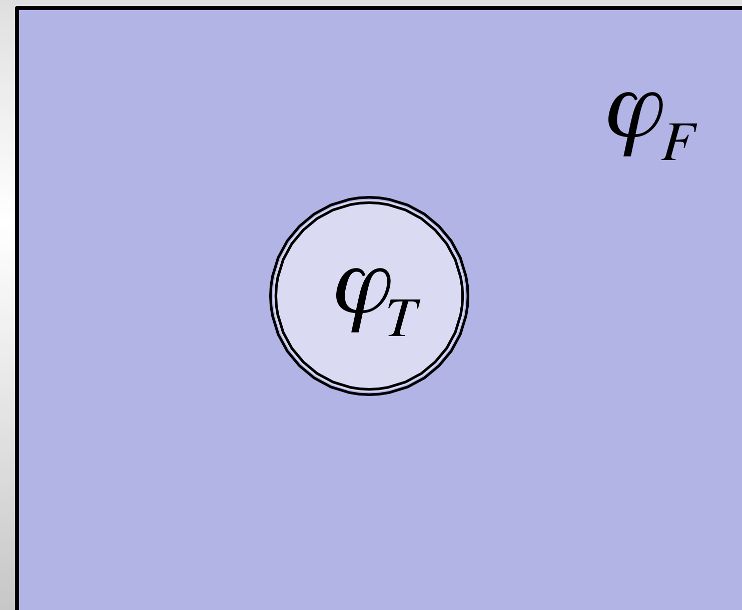
# Coleman's argument

Amplitude determined by action of Euclidean tunneling solution: “The Bounce”

$$\mathcal{B} = -\varepsilon \int d^4x\sqrt{g} + \sigma \int d^3x\sqrt{h}$$
$$\sim -\frac{\pi^2}{2}\varepsilon R^4 + 2\pi^2\sigma R^3$$

GAIN FROM VACUUM

COST OF WALL



# Coleman

Since the bounce is a solution to eqns of motion, it should be stationary under variation of R:

$$R = \frac{3\sigma}{\varepsilon} \quad , \quad \mathcal{B} = \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

Tunneling amplitude:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

(Notice, R is big, so justifies use of the “thin wall” approximation.)

# Coleman-de Luccia

Includes gravity:

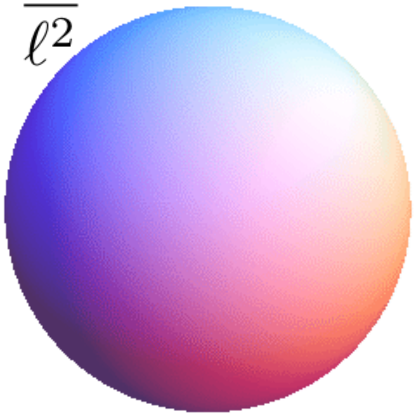
o The instanton is a solution of the Euclidean Einstein equations with a bubble of flat space separated from dS space by a thin wall.
o The wall radius is ***determined*** by the Israel junction conditions
o The action of the bounce is the difference of the action of this wall configuration and a pure de Sitter geometry.

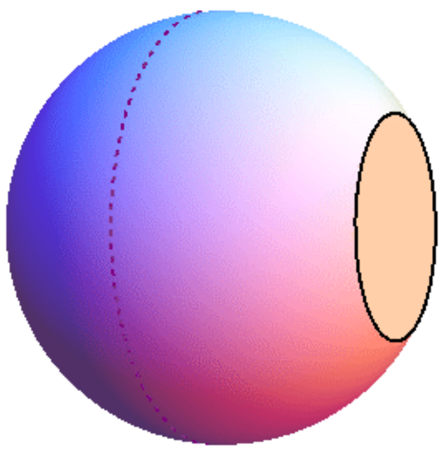
*Coleman and de Luccia, PRD21 3305 (1980)*

# CDL Instanton

Euclidean de Sitter space is a sphere, of radius $\ell$ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$



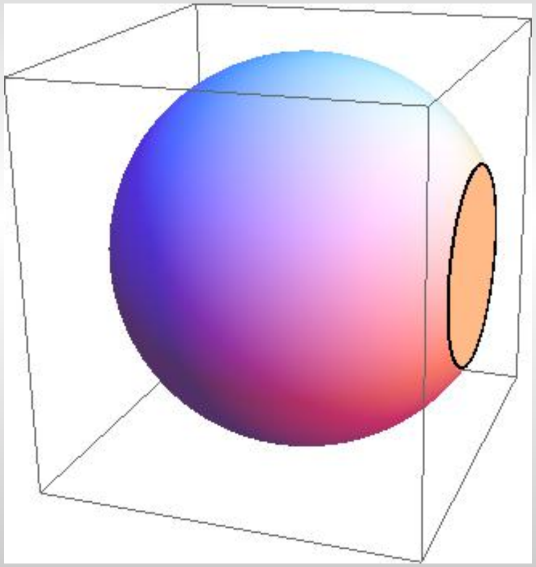


The bounce looks like a truncated sphere.

# CDL Action

To compute action, we have to integrate Ricci curvature

$$ds^2_{\rm dS} = d\chi^2 + \ell^2 \sin^2 \frac{\chi}{\ell}\, d\Omega^2_{III} \quad , \quad S_{\rm dS} = -\frac{\ell^2 \pi}{G}$$



Israel conditions give truncation radius:

$$\frac{3}{\ell}\left(\cot\chi_0 - \csc\chi_0\right) = -4\pi G\sigma$$

hence bounce action:

$$\mathcal{B} = -\frac{\Lambda}{8\pi G}\int_{\rm int} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h}$$

$$= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar\sigma^4\ell^4}{(1+4\bar\sigma^2\ell^2)^2}$$

$$\bar\sigma = 2\pi G\sigma$$

# G $\Longrightarrow$ O??

Just to check….

$$\mathcal{B} = \frac{\pi \ell^2}{G} \frac{16\bar{\sigma}^4 \ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}$$

$$\sim 256\pi^5 G^3 \sigma^4 \ell^6$$

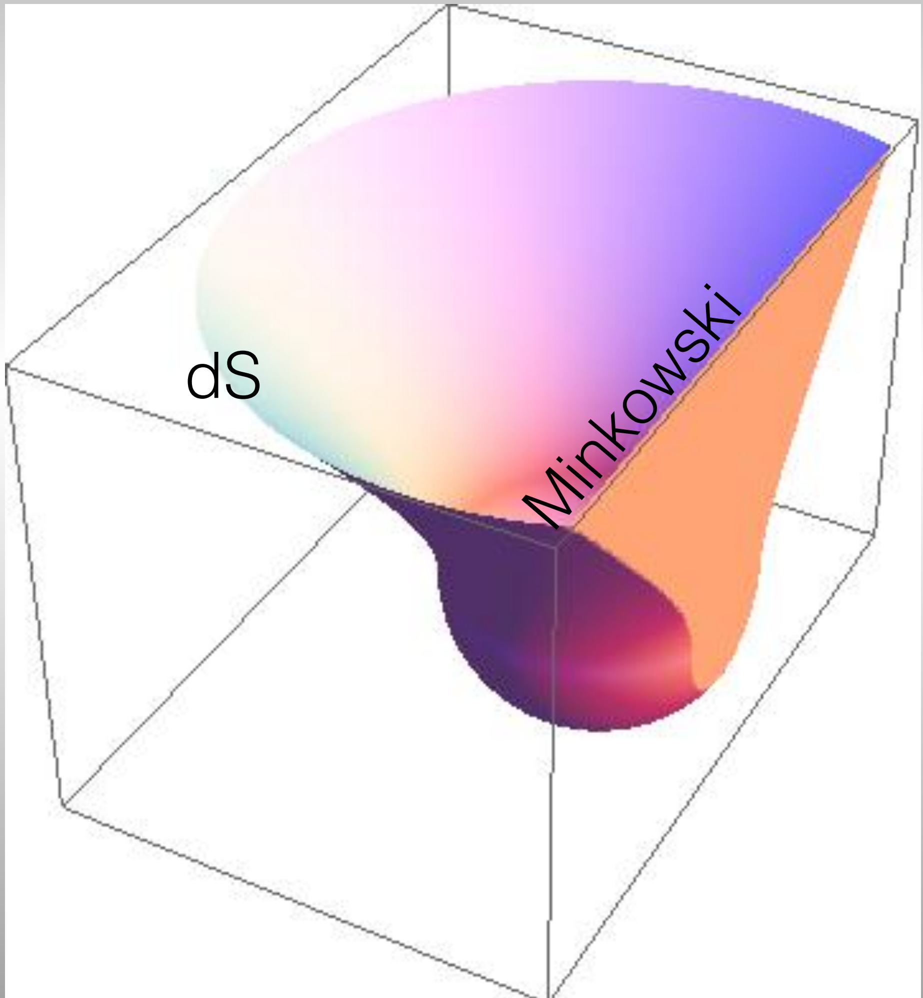
$$= 256\pi^5 G^3 \sigma^4 \left(\frac{3}{8\pi G \varepsilon}\right)^3$$

$$= \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

$$\bar{\sigma} = 2\pi G \sigma$$

$$\Lambda = \frac{3}{\ell^2}$$

# Geometrical Picture




de Sitter space is represented by a hyperboloid (sphere) in 5D Lorentzian (Euclidean) spacetime. The instanton is often represented by joining the virtual Euclidean geometry to the real Lorentzian geometry across a surface of ‘time’ symmetry.

# More on Euclidean Gravity

Although Euclideanisation is a “trick” for tunneling, there is more physics in the Euclidean section. Take the Schwarzschild black hole:

$$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

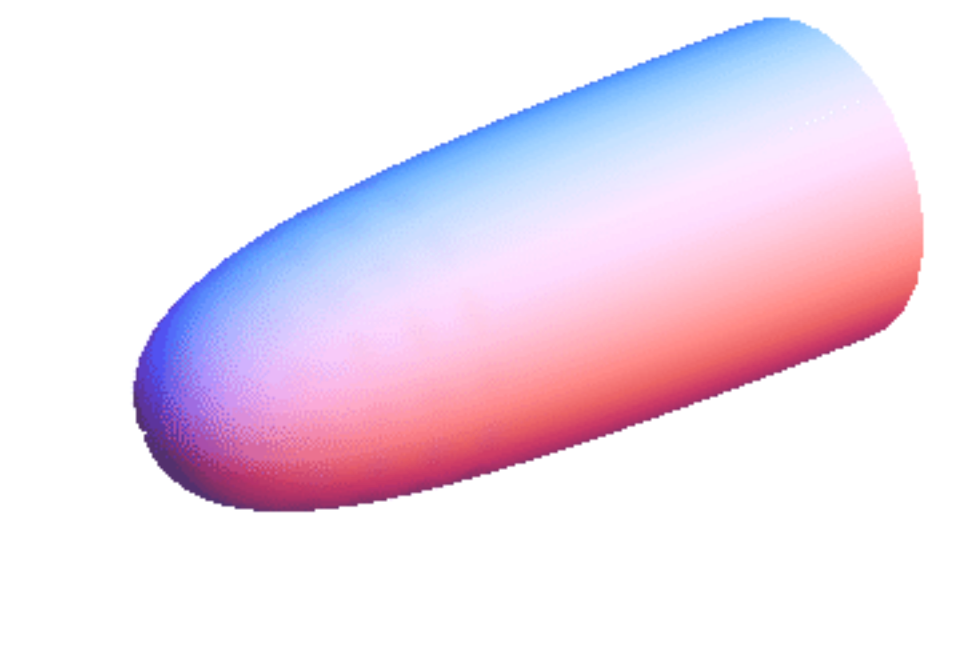
Near 2GM: $$\sim \rho^2 d\left(\frac{\tau}{4GM}\right)^2 + d\rho^2 + (2GM)^2 d\Omega_{II}^2$$

Where $$\rho^2 = 8GM(r - 2GM) \qquad \tau \sim \tau + 8\pi GM$$

# Euclidean Schwarzschild

I.e. redefining to a proper radial coordinate gives a locally flat space at 2GM. If we do not want a conical singularity, then we must make $\tau$ periodic with periodicity $8\pi GM$. Periodic Euclidean time is the smoking gun of a finite temperature field theory!

The Euclidean manifold looks like a “cigar”.



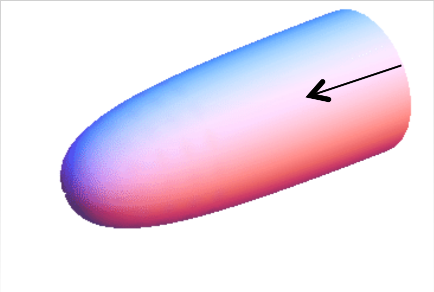
# Calculating actions

Although it looks like the action is zero, because R=0, this is not the whole story. The actual action has a boundary term (Gibbons-Hawking)

$$I_E = \int_{\mathcal{M}} \frac{R\sqrt{g}}{16\pi G} d^4x + \int_{\partial\mathcal{M}} \frac{K\sqrt{h}}{8\pi G} d^3x$$

So although the first piece is zero, the second piece isn't!

# Gibbons Hawking



$$n = -\sqrt{1 - \frac{2GM}{R}}\,\frac{\partial}{\partial r}$$

$$\nabla_a n^a = K = \frac{2}{r} n^r + \frac{GMn^r}{r(r-2GM)}$$

$$\int K\sqrt{h}d^3x = -4\pi\beta\left[2R - 3GM\right]$$

This term looks infinite, but is infinite even if M=0, so must renormalize.

$$I - I_0 = \frac{\beta M}{2}$$

# Black Holes & Tunneling

More General Bubbles

Singular Instantons

The Higgs Vacuum

# A more general look

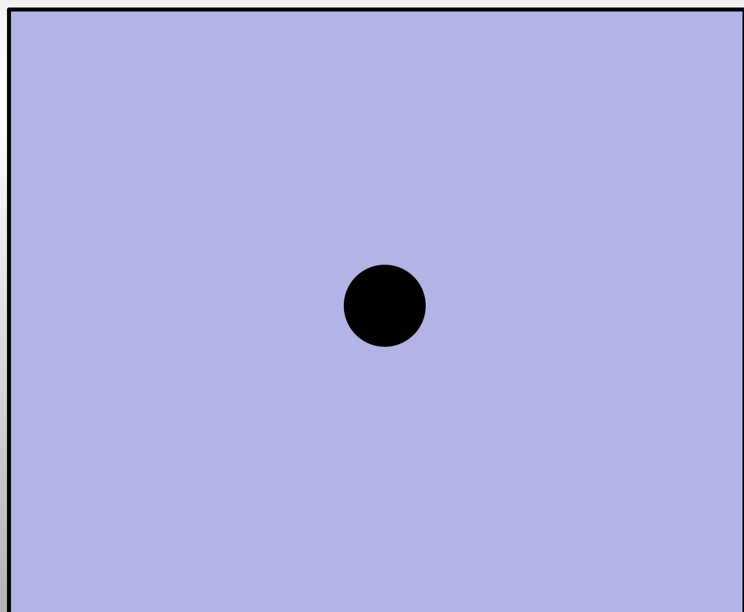
The Coleman de Luccia instanton started a trend of understanding more complex and physically realistic tunnelling scenarios, including gravity *and* nonlinear field theory.

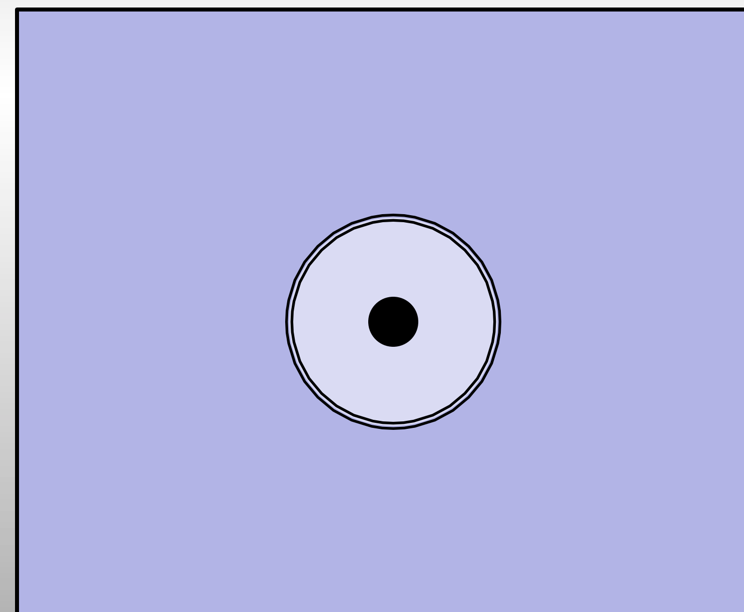
CDL still the “gold standard” in computing probability of false vacuum decay, but –

? How dependent is amplitude on homogeneity?

# Tweaking CDL

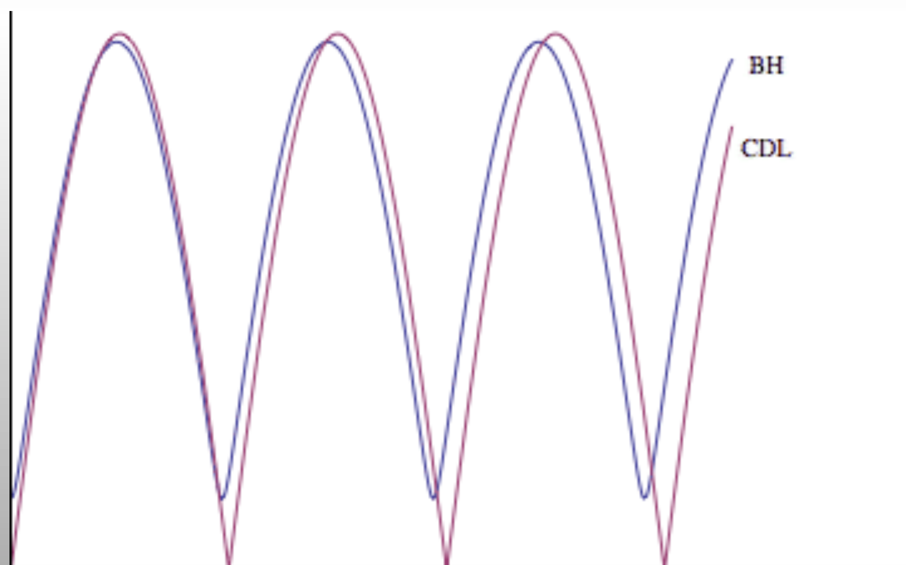
The bubble of true vacuum has a spherical symmetry, so we can add a black hole at “minimal expense”!

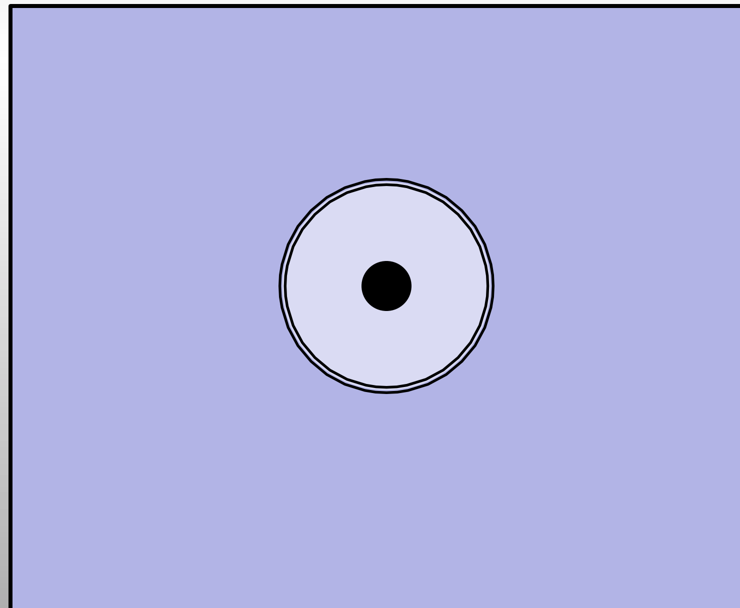




# Tweaking CDL action

The area of the bubble is still $4\pi r^2$, but the interior "volume" increases relative to r due to the curved geometry of the black hole. This adjusts the Coleman argument meaning that the bubble forms at smaller radius, and with smaller action.



# A more general bubble

The wall now will separate two different regions of spacetime, each of which solve the Einstein equations:

$$ds^2 = f(r)dt^2 \pm \left[f^{-1}(r)dr^2 + r^2 d\mathbf{x}_\kappa^2\right]$$

$$f(r) = \kappa - \frac{\Lambda}{3}r^2 - \frac{2GM}{r}$$

The regions in general have different cosmological constants, and possibly a black hole mass.

*Bowcock, Charmousis, RG: CQG17 4745 (2000)*

# Wall trajectories

Wall trajectory:

$$X^\mu = (t(\lambda), R(\lambda), \theta, \phi)$$

$$g_{tt}\dot{t}^2 \pm g_{rr}\dot{R}^2 = 1$$

Israel junction conditions determine the equation of motion:

$$\Delta K_{ab} = -4\pi G \sigma h_{ab}$$

Inputting the form of the trajectory gives a Friedmann like equation for R:

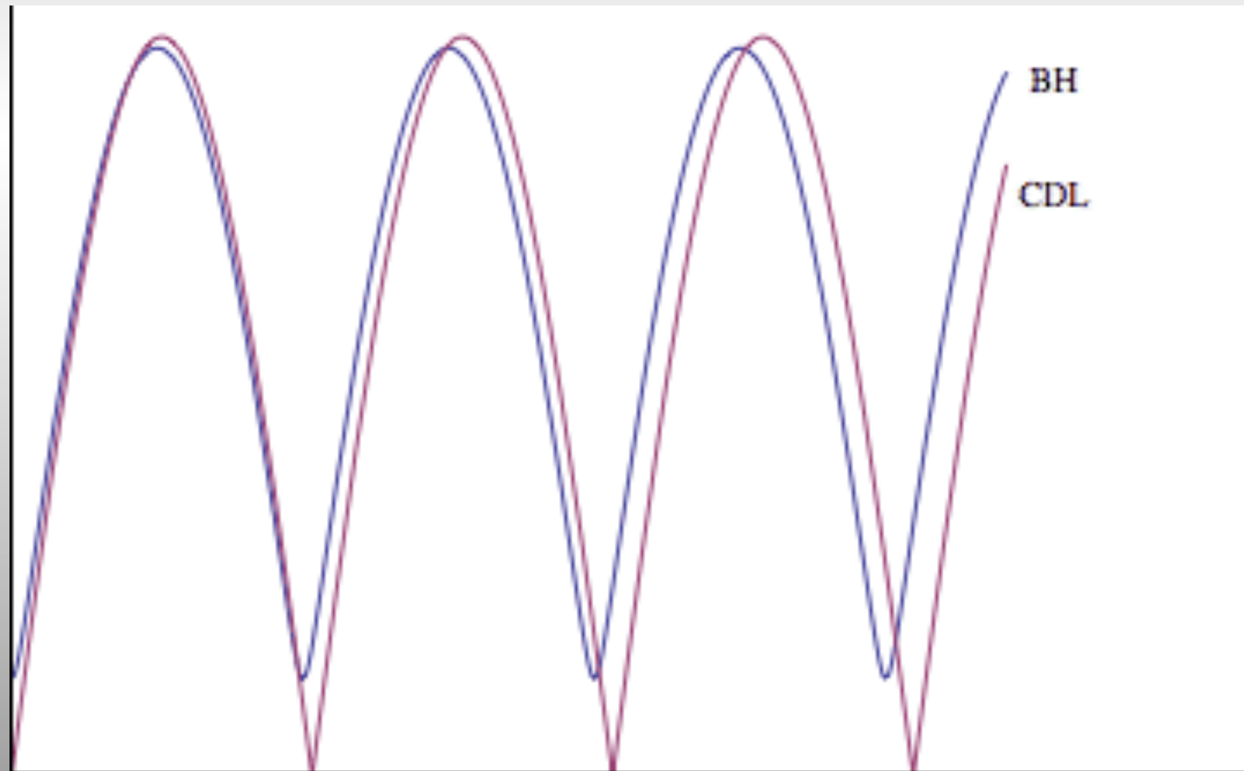
Lorentz

$$\pm \left(\frac{\dot{R}}{R}\right)^2 = \bar{\sigma}^2 - \frac{\bar{f}}{R^2} + \frac{(\Delta f)^2}{16\bar{\sigma}^2 R^4}$$

Euclid

# Adding a black hole

Black hole modifies the wall trajectory only slightly. Key difference is that wall never has zero area – it must stay outside the event horizon.

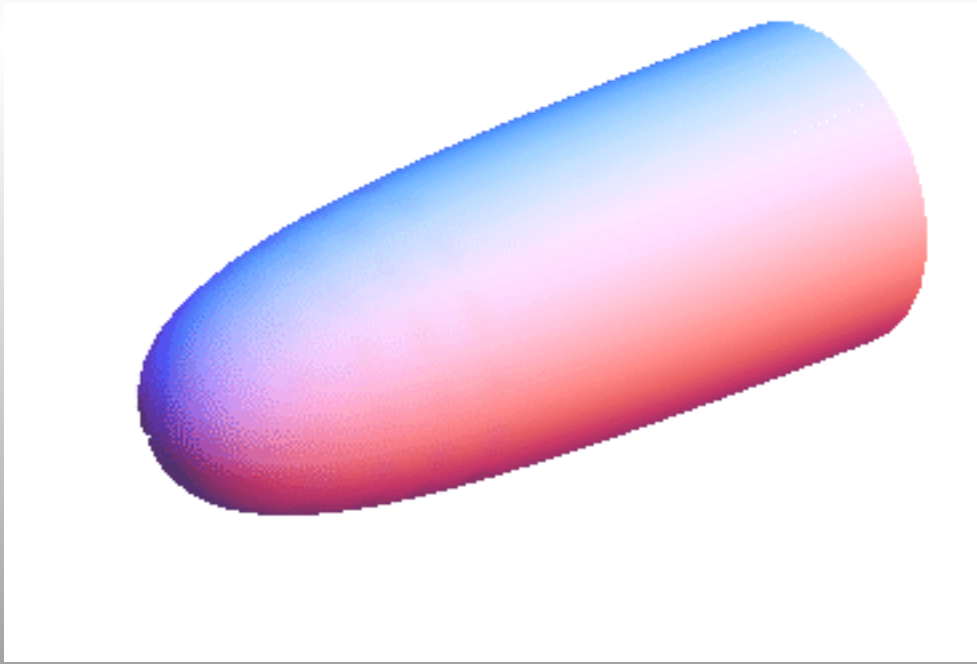


# What Changes?

- The Euclidean black hole is understood, but it is very different from Euclidean flat space – it has a periodic Euclidean time.
- The black hole distorts the geometry – the “volume” inside a bubble of radius r will now change.
- The bubble is now O(3) not O(4) symmetric, this will also change both volume inside and area of the bubble

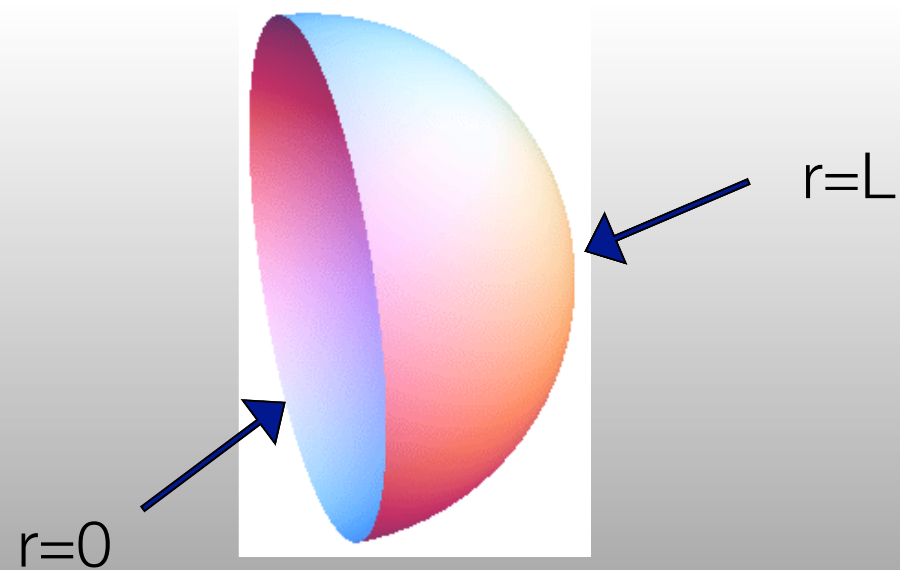
# Euclidean Black Holes

In Euclidean Schwarzschild, to make the black hole horizon regular, we must have $\tau$ periodic. This “explains” black hole temperature, but also sets a specific value, $8\pi GM$.



# Euclidean de Sitter – Static

For de Sitter in black hole coordinates, we have a "cosmological horizon", and again $\tau$ is periodic with a specific (but different!) value.

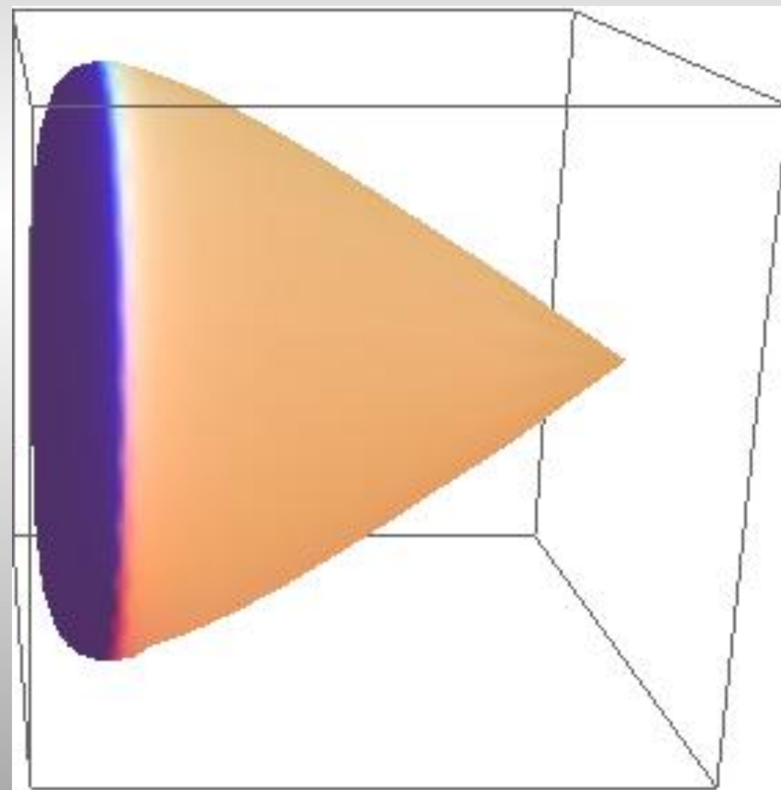


# Schwarzschild-De Sitter

Putting a black hole in de Sitter means we can never have a smooth geometry: SdS has a conical deficit/excess on at least one horizon:

$$\Delta\tau = \frac{4\pi}{|f'(r_i)|}$$

$$\frac{\Delta\tau_h}{\Delta\tau_c} = \frac{r_c(1-3\frac{r_h^2}{\ell^2})}{r_h(3\frac{r_c^2}{\ell^2}-1)} \sim \frac{\ell}{2r_h} \quad (r_h \to 0)$$



# Conical Actions

The conical deficit has a delta function in the Ricci tensor (caveat – no transverse energy momentum, metric a product space) so can compute the action:

$$ds^2 = d\rho^2 + A^2(\rho)d\chi^2 + C^2(\rho)d\Omega_{II}^2$$

Smooth out A:

$$A'(0) = 1 \quad , \quad A'(\varepsilon) = (1-\delta)$$

$$\mathcal{R} = -\frac{2A''}{A} - \frac{4C''}{C} - \frac{4A'C'}{AC} + \frac{2(1-C'^2)}{C^2} \sim -\frac{2A''}{A} - \frac{8C_2}{C_0} + \frac{2}{C_0^2} + \mathcal{O}(\rho < \varepsilon)$$

$$\int d^4x\sqrt{g}\mathcal{R} \sim (4\pi C_0^2)4\pi[A'(0) - A'(\varepsilon)] + \mathcal{O}(\varepsilon) = (4\pi C_0^2)(4\pi\delta) + \mathcal{O}(\varepsilon)$$

*(Geroch-Traschen – not!)*

# SdS Action

Calculating the action of the SdS black hole now gives an interesting result. For a general periodicity:

Cosmological Conical Bulk

$$S_{SdS} = \int d^4x\sqrt{g}(-R+2\Lambda) = -\frac{\pi(r_c^2+r_h^2)}{G} + \frac{\beta}{2G}\left[\cancel{2\pi\frac{r_c^2}{\beta_c} + 2\pi\frac{r_h^2}{\beta_h} - \frac{r_c^3-r_h^3}{\ell^2}}\right]$$

Black hole Conical

i.e. the result is independent of β

$$r_c^2 + r_h^2 + r_c r_h = \ell^2$$

(as it should be for a physically reasonable solution)

$$\beta_i = \frac{4\pi r_i}{|1-3\frac{r_i^2}{\ell^2}|}$$

# Back to Bounce

Adding in a wall adds in a contribution to the action:

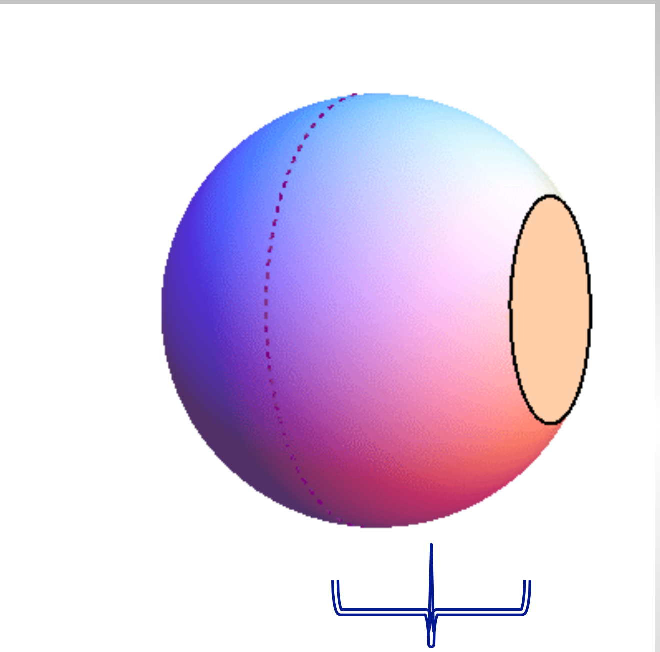
$$-\int_{\mathcal{W}} \frac{\sigma}{2} - \frac{1}{16\pi G}\int_{\mathcal{W}} \left(f'_+\dot{\tau}_+ - f'_-\dot{\tau}_-\right)$$

The general action with a black hole on each side is (details vary with Lambda)

$$\mathcal{B} = \underbrace{\frac{\pi(r_+^2 - r_-^2)}{G}}_{\text{Geometry}} \underbrace{- \frac{\bar{\sigma}}{G}\int d\lambda R^2 - \frac{1}{4G}\int d\lambda R^2 \left(f'_+\dot{\tau}_+ - f'_-\dot{\tau}_-\right)}_{\text{Bubble}}$$

# Check CDL

The usual Coleman-de Luccia solution is given in different coordinates – here we are in the "static patch" of de Sitter – or half the sphere. Must be sure it gives the right answer.



Putting in the geometry on each side of the wall gives the "Friedmann equation" for wall motion:

$$\pm\left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} = \left(\bar{\sigma} + \frac{1}{4\bar{\sigma}\ell^2}\right)^2$$

$$\bar{\sigma} = 2\pi G\sigma$$

Static patch

$$f_+ = 1 - \frac{r^2}{\ell^2}$$

$$f_- = 1$$

# CDL Wall

$$\pm\left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} = \left(\bar{\sigma} + \frac{1}{4\bar{\sigma}\ell^2}\right)^2$$

Solved by sine/cosine functions:

$$\left.\begin{aligned} R(\lambda) &= \gamma\cos\frac{\lambda}{\gamma} \\ t_-(\lambda) &= \gamma\sin\frac{\lambda}{\gamma} \\ \sqrt{\ell^2-\gamma^2}\,\tan\frac{t_+(\lambda)}{\ell} &= \gamma\sin\frac{\lambda}{\gamma} \end{aligned}\right\}$$

PERIODICITY *NOT* THE SAME AS STATIC PATCH

***HENCE CONICAL DEFICIT IN BOUNCE***

But gives same result.

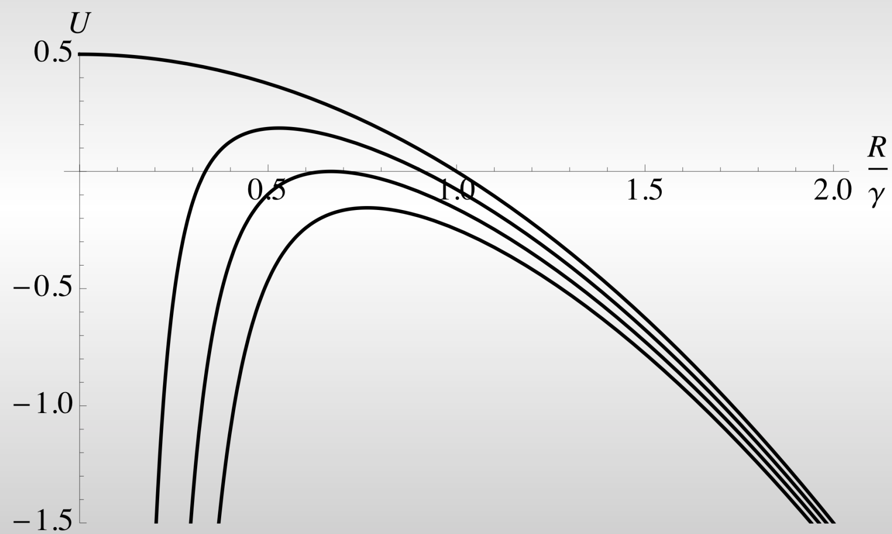
# Adding a black hole

Modifies the potential for the instanton: e.g. a Minkowski bubble inside SdS

$$\pm\left(\frac{\dot{R}}{R}\right)^2+\frac{1}{R^2}=\left(\bar{\sigma}+\frac{1}{4\bar{\sigma}\ell^2}+\frac{GM}{2\bar{\sigma}R^3}\right)^2$$

$$U(R)=\frac{1}{2}\left[1-\left(\frac{R}{\gamma}+\kappa\frac{\gamma^2}{R^2}\right)^2\right]\qquad\left(\kappa=\frac{GM}{2\bar{\sigma}\gamma^2}\right)$$

$$R=R_*=\sqrt{\frac{3GM}{2\bar{\sigma}}}\quad(\kappa=\kappa_*=4/27)$$




Find numerically (except for unstable static solution $R_*$)

*See also Hiscock, PRD35 1161 (1987)*

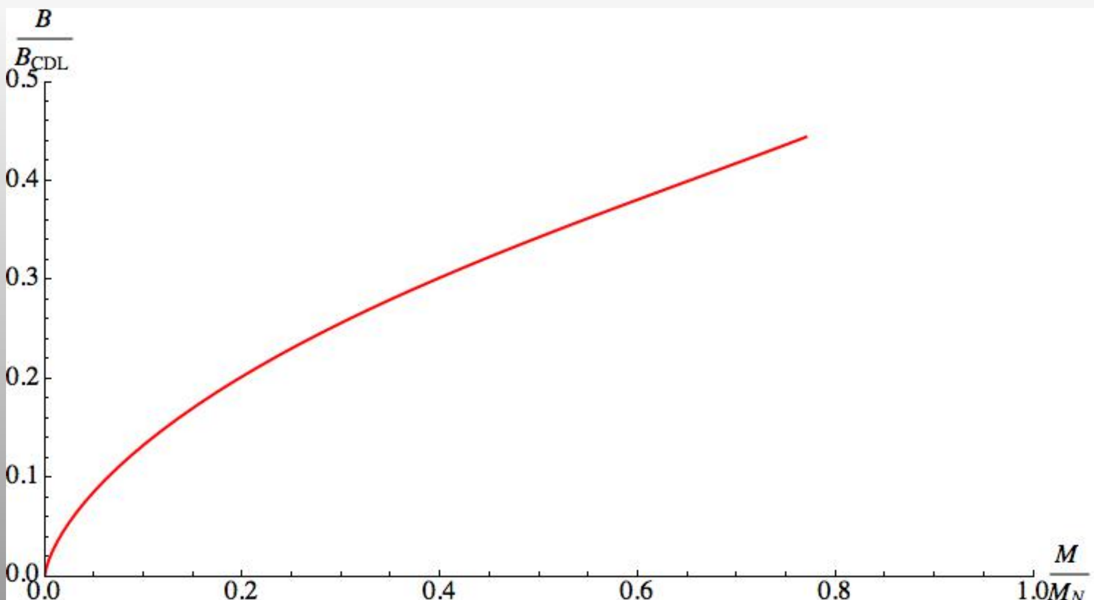
# Static solution

The static solution is the one example where we can have no conical deficit (hence another cross-check).

$$R = R_* = \sqrt{\frac{3GM}{2\bar{\sigma}}}$$

Both methods give the bounce action:

$$\mathcal{B}_* = \frac{\pi r_h^2}{G}$$

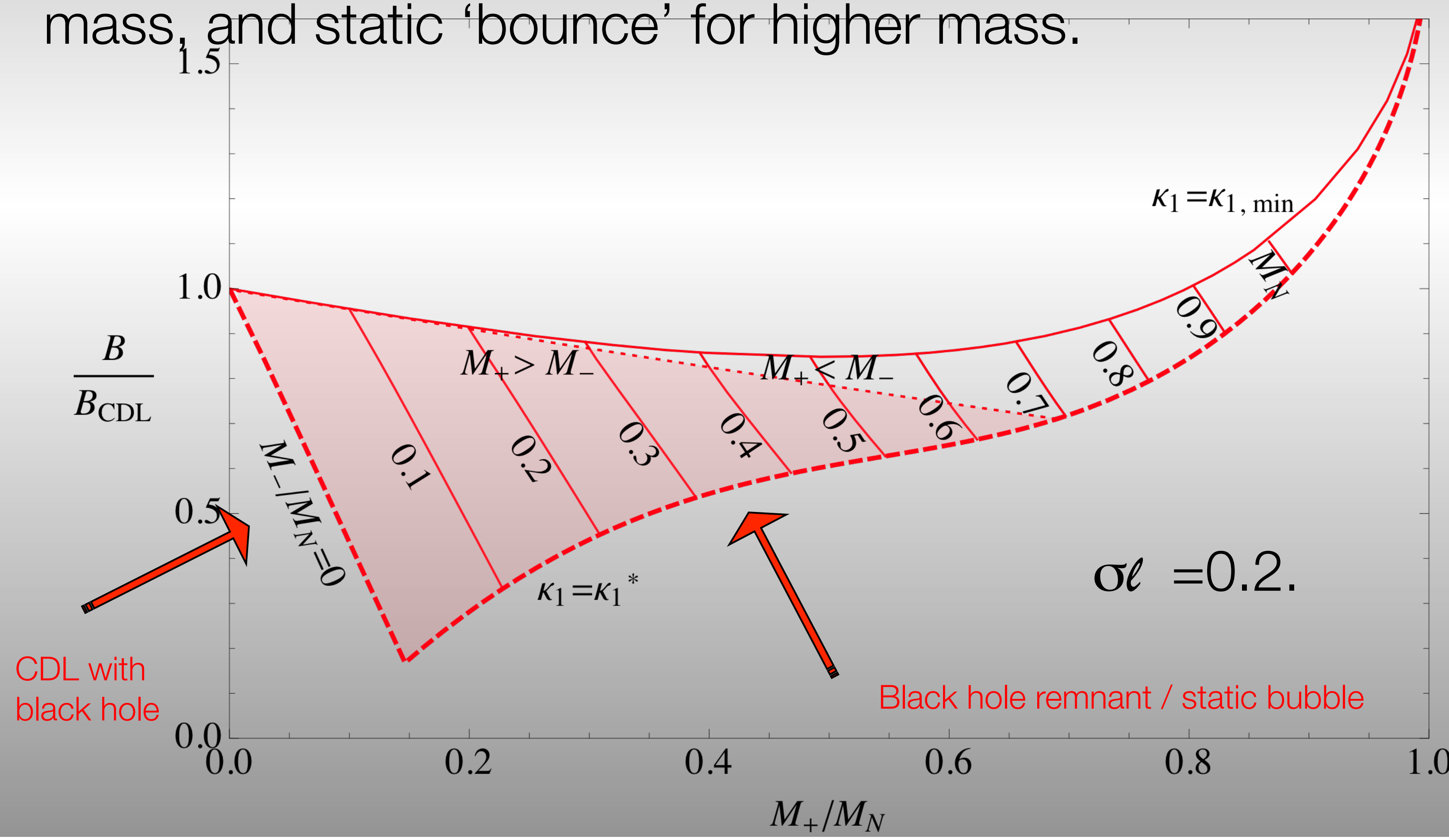



$$M_N = \frac{\ell}{3\sqrt{3}G}$$

Note how action drops!

# Expanding Bubbles

The general solutions have expanding bubbles, and have to find the wall numerically. For a given seed mass $M_+$, there is a preferred instanton with lowest action – the flat interior for low mass, and static 'bounce' for higher mass.

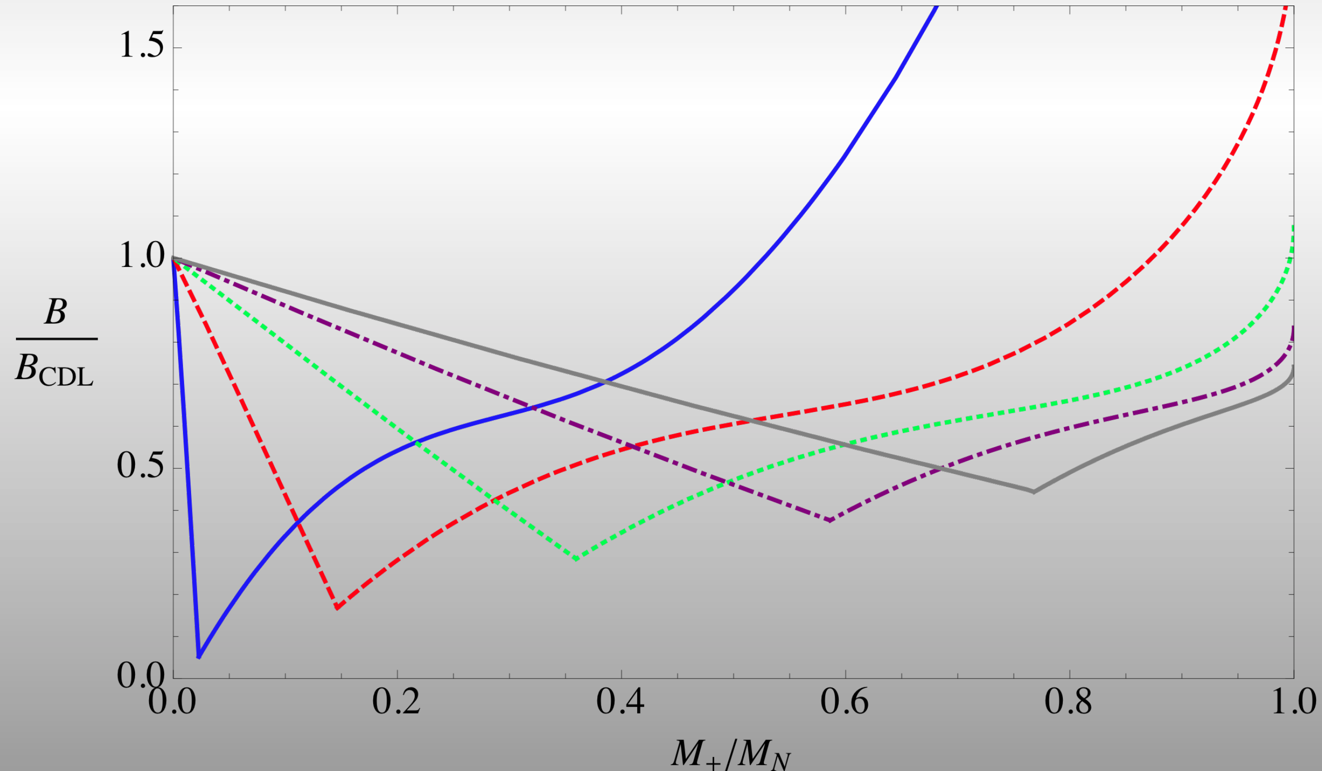


# General Instantons

The general instanton can have a remnant black hole, as well as a seed.

$$U(R) = \frac{1}{2}\left[1 - \left(\frac{R}{\gamma} + \kappa_2 \frac{\gamma^2}{R^2}\right)^2 - \kappa_1 \frac{\gamma}{R}\right]$$

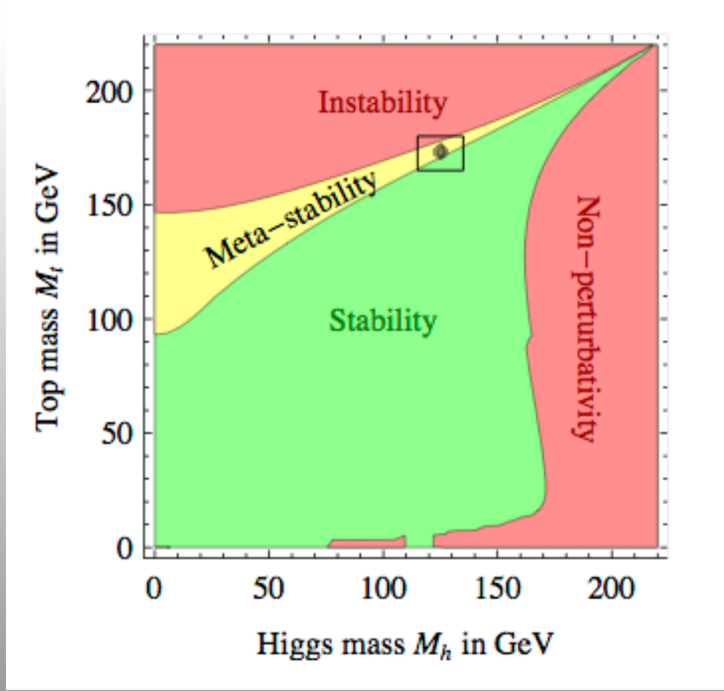
$$\kappa_1 = \frac{2GM_-}{\gamma} \quad , \quad \kappa_2 = \frac{G\Delta M}{2\bar{\sigma}\gamma^2}$$



# What about Higgs?

At large values of the Higgs field, the SM running of the quartic coupling *may* cause the potential to become negative – our vacuum becomes metastable. Natural to include quantum gravity terms in potential.

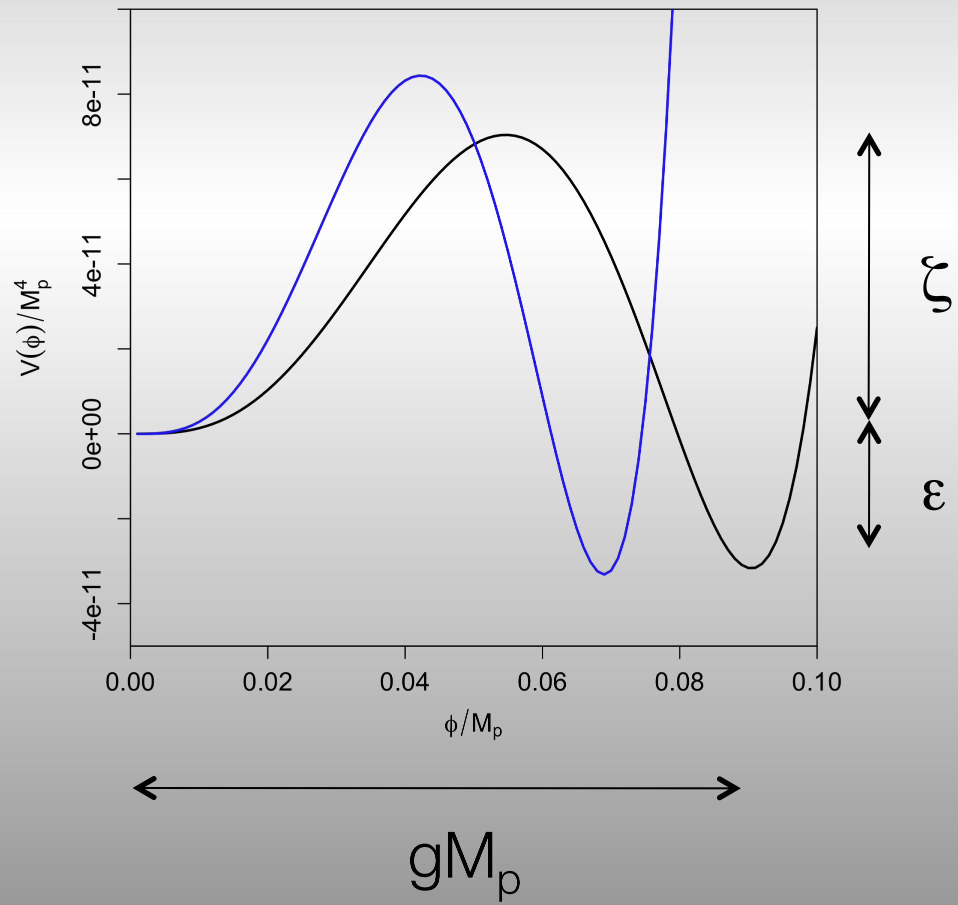
$$V(\phi) = \frac{\phi^4}{4}\left[\lambda_* + b\left(\ln\left(\frac{\phi}{\phi_*}\right)\right)^2\right] + \frac{\lambda_6}{6}\frac{\phi^6}{M_p^2} + \ldots$$

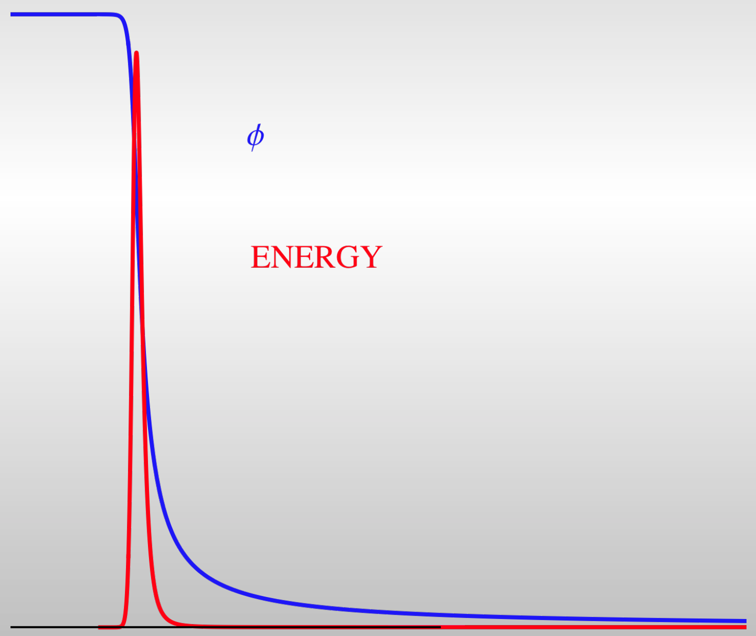



For $\lambda_6 < 0$, we can have AdS minimum at $\phi \sim M_p$. Typical CDL result gives tunnelling time longer than age of universe.

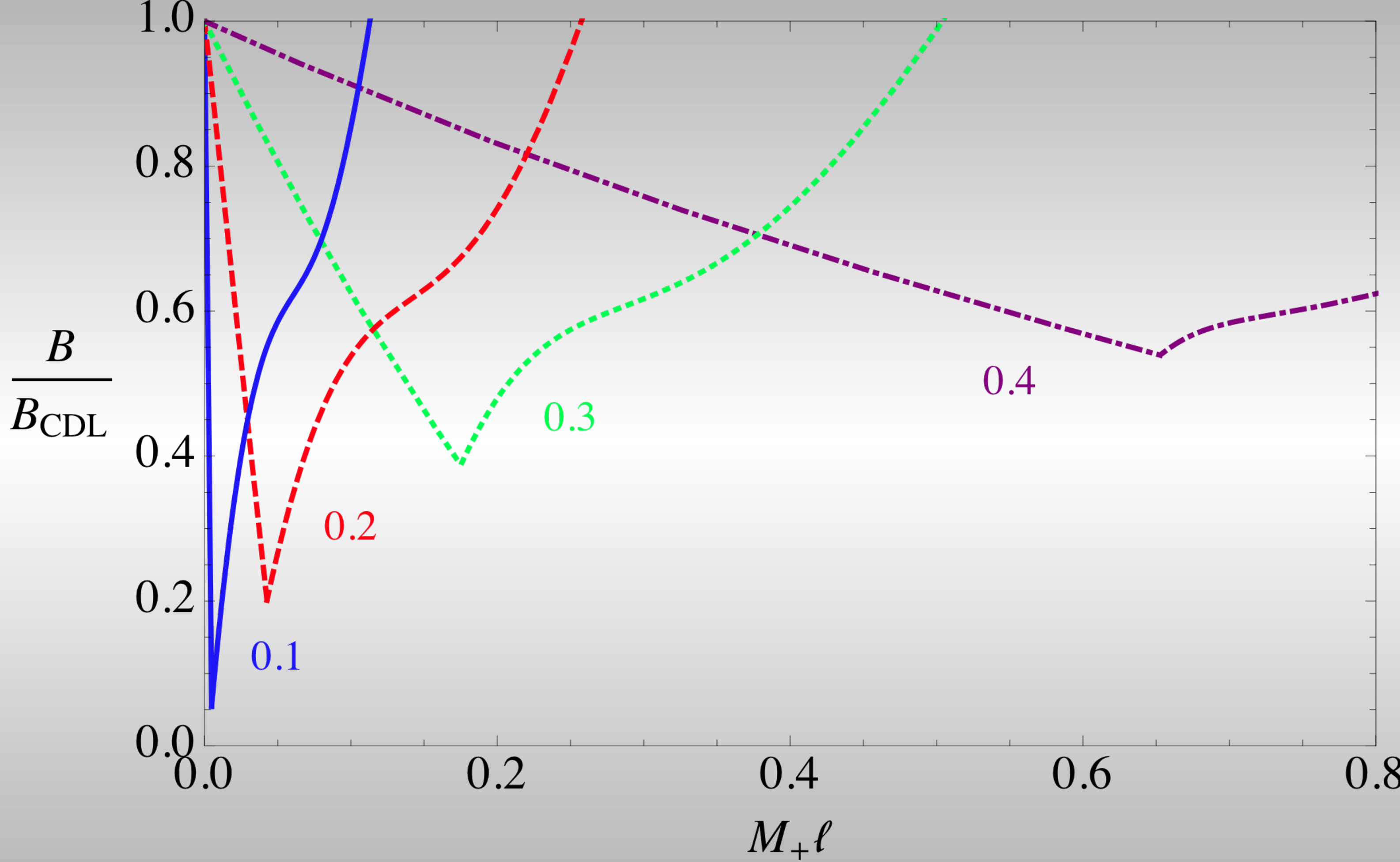
# Higgs potential?

Instanton characterized by barrier height ζ, vacuum energy ε, and VEV $gM_p$ of true vacuum. Can take thin wall limit if depth is less than height.



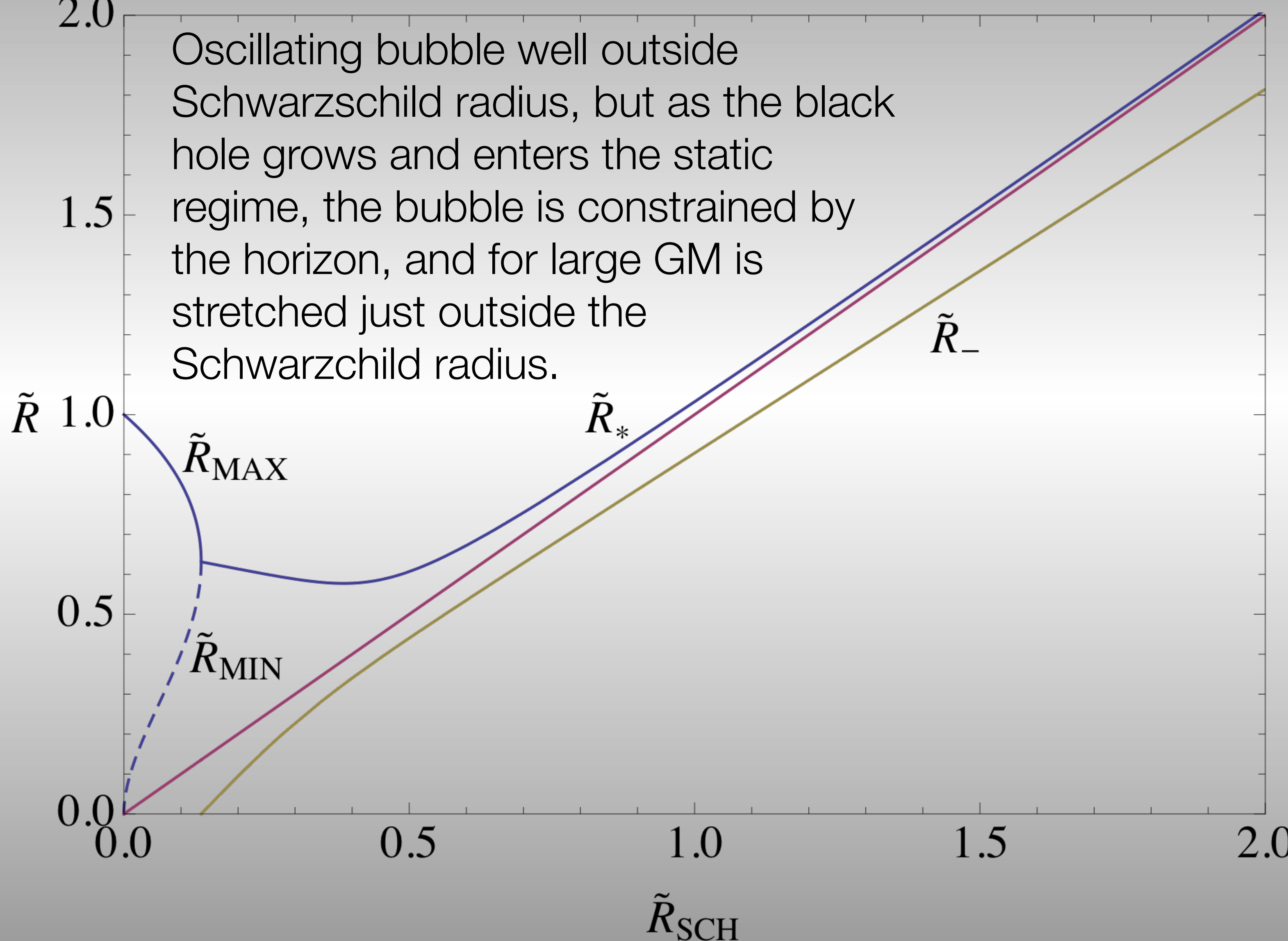



Explore instantons in thin wall approximation. CDL parameters related to parameters in Higgs potential.




Re-do analysis with black holes and AdS, very similar equations.

# Trying on for size:



# Tunneling v Evaporation:

For Hawking evaporation: $\Gamma_H \approx 3.6 \times 10^{-4} (G^2 M_*^3)^{-1}$

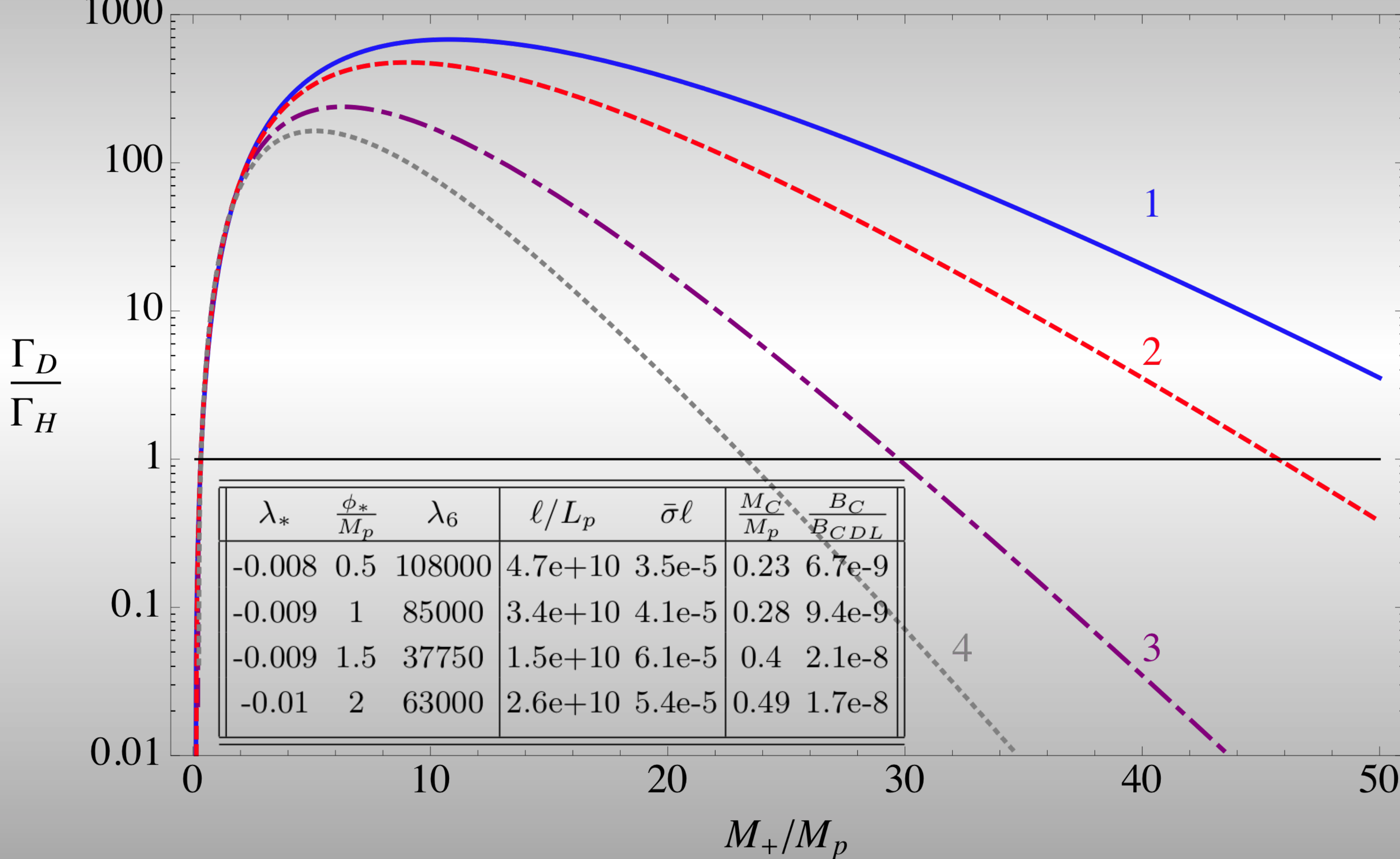
And tunneling for critical case: $\Gamma_* \approx \left(\frac{2}{G}\right)^{1/2} e^{-4\pi G M_*^2}.$

Ratio: $$\frac{\Gamma_*}{\Gamma_H} \approx 3.9 \times 10^3 (G M_*^2)^{3/2} e^{-4\pi G M_*^2}.$$

i.e. less than unity for black holes above the Planck mass.

But – assumes black hole is at critical limit!

Need to compute branching ratio as the black hole evaporates. Compare static bubble action to evaporation rate at the instantaneous value of mass.




| $\lambda_*$ | $\frac{\phi_*}{M_p}$ | $\lambda_6$ | $\ell/L_p$ | $\bar{\sigma}\ell$ | $\frac{M_C}{M_p}$ | $\frac{B_C}{B_{CDL}}$ |
|---|---|---|---|---|---|---|
| -0.008 | 0.5 | 108000 | 4.7e+10 | 3.5e-5 | 0.23 | 6.7e-9 |
| -0.009 | 1 | 85000 | 3.4e+10 | 4.1e-5 | 0.28 | 9.4e-9 |
| -0.009 | 1.5 | 37750 | 1.5e+10 | 6.1e-5 | 0.4 | 2.1e-8 |
| -0.01 | 2 | 63000 | 2.6e+10 | 5.4e-5 | 0.49 | 1.7e-8 |

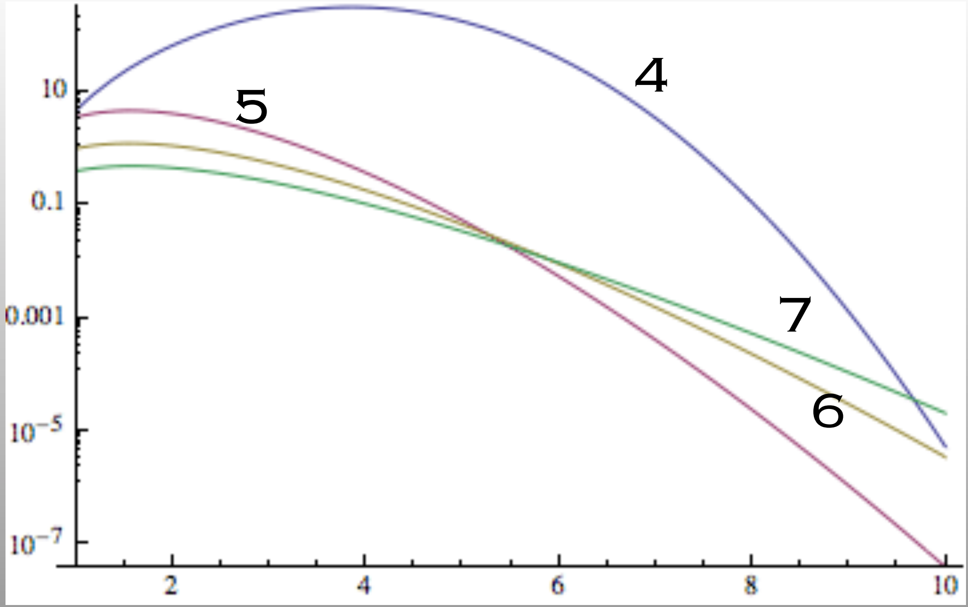
Primordial black holes will evaporate and hit these curves.

Primordial black holes start out with small enough mass to evaporate and will eventually hit these curves.

Can view as a constraint on PBH's or (weak) on corrections to the Higgs potential.

Small black holes also possible in theories with Large Extra Dimensions.

(but the branching ratio seems to drop with D)



# Summary

- Have shown how to compute the action of a singular instanton, verified for known or special cases.
- Tunneling amplitude significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether.
- Higgs vacuum decay occurs with enhanced probability, but with primordial black holes, and parameter space limited. Beyond thin wall necessary to see how general an issue this is.