CONFORMAL FRAME DEPENDENCE OF INFLATION

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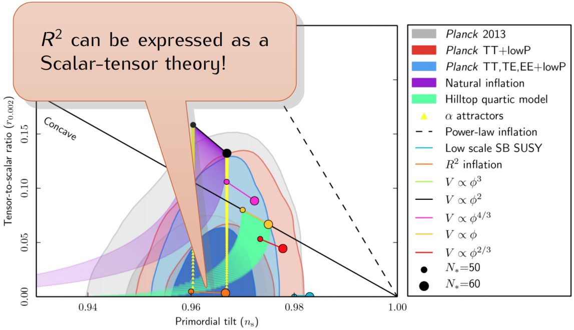
Introduction

Why do we consider scalar-tensor theory of gravity?

- Naturally arises in higher dimensional theories.
- Attractive from a renormalization point of view.
- Favoured by recent Planck observational results.



Introduction

What is the Scalar-tensor theory of gravity? It considers a scalar field non-minimally coupled to gravity:

$$S \sim \int d^4x\sqrt{-\tilde{g}}\left\{F(\phi)\tilde{R} + \tilde{L}(\phi)\right\}$$

This form of the action is called **Jordan frame**.

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This form of the action is called **Jordan frame**.

By means of a conformal transformation, i.e.

$$\tilde{g}_{\mu\nu} = F^{-1} g_{\mu\nu} \ldots$$

Frame independence

...one can bring the Jordan frame into the Einstein-Hilbert action, that is

$$S \sim \int d^4x\sqrt{-g}\,\{R + L(\phi)\}\ , \tag{1}$$

the so-called **Einstein frame**.

What is the advantatge of such a transformation?

- Very well know how to deal with EH action (and much easier!).
- **Physical observables** are in fact **frame independent** (Deruelle and Sasaki, 2011).

Frame independence

However, what about the **matter sector**?

- **Matter minimally couples to** $\tilde{g}$.
- As long as we have successful inflation in the Einstein frame we can choose the matter metric $\tilde{g}$ by a conformal transformation.
- How different can $\tilde{g}$ and g be?
- Can this matter point of view leave observational imprints?

Review: PL Inflation

We consider power-law inflation to illustrate this points. The Inflaton field ϕ in a potential $V(\phi) = V_0 e^{-\lambda\phi}$ gives rise to $(p = 2/\lambda^2)$: (Lucchin and Matarrese, 1985)

$$a = a_0 \, (t/t_0)^p \qquad \phi = \frac{2}{\lambda} \ln(t/t_0) \qquad H = p/t \qquad \epsilon = 1/p \, .$$

The curvature and tensor power spectrum under the slow-roll approximation are given by

$$\mathcal{P}_{\mathcal{R}_c}(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 = \frac{p}{8\pi^2} \frac{H_0^2}{M_{pl}^2} \left(\frac{k}{k_0}\right)^{\frac{-2}{p-1}} \, ,$$

$$\mathcal{P}_{\mathcal{T}}(k) = \frac{2}{\pi^2} \frac{H^2}{M_{pl}^2} = \frac{16}{p} \mathcal{P}_{\mathcal{R}_c}(k) \, .$$

We need $p \gg 1$ for a successful inflation. $(r = 16/p)$

Curvaton model

For simplicity, let us take a curvaton as a representative of matter. The curvaton is a scalar field χ that:

- Initially is subdominant
- Has a non-vanishing initial energy density
- Dominates after inflaton decays
- Right after decays and **contributes to the scalar power spectrum**

Curvaton model

Our curvaton is a matter field and therefore lives in the Jordan frame, i.e.

$$S_m \sim \int d^4x \sqrt{-\tilde{g}} \left(-\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \tilde{m}^2\chi^2\right) .$$

The power-spectrum for the curvaton under the sudden decay approximation is given by (Lyth and Wands, 2002)

$$\mathcal{P}_\chi(k) = r_\star \frac{\delta\chi^2}{\chi_\star^2} = r_\star \frac{\widetilde{H}^2}{(2\pi M_{pl}\chi_\star)^2} ,$$

where $r_\star$ is the energy density fraction of the curvaton at decay.

Matter point of view

Matter is coupled to the Jordan $\tilde{g}$ so our conformal transformation yields

$$\tilde{a} = F^{-1/2} a \qquad \text{and} \qquad d\tilde{t} = F^{-1/2} dt\,. \tag{2}$$

Let us take a concrete example inspired in a dilationic coupling, that is

$$F(\phi) = \mathrm{e}^{\gamma\lambda\phi/M_{pl}} = (t/t_0)^{2\gamma}\,. \tag{3}$$

Matter point of view

After integrating time, the Jordan scale factor is given by another power-law

$$\tilde{a} = \tilde{a}_0 \left(\tilde{t}/\tilde{t}_0\right)^{\tilde{p}} \qquad \tilde{H} = \tilde{p}/\tilde{t}\,, \tag{4}$$

where $\tilde{p} - 1 = \frac{p-1}{1-\gamma}$. ($\tilde{p}$ can be negative!)

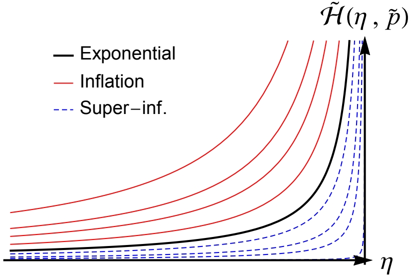

Figure: Jordan conformal hubble parameter $\tilde{\mathcal{H}}$ as a function of the conformal time η and $\tilde{p}$. For $\tilde{p} < 0$ we have super-inflation.

Jordan Power Law

The curvaton follows the Jordan power law. This time the power spectrum takes the same form but with $\tilde{p}$ instead of p. For $\tilde{p} < 0$ the **spectrum is blue!**

$$\tilde{n}_\chi - 1 = \frac{-2}{\tilde{p} - 1} . \qquad (5)$$

Such a blue tilt might induce the **formation of primordial black holes.**

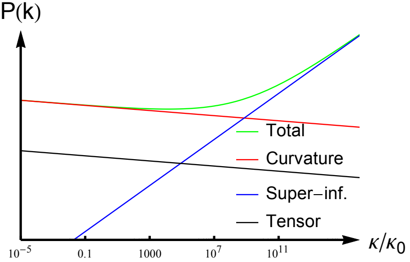

Figure: Power-spectrum for the Jordan power-law case.

Jordan Bounce

We can consider a slightly more complicated transformation, e.g.

$$F(\phi) = \left(1 + \mathrm{e}^{\frac{-\gamma\lambda}{2M_{pl}}\phi}\right)^{-2} = \left(1 + (t/t_0)^{-\gamma}\right)^{-2} . \tag{6}$$

It corresponds to a **bouncing** Jordan frame!

$$\tilde{a} \approx \begin{cases} a_0(-\tilde{t}/\tilde{t}_0)^{\tilde{p}} & |\tilde{t}| \gg \tilde{t}_0 \quad (\tilde{t} < 0) \\ a_0(\tilde{t}/\tilde{t}_0)^{p} & \tilde{t} \gg \tilde{t}_0 \end{cases} . \tag{7}$$

The singularity has been sent to $\tilde{t} \to -\infty$.

Jordan Bounce

We find a **blue tilt** at short scales that gives an **apparent suppresion**.

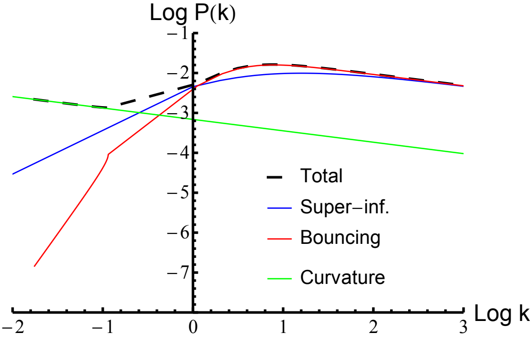

Figure: Power-spectrum for the Jordan bouncing frame.

Summary

With a simple analytic model we have shown that:

- In the scalar-tensor theory the matter point of view can be very different although we have inflation in the Einstein frame!
- Depending on which frame matter is minimally coupled, it can leave important features, e.g. to the power spectrum.
- We easily obtain a blue tilt at large scales (for the super-inf. case) and a blue tilt at short scales (for the bouncing case).