Dark Energy

RESCEU APcosPA Summer School on Cosmology and Particle Astrophysics Matsumoto city, Nagano

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Friedmann-Lemaître-Robertson-Walker (FLRW) universes:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

$$d\ell^2 = \frac{dr^2}{1 - k\, r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

k = 0, 1, -1 flat, closed, open spatial sections

If gravity is described by General Relativity (GR):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

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Comoving perfect fluids:

$$T_{\mu\nu} = (\rho + p)\, U_\mu\, U_\nu - p\, g_{\mu\nu}$$

$U_0 = U^0 = 1$, spatial components vanish .

$$\begin{aligned} \frac{d\rho}{dt} &= -3H(\rho + p) \\ &= -3H(1 + w)\, \rho \end{aligned}$$

$$w \equiv \frac{p}{\rho}$$

$w = constant \Rightarrow \rho \propto a^{-3(1+w)}$

$$w_m = 0 \Rightarrow \rho_m \propto a^{-3}$$

$$w_r = \frac{1}{3} \Rightarrow \rho_m \propto a^{-4}$$

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Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \sum_i \rho_i$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i)$$

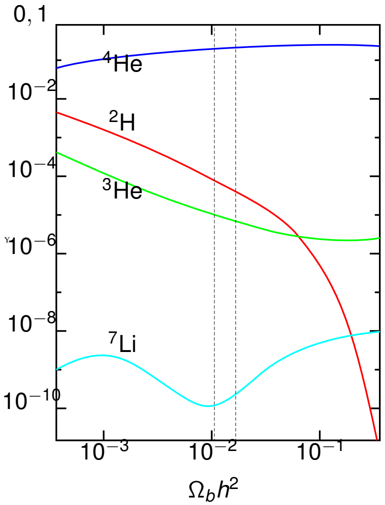
$$\Omega_i = \frac{\rho_i}{\rho_{cr}} \qquad H^2 \equiv \frac{8\pi G}{3}\,\rho_{cr} \qquad \Omega_k = -\frac{k}{a^2 H^2}$$

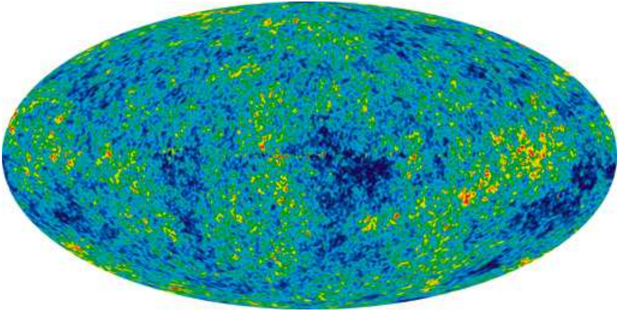
$$\sum_i \Omega_i + \Omega_k = 1$$

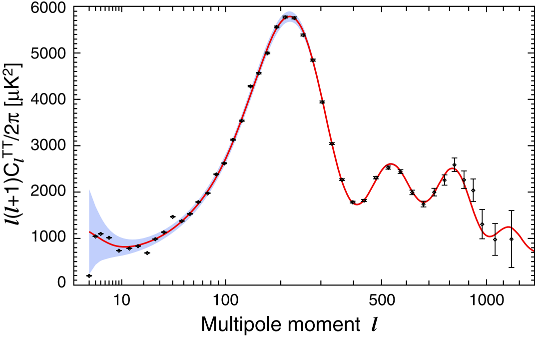
$$q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2}\sum_i \Omega_i (1 + 3w_i)$$

$$w_m = 0, \quad w_r = \tfrac{1}{3} > 0 \quad \Rightarrow \quad \ddot{a} < 0$$

$$\frac{\dot{a}}{a} \equiv H \qquad H_0 = 67 \ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$


0, 1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
^{4}He
^{2}H
^{3}He
^{7}Li
10^{-3}
10^{-2}
10^{-1}
$\Omega_b h^2$

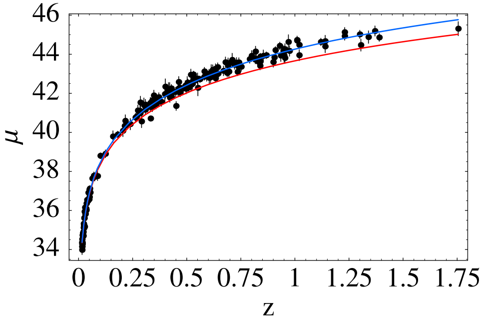



6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l^{TT}/2\pi$ [μK^2]
10
100
500
1000
Multipole moment l

Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$\mathcal{F} = \frac{L}{4\pi {d_L}^2}$ $\qquad m - M = 5 \log d_L + 25$

$$d_L(z) = a_0\,(1+z)\,r = a_0\,(1+z)\,\mathcal{S}\left(\frac{c}{a_0}\int_0^z \frac{dz'}{H(z')}\right)$$



$\Lambda CDM \quad \Omega_{m,0} = 0.3 \quad \Omega_{k,0} = 0$ EdS

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Expansion no longer as in standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad @ \quad z \sim 0.5$$

Dark Energy puzzle:
What is the origin of this accelerated expansion ?

We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

A new vision has emerged supported by many observations: SNIa, CMB, BAO,...

Introduce Dark Energy (DE)

For $k = 0,\ z \ll z_{\rm eq}$:

$$\begin{aligned} 3H^2 &= 8\pi G\,(\rho_m + \rho_{DE}) \\ -2\dot{H} &= 8\pi G\,(\rho_m + \rho_{DE} + p_{DE}) \end{aligned}$$

$$h^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\,f(z)$$

$$f(z) = \exp\left[3\int_0^z dz'\,\frac{1+w_{DE}(z')}{1+z'}\right]$$

$$w_{DE}(z) = \frac{\frac{1+z}{3}\frac{d\ln H^2}{dz} - 1}{1 - \Omega_{m,0}\,\frac{H_0^2}{H^2}\,(1+z)^3}$$

$$d_L(z) \quad\Rightarrow\quad H(z) \quad\Rightarrow\quad w_{DE}(z)$$

Large errors!

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"Old" simple solution: cosmological constant Λ
"...My greatest blunder..." A. Einstein

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$

...But consistent with observations

$w_\Lambda = -1$, and ρ_Λ is exactly constant

Other models have generically: $w_{DE}(z)$!!

Interesting questions:

Is ρ_{DE} constant / Is $w_{DE} = -1$?
$w_{DE}(z)$?

Is $\Omega_{DE} \ll 1$ at $z \gg 1$?

Coupling in the dark sector ?

Is DE connected to dark matter ?

Is DE a perfect fluid ?

Is gravity described by GR ?

Is our universe homogeneous ?

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Early DE (k=0)

$$\Omega_{DE}(x) = \frac{\Omega_{DE,0} - \Omega^e_{DE}\left(1 - x^{-3w_0}\right)}{\Omega_{DE,0} + \Omega_{m,0}\, x^{3w_0}} + \Omega^e_{DE}\left(1 - x^{-3w_0}\right)$$

$x \equiv \frac{a}{a_0}$ $\quad$ $w_0 < 0$ $\quad$ $\Omega^e_{DE} = \mathrm{const}$ $\quad$ $w_0 < 0$

$\Omega_{DE}(x) \to \Omega_{DE,0}$ for $x \to 1$

$\Omega_{DE}(x) \to \Omega^e_{DE}$ for $x \to 0$

$w_{DE,0} = w_0$

$\Omega_{DE}(x) = \Omega_\Lambda(x)$ for $\Omega^e_{DE} = 0, \;\; w_0 = -1$

(Generalized) Chaplygin gas

$$p = -\frac{A}{\rho^{\alpha}} \qquad A > 0, \;\; \alpha > 0$$

$$\Rightarrow \rho(a) = \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}}$$

B is an arbitrary integration constant

$$w(a) = -\left(1 + \frac{B}{A\, a^{3(1+\alpha)}} \right)^{-1}$$

$w \to 0$ for $a \to 0$
$w \to -1$ for $a \to \infty$

Genuine barotropic equation of state $p(\rho)$

$$c_s^2 = -\alpha\, w \qquad \frac{dw}{d\ln a} = -3(1+w)(c_s^2 - w)$$

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- Quintessence: (minimally coupled) scalar field $\phi(t)$

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \, \partial\phi\phi - V(\phi) \right)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}$$

$$-1 \le w_\phi \le 1 \Leftrightarrow \rho_\phi + p_\phi \ge 0$$

No “phantom”!

Dynamical properties:
attractors, thawing and freezing models
Many generalizations: DBI, tachyons, k-essence,...

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- Ratra-Peebles model

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}} \qquad \alpha > 0$$

"Quintessence" field ϕ is subdominant in the early universe:

$$\rho_\phi \ll \rho_B \qquad a(t) \propto t^q \qquad q = \frac{2}{3(1+w_B)}$$

Equation of motion (Klein-Gordon equation)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

has solution:

$$\phi \propto t^p, \qquad p = \frac{2}{2+\alpha}$$

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- Quintessence field is catching up:

$$\rho_\phi \propto t^{2p-2} \propto a^{-\frac{\alpha}{q}\frac{p}{q}}$$

$$\frac{\rho_\phi}{\rho_B} \propto t^{2p} \qquad \rho_B \propto t^{-2}$$

Can also be understood from the behaviour of EoS parameter w_ϕ

$$w_\phi = \frac{\alpha\, w_B - 2}{\alpha + 2} < w_B$$
$$= -1 + \frac{1 + w_B}{\frac{2+\alpha}{\alpha}}$$

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- Reconstruction of the quintessence potential $V(\phi)$

Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\dot{H} = -4\pi G\left(\rho_m + \dot{\phi}^2\right)$$

$$\begin{aligned}\frac{8\pi G}{3}V &= H^2 - \Omega_{m,0}H_0^2(1+z)^3 - \frac{4\pi G}{3}\dot{\phi}^2 \\ &= H^2 - \frac{1}{2}\Omega_{m,0}H_0^2(1+z)^3 + \frac{\dot{H}}{3}\end{aligned} \tag{1}$$

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$$8\pi\ G\ V = 3H^2 - \frac{3}{2}\Omega_{m,0} H_0^2 (1+z)^3 - \frac{(1+z)}{2}\ \frac{dH^2}{dz}$$

$$4\pi G (1+z)^2\ H^2 \left(\frac{d\phi}{dz}\right)^2 = \frac{(1+z)}{2}\ \frac{dH^2}{dz} - \frac{3}{2}\Omega_{m,0} H_0^2 (1+z)^3$$

Reconstruction from the background expansion:

$$D_L(z) \Rightarrow \frac{1}{H(z)} = \left(\frac{D_L(z)}{1+z}\right)' + \Omega_{m,0} \Rightarrow V(z) \Rightarrow V(\phi - \phi_0)$$

Reconstruction also possible using perturbations

- Another scalar field model (Tachyon):

$$S_T = \int d^4x\sqrt{-g}\, L_T = \int d^4x\sqrt{-g}\left[-V(T)\,\sqrt{1-g^{\mu\nu}\partial_\mu T\,\partial_\nu T}\right]$$

$$T_{\mu,\nu} = V(T)\,\frac{\partial_\mu T\,\partial_\nu T}{\sqrt{1-g^{\mu\nu}\partial_\mu T\,\partial_\nu T}} - g_{\mu,\nu}\,L_T$$

$$\text{FLRW} \;:\; p = -V\sqrt{1-\dot{T}^2} \qquad \rho = \frac{V}{\sqrt{1-\dot{T}^2}}$$

Equation of motion:

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0$$

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- In general:

$$S_\phi = \int d^4x \sqrt{-g}\, L(X, \phi) \qquad X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi$$

$$\begin{aligned} T\mu\nu &= \frac{\partial L}{\partial X} \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu}\, L \\ &\equiv (p + \rho)\, U_\mu\, U_\nu - g_{\mu\nu}\, p \end{aligned}$$

$$p = L$$

$$\rho = 2\, X \frac{\partial L}{\partial X} - L$$

$$U_\mu = \frac{\partial_\mu \phi}{\sqrt{2\, X}}$$

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- Dynamical system analysis
  Introduce dimensionless variables

$$x^2 = \frac{\frac{1}{2}\dot{\phi}^2}{\rho_{\rm cr}} \qquad y^2 = \frac{V}{\rho_{\rm cr}} \qquad \rho_{\rm cr} = \frac{3\ H^2}{8\pi\ G} \equiv 3\ M_p^2\ H^2$$

$$\frac{dx}{dN} = F_1(x,\ y;\ w_B,\ \lambda) \qquad \lambda \equiv -M_p\ \frac{V'}{V}$$

$$\frac{dy}{dN} = F_2(x,\ y;\ w_B,\ \lambda) \qquad N \equiv \ln a$$

$\lambda =$ constant corresponds to exponential potential

$$V = V_0\ \exp(-\lambda\ \frac{\phi}{M_p})$$

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- Critical points:

$$F_1(x_c, y_c) = 0 \qquad F_2(x_c, y_c) = 0$$

We linearize around the critical points:

$$\begin{aligned} F_i(x,y) &= \frac{\partial F_i}{\partial x}(x_c, y_c)\,(x - x_c) + \frac{\partial F_i}{\partial y}(x_c, y_c)\,(y - y_c) \\ &\equiv M_{i1}\,\delta x + M_{i2}\,\delta y \end{aligned}$$

$$\begin{aligned} \frac{d\delta x}{dN} &= M_{11}\,\delta x + M_{12}\,\delta y \\ \frac{d\delta y}{dN} &= M_{21}\,\delta x + M_{22}\,\delta y \end{aligned}$$

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- Eignevalues $\omega_1,\ \omega_2$ real and nondegenerate:

  1) Same sign $\Rightarrow$ node
  Both negative $\Rightarrow$ stable, otherwise unstable

  2) Opposite sign $\Rightarrow$ saddle

- Eigenvalues $a \pm ib$ are complex conjugate:
  Stable spiral if $a < 0$

- Eigenvalues are real, degenerate:
  .....

$$\dot{H} = -\frac{3}{2}\, H^2(1 + w_{\rm eff}) \qquad w_{\rm eff} = \sum_i w_i\, \Omega_i$$

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- Eigenvalues are real, degenerate:
  .....

$$\dot{H} = -\frac{3}{2}\,H^2(1 + w_{\rm eff}) \qquad w_{\rm eff} = \sum_i w_i\,\Omega_i$$

- Critical points:

1) $(x_c, y_c) = (0, 0)$
$\Omega_\phi = 0 \qquad w_B = w_{\rm eff}, \;\; w_\phi$ not determined
$\omega_1 = -\frac{3(1-w_B)}{2}, \;\; \omega_2 = \frac{3(1+w_B)}{2}$
It is a saddle

2) $(x_c, y_c) = \left(\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}\right)$
$\Omega_\phi = 1, \;\; w_{\rm eff} = w_\phi = -1 + \frac{\lambda^2}{3}$
$\omega_1 = \frac{\lambda^2 - 6}{2}, \;\; \omega_2 = \lambda^2 - 3(1 + w_B)$
Stable for $\lambda^2 < 3(1 + w_B)$
Accelerated expansion for $\lambda^2 < 2$

3) $(x_c, y_c) = \left(\sqrt{\frac{3}{2}} \frac{1+w_B}{\lambda}, \;\; \sqrt{\frac{3}{2}} \frac{\sqrt{1-w_B^2}}{\lambda}\right)$
$\Omega_\phi = \frac{3(1+w_B)}{\lambda^2} \qquad w_{\rm eff} = w_\phi = w_B$
Scaling solution, stable for $\lambda^2 > 3(1 + w_B)$

4) $(x_c, y_c) = (\pm 1, 0) \quad \Omega_\phi = 1 \quad w_{\rm eff} = w_\phi = 1$

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- Generically λ is not constant!

$$\frac{dw_\phi}{dN} = (w_\phi - 1)\left(3(1+w_\phi) - \lambda\sqrt{3(1+w_\phi)\,\Omega_\phi}\right)$$

$$\frac{d\ln\Omega_\phi}{dN} = -3(w_\phi - w_B)\,(1-\Omega_\phi)$$

$$\frac{d\lambda}{dN} = -\sqrt{3(1+w_\phi)\,\Omega_\phi}\,(\Gamma - 1)\,\lambda^2 \qquad \Gamma \equiv \frac{V\,V''}{V'^2}$$

A “tracker” solution attracts evolutions with various initial conditions:

$$\Omega_\phi = \frac{3(1+w_\phi)}{\lambda^2} \qquad \Gamma > 1$$

$$\Rightarrow w_\phi = \text{constant}$$

$$\Rightarrow \frac{d\ln\Omega_\phi}{dN} = -2\,\frac{\frac{d\lambda}{dN}}{\lambda}$$

$$\Rightarrow w_\phi = \frac{w_B - 2(\Gamma-1)}{2\Gamma-1} \quad \text{for} \quad \Omega_\phi \ll 1$$

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- Ratra-Peebles model:

  $$\Gamma = 1 + \frac{1}{\alpha} > 1$$

  $$\Rightarrow w_\phi = \frac{\alpha\ w_B - 2}{\alpha + 2}$$

  From observations $w_\phi < -0.964$ or $\alpha < 0.075$ at $2 - \sigma$

- Modified gravity DE models
  Maybe gravity differs from GR on large scales ?
  Can this explain the accelerated expansion without introducing a new DE component ?

We keep the RW metric

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

We get modified Friedmann equations
They can be often recast in an "Einsteinian" way

The growth of perturbations get modified as well!
Crucial probe of modified gravity models

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- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$

- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

-

$$V = -G_{\text{eff}}\,\frac{M_1\ M_2}{r} \qquad \text{massless}\ \ \Phi\ \ \text{field}$$

$$G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD} + 3}\right) \qquad G_N = \frac{G_*}{F}$$

- $G_{\text{eff},0} \simeq G_{N,0} \simeq G$

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- $L = \frac{1}{16\pi G_*}\Big(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\Big) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad\qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad\qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad\qquad \omega_{BD,0} > 4 \times 10^4$$

- $$V = -G_{\rm eff}\,\frac{M_1\ M_2}{r} \qquad\qquad \text{massless}\quad \Phi \quad \text{field}$$

$$G_{\rm eff} = G_N\left(1 + \frac{1}{2\omega_{BD} + 3}\right) \qquad\qquad G_N = \frac{G_*}{F}$$

- $G_{\rm eff,0} \simeq G_{N,0} \simeq G$

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- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad\qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad\qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad\qquad \omega_{BD,0} > 4 \times 10^4$$

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- Modified background equations

$$3FH^2 = 8\pi G_* \rho_m + \frac{\dot\Phi^2}{2} + U - 3H\dot F$$

$$-2F\dot H = 8\pi G_* \rho_m + \dot\Phi^2 + \ddot F - H\dot F$$

Define ρ_{DE} and p_{DE}:

$$3H^2 = 8\pi G_{N,0}\,(\rho_m + \rho_{DE})$$
$$-2\dot H = 8\pi G_{N,0}\,(\rho_m + \rho_{DE} + p_{DE})$$

- $\frac{dh^2}{dz} < 3\,\Omega_{m,0}\,(1+z)^2 + 2\,\Omega_{k,0}\,(1+z) \iff$ phantom

Possible in scalar-tensor models
$8\pi\, G_*\,(\rho_{DE} + p_{DE}) = \dot\Phi^2 + \ddot F - H\dot F + 2(F - F_0)\,\dot H$

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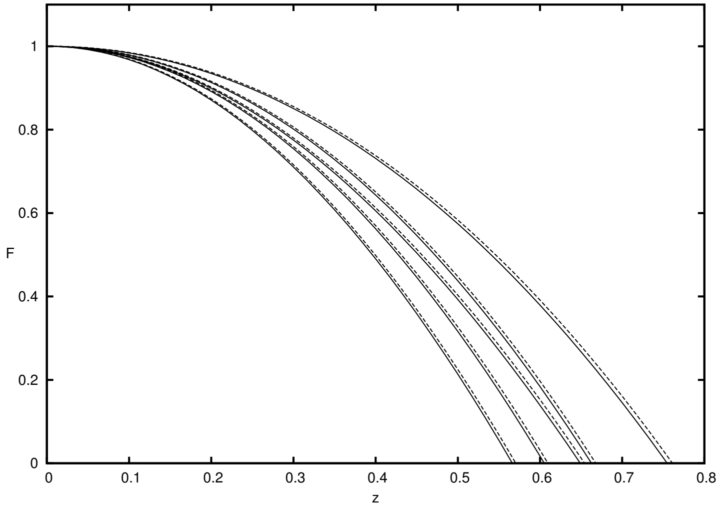

Figure: The function $\frac{F}{F_0}$ is shown for several models with vanishing potential $U = 0$. We have from left to right the following equation of state parameter w_{DE}: -2, -1.5, polynomial expression, -1, -0.5.

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Growth of matter perturbations is modified:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

$$h^2\ \delta''_m + \left(\frac{(h^2)'}{2} - \frac{h^2}{1+z}\right)\delta'_m = \frac{3}{2}(1+z)\frac{G_{\rm eff}}{G}\ \Omega_{m,0}\ \delta_m$$

Growth of perturbations must be consistent with background expansion!

$$h^2 = \frac{(1+z)^2}{\delta'^2}\left[\delta_0'^2 + 3\Omega_{m,0}\int_0^z \frac{G_{\rm eff}}{G}\frac{\delta'\delta}{1+z'}dz'\right]$$

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- Technical details

$$ds^2 = (1+2\phi)dt^2 - a^2\,(1-2\psi)d\mathbf{x}$$

$$\dot{V} = \phi$$

$$\dot{\delta}_m = -\frac{k^2}{a^2}\,V + 3(\dot{HV}) + 3\dot{\psi}$$

$$\phi = \psi - \frac{\delta F}{F}$$

In quintessence: $\phi = \psi$!

V: peculiar velocity potential $\qquad \delta_m = \frac{\delta\rho_m}{\rho_m} + 3HV$

$\phi,\ \psi,\ \delta_m$ are gauge-invariant quantities

In quasi static limit:

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\,\phi = 0$$

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- Perturbed dilaton equation of motion:

$$\delta\Phi = (\phi - 2\psi)\,\frac{dF}{d\Phi} = -\phi\,\sqrt{F}\,\frac{\sqrt{\omega_{BD}}}{2+\omega_{BD}}$$

Some small clustering in contrast to quintessence ($\frac{k^2}{a^2}\,\delta\varphi \propto \phi$)

- Combination of the perturbed Einstein equations:

$$\frac{k^2}{a^2}\,\phi = 4\pi G_{\rm eff}\,\rho_m\,\delta_m$$

$$\Rightarrow \ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\,\rho_m\,\delta_m = 0$$

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$$L = -\frac{R}{16\pi G_*} + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) + L_m\left[\Psi_m; A^2(\phi)\, g_{\mu\nu}\right]$$

$$A^2 = e^{2\beta\phi/M_{PL}} \qquad\qquad V = M^4\ e^{\left(\frac{M}{\phi}\right)^n}$$

$$M \ll \phi \ll M_{PL} \rightarrow V \text{ is like } \Lambda!$$

Growth of perturbations is modified:

$$G_{\rm eff}(z, k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r}\ \left(1 + 2\,\beta^2\ e^{-m_\phi r}\right)$$

In this model, m_ϕ is too large, no influence on cosmological scales!

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- Interacting dark sector

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\alpha\, \dot{\phi}\, \rho_m$$
$$\dot{\rho}_m + 3H\,\rho_m = \alpha\, \dot{\phi}\, \rho_m$$

$\alpha \equiv \frac{d \ln A}{d\phi}$

$$\ddot{\phi} + 3H\dot{\phi} = -V' - \alpha\,\rho_m = -V'_{\rm eff}$$
$$V_{\rm eff} \equiv V + \rho_m \qquad \rho_m \propto A$$

$$\Rightarrow \text{Chameleon mechanism}$$

Coupling to dark matter only?

$A_b = 0 \qquad A_{dm} \neq 0$

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- $f(R)$ modified gravity DE models: $R \to f(R)$
  Basic idea: No new component, just change gravity!

  Many models (e.g. $R + \frac{\mu^2}{R}$) lead to unviable cosmic expansion with $a \sim t^{\frac{2}{3}} \to a \sim t^{\frac{1}{2}}$

  Some interesting viable $f(R)$ models still remain:
  $f(R) = R - \lambda R_c \, f_1(x) \qquad x \equiv R/R_c$
  e.g. $R - \lambda R_c \left(1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right), \; n, \lambda > 0 (n \geq 2)$

$$F \equiv \frac{df(R)}{dR} > 0, \; \frac{dF(R)}{dR} > 0$$

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$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

$$G_{\rm eff} = \frac{G_*}{F}\ \left(1 + \frac{1}{3}\ \frac{\frac{k^2}{a^2 m^2}}{1 + \frac{k^2}{a^2 m^2}}\right) \qquad \frac{F}{F'} \equiv 3\ m^2$$

$$G_{\rm eff} = G_{\rm eff}(z, k) \Leftrightarrow V(r) = -\frac{G_*}{F}\frac{M_1\ M_2}{r}\ \left(1 + \tfrac{1}{3}\ e^{-mr}\right)$$

Deep in matter-domination and $R \gg R_c$:

$$\begin{aligned} G_{\rm eff} &= \frac{4}{3}\ G \qquad \delta_m \propto a^{\frac{\sqrt{33}-1}{4}} \qquad k \gg am \\ &= G \qquad \delta_m \propto a \qquad k \ll am \end{aligned}$$

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- galileon model

  modified Friedmann eqs with “effective” $\rho_{DE}(\phi, \dot{\phi})$

  $w_{DE,0} = -1$, w_{DE} can be < -1

  $G_{\rm eff}(z) \;\; \Rightarrow$ signature in the perturbation growth

  Laboratory and solar system constraints: Vainshtein mechanism

- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2}\left(1 - 3\ w_{\rm eff}\right) f = \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$.
Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
It can be very different in modified gravity models!

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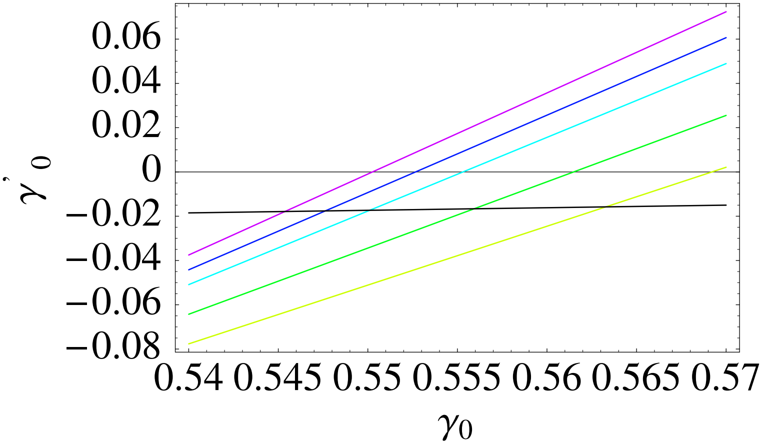


$\Omega_{m,0} = 0.3$ and various values of $w_{DE,0}$
We have from top to bottom:
$w_{DE,0} = -1.4,\ -1.3,\ -1.2,\ -1,\ -0.8.$
The black line gives the **true** value of γ_0 realized:
same non vanishing $\gamma'_0 \approx -0.02$.

Exact results if γ is constant

Requiring $w_{DE} \leq 0$ deep in the matter era

$$\Rightarrow 0.5 < \gamma \leq 0.6$$

$\gamma = 0.5$ is singular

$\gamma = 0.6$: DE behaves like dust

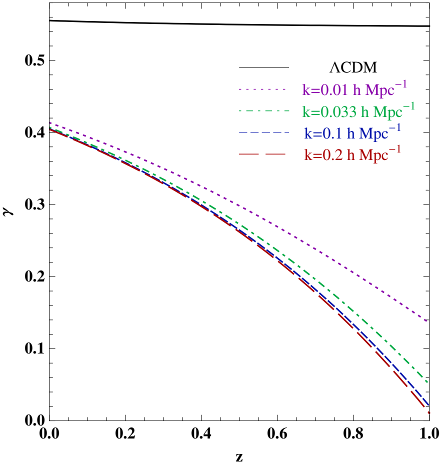
$\gamma = \frac{6}{11} = 0.545454$ if $DE = \Lambda$

$0.5 < \gamma < \frac{6}{11}$: phantom regime

Deep in the DE dominated era: phantom regime

$\Rightarrow$ Cannot be realized by quintessence!

$$f(R) = R - \lambda R_c \frac{x^{2n}}{x^{2n}+1} \qquad x \equiv \frac{R}{R_c}$$



$$n = 1, \quad \lambda = 1.55$$

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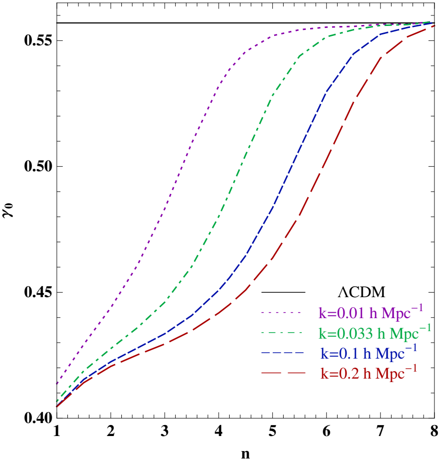

$$\lambda = 1.55$$

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- We need complementary probes:

  Supernovae

  Clusters

  Weak lensing

  Baryon Acoustic Oscillations

  Cosmic Microwave Background

  Gamma Ray Bursts?

  Gravitational waves?

  Redshift drift?

- ...theoretical tools: parametrizations, Fisher matrices, Bayesian approach,..

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- parametrizations: assume for some physical quantity a functional dependence depending on a restricted number of parameters

$$\text{e.g.}\quad w_{DE}(z) = (-1+\alpha) + \beta\,(1-x) \equiv w_0 + w_a\,(1-x)$$

$$\Leftrightarrow w_{DE}(z) = (-1+\alpha_p) + \beta\,(x_p - x)$$

$$f(z) = (1+z)^{3(\alpha+\beta)} \mathrm{e}^{-3\beta\frac{z}{1+z}}$$

Construct a quantity which assumes a special value, typically for Λ

$$\text{e.g.}\quad Om(z) = \frac{h^2(z)-1}{(1+z)^3-1}$$

$Om(z) = \Omega_{m,0} =$ constant for ΛCDM

$Om(z)$ evolves for all other DE models!

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Union2.1 set, 580 SNIa, $0.015 \le z \le 1.414$

wCDM : $w = -0.985^{+0.071}_{-0.077}$ (1σ C.L.)

Suzuki N.et al., ApJ 746,85 (2012), arXiv:1105.3470

7-year WMAP

Shift parameter:

$$\mathcal{R} = \sqrt{\Omega_{m,0}} \int_0^{z_{rec}} \frac{dz}{h(z)} = 1.725 \pm 0.018$$

E. Komatsu et al.,ApJ Suppl. 192, 18 (2011)

Baryon Acoustic Oscillations:

$$D_V(z) = H_0^{-1} \left[\frac{z}{h(z)} \left(\int_0^z \frac{dz'}{h(z')} \right)^2 \right]^{1/3}$$

for $z = 0.106, 0.2, 0.35, 0.44, 0.6, 0.73$

From the following galaxy surveys:
SDSS DR7,W. Percival et al., MNRAS 401,2148 (2010)
WiggleZ, C. Blake et al., MNRAS 415, 2892 (2011); MNRAS 418, 1707 (2011)
6dF GS, F. Beutler et al., MNRAS 416, 3017 (2011).

SDSS-3BOSS: L. Anderson et al., arXiv:1303.4666
$D_A(0.57) = 1408 \pm 45$ Mpc and $H(0.57) = 92.9 \pm 7.8$ km/s/Mpc

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Planck results, arXiv:1303.5076

Planck+WP+BAO

$w = -1.13^{+0.24}_{-0.25} \quad (2\sigma \text{ C.L.})$

Planck+WP+Union2.1

$w = -1.09 \pm 0.17 \quad (2\sigma \text{ C.L.})$

Planck+WP+SNLS

$w = -1.13^{+0.13}_{-0.14} \quad (2\sigma \text{ C.L.})$

Planck+WP+H_0

$w = -1.24^{+0.18}_{-0.19} \quad (2\sigma \text{ C.L.})$

Planck+WP+BAO

$w_0 = -1.04^{+0.72}_{-0.69} \qquad w_a < 1.32 \quad (2\sigma \text{ C.L.})$

$w(a) = w_0 + w_a(1-a)$

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BAO

Acoustic scale at various $z \Rightarrow D_A(z)$ and $H(z)$

BAO forecasts for a Full-Sky survey

$z_{\min}$	$z_{\max}$	Vol	% Err $D_A(z)$	% Err $H(z)$	Ω_Λ	σ_w
0.00	0.15	0.33	2.8	4.9	0.708	0.64
0.15	0.32	2.62	0.95	1.7	0.616	0.088
0.32	0.51	7.89	0.53	0.96	0.515	0.036
0.51	0.73	16.5	0.35	0.63	0.413	0.021
0.73	0.99	28.4	0.26	0.46	0.318	0.015
0.99	1.28	42.9	0.21	0.36	0.236	0.013
1.28	1.62	59.0	0.17	0.28	0.170	0.012
1.62	2.00	75.8	0.14	0.24	0.119	0.013
2.00	2.44	92.3	0.13	0.21	0.082	0.014
2.44	2.95	108	0.12	0.18	0.056	0.016
2.95	3.53	121	0.11	0.17	0.038	0.020
3.53	4.20	133	0.10	0.15	0.025	0.025
4.20	4.96	142	0.10	0.15	0.017	0.033

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The art of inducing accelerated expansion

Observations seems to close on ΛCDM....
Smooth component in GR with $w \approx -1$
Λ? $w(z)$? $w < -1$?
Clustering?

Modifying gravity ?
Modified growth of perturbations

Challenging cosmological framework ?
Future of our universe?

Observations will single out viable models
A new vision with challenging questions!

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