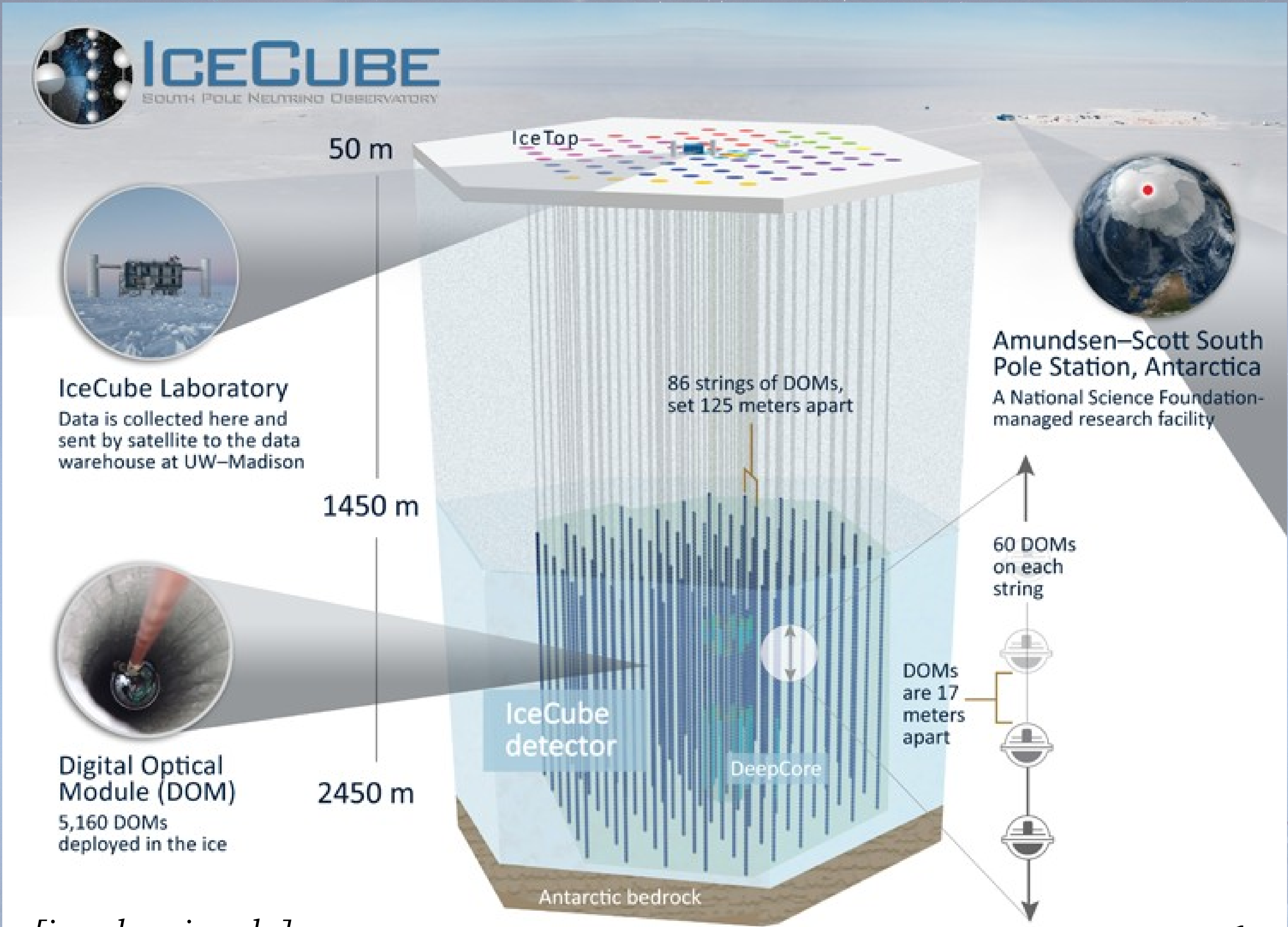
Analysis of the recent IceCube results on the astrophysical neutrino flavour ratio

Mariia Redchuk
NCTU Institute of Physics, Taiwan

Work in progress with:
Kwang-Chang Lai
Wei-Hao Lai
Guey-Lin Lin
Tsung-Che Liu



[icecube.wisc.edu]

Sources of astrophysical neutrinos

$$\pi^{+} \longrightarrow \mu^{+} + \nu_{\mu}$$

$$\mu^{+} \longrightarrow \bar{\nu}_{\mu} + e^{+} + \nu_{e}$$

Pion source: muon decays quickly → comparable energy → 1:2:0 → (1/3, 2/3, 0)

Muon damped source: muon interacts before decaying → negligible neutrino energy → (0,1,0)

$$n \rightarrow p + e^{-} + \bar{\nu}_{e}$$

Neutron beta decay source: (1,0,0)

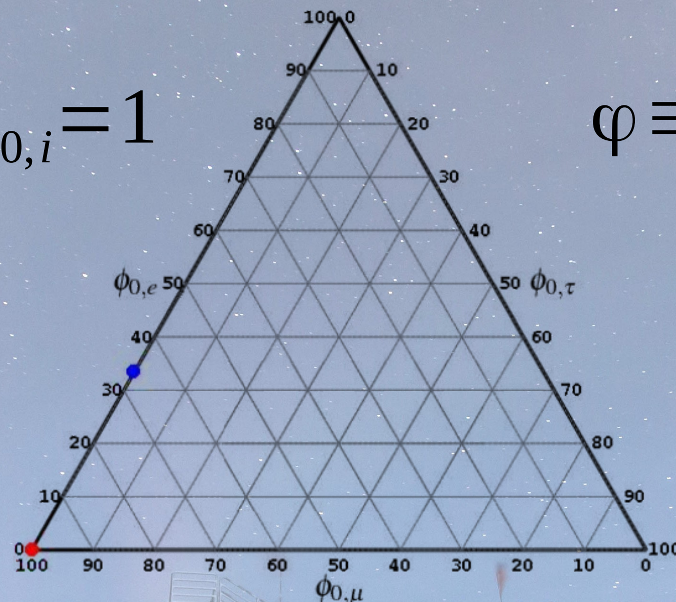
Basic Linear Algebra

Initial flux (normalized)

$$\varphi_0 \equiv \begin{pmatrix} \varphi_{e,0} \\ \varphi_{\mu,0} \\ \varphi_{\tau,0} \end{pmatrix}, \quad \sum_{i=1}^{3} \varphi_{0,i} = 1$$

Flux measured on Earth

$$\varphi \equiv \begin{pmatrix} \varphi_{e} \\ \varphi_{\mu} \\ \varphi_{\tau} \end{pmatrix}, \quad \sum_{i=1}^{3} \varphi_{i} = 1$$



Example:

$$\varphi_e = \varphi_{e,0} \times P_{ee} + \varphi_{\mu,0} P_{e\mu} + \varphi_{\tau,0} P_{e\tau}$$

$$P_{\alpha\beta} = P(\nu_\beta \rightarrow \nu_\alpha)$$

$$P_{e\mu} = P_{12} = P(\nu_\mu \rightarrow \nu_e)$$

$$\vec{\varphi} = \boldsymbol{P} \vec{\varphi}_0$$

Do you even mix?

	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$31.29 \to 35.91$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \to 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \to 53.3$	$38.3 \to 53.3$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \to 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$7.87 \to 9.11$
$\delta_{CP}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.02 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$	$\begin{bmatrix}+2.325 \to +2.599 \\ -2.590 \to -2.307\end{bmatrix}$

[1] M. C. Gonzalez-Garcia et al, arXiv: 1409.5439

Do you even mix?

$\theta_{12,}\,\theta_{23,}\,\theta_{13,}\,\delta_{CP}$ → PMNS matrix U → Transition matrix P

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Example: various decay models →

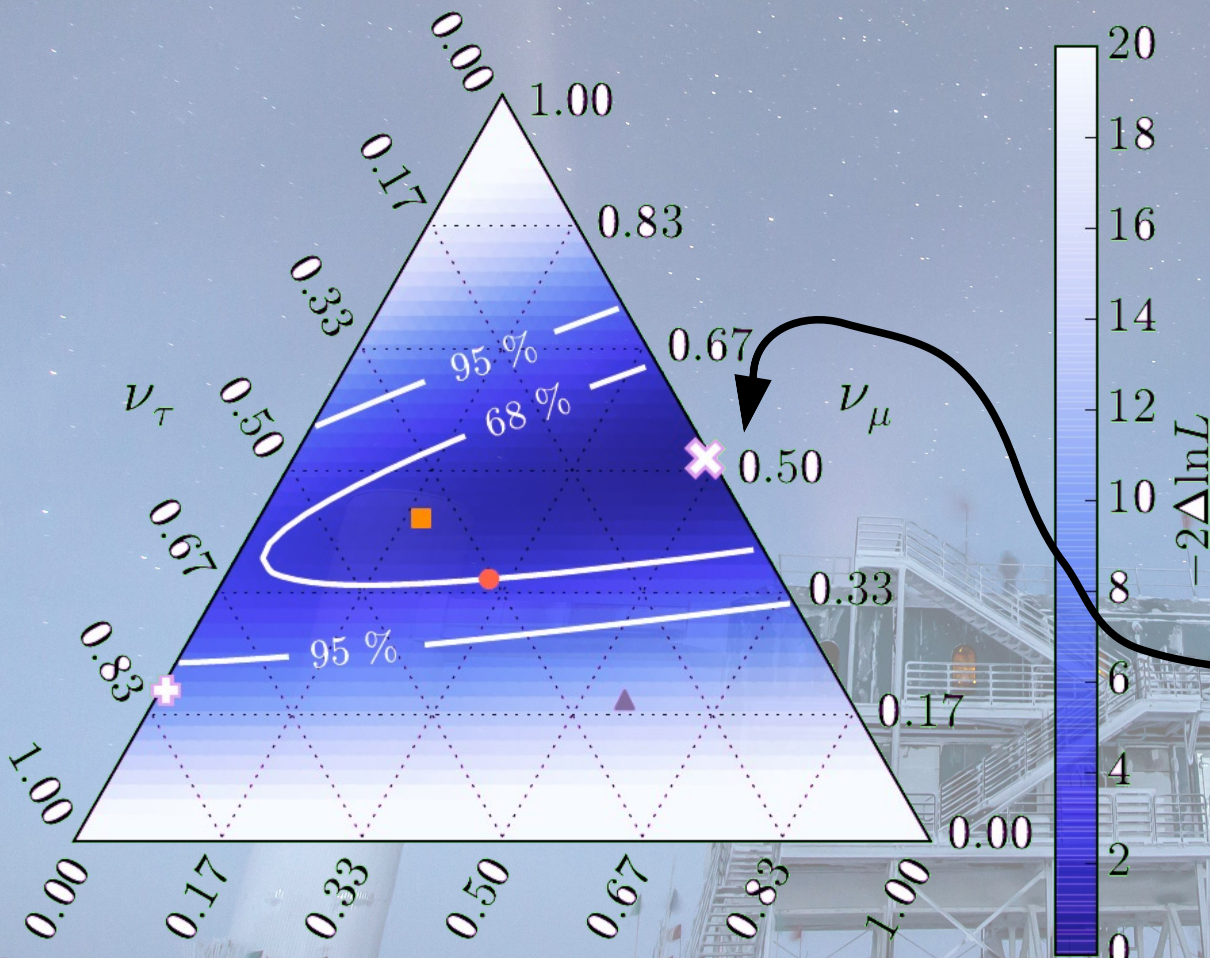
$$P_{\alpha\beta} = \sum_{f\ \text{stable}} \left(|U_{\alpha f}|^2 + \sum_{i\ \text{unstable}} |U_{\alpha i}|^2 \, \text{Br}_{i \to f} \right) |U_{\beta f}|^2 \,.$$

[3] Michele Maltoni, Walter Winter, JHEP07(2008)064

Example: oscillation →
(all stable $\text{Br}_{i \to f} = 0$)

$$P^{\text{osc}}_{\alpha\beta} = \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2,$$

Flavor Ratio of Astrophysical Neutrinos above 35 TeV in IceCube



Neutrino energy range:
35 TeV – 1.9 PeV

Best-fit composition:
(0.5 : 0.5 : 0)

[2] M. G. Aartsen, K. Abraham et al., arXiv: 1507.03991

Is this basis convenient?

Let's see...

1. Values of P are not useful for model analysis

$$P_{\alpha\beta} = P\left(\nu_\beta \rightarrow \nu_\alpha\right)$$

2. Flux non-conservation becomes messy

$$\vec{\varphi} = \boldsymbol{P}\,\vec{\varphi}_0 \rightarrow \frac{1}{\sum_{i=1}^{3} \varphi_i}\,\vec{\varphi}$$

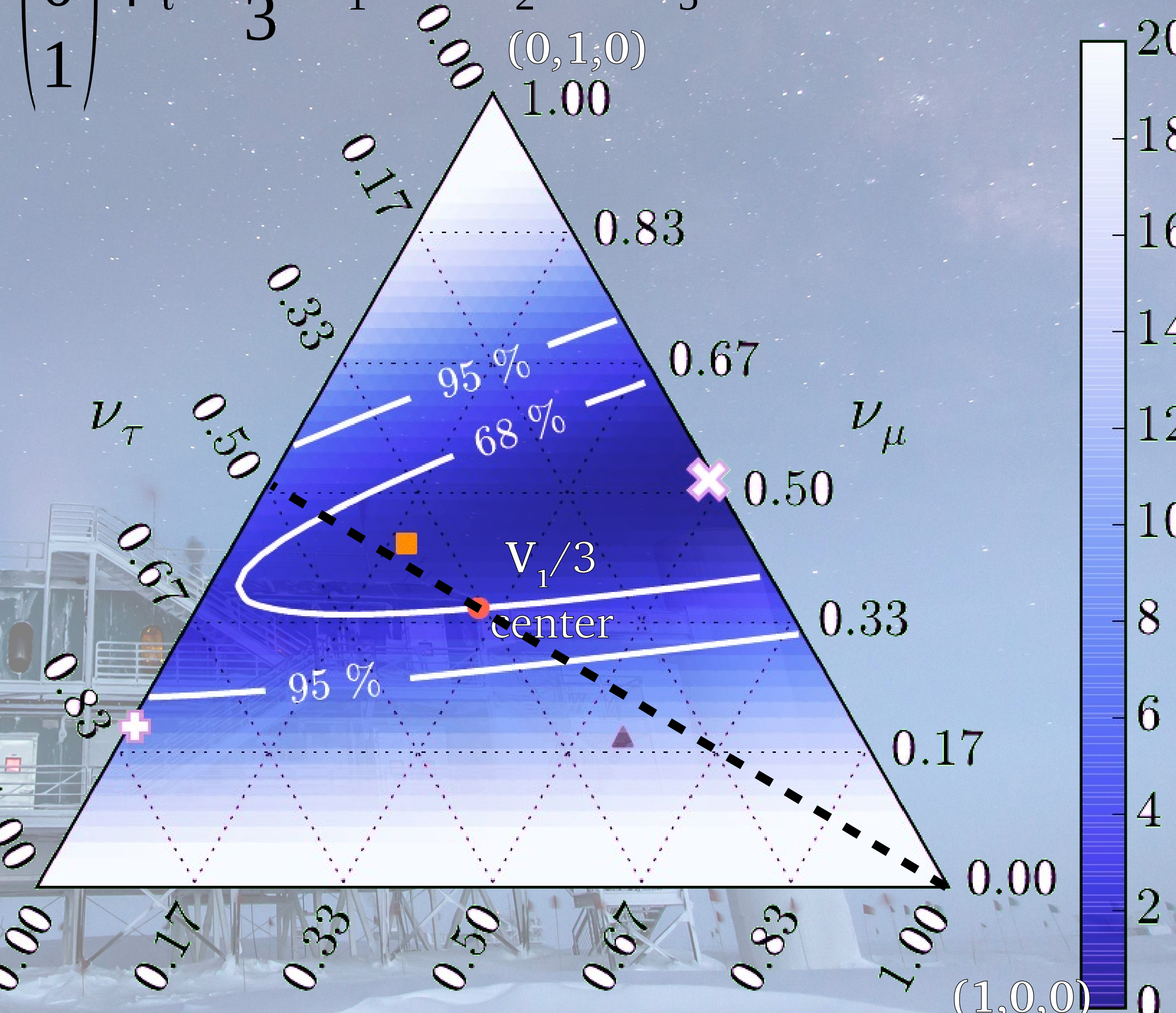
How about...

$$\vec{\varphi}_0 = \begin{pmatrix}1\\0\\0\end{pmatrix}\varphi_e + \begin{pmatrix}0\\1\\0\end{pmatrix}\varphi_\mu + \begin{pmatrix}0\\0\\1\end{pmatrix}\varphi_\tau = \frac{1}{3}\vec{V}_1 + a\vec{V}_2 + b\vec{V}_3$$

$$\boldsymbol{V_1} = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\boldsymbol{V_2} = \begin{pmatrix}0\\-1\\1\end{pmatrix}$$

$$\boldsymbol{V_3} = \begin{pmatrix}2\\-1\\-1\end{pmatrix}$$



[3] Kwang-Chang Lai, Guey-Lin Lin and T. C. Liu, arXiv: 1004.1583

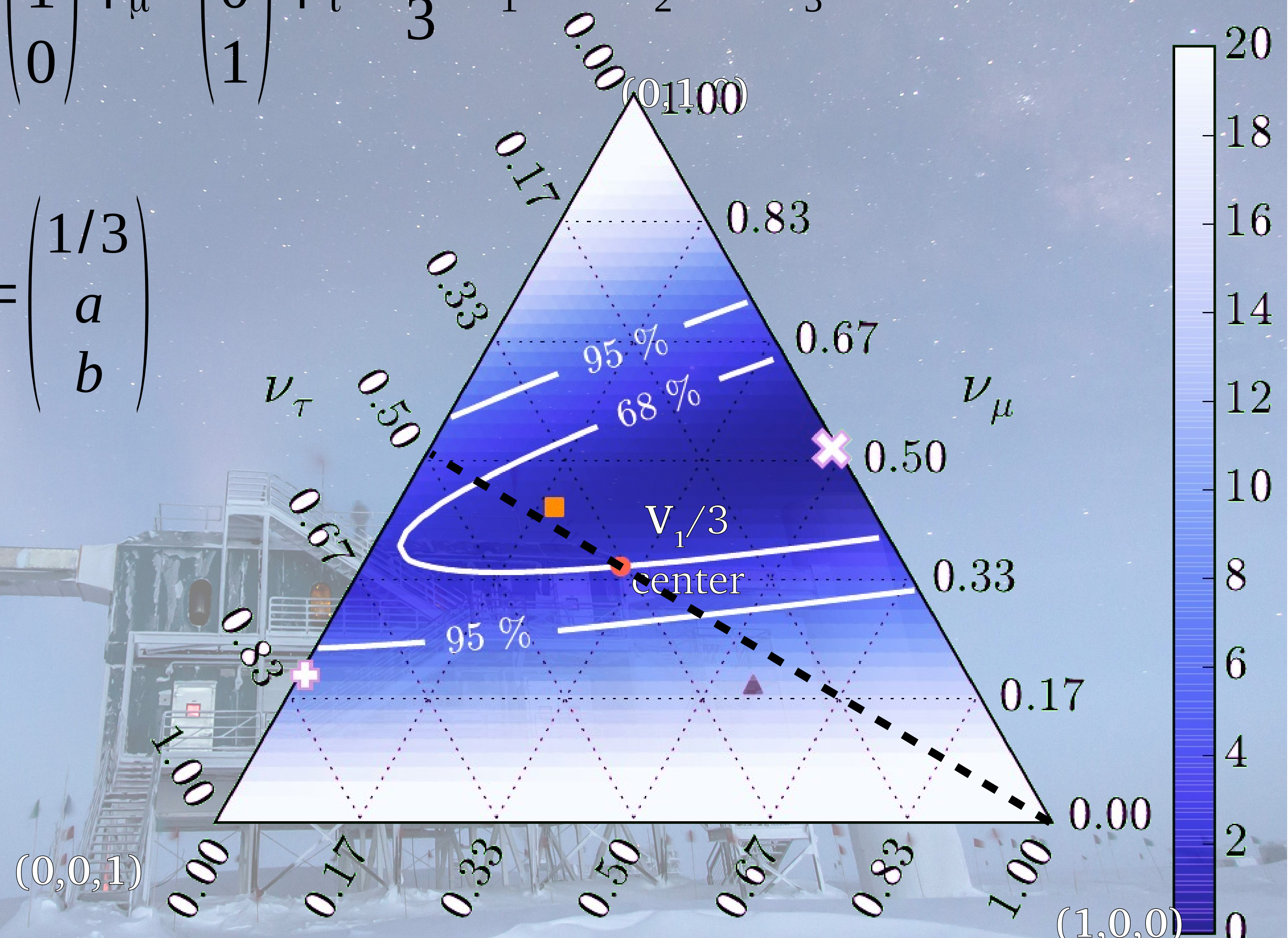
How about…

$$\vec{\varphi}_0 = \begin{pmatrix}1\\0\\0\end{pmatrix}\varphi_e + \begin{pmatrix}0\\1\\0\end{pmatrix}\varphi_\mu + \begin{pmatrix}0\\0\\1\end{pmatrix}\varphi_\tau = \frac{1}{3}\vec{V}_1 + a\vec{V}_2 + b\vec{V}_3$$

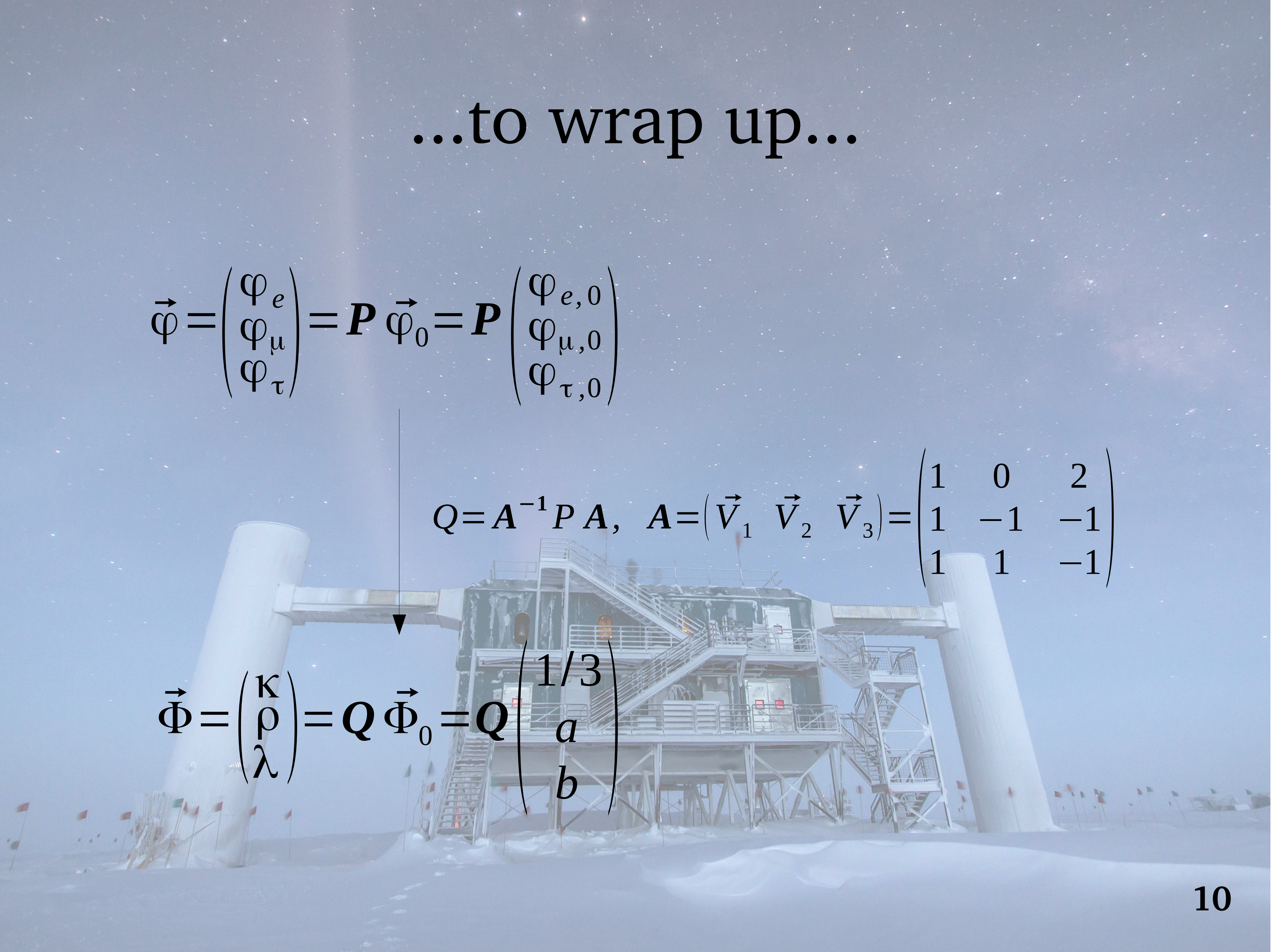
$$[\vec{\varphi}_0]_V = \boldsymbol{A}\vec{\varphi}_0 \equiv \Phi_0 = \begin{pmatrix}1/3\\a\\b\end{pmatrix}$$

$$[P]_V = \boldsymbol{A}^{-1} P \boldsymbol{A} \equiv \boldsymbol{Q}$$

$$\boldsymbol{A} = \begin{pmatrix}\vec{V}_1 & \vec{V}_2 & \vec{V}_3\end{pmatrix} = \begin{pmatrix}1 & 0 & 2\\1 & -1 & -1\\1 & 1 & -1\end{pmatrix}$$



[3] Kwang-Chang Lai, Guey-Lin Lin and T. C. Liu, arXiv: 1004.1583



...to wrap up...

$$\vec{\varphi} = \begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} = \boldsymbol{P}\,\vec{\varphi}_0 = \boldsymbol{P} \begin{pmatrix} \varphi_{e,0} \\ \varphi_{\mu,0} \\ \varphi_{\tau,0} \end{pmatrix}$$

$$Q = \boldsymbol{A}^{-1} P\,\boldsymbol{A}, \quad \boldsymbol{A} = \begin{pmatrix} \vec{V}_1 & \vec{V}_2 & \vec{V}_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\vec{\Phi} = \begin{pmatrix} \kappa \\ \rho \\ \lambda \end{pmatrix} = \boldsymbol{Q}\,\vec{\Phi}_0 = \boldsymbol{Q} \begin{pmatrix} 1/3 \\ a \\ b \end{pmatrix}$$

Is this basis convenient?

1. Transition models

$$P_{\alpha\beta} = P(\nu_\beta \rightarrow \nu_\alpha) \dashrightarrow \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix}$$

Flux conservation: $=1$, $=0$, $=0$

$\mu-\tau$ symmetry

Distinguishing different models

$$3\kappa = \varphi_e + \varphi_\mu + \varphi_\tau$$

$$\varphi_e, \varphi_\mu, \varphi_\tau \dashrightarrow \lambda = \frac{\varphi_e}{3} - \frac{\varphi_\tau + \varphi_\mu}{6} = \frac{\varphi_e}{2} - \frac{1}{6}$$

$$\rho = \frac{\varphi_\tau - \varphi_\mu}{2}$$

2. Flux conervation

$$\begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} \rightarrow \frac{1}{\varphi_e + \varphi_\mu + \varphi_\tau} \begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} \dashrightarrow \begin{pmatrix} \kappa \\ \rho \\ \lambda \end{pmatrix} \rightarrow \frac{1}{3\kappa} \begin{pmatrix} \kappa \\ \rho \\ \lambda \end{pmatrix}$$

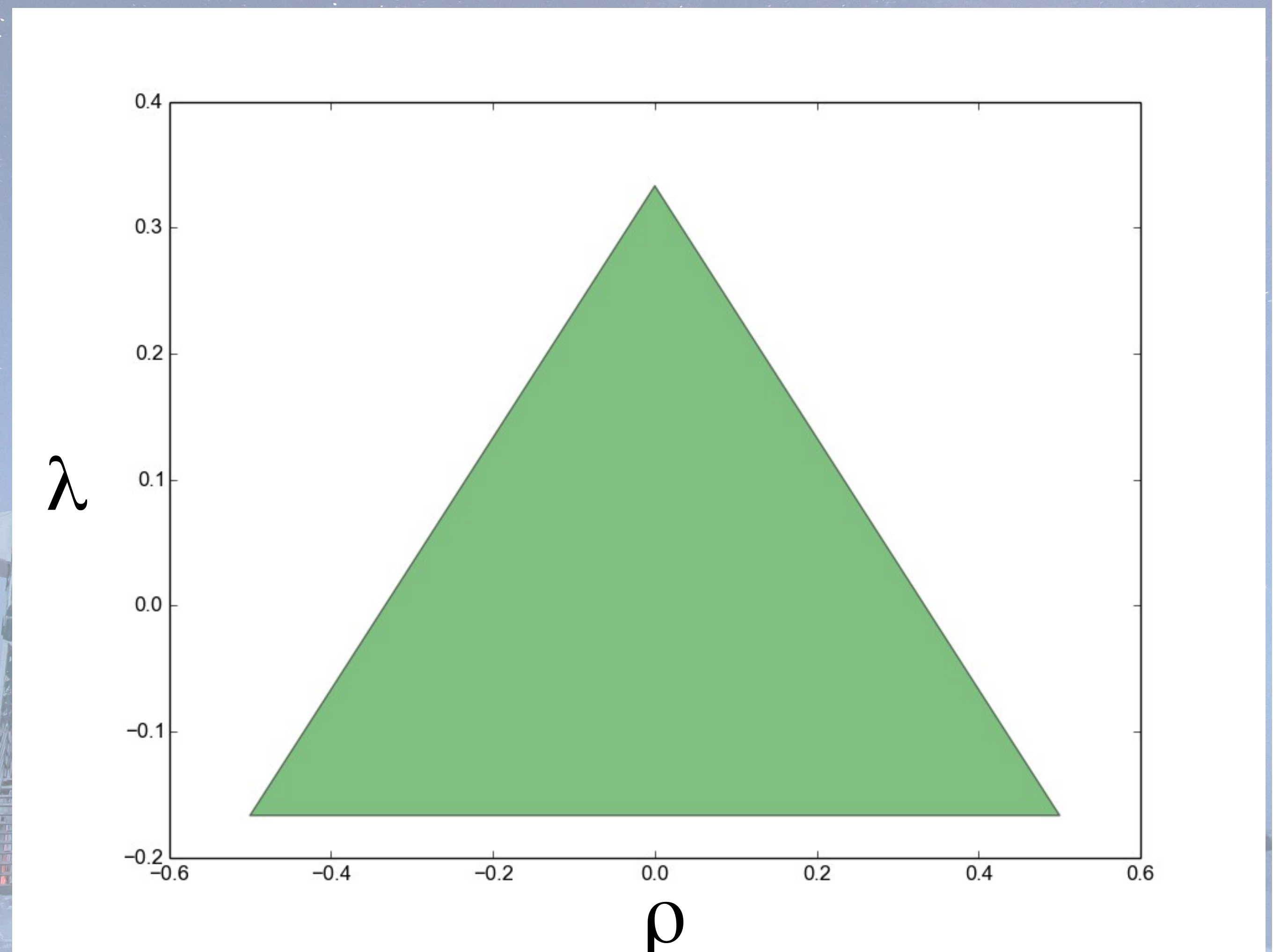
Same, but more physical

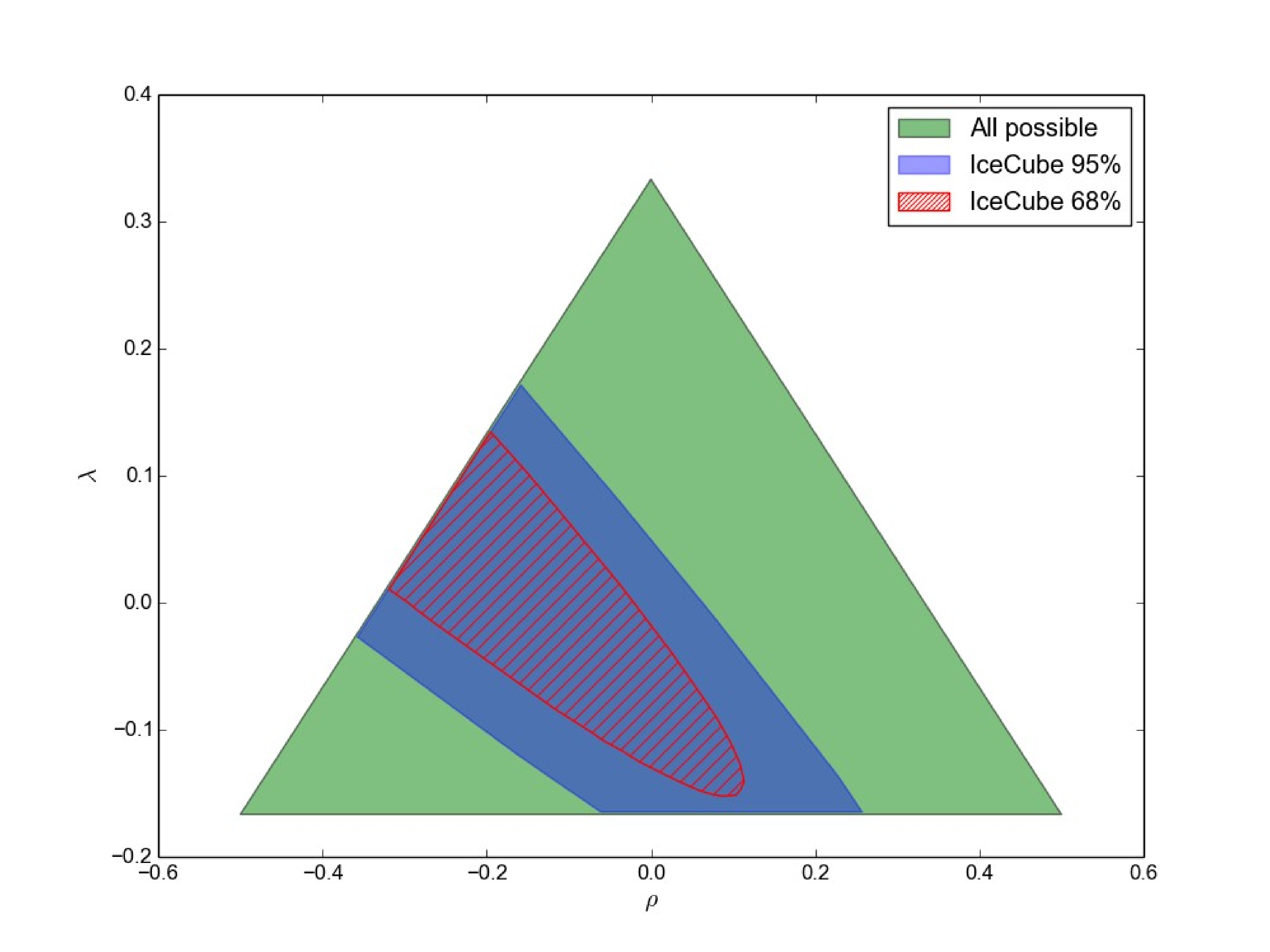
Recall:

$$\lambda = \frac{\varphi_e}{3} - \frac{\varphi_\tau + \varphi_\mu}{6} = \frac{\varphi_e}{2} - \frac{1}{6}$$

$$\rho = \frac{\varphi_\tau - \varphi_\mu}{2}$$

Linear transformation



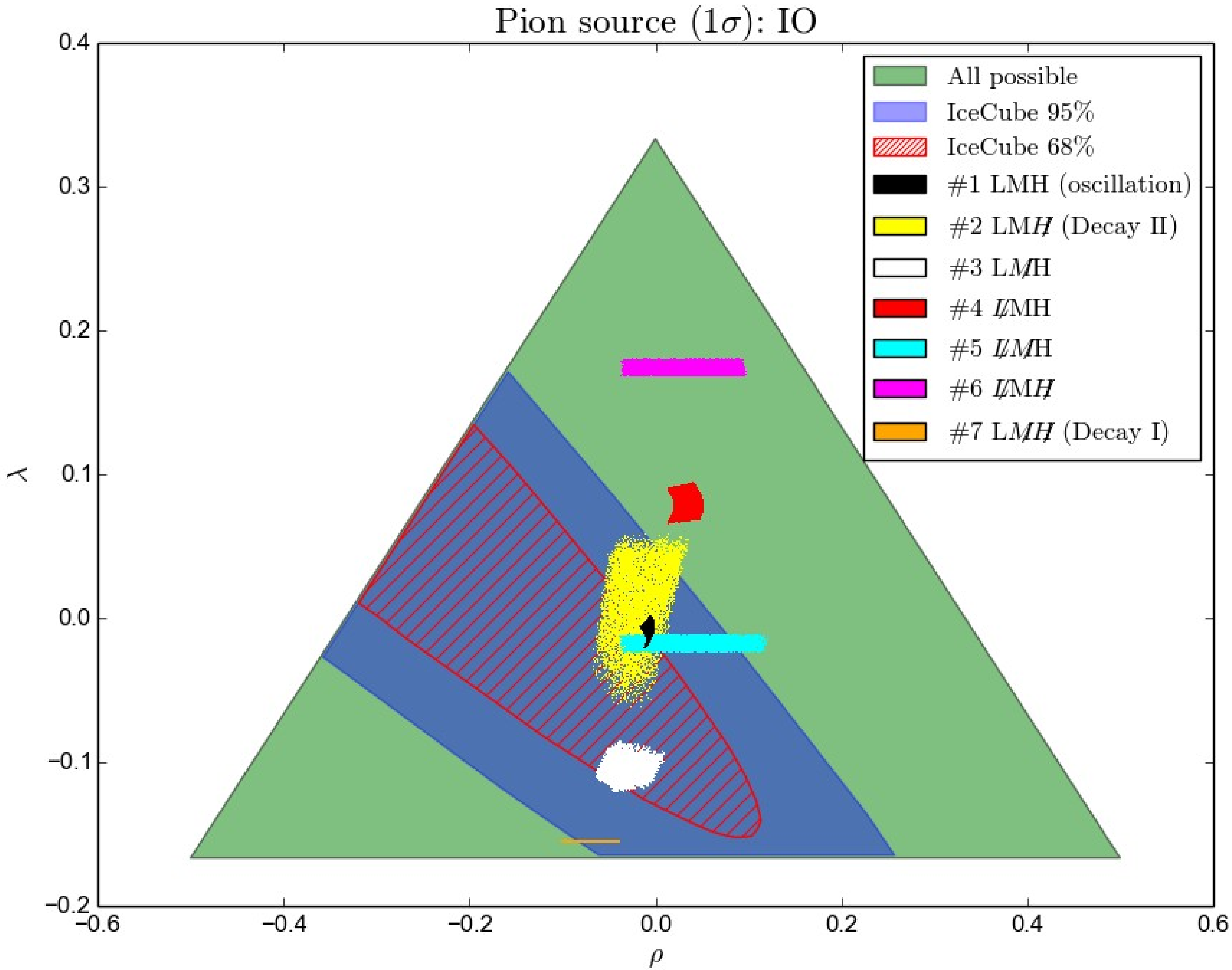


All possible
IceCube 95%
IceCube 68%
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
λ
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
ρ

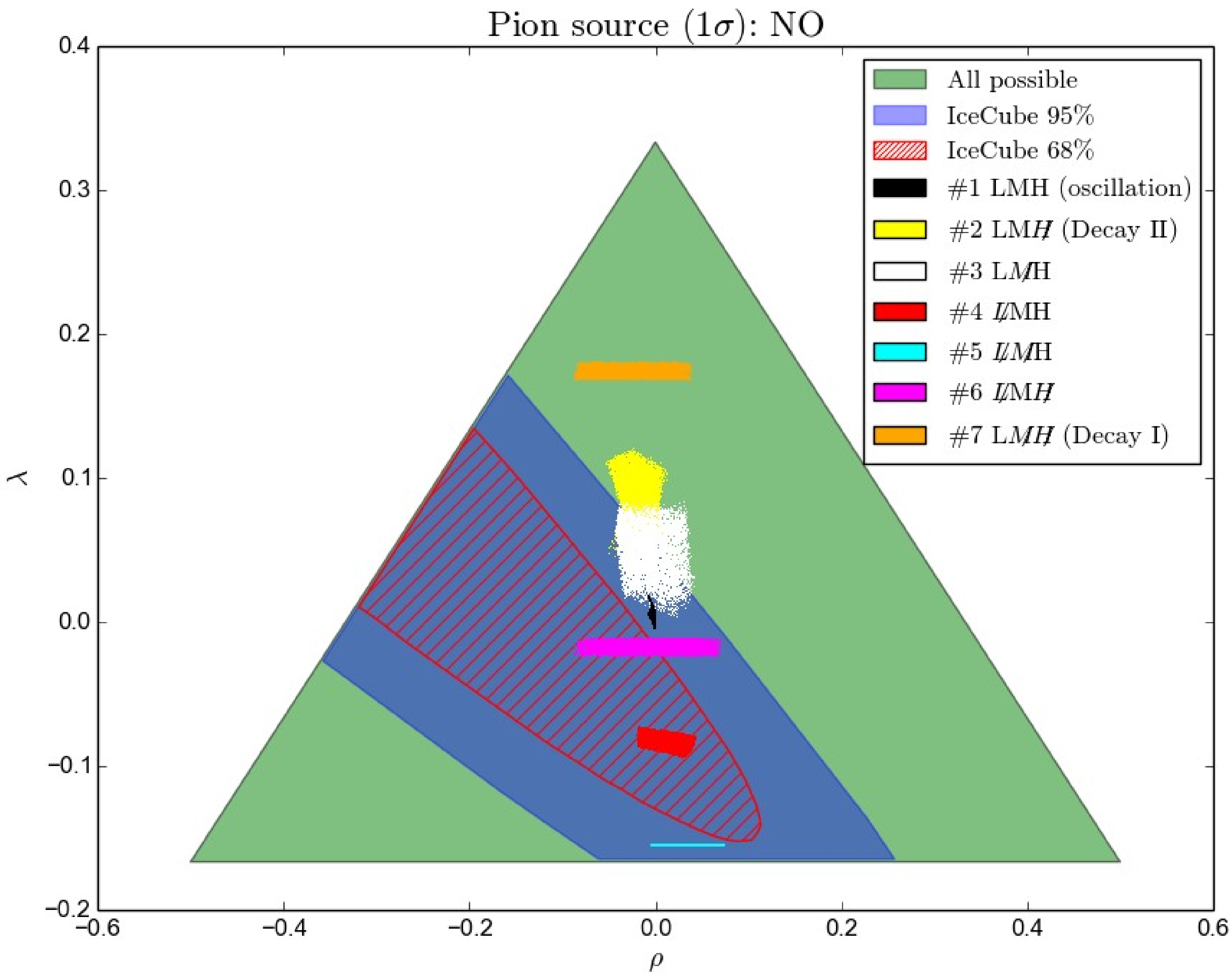
$$P_{\alpha\beta} = \sum_{f \text{ stable}} \left(|U_{\alpha f}|^2 + \sum_{i \text{ unstable}} |U_{\alpha i}|^2 \, \mathrm{Br}_{i \to f} \right) |U_{\beta f}|^2 \, .$$

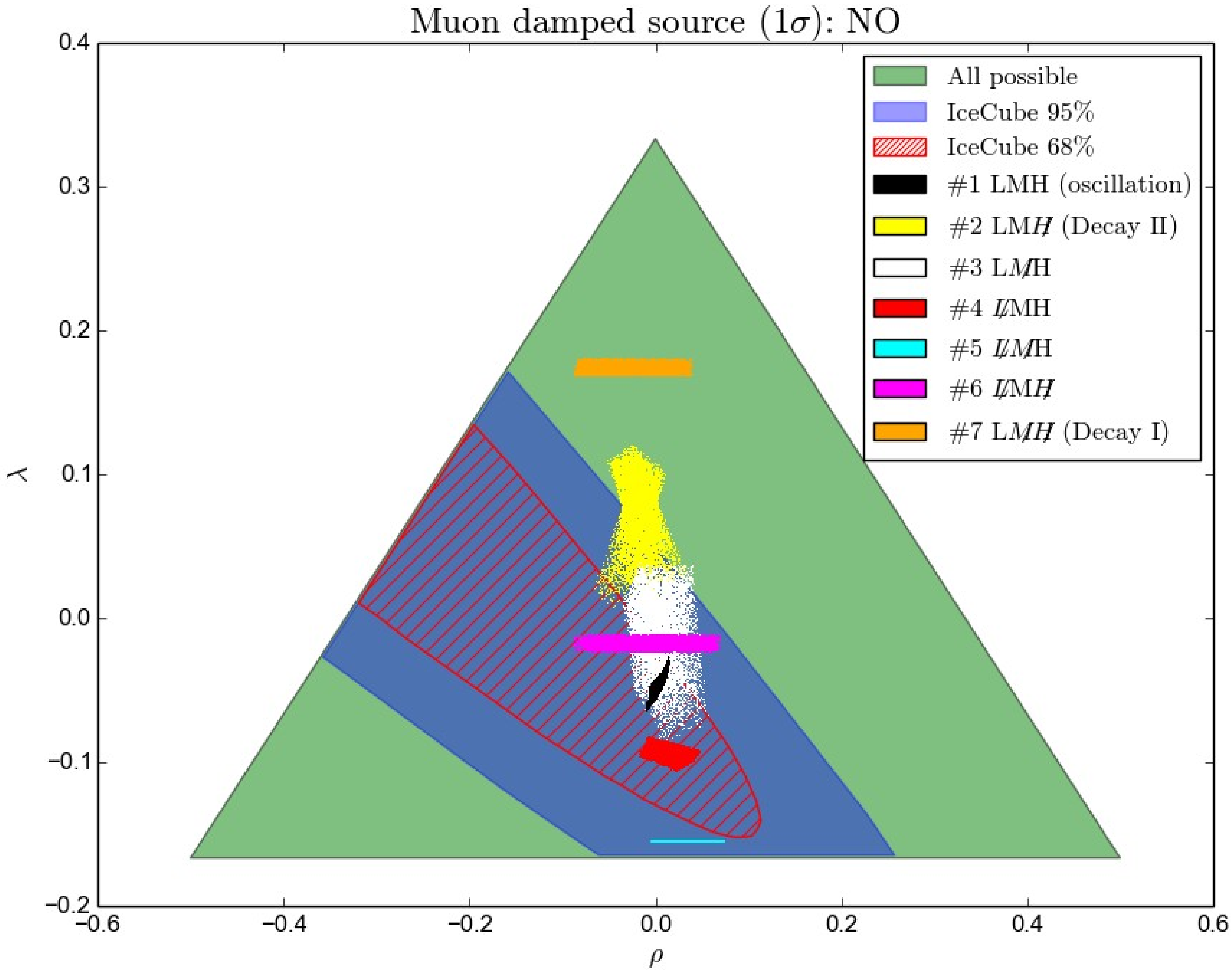
Branchings ratios			\$1 123	\$2 $12\not{3}$	\$3 $1\not{2}3$	\$4 $\not{1}23$	\$5 $\not{1}\not{2}3$	\$6 $\not{1}2\not{3}$	\$7 $1\not{2}\not{3}$	\$8 $\not{1}\not{2}\not{3}$
#1 LMH	–	–	❸ ❷ ❶ ❷ ❶ ❸	–	–	–	–	–	–	–
#2 LM$\not{\mathrm{H}}$	$\mathrm{Br}_{\mathrm{H}\to\mathrm{M}} = a$, $\mathrm{Br}_{\mathrm{H}\to\mathrm{L}} = b$ $\mathrm{Br}_{\mathrm{H}\to\mathrm{I}} = 1 - a - b$	$0 \leq a \leq 1$ $0 \leq b \leq 1 - a$	–	③ ❷ ❶	② ❶ ❸	–	–	–	–	–
#3 L$\not{\mathrm{M}}$H	$\mathrm{Br}_{\mathrm{M}\to\mathrm{L}} = a$, $\mathrm{Br}_{\mathrm{M}\to\mathrm{I}} = 1 - a$	$0 \leq a \leq 1$	–	–	❸ ② ❶	❷ ① ❸	–	–	–	–
#4 $\not{\mathrm{L}}$MH	$\mathrm{Br}_{\mathrm{L}\to\mathrm{I}} = 1$	–	–	❷ ❶ ③	–	❸ ❷ ①	–	–	–	–
#5 $\not{\mathrm{L}}\not{\mathrm{M}}$H	$\mathrm{Br}_{\mathrm{M}\to\mathrm{I}} = 1$ $\mathrm{Br}_{\mathrm{L}\to\mathrm{I}} = 1$	–	–	–	–	–	❸ ② ①	❷ ① ③	–	–
#6 $\not{\mathrm{L}}$M$\not{\mathrm{H}}$	$\mathrm{Br}_{\mathrm{H}\to\mathrm{M}} = a$, $\mathrm{Br}_{\mathrm{H}\to\mathrm{I}} = 1 - a$ $\mathrm{Br}_{\mathrm{L}\to\mathrm{I}} = 1$	$0 \leq a \leq 1$	–	–	–	–	–	③ ❷ ①	② ❶ ③	–
#7 L$\not{\mathrm{M}}\not{\mathrm{H}}$	$\mathrm{Br}_{\mathrm{H}\to\mathrm{L}} = a$, $\mathrm{Br}_{\mathrm{H}\to\mathrm{I}} = 1 - a$ $\mathrm{Br}_{\mathrm{M}\to\mathrm{L}} = b$, $\mathrm{Br}_{\mathrm{M}\to\mathrm{I}} = 1 - b$	$0 \leq a \leq 1$ $0 \leq b \leq 1$	–	–	–	–	② ① ❸	–	③ ② ❶	–
#8 $\not{\mathrm{L}}\not{\mathrm{M}}\not{\mathrm{H}}$	Not relevant, since no neutrinos observed		–	–	–	–	–	–	–	③ ② ① ② ① ③

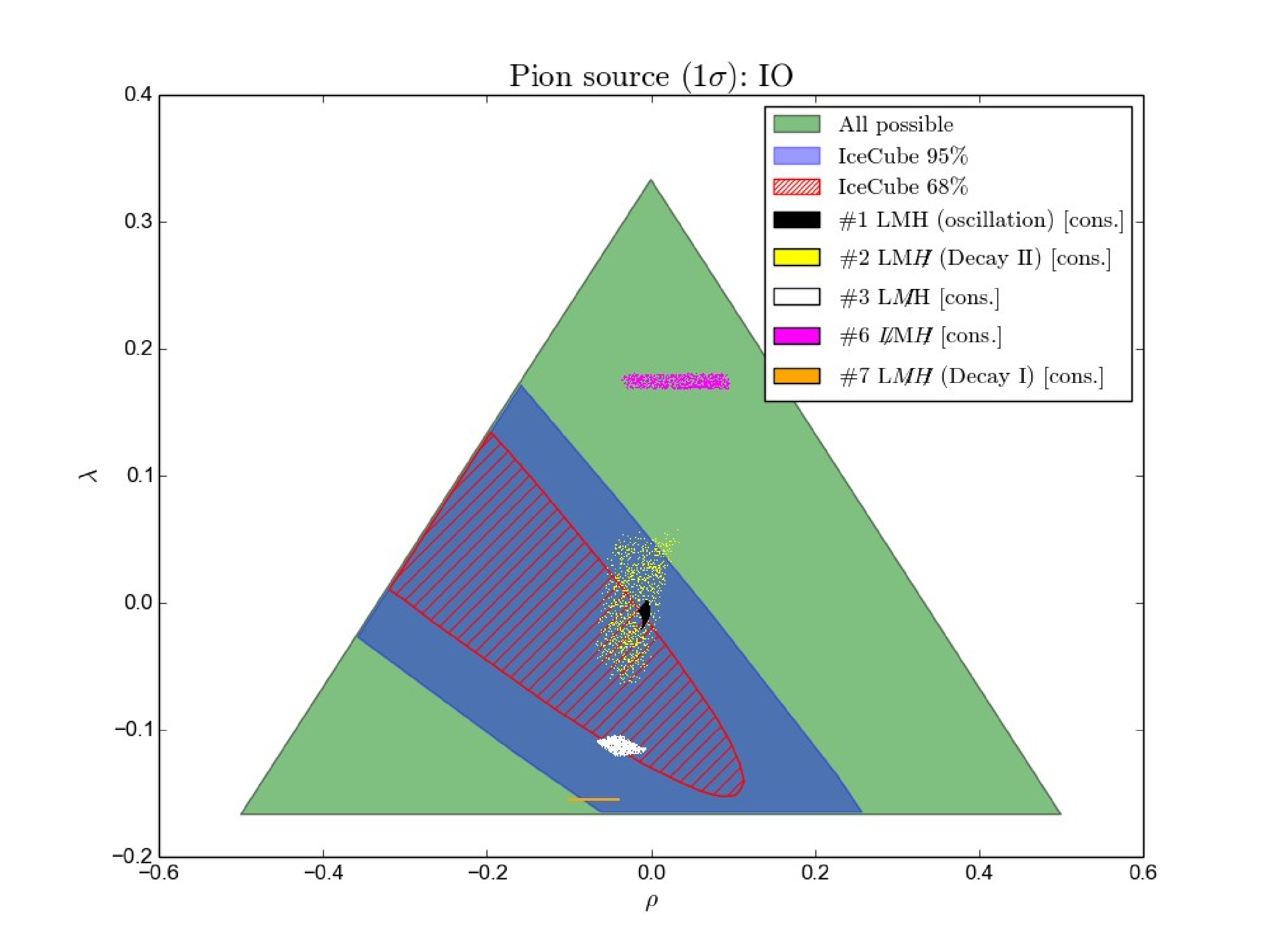
[4] Michele Maltoni, Walter Winter, JHEP07(2008)064



Pion source (1σ): IO
All possible
IceCube 95%
IceCube 68%
#1 LMH (oscillation)
#2 LMH̸ (Decay II)
#3 LM̸H
#4 L̸MH
#5 L̸M̸H
#6 L̸MH̸
#7 LM̸H̸ (Decay I)
λ
ρ
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6


Pion source (1σ): NO
All possible
IceCube 95%
IceCube 68%
#1 LMH (oscillation)
#2 LMH (Decay II)
#3 LMH
#4 LMH
#5 LMH
#6 LMH
#7 LMH (Decay I)
λ
ρ
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6

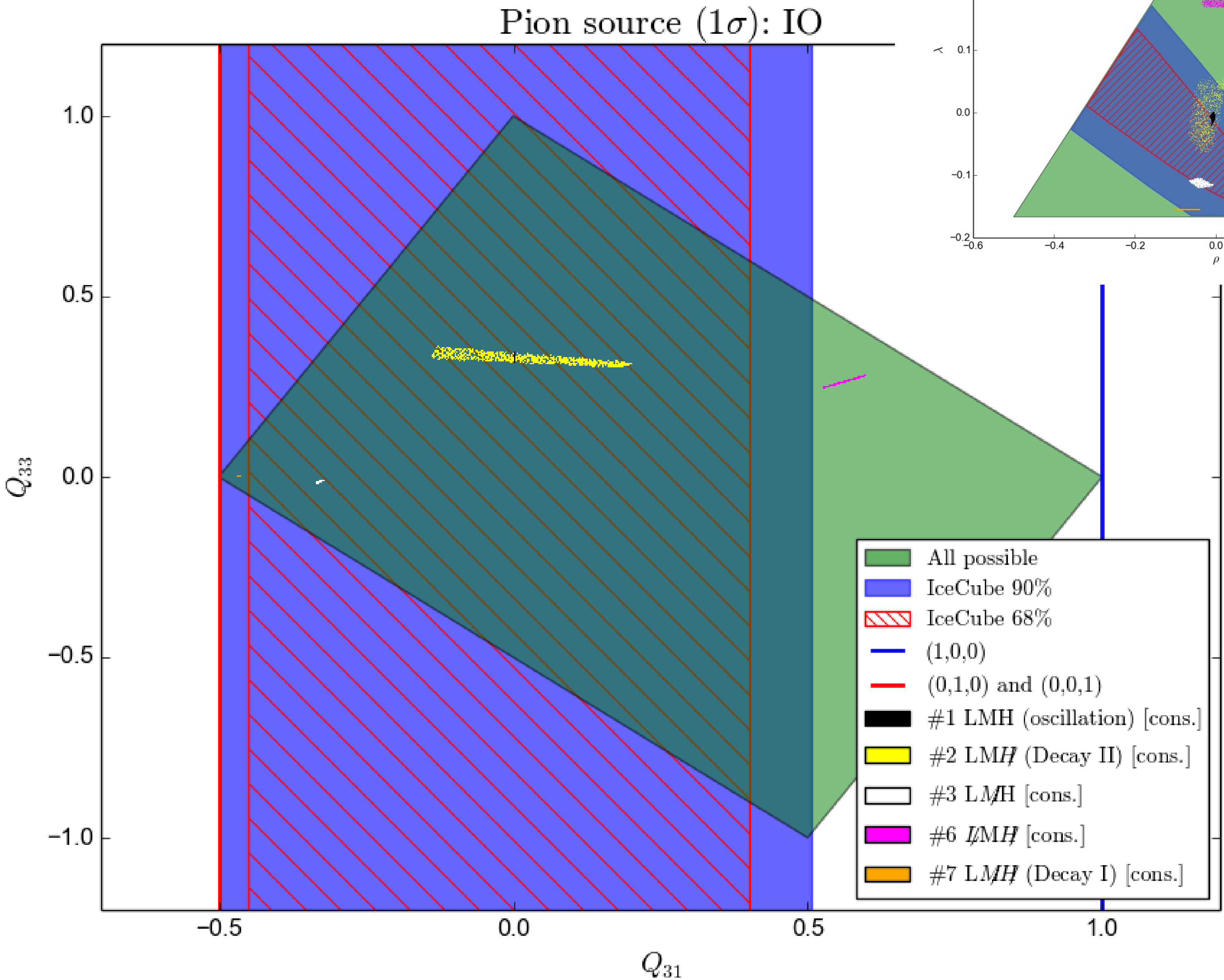

Muon damped source (1σ): NO
All possible
IceCube 95%
IceCube 68%
#1 LMH (oscillation)
#2 LMH̸ (Decay II)
#3 LM̸H
#4 L̸MH
#5 L̸M̸H
#6 L̸MH̸
#7 LM̸H̸ (Decay I)
λ
ρ
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6



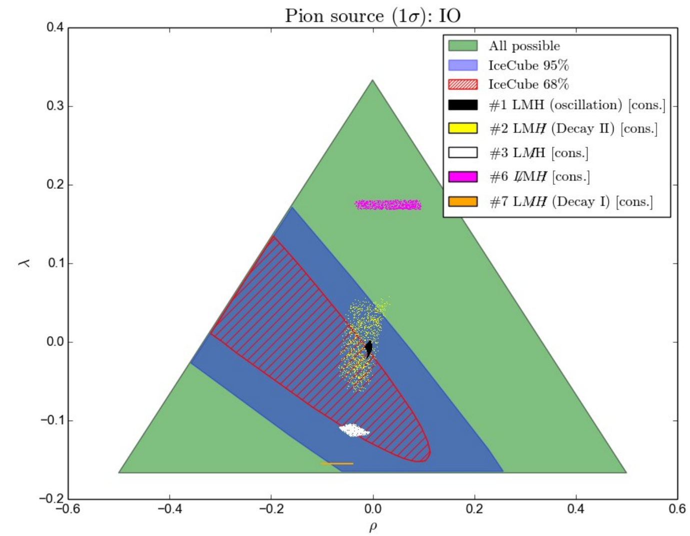
$$\begin{pmatrix}\kappa\\ \rho\\ \lambda\end{pmatrix} = \boldsymbol{Q}\begin{pmatrix}1/3\\ a\\ b\end{pmatrix}$$

$$\lambda = \frac{Q_{31}}{3} + aQ_{32} + b\,Q_{33}$$

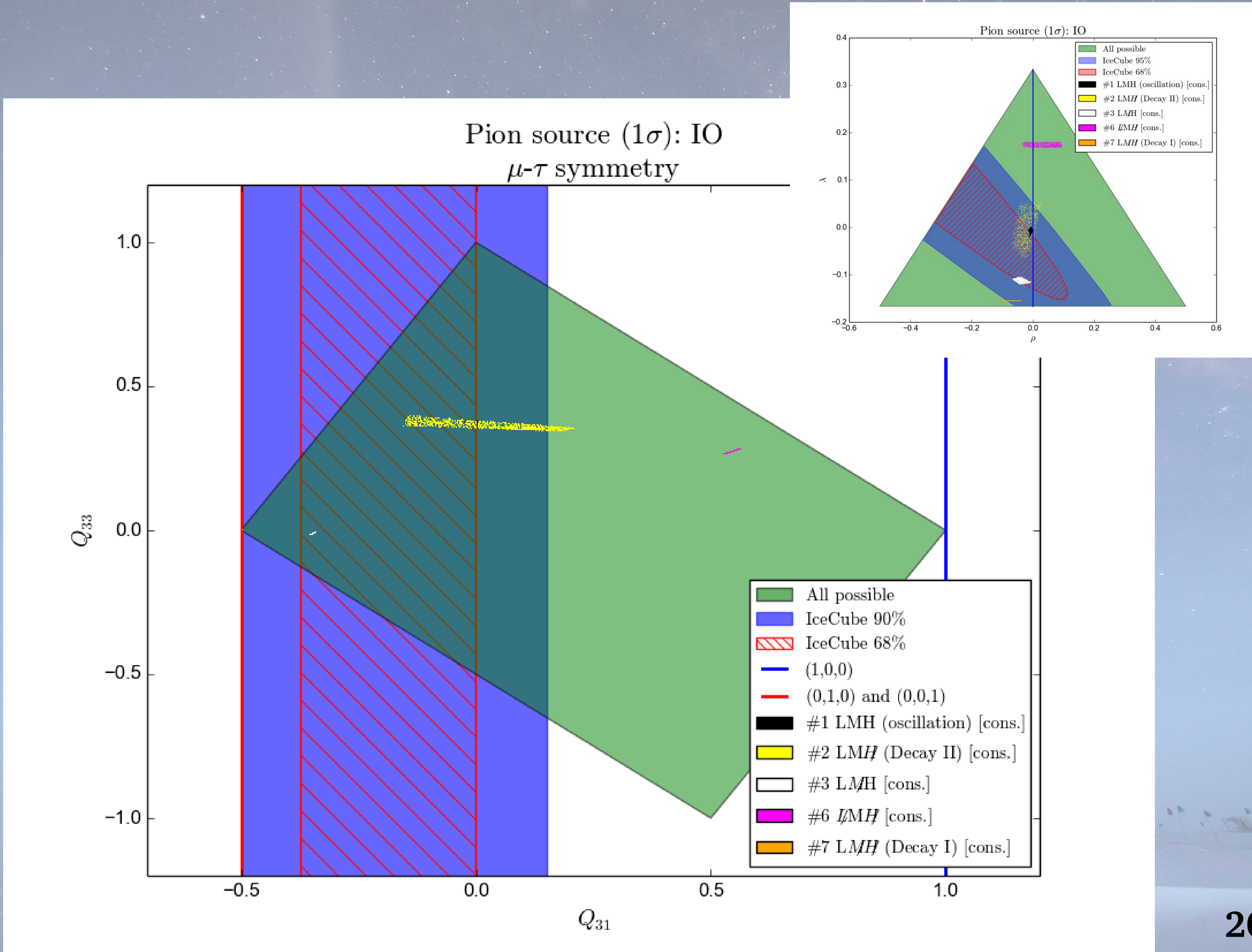
→ $\mu - \tau$ symmetry $\left(Q_{32} = 0 \text{ or } \rho = 0\right)$: $Q_{33} = \frac{\lambda}{b} - \frac{Q_{31}}{3b}$

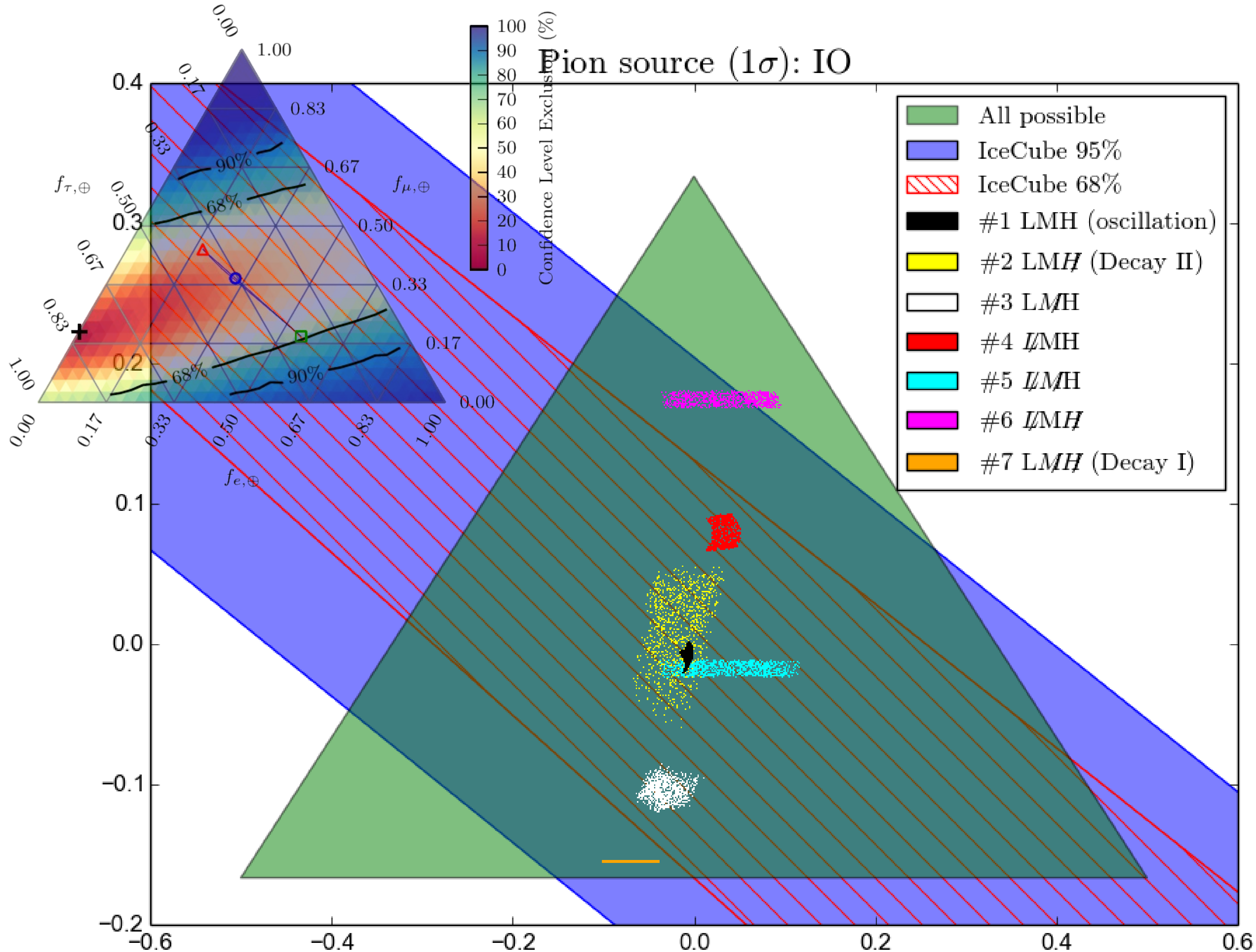


Pion source (1σ): IO
1.0
0.5
0.0
−0.5
−1.0
Q_{33}
−0.5
0.0
0.5
1.0
Q_{31}
All possible
IceCube 90%
IceCube 68%
(1,0,0)
(0,1,0) and (0,0,1)
#1 LMH (oscillation) [cons.]
#2 LMH̸ (Decay II) [cons.]
#3 LM̸H [cons.]
#6 L̸MH̸ [cons.]
#7 LM̸H̸ (Decay I) [cons.]

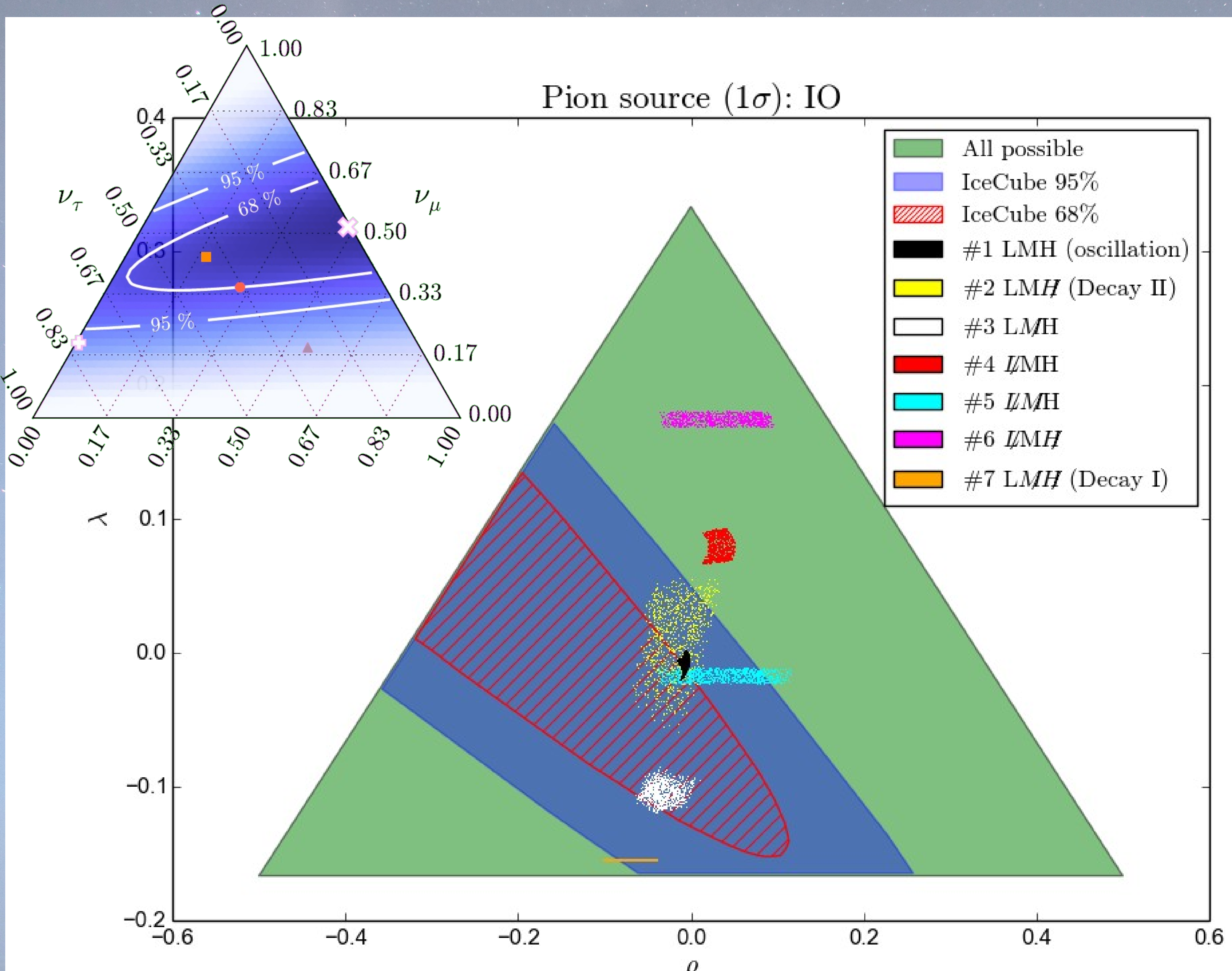


Pion source (1σ): IO
All possible
IceCube 95%
IceCube 68%
#1 LMH (oscillation) [cons.]
#2 LMH̸ (Decay II) [cons.]
#3 LM̸H [cons.]
#6 L̸MH̸ [cons.]
#7 LM̸H̸ (Decay I) [cons.]
λ
ρ


Pion source (1σ): IO
μ-τ symmetry
All possible
IceCube 90%
IceCube 68%
(1,0,0)
(0,1,0) and (0,0,1)
#1 LMH (oscillation) [cons.]
#2 LMH (Decay II) [cons.]
#3 LMH [cons.]
#6 LMH [cons.]
#7 LMH (Decay I) [cons.]
Q33
Q31
Pion source (1σ): IO
All possible
IceCube 95%
IceCube 68%
#1 LMH (oscillation) [cons.]
#2 LMH (Decay II) [cons.]
#3 LMH [cons.]
#6 LMH [cons.]
#7 LMH (Decay I) [cons.]
λ
ρ



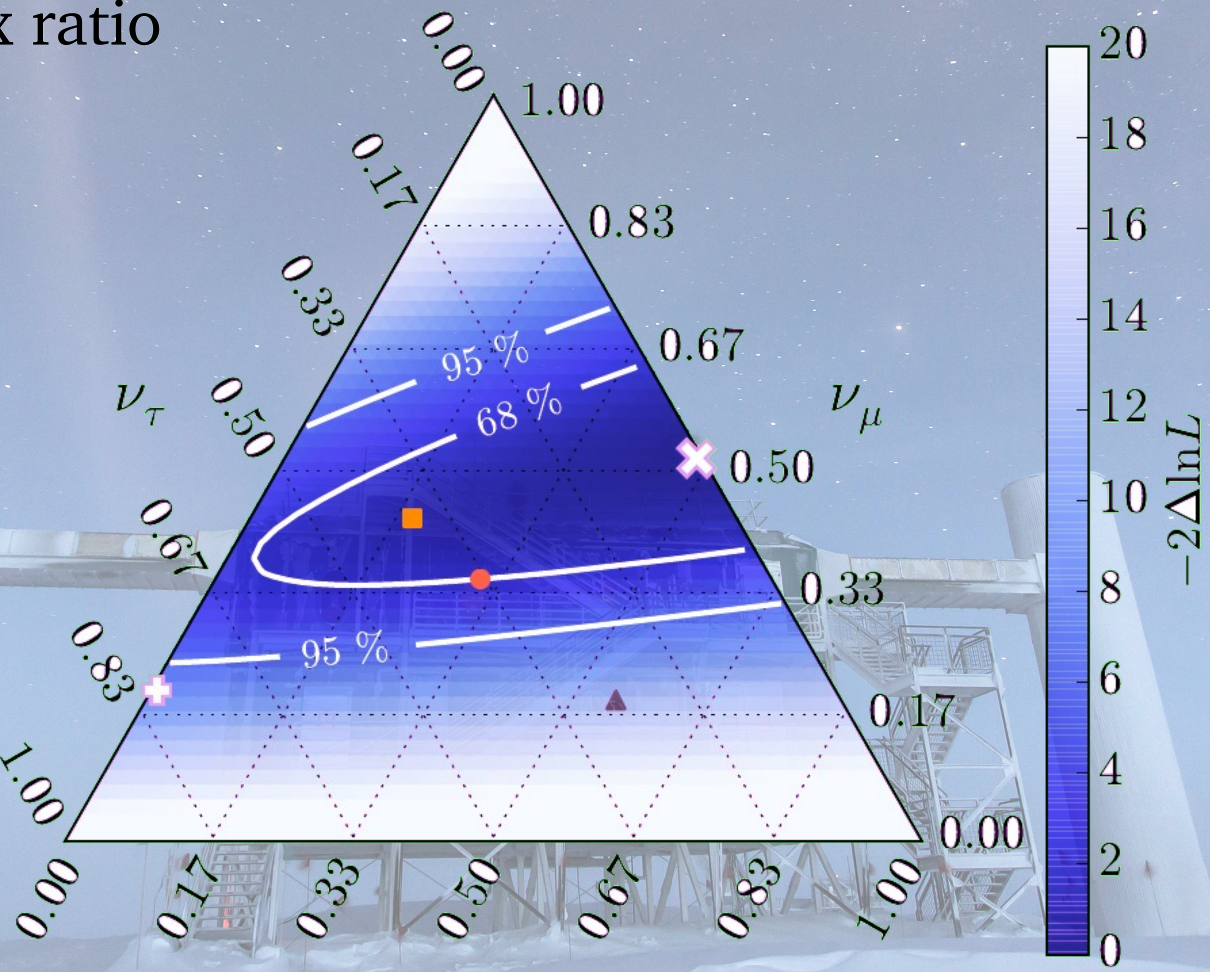
[5] M. G. Aartsen, M. Ackermann et al, arXiv: 1502.03376



See reference [1]

Summary

* The most recent IceCube results on the astrophysical neutrino flux ratio

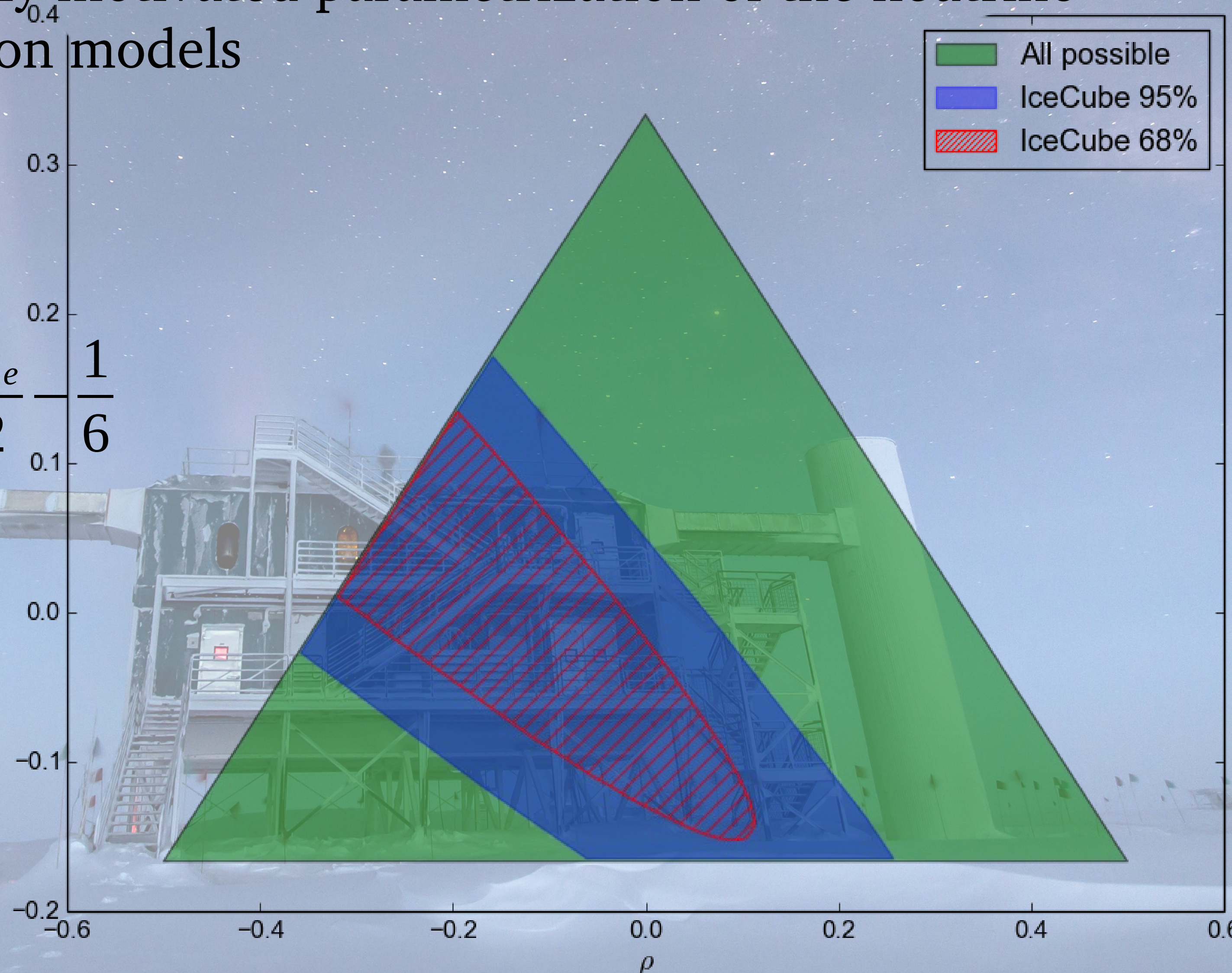


Summary

* New physically motivated parametrization of the neutrino flavour transition models

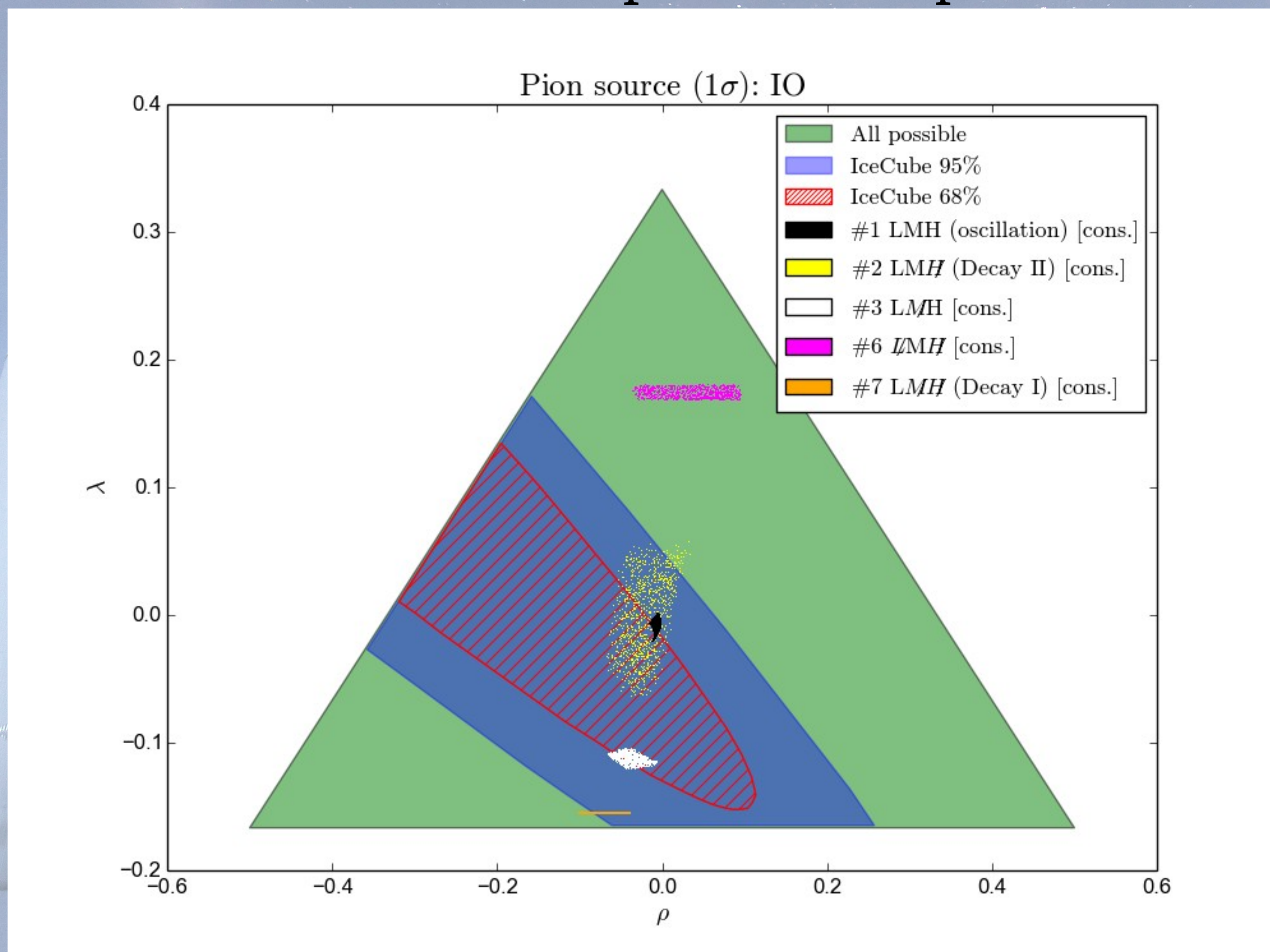
$$\lambda = \frac{\varphi_e}{3} - \frac{\varphi_\tau + \varphi_\mu}{6} = \frac{\varphi_e}{2} - \frac{1}{6}$$

$$\rho = \frac{\varphi_\tau - \varphi_\mu}{2}$$



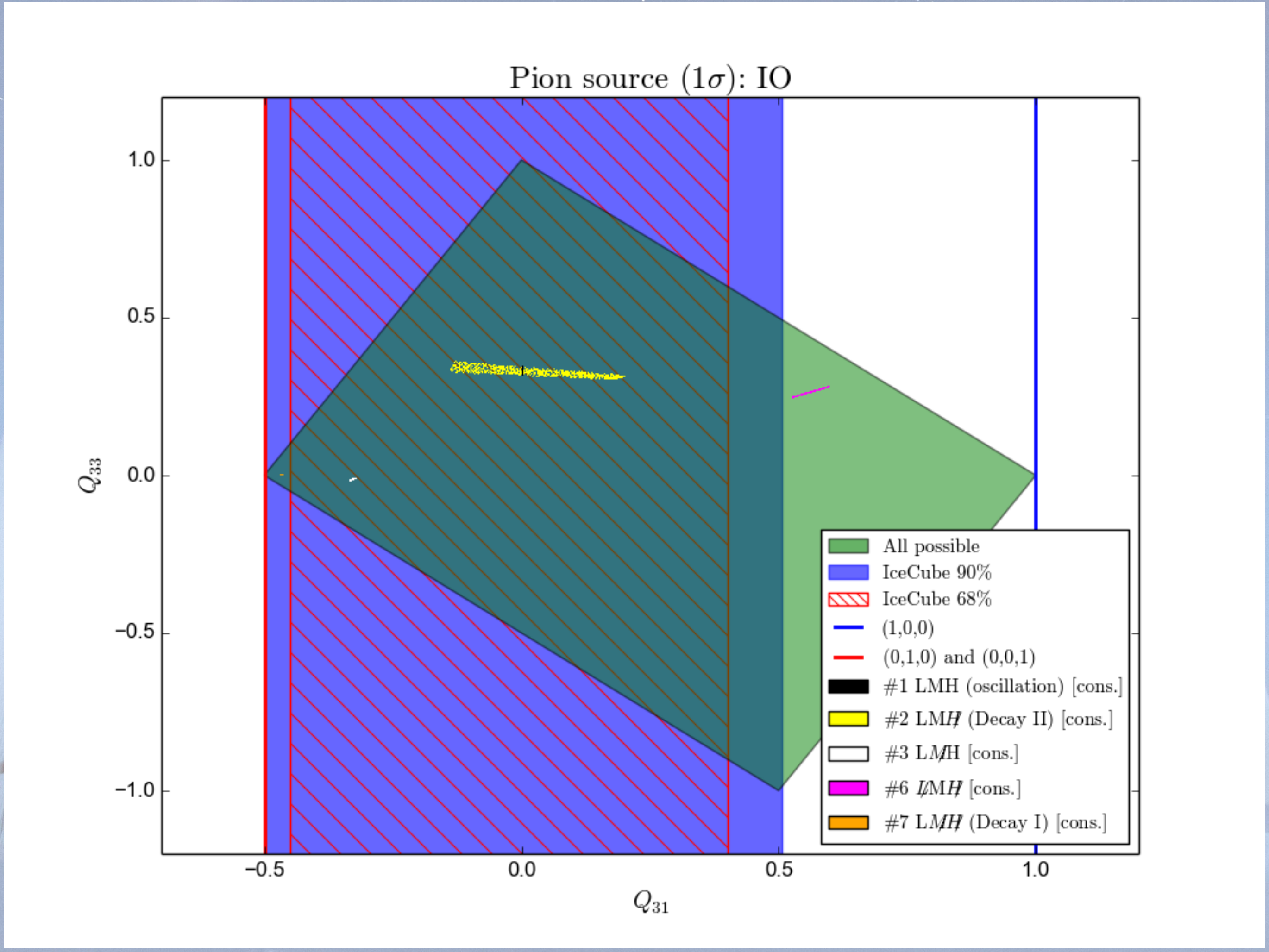
Summary

* Resulting analysis of the constraints imposed by the IceCube measurements on the parameter space.



Summary

* Resulting analysis of the constraints imposed by the IceCube measurements on the parameter space.



References

- [1] M. C. Gonzalez-Garcia et al, *Updated fit to three neutrino mixing: status of leptonic CP*
- [2] M. G. Aartsen, K. Abraham et al., *A COMBINED MAXIMUM-LIKELIHOOD ANALYSIS OF THE HIGH-ENERGY ASTROPHYSICAL NEUTRINO* FLUX MEASURED WITH ICECUBE
- [3] Kwang-Chang Lai, Guey-Lin Lin and T. C. Liu, *Flavor Transition Mechanisms of Propagating Astrophysical Neutrinos - A Model Independent Parametrizations*
- [4] Michele Maltoni, Walter Winter, *Testing neutrino flavor mixing plus decay with neutrino telescope violation*
- [5] M. G. Aartsen, M. Ackermann et al, *Flavor Ratio of Astrophysical Neutrinos above 35 TeV in IceCube*

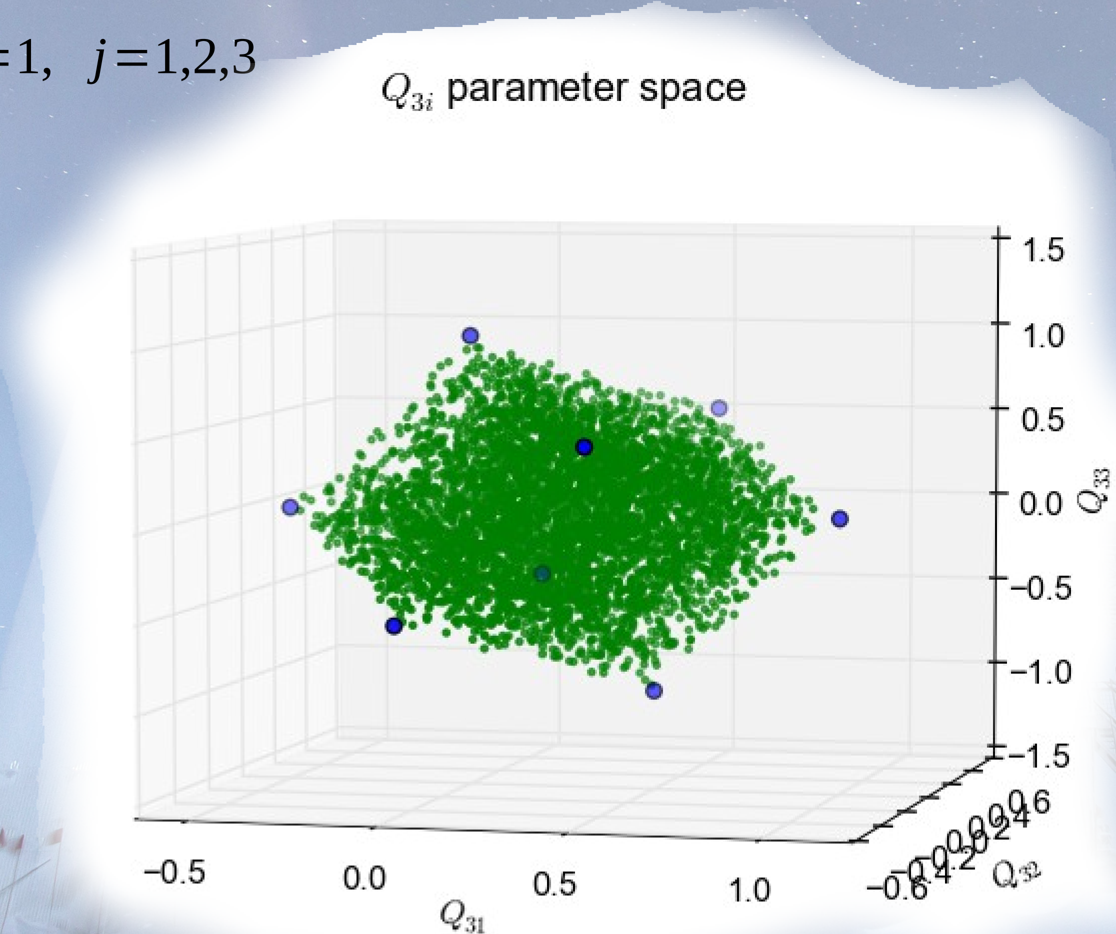
Extra: Q restrictions

$$P(\nu_\alpha \rightarrow \nu_e) + P(\nu_\alpha \rightarrow \nu_\mu) + P(\nu_\alpha \rightarrow \nu_\tau) = P_{1\alpha} + P_{2\alpha} + P_{3\alpha} = 1$$

i.e.

$$\sum_{i=1}^{3} P_{ij} = 1, \quad j = 1,2,3$$

→ The values of the Q-matrix are constrained as well



Extra: Q restrictions

$$P(\nu_\alpha \rightarrow \nu_e) + P(\nu_\alpha \rightarrow \nu_\mu) + P(\nu_\alpha \rightarrow \nu_\tau) = P_{1\alpha} + P_{2\alpha} + P_{3\alpha} = 1$$

i.e.

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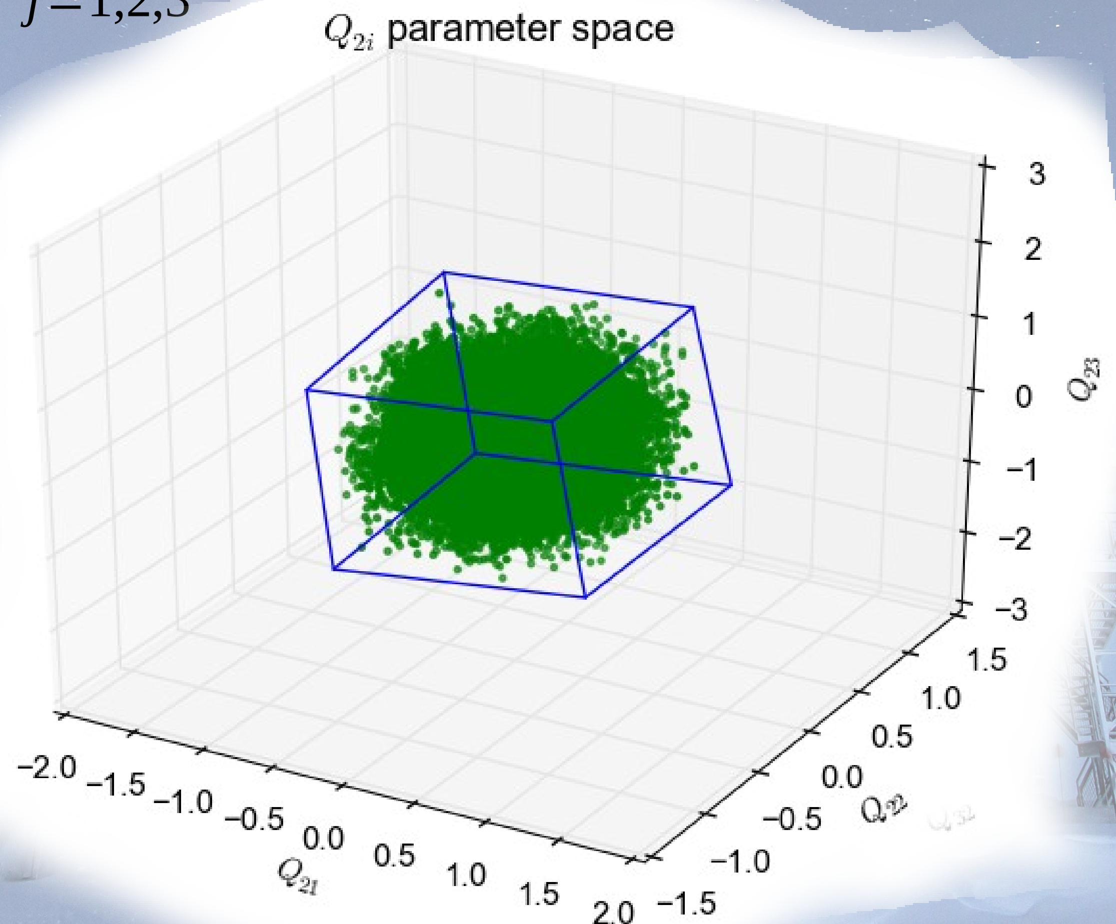
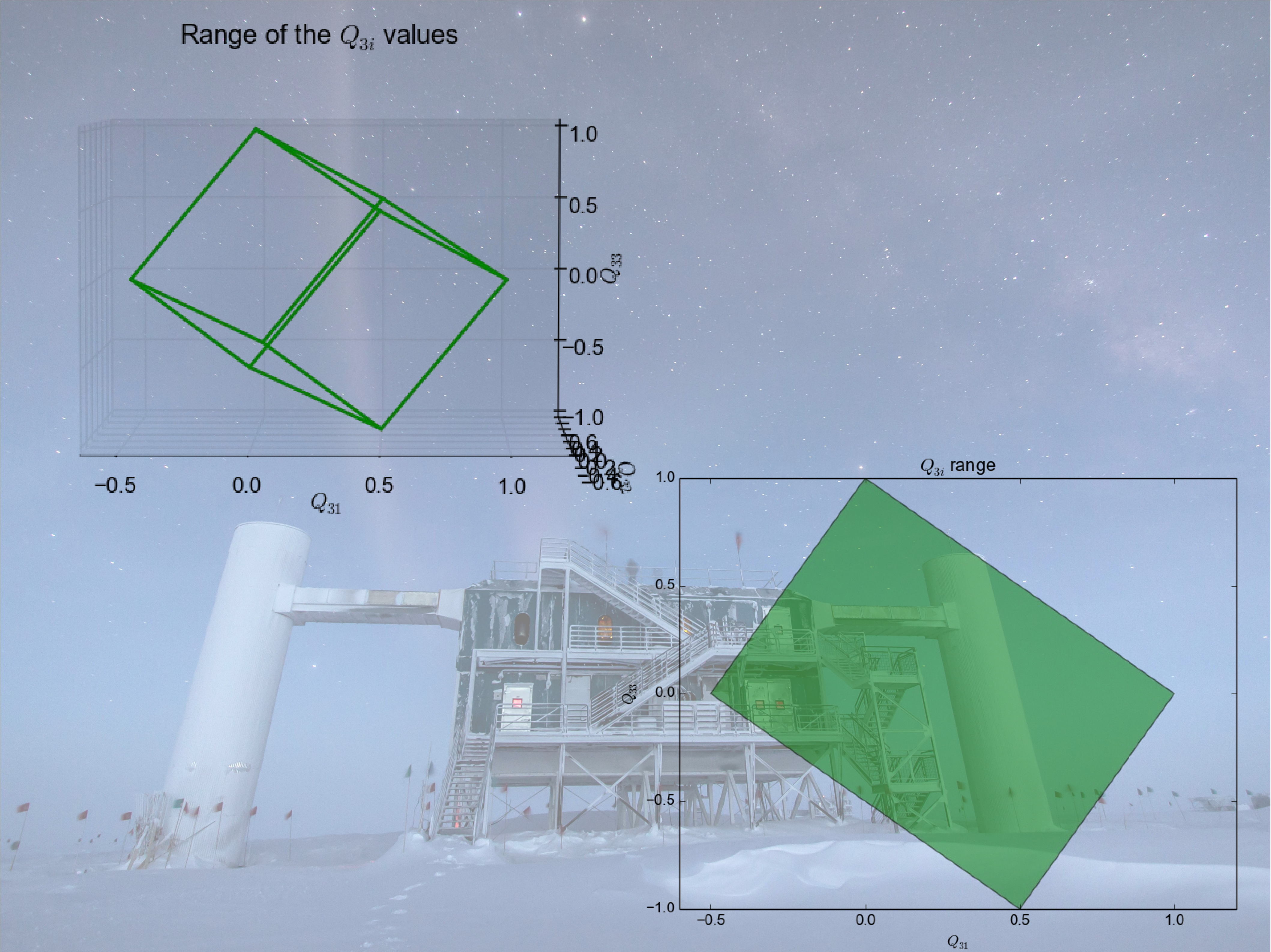
→ The values of the Q-matrix are constrained as well





Range of the Q_{3i} values
1.0
0.5
0.0
−0.5
−1.0
Q_{33}
−0.5
0.0
0.5
1.0
Q_{31}
Q_{3i} range
1.0
0.5
0.0
−0.5
−1.0
Q_{33}
−0.5
0.0
0.5
1.0
Q_{31}