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CMB bispectrum

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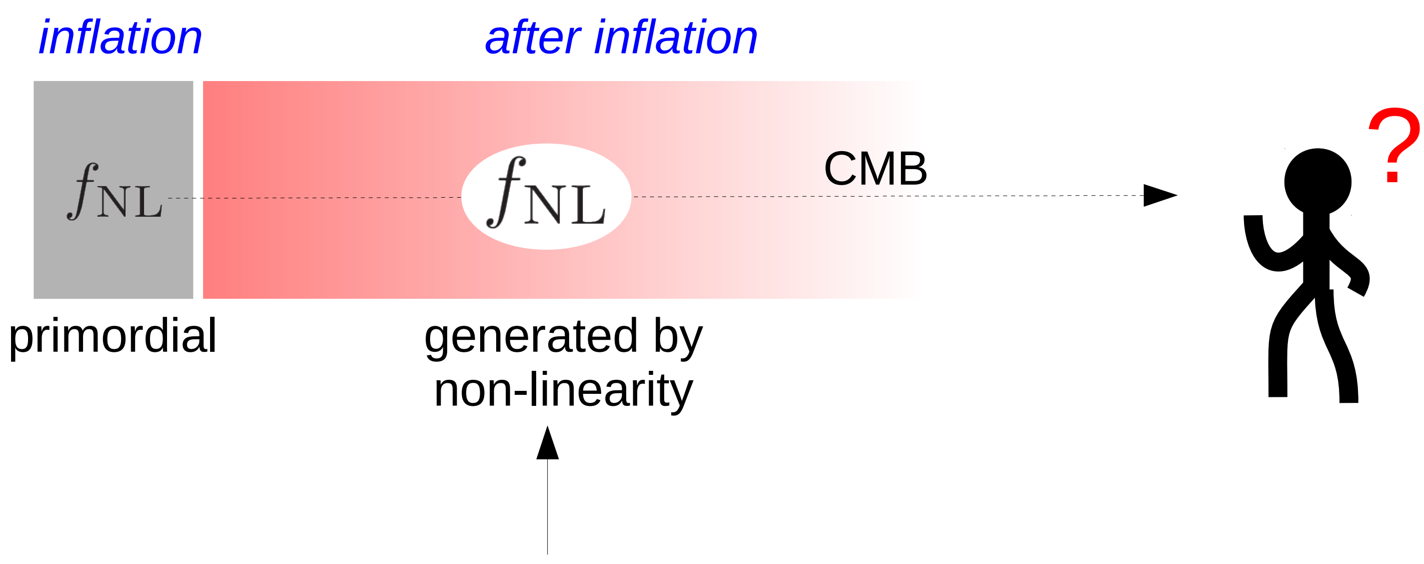
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

... parameterised by $f_{\rm NL}$



Unfortunately dominant ...

How large ?

Basic equations

Dodelson, "Modern cosmology", (Academic press)
Matsubara, "Uchuron no Butsuri" (Tokyo Univ.)

Photon/Neutrino

$$f_{\gamma,\nu}(\mathbf{x},\mathbf{p},\eta) = \left[\exp\left\{\frac{p}{T_{\gamma,\nu}(\eta)\left[1+\Theta_{\mathrm{T,N}}(\mathbf{x},\hat{p},\eta)\right]}\right\} \pm 1\right]^{-1}$$

$$\frac{df_{\gamma,\nu}}{d\eta} \equiv \frac{\partial f_{\gamma,\nu}}{\partial \eta} - \frac{a'}{a} E \frac{\partial f_{\gamma,\nu}}{\partial E} = \frac{\mathcal{C}[f_{\gamma,\nu}]}{E}$$

← Collision term of Thomson scattering (only for photons)

CDM/Baryon

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + g_{\mu\nu} p$$

$$\nabla^\mu \delta T_{\mu\nu} = 0$$

← Photon's Thomson scattering term is derived from Boltzmann eq. of baryons.

Gravity

$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij}dx^i dx^j\right]$$

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

1st-order perturbation equations

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Photon temperature

$$\begin{cases} \dot{\Theta}_0^{\mathrm{T}} = -k\Theta_1^{\mathrm{T}} - \dot{\Phi} \\ \dot{\Theta}_1^{\mathrm{T}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{T}} + \Theta_0^{\mathrm{T}}) + \dot{\tau}\left(\Theta_1^{\mathrm{T}} + \frac{1}{3}v_b\right) + \frac{1}{3}k\Psi \\ \dot{\Theta}_2^{\mathrm{T}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{T}} + 2\Theta_1^{\mathrm{T}}) + \dot{\tau}\left(\Theta_2^{\mathrm{T}} - \frac{1}{10}\Pi\right) \\ \dot{\Theta}_\ell^{\mathrm{T}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{T}} + \ell\Theta_{\ell-1}^{\mathrm{T}}\right] + \dot{\tau}\Theta_\ell^{\mathrm{T}} \end{cases}$$

Photon polarisation

$$\begin{cases} \dot{\Theta}_0^{\mathrm{P}} = -k\Theta_1^{\mathrm{P}} + \dot{\tau}\left(\Theta_0^{\mathrm{P}} - \frac{1}{2}\Pi\right) \\ \dot{\Theta}_1^{\mathrm{P}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{P}} + \Theta_0^{\mathrm{P}}) + \dot{\tau}\Theta_1^{\mathrm{P}} \\ \dot{\Theta}_2^{\mathrm{P}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{P}} + 2\Theta_1^{\mathrm{P}}) + \dot{\tau}\left(\Theta_2^{\mathrm{P}} - \frac{1}{10}\Pi\right) \\ \dot{\Theta}_\ell^{\mathrm{P}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{P}} + \ell\Theta_{\ell-1}^{\mathrm{P}}\right] + \dot{\tau}\Theta_\ell^{\mathrm{P}} \end{cases}$$

Massless neutrino temperature

$$\begin{cases} \dot{\Theta}_0^{\mathrm{N}} = -k\Theta_1^{\mathrm{N}} - \dot{\Phi} \\ \dot{\Theta}_1^{\mathrm{N}} = \frac{1}{3}k(-2\Theta_2^{\mathrm{N}} + \Theta_0^{\mathrm{N}}) + \frac{1}{3}k\Psi \\ \dot{\Theta}_2^{\mathrm{N}} = \frac{1}{5}k(-3\Theta_3^{\mathrm{N}} + 2\Theta_1^{\mathrm{N}}) \\ \dot{\Theta}_\ell^{\mathrm{N}} = \frac{1}{2\ell+1}k\left[-(\ell+1)\Theta_{\ell+1}^{\mathrm{N}} + \ell\Theta_{\ell-1}^{\mathrm{N}}\right] \end{cases}$$

CDM, baryon

$$\begin{cases} \dot{\delta}_{\mathrm{c}} = -ikv_{\mathrm{c}} - 3\dot{\Phi} \\ \dot{\delta}_{\mathrm{b}} = -ikv_{\mathrm{b}} - 3\dot{\Phi} \\ \dot{v}_{\mathrm{c}} = -\mathcal{H}v_{\mathrm{c}} - ik\Psi \\ \dot{v}_{\mathrm{b}} = -\mathcal{H}v_{\mathrm{b}} - ik\Psi + \frac{\dot{\tau}}{R}(v_{\mathrm{b}} + 3i\Theta_{\mathrm{T}1}) \end{cases}$$

Gravity
(conformal Newtonian gauge)

$$\dot{\Phi} = -\frac{k^2}{3\mathcal{H}}\Phi + \mathcal{H}\Psi + \frac{a^2\mathcal{H}_0^2}{2\mathcal{H}}\delta\Omega_0$$

$$\left(\Psi = -\Phi - 12\frac{a^2\mathcal{H}_0^2}{k^2}\Omega_r\Theta_{r,2}\right)$$

Line-of-sight formula

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Boltzmann equation

$$\dot{\Theta}^{\mathrm{T}} + ik\mu\Theta^{\mathrm{T}} = -\dot{\Phi} - ik\mu\Psi$$
$$-\dot{\tau}\left[\Theta_0^{\mathrm{T}} - \Theta^{\mathrm{T}} + \mu v_{\mathrm{b}} - \frac{1}{2}\mathcal{P}_2(\mu)(\Theta_2^{\mathrm{T}} + \Theta_0^{\mathrm{P}} + \Theta_2^{\mathrm{P}})\right]$$

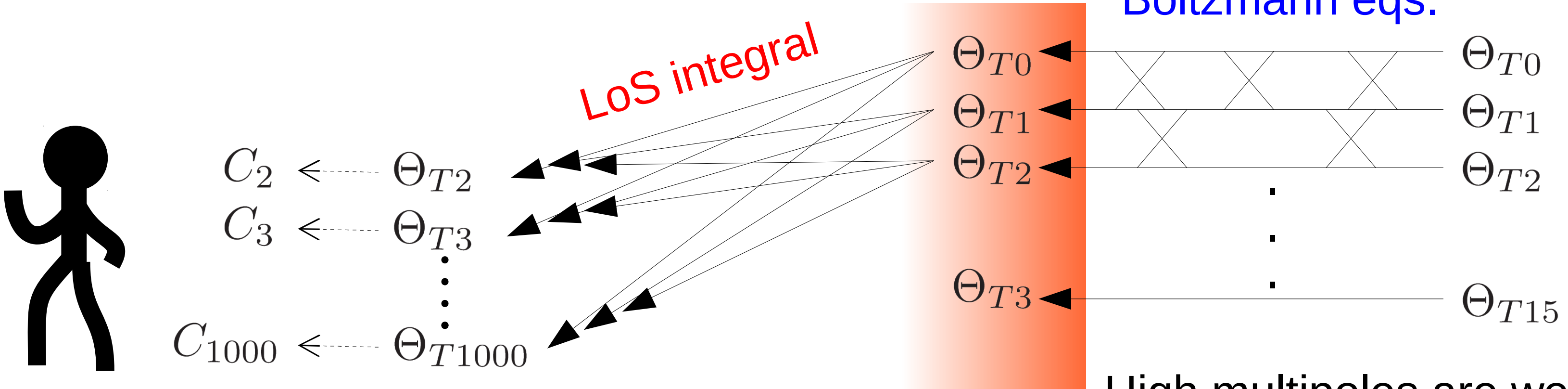
Integral form

$$\Theta_\ell^{\mathrm{T}}(k,\eta_0) = \int_0^{\eta_0} d\eta\, {\color{red}S(k,\eta)}\, j_\ell[k(\eta_0-\eta)]$$

Seljak, Zaldarriaga, APJ 469 (1996) 437

Highly suppressed before LSS

directly solving
Boltzmann eqs.



High multipoles are well suppressed by tight-coupling between baryons-photons

Initial conditions

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Adiabatic initial condition

Ma, Bertschinger, APJ 455 (1995) 7

Unchanged potential,
Radiation dominant,
Similarly fluctuated,
Superhorizon,
Tight-coupling,
Negligible photon's quadrupole,
Neutrino's.quadrupole

$$\delta_c = \delta_b = 3\Theta_{T0} = 3\Theta_{N0} = -\frac{1}{2}\Psi$$

$$v_b = v_c = -3\Theta_{T1} = -3\Theta_{N1} = -\frac{k}{2\mathcal{H}}\Psi$$

$$\Phi = -\left(1 + \frac{2}{5}f_\nu\right)\Psi$$

$$\Psi = -\frac{10}{15 + 4f_\nu}\zeta$$

massless neutrino fraction : $f_\nu = \rho_\nu / \rho_R$

primordial curvature perturbation

Recombination history

$\dot{\tau} = -X_e(\eta) n_{\mathrm{p}} \sigma_{\mathrm{T}} a$ ..... found by solving eqs describing chemical evolution of H, He ions.

Peebles-Weinberg scheme or Seager's scheme (recfast)

Reionisation

Use a simple ansatz so that $X_e(\eta) = 0 \to 1$ and satisfy $\tau = (8.9 \pm 3.2) \times 10^{-2}$ by Planck.

Angular power spectrum (1st-order temp.)

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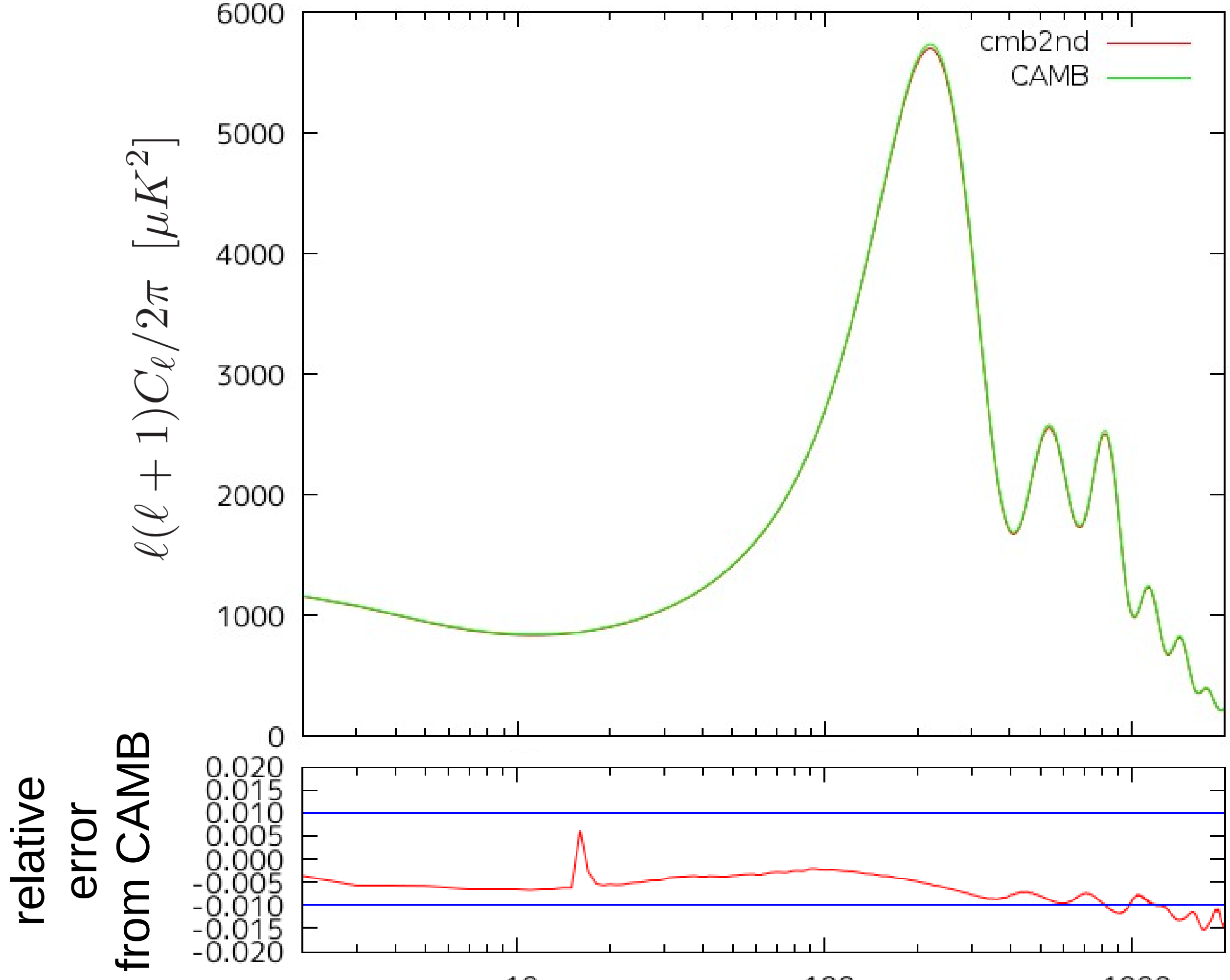
$$C_\ell^{\rm TT} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{\Theta_\ell^{\rm T}}(k,\eta)^2 P_\zeta(k,\eta_{\rm in})$$



parameters

$$\Delta_\zeta^2(k_{\rm pivot}) = 2.46\times 10^{-9}$$
$$k_{\rm pivot} = 0.002\ {\rm Mpc}^{-1}$$
$$n_s = 0.96$$
$$h^2\Omega_{\rm CDM} = 0.114$$
$$h^2\Omega_{\rm B} = 0.0226$$
$$h = 0.7$$
$$\tau = 0.0876$$
$$Y_p = 0.24$$
$$N_{\rm eff} = 3.04$$
$$T_0 = 2.725\ {\rm K}$$

existing codes

CMBFAST : Seljak, Zaldarriaga, APJ469 (1996) 437
CAMB : Lewis, Challinor, APJ538 (2000) 473
CLASS II : Blas, Lesgourgues, Tram, JCAP 1107 (2011) 034
CosmoLib : Huang, JCAP 1206 (2012) 012

Angular power spectrum (1st-order E-pol.)

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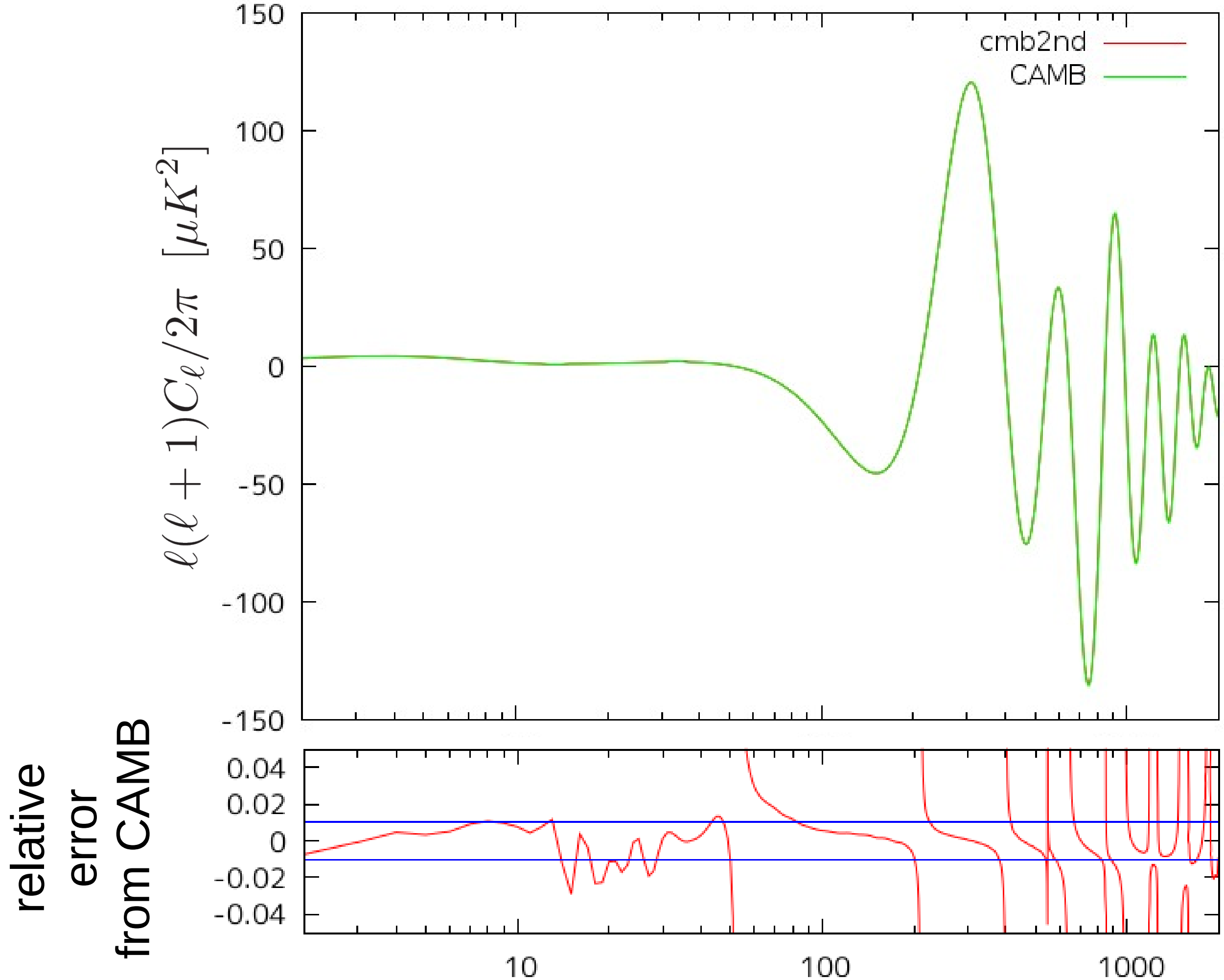
$$C_\ell^{\rm TE} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{\Theta_\ell^{\rm T}}(k,\eta)\mathcal{T}_{\Theta_\ell^{\rm E}}(k,\eta)P_\zeta(k,\eta_{\rm in})$$



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Angular power spectrum (1st-order E-pol.)

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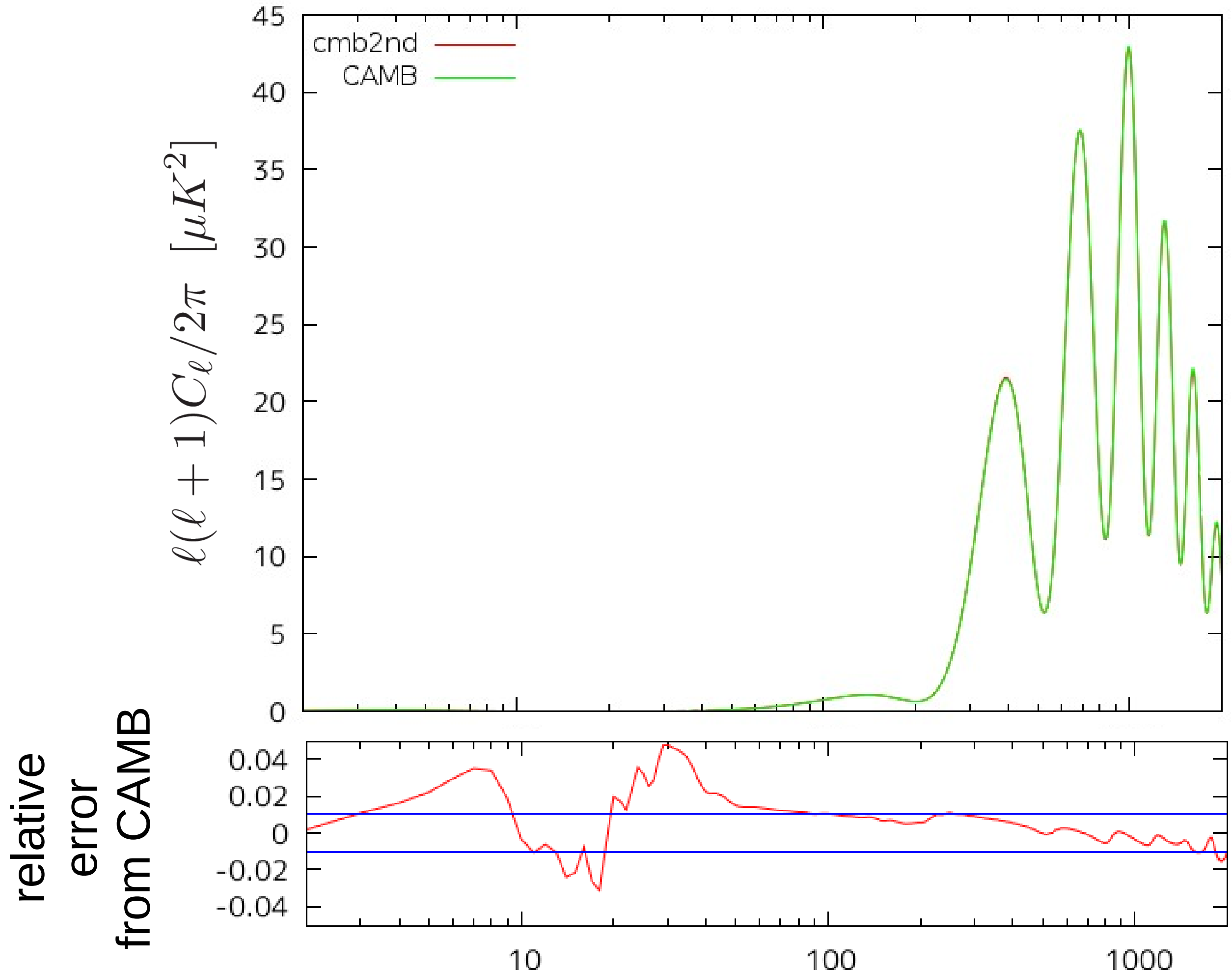
$$C_\ell^{\rm EE} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \mathcal{T}_{\Theta_\ell^{\rm E}}(k,\eta)^2 P_\zeta(k,\eta_{\rm in})$$



<u>parameters</u>

$$\Delta_\zeta^2(k_{\rm pivot}) = 2.46\times 10^{-9}$$
$$k_{\rm pivot} = 0.002\ {\rm Mpc}^{-1}$$
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Go to 2nd-order

Line-of-sight is bending by the gravity potential --> 'Curve-of-sight'

$$\Theta_\ell^{\mathrm{T}(1)}(k,\eta_0) = \int_0^{\eta_0} S(k,\eta) j_\ell[k(\eta_0-\eta)]\, d\eta$$

$$\Theta_\ell^{\mathrm{T}(2)}(k,\eta_0) = \int S(\Theta_{\mathrm{T}}^{(1)}, \Psi^{(1)}, \Phi^{(1)}, \cdots) \times T(\Psi^{(1)}, \Phi^{(1)}, \cdots)\, d\eta$$

Fluctuations on/after LSS

Bending effect
by gravity perturbations

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]

(cf. Fidler, Koyama, Pettinari, arXiv:1409.2461)

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]

Curve-of-sight formula

$$\delta I^{(\mathrm{II})} = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \mathcal{T}^{(\mathrm{II})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{n}_{\mathrm{obs}}) \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)$$

$$\mathcal{T}^{(\mathrm{II})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{n}_{\mathrm{obs}}) = F(\mathbf{n}_{\mathrm{obs}}) \int_0^{\eta_0} d\eta' \, F_S(\hat{k}_1) S(k_1, \eta') e^{i\mathbf{k}_1 \cdot \mathbf{n}_{\mathrm{obs}}(\eta_0 - \eta')}$$

$$\times \int d\eta_1 \, F_T(\hat{k}_2) T(k_2, \eta_1, \eta') e^{i\mathbf{k}_2 \cdot \mathbf{n}_{\mathrm{obs}}(\eta_0 - \eta_1)}$$

Fluctuations on/after LSS

Gravitational effects transmogrifying “line” into “curve”

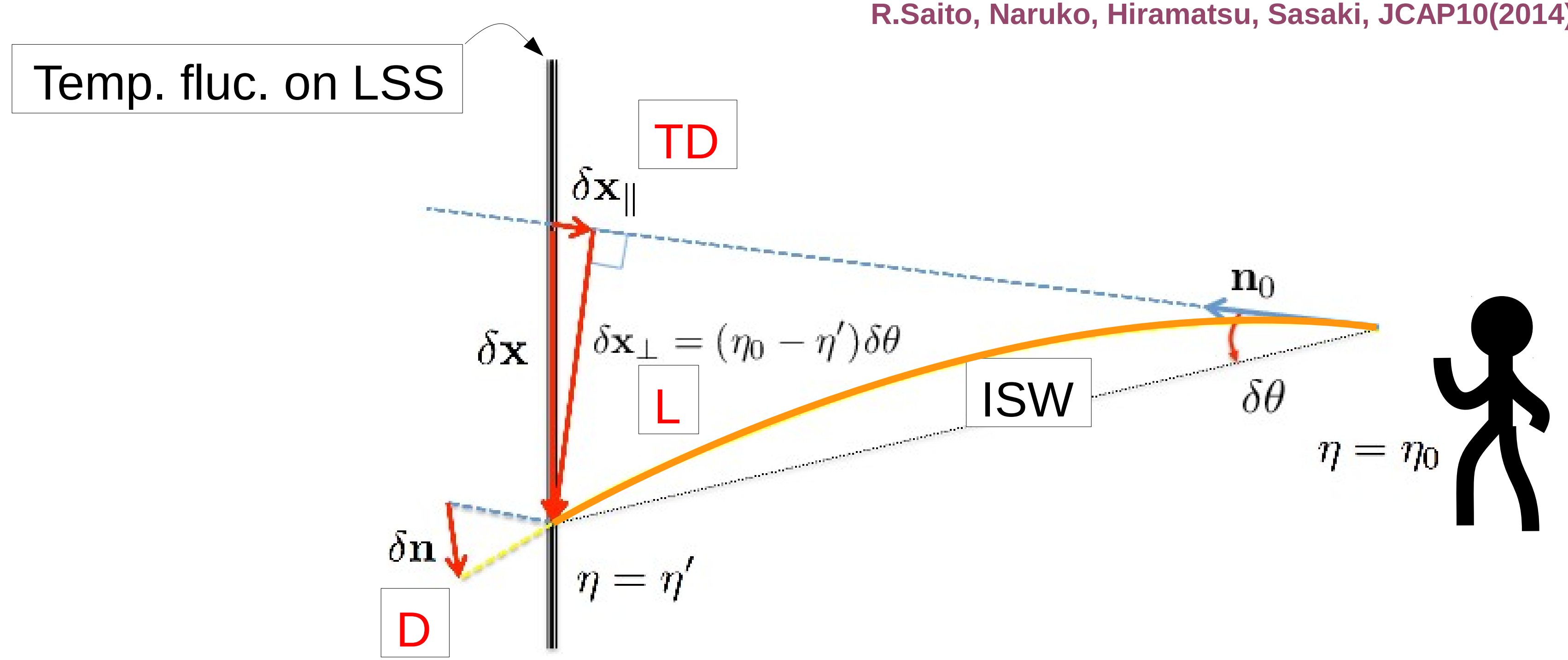
Line-of-sight formula

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk \, k^2 \mathcal{T}_{\Theta_\ell}(k, \eta)^2 P_\zeta(k, \eta_{\mathrm{in}})$$

$$\mathcal{T}_{\Theta_\ell}(k, \eta_0) = \int_0^{\eta_0} d\eta \, S(k, \eta) j_\ell[k(\eta_0 - \eta)]$$

2nd-order line(curve)-of-sight formula

R.Saito, Naruko, Hiramatsu, Sasaki, JCAP10(2014)051 [arXiv:1409.2464]



We found 7 combinations in this formula,

[Fluc. on LSS] x [gravitational]

- Source x ISW
- Source x Lensing
- Source x Time-delay
- Source x Deflection

[gravitational] x [gravitational]

- ISW x ISW
- ISW x Lensing
- ISW x Time-delay

'Curve'-of-sight formula example

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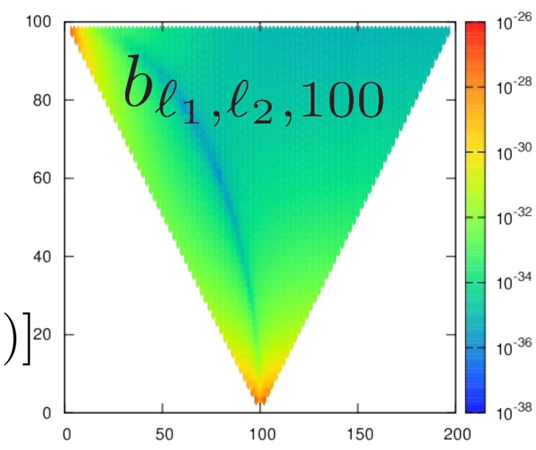
$$\langle \Theta_{\rm T}(\mathbf{k}_1)\Theta_{\rm T}(\mathbf{k}_2)\Theta_{\rm T}(\mathbf{k}_3)\rangle \sim \langle \Theta_{\rm T}^{(2)}(\mathbf{k}_1)\Theta_{\rm T}^{(1)}(\mathbf{k}_2)\Theta_{\rm T}^{(1)}(\mathbf{k}_3)\rangle \longrightarrow b_{\ell_1\ell_2\ell_3}$$

$$b^{S\times T}_{\ell_1\ell_2\ell_3} = F_{\ell_1\ell_2\ell_3}\int_0^{\eta_0} d\eta'\, b^S_{\ell_1}(\eta')b^T_{\ell_2}(\eta') + 5 \text{ perms.}$$

$$b^S_{\ell_1}(\eta') = \frac{2}{\pi}\int dk_1\, k_1^2 P_\zeta(k_1)\mathcal{T}_{\ell_1}(k_1)S(k_1,\eta')j_{\ell_1}[k_1(\eta_0-\eta')]$$

$$b^T_{\ell_2}(\eta') = \frac{2}{\pi}\int_{\eta'}^{\eta_0} d\eta_1 \int dk_2\, k_2^2 P_\zeta(k_2)\mathcal{T}_{\ell_2}(k_2)T(k_2,\eta_1,\eta')j_{\ell_2}[k_2(\eta_0-\eta_1)]$$



Source x Lensing

$$F_{\ell_1\ell_2\ell_3} = \frac{\ell_3(\ell_3+1)-\ell_1(\ell_1+1)-\ell_2(\ell_2+1)}{2}$$

$$S(k_1,\eta') = \frac{1}{\eta_0-\eta'}\left[4k_1 g(\eta')\left[\Theta_{T0}+\Psi\right] + 4\frac{d}{d\eta'}\left(\frac{g(\eta')v_{\rm b}}{k_1}\right) + 4\mathcal{P}_2\left(\frac{1}{ik_1}\frac{d}{d\eta'}\right)[g(\eta')\Pi]\right]$$

$$T(k_2,\eta_1,\eta') = \left(\frac{\eta_1-\eta'}{\eta_0-\eta_1}\right)[\Psi(k_2,\eta_1)-\Phi(k_2,\eta_1)]$$

Non-Gaussianity parameter

$$B_\Theta(k_1, k_2, k_3) = \sum_i f_{\rm NL}^{(i)} B^{(i)}(k_1, k_2, k_3)$$

functional bases (templates)

Local type

Expanding spatial curvature perturbation up to 2nd-order,

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{\rm NL}(\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2\rangle)$$

Gangui et al., APJ 430 (1994) 447
Verde et al., MNRAS 313 (2000) L141
Komatsu, Spergel, PRD63 (2001) 063002

(assumption : 1st-order is Gaussian) "local" interaction

we have its bispectrum straightfowardly,

$$B_\Phi^{\rm local}(k_1,k_2,k_3) = 2f_{\rm NL}^{\rm local}\left[P_\Phi(k_1)P_\Phi(k_2) + 2\text{ perms}\right]$$

Other types

$$B_\Phi^{\rm equil} = 6f_{\rm NL}^{\rm equil}\Big[-\{P_\Phi(k_1)P_\Phi(k_2) + 2\text{ perms}\} - 2P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} + P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\text{ perms}\Big]$$

Babich et al., JCAP 0408 (2004) 009

$$B_\Phi^{\rm ortho} = 6f_{\rm NL}^{\rm ortho}\Big[-3\{P_\Phi(k_1)P_\Phi(k_2) + 2\text{ perms}\} - 8P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} + 3\left\{P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\text{ perms}\right\}\Big]$$

Senatore et al., JCAP 1001 (2010) 028

$$B_\Phi^{\rm folded} = 6f_{\rm NL}^{\rm folded}\Big[\{P_\Phi(k_1)P_\Phi(k_2) + 2\text{ perms}\} + 3P_\Phi(k_1)^{2/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3)^{2/3} - \left\{P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\text{ perms}\right\}\Big]$$

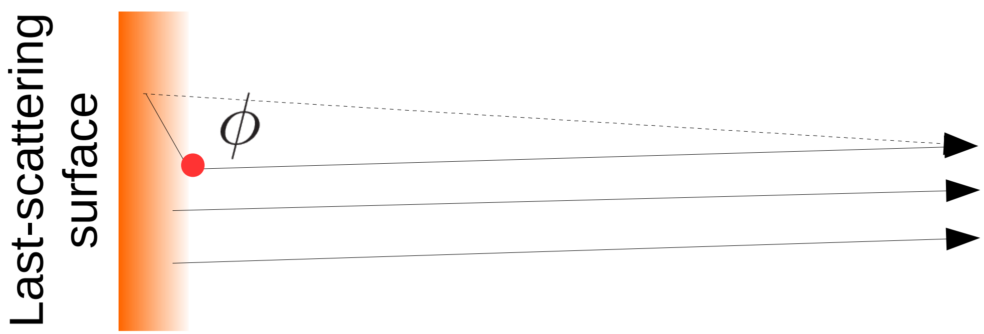
Chen et al., JCAP 0701 (2007) 002

Past results on CMB lensing

Dominant contribution is CMB lensing.
That corresponds to [Source x Lensing] + [ISW x Lensing] in our COS formula.

Remapping scheme

Taking into account scattering once at LSS by gravity potential.



$$\widetilde{\Theta}_T(\mathbf{n}) \equiv \Theta_T(\mathbf{n} + \nabla\phi)$$

$$\Theta_T(\mathbf{n})$$

$$\Theta_T(\mathbf{n})$$

Goldberg, Spergel, PRD 59 (1999) 103002
Hu, PRD 62 (2000) 043007
Zaldarriaga, PRD 62 (2000) 063510
Review : Lewis, Challinor, PR 429 (2006) 1

Lensing potential

$$\phi(\mathbf{n}) = -2 \int dD\, g_\phi(D) \Psi(\mathbf{x}, D)$$

Taylor expansion

$$\widetilde{\Theta}(\mathbf{n}) \approx \Theta(\mathbf{n}) + \nabla_i \phi(\mathbf{n}) \nabla^i \Theta(\mathbf{n}) + \cdots$$

Leading contribution to lensing bispectrum

$$B_{\ell_1 \ell_2 \ell_3} \approx \langle \widetilde{\Theta}^T_{\ell_1} \Theta^T_{\ell_2} \Theta^T_{\ell_3} \rangle \longrightarrow f_{\rm NL}^{\rm local} = 9.3$$

Hanson et al., PRD 80 (2009) 083004

Past results on CMB lensing

Takashi Hiramatsu

Dominant contribution is CMB lensing.
That corresponds to [Source x Lensing] + [ISW x Lensing] in our COS formula.

Remapping scheme

$$f_{\rm NL}^{\rm local} = 9.3$$

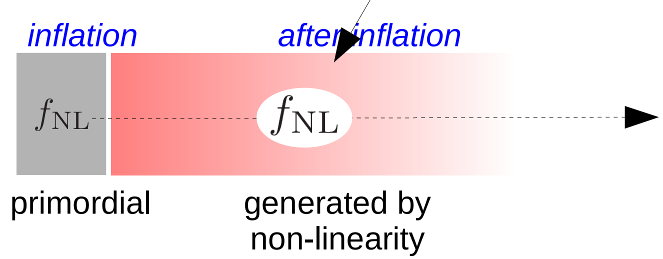
Instrumental noise
+ data masking

$$f_{\rm NL}^{\rm local} \sim 7$$

Observational constraints by Planck

$$f_{\rm NL}^{\rm pri,local} = 2.7 \pm 5.7$$
$$f_{\rm NL}^{\rm pri,equil} = -16 \pm 70$$
$$f_{\rm NL}^{\rm pri,ortho} = -34 \pm 33$$

(68% confidence level)



$$f_{\rm NL}^{\rm local} = 10.2 \pm 5.7$$
$$f_{\rm NL}^{\rm equil} = -13 \pm 70$$
$$f_{\rm NL}^{\rm ortho} = -56 \pm 33$$

Planck collaboration, arXiv:1502.01592

Using the least-square method, we determine the fitting parameter $f_{\rm NL}^{(i)}$ so that

$$\chi^2 \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3}^{\ell_{\rm max}} \frac{\left(B_{\ell_1\ell_2\ell_3} - \sum_i f_{\rm NL}^{(i)} B_{\ell_1\ell_2\ell_3}^{(i)}\right)^2}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

is minimised.

- local-type
- equilateral-type
- orthogonal-type

$$F^{ij} \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B_{\ell_1\ell_2\ell_3}^{(i)} B_{\ell_1\ell_2\ell_3}^{(j)}}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

$$G^{j} \equiv \sum_{2\le\ell_1\le\ell_2\le\ell_3} \frac{B_{\ell_1\ell_2\ell_3} B_{\ell_1\ell_2\ell_3}^{(j)}}{\sigma_{\ell_1\ell_2\ell_3}^2}$$

$$f_{\rm NL}^{(i)} = (F^{-1})^{ij} G^j$$

Komatsu, Spergel, PRD63 (2001) 063002

$f_{\rm NL}$ from curve-of-sight formula

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(Single-template fitting)

	Local	Equilateral	Orthogonal
Source x ISW	-0.012	1.1	0.050
Source x Lensing	8.2	-0.32	-24
Source x Time-delay	0.24	0.38	-0.28
Source x Deflection	-0.026	-0.23	0.52
ISW x ISW	-0.000039	0.047	0.010
ISW x Lensing	0.076	0.14	-0.56
ISW x Time-delay	-0.15	-0.13	0.098

m230c

- Lensing effect ([Src x Lens] + [ISW x Lens]) dominates as expected.

- Remapping scheme predicts $f_{\rm NL}^{\rm local} \sim 9.3, \; f_{\rm NL}^{\rm equil} \sim -2.4$ by single-template fitting.

- 'Local' looks fine, but 'Equilateral' is smaller than expected. 'Equilateral' needs high-precision calculation.

Summary : current status

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- 1st-order completed !! (TT, TE, EE for scalar)

- Implemented “curve”-of-sight formulas (2nd-order line-of-sight) for scalar contributions of temperature fluctuations (TTT.)

- Implemented KS bispectrum estimator.

- Implemented 2nd-order Boltzmann equations only for gravity and matter. (skipped today)

- Implemented remapping approximation. (skipped today)

Summary : to-do and application ?

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To-do

- Implement pure 2nd-order Boltzmann equations for radiation
- Verifying the small $f_{\rm NL}^{\rm equil}$
- Our remapping approximation still has some errors ?

$$b^{S\times T}_{\ell_1\ell_2\ell_3} \sim \int d\eta'\, b^S_{\ell_1}(\eta') b^T_{\ell_2}(\eta') \longrightarrow b^T_{\ell_2}(\eta_{\rm LSS}) \int d\eta'\, b^S_{\ell_1}(\eta')$$

Applications ?

- 2nd-order gravitational waves, magnetic field from [1st-order]^2
- curve-of-sight for polarisation
- curve-of-sight for [Scalar] x [Tensor] & [Tensor] x [Tensor]
- y-distortion to photon's distribution function ?