Multi-disformal invariance of nonlinear primordial perturbations

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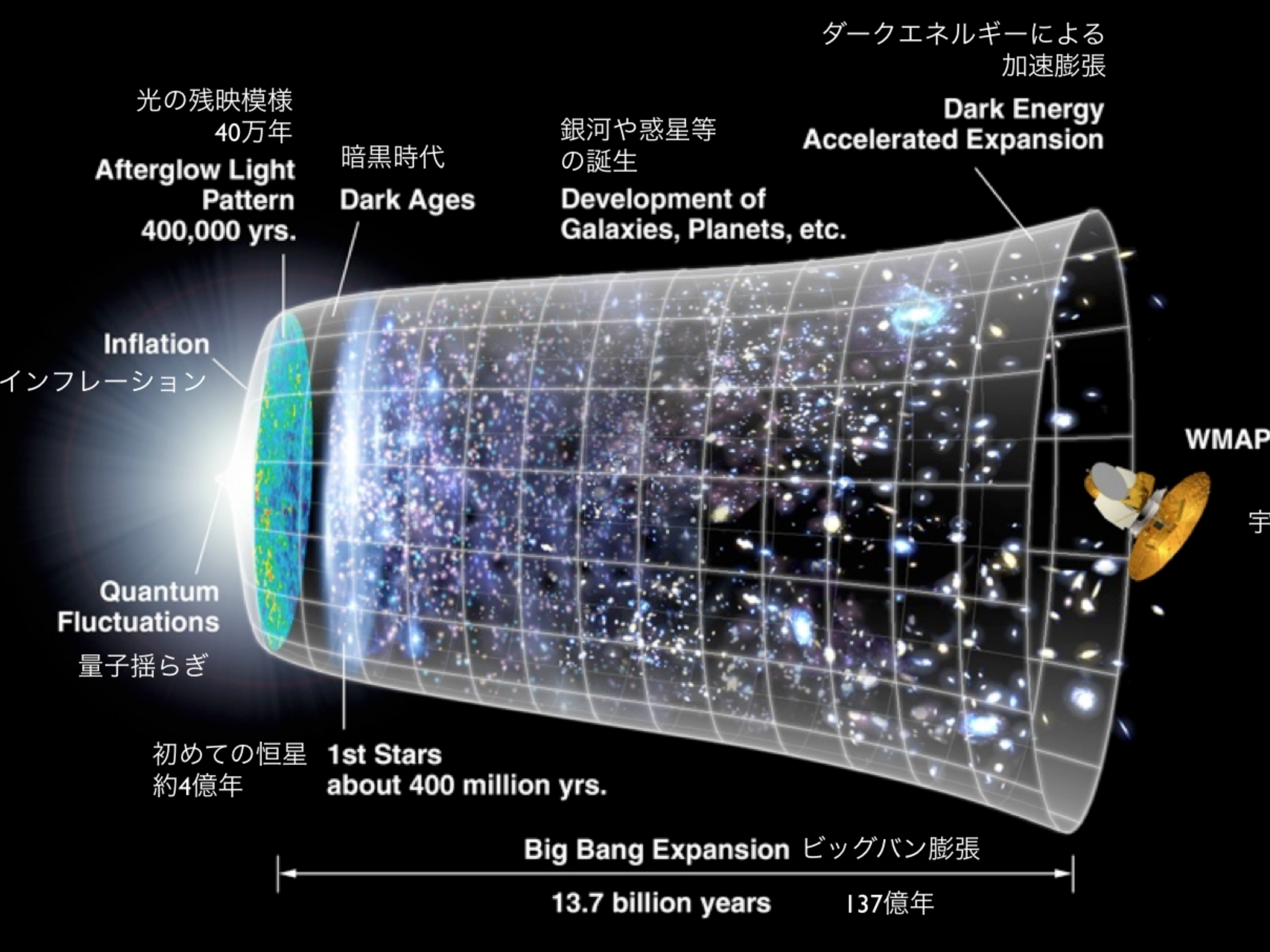
ダークエネルギーによる
加速膨張
Dark Energy
Accelerated Expansion
光の残映模様
40万年
Afterglow Light
Pattern
400,000 yrs.
暗黒時代
Dark Ages
銀河や惑星等
の誕生
Development of
Galaxies, Planets, etc.
Inflation
インフレーション
WMAP
Quantum
Fluctuations
量子揺らぎ
初めての恒星
約4億年
1st Stars
about 400 million yrs.
Big Bang Expansion ビッグバン膨張
13.7 billion years
137億年

Introduction & goals

- anti-correlation of TE spectrum -> **super-H curvature perturbations**
- (future) BB spectrum -> **super-H gravitational-wave perturbations**
- The primordial linear tensor power spectrum from inflation can be always cast into the standard form at leading order in derivatives with suitable conformal and *disformal transformations* in EFT of inflation. [Creminelli et al 1407.8439]
- Invariance of the curvature perturbation under *disformal transformations* has been shown at linear order. [Minamitsuji 1409.1566, Tsujikawa 1412.6210]

- We extend the invariance of the curvature and GW perturbations to **fully nonlinear order**.
- We further show the invariance under a new type of disformal transformation, dubbed **multi-disformal transformation**, generated by a multi-component scalar field.

Decomposing spacetime

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

α is the lapse function, β^i is the shift vector

$$\hat{\gamma}_{ij} = a^2(t) e^{2\psi} \gamma_{ij} \,, \qquad \det \gamma_{ij} = 1$$

$$\chi \equiv -\frac{3}{4} \triangle^{-1} \left\{ \partial^i \left[e^{-3\psi} \partial^j \left(e^{3\psi} (\gamma_{ij} - \delta_{ij}) \right) \right] \right\}$$

$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

Decomposing spacetime and curvature pert'n

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

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$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

The uniform φ slicing
(comoving slicing if φ dominates):

$$\phi = \phi(t)$$

$$\mathfrak{R}_c \equiv \psi_c + \chi_c/3$$

$\mathcal{R}_c$ is the (comoving) curvature perturbation at linear order and $\mathfrak{R}_c$ its non-linear generalization

Decomposing spacetime and GW pert'n

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

α is the lapse function, β^i is the shift vector

$$\hat{\gamma}_{ij} = a^2(t)e^{2\psi}\gamma_{ij}\,, \qquad \det\gamma_{ij} = 1$$

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$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

The uniform ϕ slicing: $\phi = \phi(t)$

$$\partial^j \gamma_{ij}^{\mathrm{TT}} = 0$$

$\gamma_{ij}^{\mathrm{TT}}$ is independent of the time-slicing condition at linear order but is slice-dependent at higher orders.

General disformal transformation

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} + B(\phi\,, X)\, \partial_\mu\phi\partial_\nu\phi\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A = 1, B ≠ 0: **Disformal transformation**
A ≠ 1, B = 0: **Conformal transformation**

Invariance of nonlinear perturbations

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} + B(\phi\,, X)\, \partial_\mu\phi\partial_\nu\phi\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A = 1, B ≠ 0: **Disformal transformation**

The uniform ϕ slicing: $\phi = \phi(t)$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B\,\dot{\phi}^2\,\delta_\mu{}^0\delta_\nu{}^0$$

Only **the lapse function** is affected by the disformal transformation!

$$\tilde{\alpha}^2 = \alpha^2 - B\dot{\phi}^2$$

Thus, the spatial metric & shift vector are invariant to fully nonlinear order -> Invariance of curvature and GW perturbations

Invariance of nonlinear perturbations

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} + B(\phi\,, X)\, \partial_\mu\phi\partial_\nu\phi\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A ≠ 1, B = 0: **Conformal transformation**

The uniform ϕ slicing: $\phi = \phi(t)$

$$\tilde{g}_{\mu\nu} = A\, g_{\mu\nu} = \bar{A}\, g_{\mu\nu} + \delta A\, \bar{g}_{\mu\nu} + \delta A\, \delta g_{\mu\nu}$$

$$\mathcal{R} \to \mathcal{R} + \delta A/(2\bar{A}) + \cdots$$

Unimodular part (GW) is invariant to fully nonlinear order

δA is sourced by δX, but is vanishing on large scales b/c $\dot{\mathcal{R}}_c \propto H\delta\alpha_c$

$$X_c = \frac{1}{2}\frac{\dot{\phi}^2(t)}{\alpha_c^2(t\,,\boldsymbol{x})} = \frac{1}{2}\dot{\phi}^2(t) - \delta\alpha_c(t\,,\boldsymbol{x})\dot{\phi}^2(t) + \cdots$$

$$\delta\alpha_c = \mathcal{O}(\varepsilon^2)$$

ε represents the terms of 1st order in spatial deriv's.

Curvature pert'n is nonlinearly invariant on super-H scales

Transformation of linear MS equations

GR: $$\frac{1}{z^2}\frac{1}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{z^2}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+c_s^2k^2\mathcal{R}_c=0\,,\qquad z\equiv a\frac{\phi'}{\mathcal{H}}$$

α_0 is the background value of the lapse function and c_s is the sound velocity

Under disformal transformation, $\tilde{\alpha}^2=\alpha^2-B\dot{\phi}^2$, there are **two ways to interpret**:

$$\Rightarrow\quad\frac{1}{z^2}\frac{1}{\tilde{\alpha}_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{z^2}{\tilde{\alpha}_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+c_s^2k^2\mathcal{R}_c=0\qquad\cdots①$$

It takes **the same form** if $\mathrm{d}\tau=\alpha_0\mathrm{d}t$ and $\mathrm{d}\tilde{\tau}=\tilde{\alpha}_0\mathrm{d}t\ (\mathrm{d}t=a\mathrm{d}\eta)$

On the other hand $$\Rightarrow\quad\frac{1}{\tilde{z}^2}\frac{1}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\tilde{z}^2}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+\tilde{c}_s^2k^2\mathcal{R}_c=0\,,\qquad\cdots②$$

$$\tilde{c}_s\equiv\frac{\tilde{\alpha}_0}{\alpha_0}c_s\,,\qquad\tilde{z}\equiv\sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z$$

It can be interpreted as **the one in modified gravity** with this redefinition of c_s and z.

Transformation of nonlinear equations in spatial gradient expansion

GR [Takamizu et al]:

$$\frac{1}{z^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{z^2}{\alpha_0}\frac{\partial}{\partial\eta}\mathfrak{R}_c\right)+\frac{c_s^2}{4}\,{}^{(3)}R[e^{2\psi}\gamma_{ij}]=\mathcal{O}(\varepsilon^4)$$

$$\psi=\mathfrak{R}_c+\mathcal{O}(\epsilon^2)\ \text{and}\ {}^{(3)}R\ \text{is the spatial scalar curvature}$$

By the same reasoning as in the linear case, it takes **the same form if the proper time is rescaled**, or is interpreted as **the one in modified gravity with rescaled c_s and z**.

$$\tilde{c}_s\equiv\frac{\tilde{\alpha}_0}{\alpha_0}c_s\,,\qquad\qquad \tilde{z}\equiv\sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z$$

Transformation of nonlinear equations in spatial gradient expansion

GR:

$$\frac{1}{z_t^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{z_t^2}{\alpha_0}\frac{\partial}{\partial\eta}\gamma_{ij}^{\rm TT}\right)+\frac{1}{4}\left(e^{-2\psi}\,{}^{(3)}R_{ij}\left[e^{2\psi}\gamma_{ij}\right]\right)^{\rm TT}=\mathcal{O}(\varepsilon^4)$$

$z_t \equiv a$ and $(\cdots)^{\rm TT}$ denotes the transverse-traceless projection

By the same reasoning as in the linear case, it takes **the same form if the proper time is rescaled**, or is interpreted as **the one in modified gravity with rescaled c_s and z**.

$$\tilde{c}_t \equiv \frac{\tilde{\alpha}_0}{\alpha_0}\,, \qquad \tilde{z}_t \equiv \sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z_t = \sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}a$$

$$\frac{1}{\tilde{z}_t^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{\tilde{z}_t^2}{\alpha_0}\frac{\partial}{\partial\eta}\gamma_{ij}^{\rm TT}\right)+\frac{\tilde{c}_t^2}{4}\left(e^{-2\psi}\,{}^{(3)}R_{ij}\left[e^{2\psi}\gamma_{ij}\right]\right)^{\rm TT}=\mathcal{O}(\varepsilon^4)$$

Multi-disformal transformation

Suppose there are $\mathcal{N}$ component scalar field, ϕ^I $(I = 1, \cdots, \mathcal{N})$.

$$\tilde{g}_{\mu\nu} = A(\phi^I, X^{IJ})\, g_{\mu\nu} + B_{KL}(\phi^I, X^{IJ})\, \partial_\mu \phi^K \partial_\nu \phi^L \,,$$

$$X^{IJ} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J$$

Adiabatic limit: $\phi^I = \phi^I(\varphi)$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL}\left[\phi^I(\varphi)\,, X^{IJ}(\varphi\,, \partial\varphi)\right] (\phi^K)'(\phi^L)' \partial_\mu \varphi \partial_\nu \varphi \,, \qquad (\phi^I)' \equiv \frac{\mathrm{d}\phi^I}{\mathrm{d}\varphi}$$

The uniform φ slicing:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL}\left[\phi^I(\varphi)\,, X^{IJ}(\varphi\,, \dot{\varphi}/\alpha)\right] (\phi^K)'(\phi^L)' \dot{\varphi}^2 \delta_\mu{}^0 \delta_\nu{}^0$$

Since the multi-disformal transformation only affects **the lapse function**, we can apply the same argument as before!

Summary

- The curvature and tensor perturbations on the uniform ϕ slicing are **fully nonlinearly invariant** under the disformal transformation.
- The same conclusion can be drawn for a multi-component extension of the disformal transformation, dubbed **multi-disformal transformation**, on the uniform φ slicing in the adiabatic limit.
- Once a 2nd order differential eq. is obtained in **modified gravity or EFT**, one can map it into the same form as the one in GR by a suitable disformal transformation.