RESCEU APCosPA Summer School on Cosmology & Particle Astrophysics

Cosmology with spacetime curvature induced negative potentials

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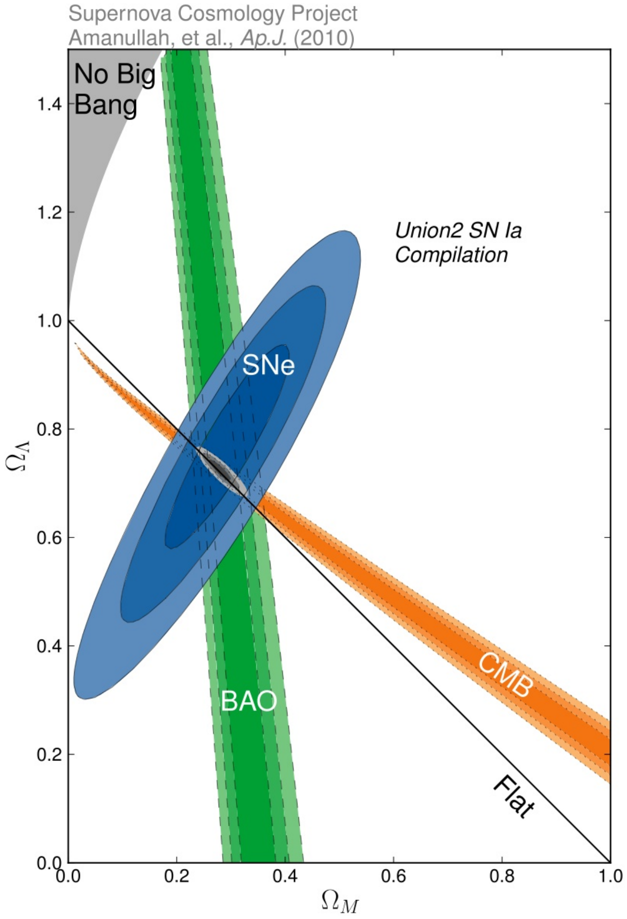
Evidence of dark energy

- Supernovae
- Cosmic Microwave Background
- Baryon Acoustic Oscillations
- Large Scale Structure

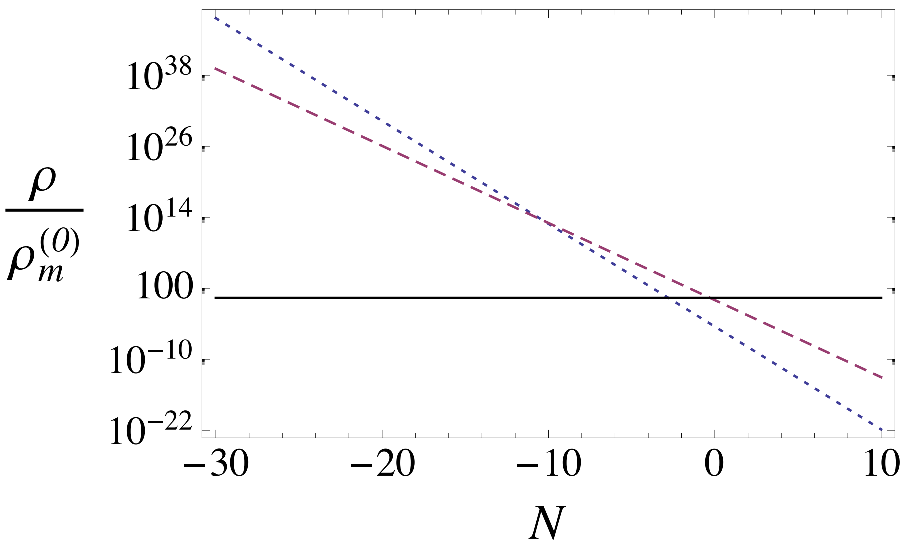
The ΛCDM model is consistent with a large number of experiments.

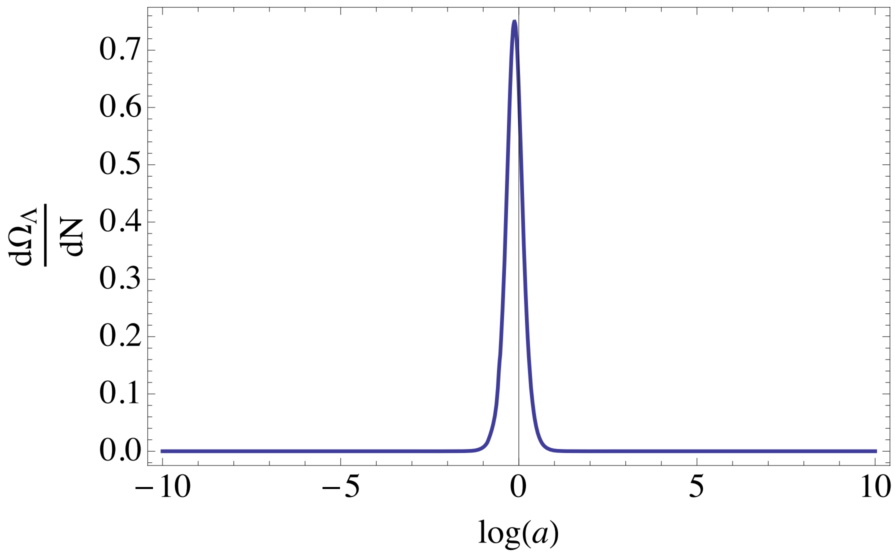


The cosmic coincidence problem

$$\left(\frac{\rho_{\mathrm{DE}}^{(0)}}{\rho_m^{(0)}}\right) \sim O(1)$$

In the ΛCDM model: $\rho_\Lambda \propto a^0, \quad \rho_m \propto a^{-3}$





The cosmic coincidence problem has different insight in scalar fields dark energy.

Cosmological scaling solutions

during the epochs of radiation domination (RD) or matter domination (MD).

$$H=\frac{2}{m}\frac{1}{t-t_0}, \quad a=a_0(t-t_0)^{2/m}, \quad \omega_B=\frac{m}{3}-1. \qquad \rho_M = \rho_M^{(0)}\left(\frac{a}{a_0}\right)^{-m}$$

$$\ddot{\phi}+\frac{6}{m}\frac{1}{t}\dot{\phi}+\frac{dV(\phi)}{d\phi}=0$$

Searching for solutions such that $\rho_\phi = \rho_\phi^{(0)}\left(\frac{a}{a_0}\right)^{-n}$

$$V(\phi)=\left(1-\frac{n}{6}\right)\rho_\phi^{(0)}e^{\lambda(\phi-\phi_0)}, \quad m=n$$

Ratra & Peebles (1988)

Wands, Copeland & Liddle (1993)

$$V(\phi)=\left(1-\frac{n}{6}\right)\rho_\phi^{(0)}\left(\frac{m-n}{2C}\right)^{\frac{-2n}{m-n}}(\phi-\phi_0)^{\frac{-2n}{m-n}}, \quad m\neq n$$

Liddle & Scherrer (1999) Uzan (1999)

Cosmological scaling solutions

with non-minimal coupling to gravity. Uzan (1999)

$$V_{\text{eff}}(\phi) = V(\phi) - \frac{1}{2}\xi R \phi^2$$

$$\ddot{\phi} + \frac{6}{m}\frac{1}{t}\dot{\phi} + \frac{12}{m}\left(\frac{4}{m} - 1\right)\frac{\xi}{t^2}\phi + \frac{dV(\phi)}{d\phi} = 0$$

Scaling solutions are found in both power-law and exponential potentials.

In the case where the non-minimal coupling term dominates the potential:

$$V_{\text{eff}}(\phi) = -\frac{1}{2}\xi R \phi^2$$

$$\ddot{\phi} + \frac{6}{m}\frac{1}{t}\dot{\phi} + \frac{12}{m}\left(\frac{4}{m} - 1\right)\frac{\xi}{t^2}\phi = 0$$

$$\Rightarrow \quad \phi(t) = C_1 t^{\beta_1} + C_2 t^{\beta_2}$$

Cosmological scaling solutions

with negative potentials induced by the non-minimal coupling to gravity.

$$V_{\rm eff}(\phi) = -\frac{1}{2}\xi R\phi^2$$

$$\beta_\pm(w,\xi) = -\frac{3}{4}(1-w) \pm \sqrt{\frac{9}{16}(1-w)^2 + 3\xi(1-3w)}$$

$$\frac{d^2\phi}{dN^2} + \frac{3}{2}(1-w)\frac{d\phi}{dN} - 3\xi\,(1-3w)\,\phi = 0$$

$$\boxed{\phi(N) = C_+ e^{\beta_+ N} + C_- e^{\beta_- N}}$$

The effective potential becomes negative for $\xi > 0$

$$V_{\rm eff}(\phi) = -\frac{3\xi}{2\kappa^2}H^2\phi^2 < 0 \quad ({\rm MD})$$

$\rho_\phi < 0 \quad (\rho_{\rm tot} > 0)$

➔ be excluded by Uzan

$$w_\phi(w,\xi) = w - \frac{2}{3}\beta_+(w,\xi)$$

Cosmological scaling solutions with flat potentials.

$\xi > 0$

$$V_{\rm eff}(\phi) = V_0 - \frac{1}{2}\xi R\phi^2 \qquad V_0 \sim M_{\rm p}^2 H_0^2 \sim 10^{-47}\,{\rm GeV}^4$$

Scaling phase (RD & MD):

$$\phi_1 = C_1 e^{\beta_1 N} \qquad \rho_\phi = \frac{1}{\kappa^2}\left(\frac{\beta_1^2}{2} - 3\xi - 6\xi\beta_1\right) H^2\phi_1^2 < 0$$

$$\beta_1(w,\xi) = -\frac{3}{4}(1-w) + \sqrt{\frac{9}{16}(1-w)^2 + 3\xi(1-3w)} \qquad w_\phi(0,\xi) = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{4}{3}\xi},$$

De Sitter expansion: $0 < \xi < 3/2$

$$\phi_2 = C_2 e^{\beta_2 N} \qquad \rho_\phi = V_0 > 0$$

$$\beta_2 = -\frac{3}{2} + \sqrt{\frac{9}{4} + 12\xi}, \qquad w_\phi = -1$$

Asymptotic universe:

$$\phi_3 = C_3 e^{\beta_3 N} \qquad \rho_\phi = V_0 + \frac{1}{\kappa^2}\left(\frac{\beta_3^2}{2} - 3\xi - 6\xi\beta_3\right) H^2\phi_3^2 > 0$$

$$\beta_3 = -\frac{3}{4}(1-w_f) + \sqrt{\frac{9}{16}(1-w_f)^2 + 3\xi(1-3w_f)}. \qquad w_f = \frac{-3+2\xi}{3(1+2\xi)}, \quad \text{and} \quad \beta_3 = \frac{4\xi}{1+2\xi}.$$

Cosmological scaling solutions with flat potentials.

$\xi > 0$

$$V_{\rm eff}(\phi) = V_0 - \frac{1}{2}\xi R\phi^2$$

$$V_0 \sim M_{\rm p}^2 H_0^2 \sim 10^{-47}\,{\rm GeV}^4$$

Scaling phase (RD & MD):

$$\phi_1 = C_1 e^{\beta_1 N}$$

$$\beta_1(w,\xi) = -\frac{3}{4}(1-w) + \sqrt{\frac{9}{16}(1-w)^2 + 3\xi(1-3w)}$$

$$\boxed{\rho_\phi = \frac{1}{\kappa^2}\left(\frac{\beta_1^2}{2} - 3\xi - 6\xi\beta_1\right) H^2\phi_1^2 < 0}$$

$$w_\phi(0,\xi) = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{4}{3}\xi},$$

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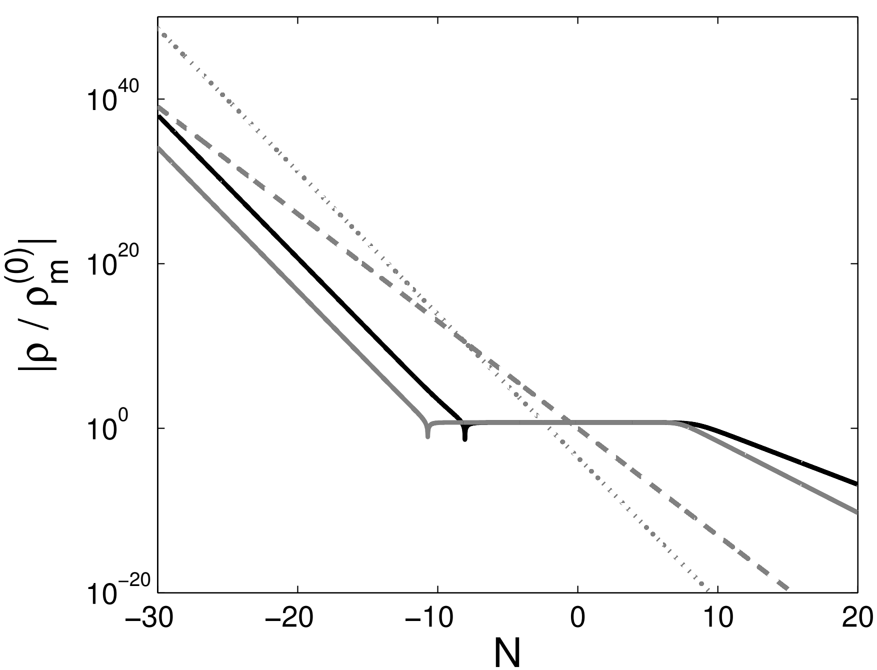
$$\rho_\phi = V_0 + \frac{1}{\kappa^2}\left(\frac{\beta_3^2}{2} - 3\xi - 6\xi\beta_3\right) H^2\phi_3^2 > 0$$

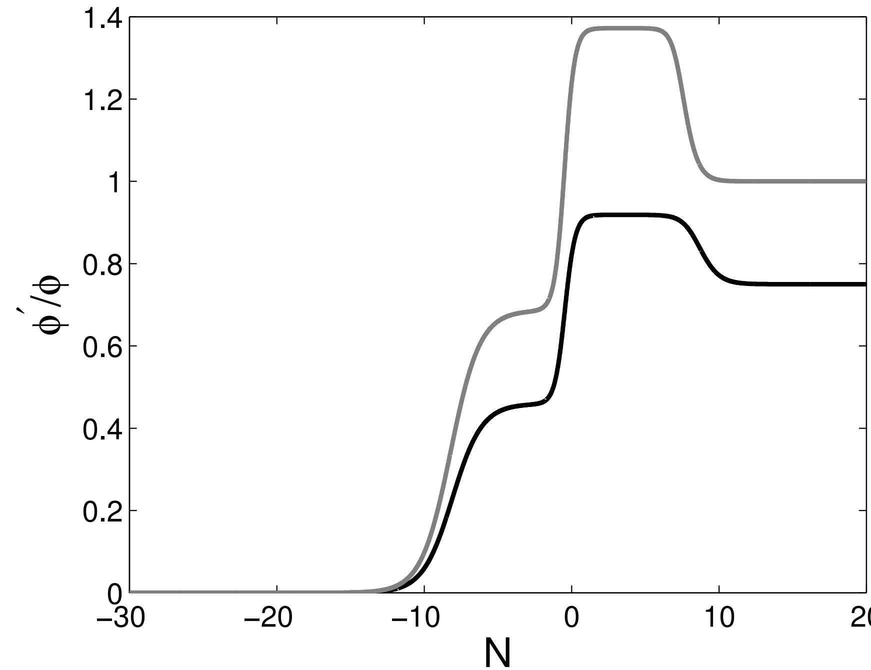
$$w_f = \frac{-3+2\xi}{3(1+2\xi)}, \quad \text{and} \quad \beta_3 = \frac{4\xi}{1+2\xi}.$$

Cosmological scaling solutions with flat potentials.

$$V_{\mathrm{eff}}(\phi) = V_0 - \frac{1}{2}\xi R \phi^2$$

gray: $\xi = 0.5$
black: $\xi = 0.3$





$$\beta_i = \frac{\phi'}{\phi}$$

Stability with negative energy density

$\xi > 0$

$$\phi(x) = \phi(\tau) + \delta\phi(\vec{x}) \quad \Rightarrow \quad \delta\mathcal{H} = \frac{1}{2}(\delta\phi')^2 - \frac{1}{2}(k^2 + V_{\phi\phi} - \frac{1}{2}\xi R)(\delta\phi)^2$$

Linear perturbation in Newtonian gauge: $(\Omega_\phi \ll 1)$

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)\,d\tau^2 + (1-2\Phi\delta_{ij} + D_{ij})\,dx^i dx^j\right]$$

$$\Rightarrow \quad \Phi'' + 3\mathcal{H}(1+c_s^2)\Phi' + \left(c_s^2 k^2 + 3\mathcal{H}c_s^2 + \mathcal{H}^2 + 2\mathcal{H}'\right)\Phi = 0$$

On super-horizon scales where $k \ll \mathcal{H} = aH$

$$\Phi = C_a + C_b \int d\tau / a^{3(1+c_s^2)} \quad \rightarrow \text{const.}$$

$$\left.\begin{aligned} \delta\phi'' + 2\mathcal{H}\delta\phi' - 6\xi(\mathcal{H}' + \mathcal{H}^2)\delta\phi = 0, \\ \phi'' + 2\mathcal{H}\phi' - 6\xi(\mathcal{H}' + \mathcal{H}^2)\phi = 0. \end{aligned}\right\} \quad \frac{\delta\phi}{\phi} \sim \text{const.} \quad \Rightarrow \quad \frac{\delta\rho_\phi}{\rho_\phi} \sim \text{const.}$$

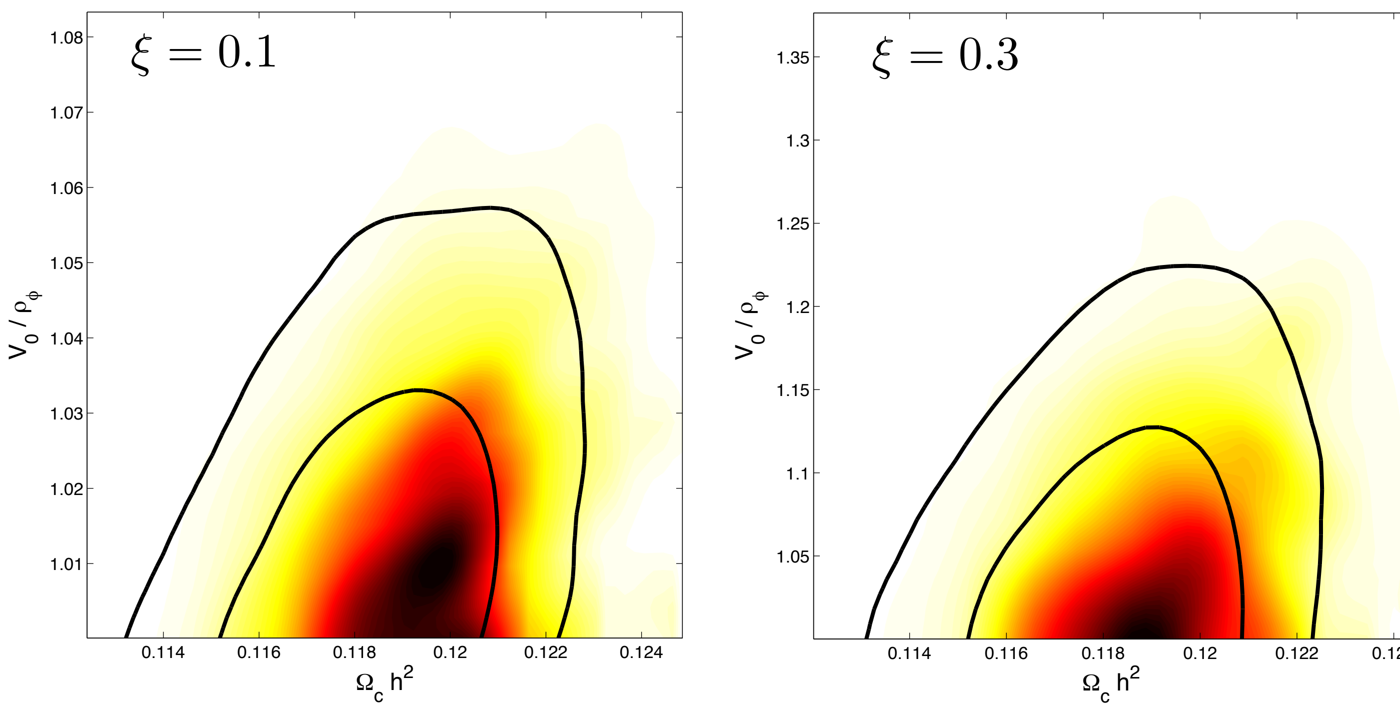
Observational constraints with flat potentials.

TABLE I. Priors for cosmological parameters.

Parameter	Prior
Baryon density	$0.5 < 100\Omega_b h^2 < 10$
CDM density	$10^{-3} < \Omega_c h^2 < 0.99$
Neutrino mass	$0.01 < \Sigma m_\nu < 2$ eV
Spectral index	$0.9 < n_s < 1.1$
Tensor-to-Scalar ratio	$0 < r < 1$

TABLE II. Constraints on cosmological parameters (95% C.L.)

Parameter	$\xi = 0.1$	$\xi = 0.3$	ΛCDM
Baryon density	$100\Omega_b h^2 = 2.21 \pm 0.05$	$100\Omega_b h^2 = 2.21^{+0.05}_{-0.01}$	$100\Omega_b h^2 = 2.22^{+0.04}_{-0.05}$
CDM density	$\Omega_c h^2 = 0.119^{+0.003}_{-0.004}$	$\Omega_c h^2 = 0.118^{+0.004}_{-0.003}$	$\Omega_c h^2 = 0.118 \pm 0.003$
Neutrino mass	$\Sigma m_\nu < 0.245$ eV	$\Sigma m_\nu < 0.245$ eV	$\Sigma m_\nu < 0.211$ eV
Spectral index	$n_s = 0.964 \pm 0.011$	$n_s = 0.964^{+0.011}_{-0.012}$	$n_s = 0.963^{+0.012}_{-0.009}$
Tensor-to-Scalar ratio	$r < 0.116$	$r < 0.118$	$r < 0.125$
Potential	$V_0/\rho_\phi = 1.018^{+0.028}_{-0.018}$	$V_0/\rho_\phi = 1.066^{+0.112}_{-0.066}$	–



Summary for the moment

We study the cosmological evolution of a scalar field with negative (effective) potentials induced by the non-minimal coupling to gravity.

For a constant potential, the negative potential exhibits an unique scaling solution that gives negative energy density to the scalar field when $\xi > 0$.

We show that the negative energy density does not leads to instability on super-horizon scales, and that the asymptotic universe is of a power-law expansion.

Observational constraints to the curvature-induced negative potential are investigated.