CMB μ distortion and primordial gravitational waves

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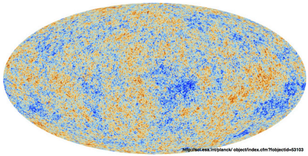
Outline

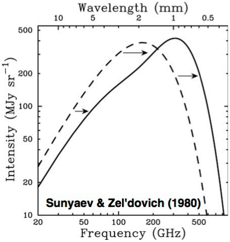
Information of Primordial fluctuations in the CMB

CMB as observables of primordial fluctuations

There are two approaches:

- Temperature anisotropies ($k < 0.1\text{Mpc}^{-1}$)
  - Local Planck distributions deviate from the averaged distribution
  - Direct detection of spacial fluctuation (Already detected)
- Spectrum distortions ($1\text{Mpc}^{-1} < k < 10^4\text{Mpc}^{-1}$)
  - Erased inhomogeneities form some distortions to the spectrum
  - Indirect detection of spatial fluctuation (Not yet detected)





Classification of distortions and Generation mechanisms

Classification of distortions

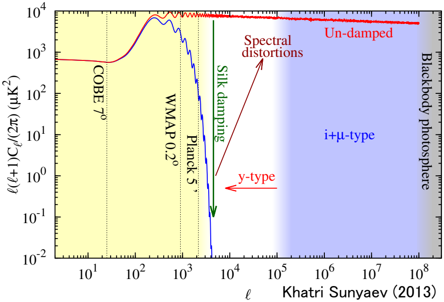
- Thermal distortion to Planck distribution $\rightarrow$ μ type distortion
  i.e. Chemical potential of general Bose distribution
- Non-thermal distortion to thermal distribution $\rightarrow$ y type distortion

Steps to generate μ distortion

1. Photon production:
   e.g. Acoustic damping, PBHs evaporation, darkmatter pair annihilation etc...
2. Thermalization under number conserving process
   e.g. Compton scattering $(10^5 < z < 10^7)$

Constraints on Primordial fluctuations by μ distortion

- Scalar case: Hu et.al (1994),Chluba et.al (2012) etc...
  - Primordial curvature (temperature) perturbation $\rightarrow$Acoustic damping $\rightarrow \mu \sim 10^{-8}$ for $n_s = 1,\ A_\zeta \sim 10^{-9}$



Khatri Sunyaev (2013)

- Tensor case: Our study
  - No acoustic damping (Tensor does not couple to baryon)
  - Primordial GW $\rightarrow$ Temperature perturbation $\rightarrow$ Thomson scattering

Goals

1. How primordial gravitational waves (GW) is related to the deviations?
2. Can we get some constraints on primordial GW from observations of CMB distortions?

Formulation of μ originate from GW

Integral solution

$$\langle\mu\rangle = -1.4 \times 4 \int_{\eta_0}^{\eta} d\eta' \mathcal{J}_{DC}(\eta') \langle {\Theta^T}_\gamma \dot{\Theta^T}_\gamma \rangle \quad (\spadesuit)$$

- Θ_γ^T: Tensor-type dimensionless temperature perturbations originate from Primordial GW
- $\mathcal{J}_{DC}(\eta')$is a window function which suppresses μ by double Compton scattering and contains t_μ
- The Ensemble average is related to tensor power spectrum $\mathcal{P}_T$

Boltzmann-Einstein system

1. Linear Boltzmann equation for tensor type temperature perturbation:

$$\dot{\Theta}_\gamma^T = -n_e \sigma_T (\Theta_\gamma^T - \Lambda) = \partial_\eta \Theta_\gamma^T + ik\lambda\Theta_\gamma^T + \frac{1}{2}\partial_\eta h$$

- n_e is electron number density and σ_T is Thomson scattering cross section
- $\Theta_\gamma^T = (1-\lambda^2)\sum_{l=1}^{\infty}(-i)^l(2l+1)P_l(\lambda)\Theta_{\gamma l}^T$
- $\Lambda = \left[\frac{3}{70}\Theta_{\gamma 4}^T + \frac{1}{7}\Theta_{\gamma 2}^T + \frac{1}{10}\Theta_{\gamma 0}^T - \frac{3}{70}\Theta_{\gamma 4}^{PT} + \frac{6}{7}\Theta_{\gamma 2}^{PT} - \frac{3}{5}\Theta_{\gamma 0}^{PT}\right]$

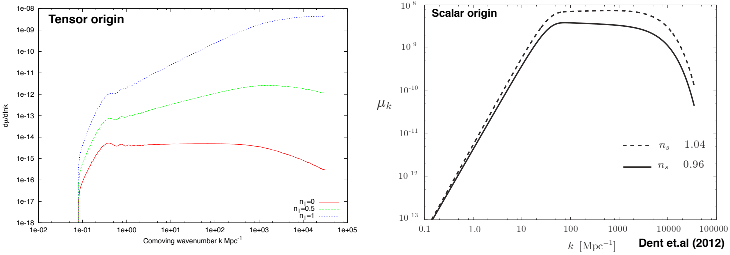
2. Substitute these to (♠)

μ distortion generated by primordial GW

$$\mu = 1.4 \cdot 4 \int_{\eta_0}^{\eta} d\eta' \int d\ln k \frac{\mathcal{P}_T(k)}{4} \mathcal{J}_{DC}(\eta')\dot{\tau}\left[\frac{12}{25}\Theta_{\gamma 0}^T\Theta_{\gamma 0}^T + \cdots\cdots\right]$$

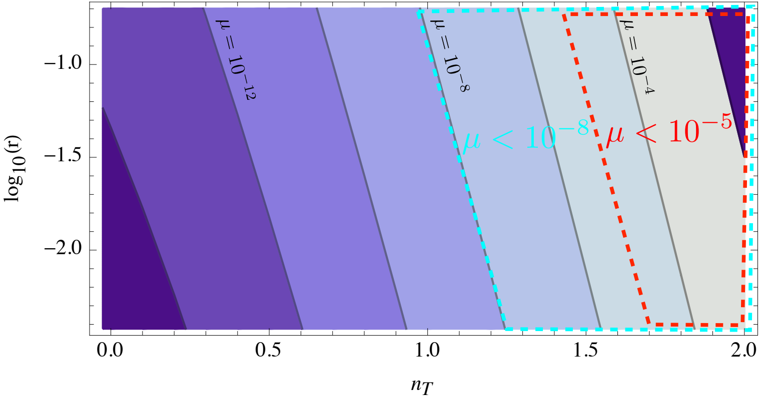
Scale dependence of μ-distortion

- $d\mu/d\ln k$ with different spectrum index is drawn bellow
- Larger spectrum index, larger contribution
- Compare to the scalar case (right figure), contribution from larger scale is bigger since Thomson scattering smoothes larger scale than Acoustic damping does!



Constraints on tensor-to-scalar ratio and spectrum index

- $\mu < 10^{-5}$ (95% CL) is a constraint today by COBE FIRAS
- $\mu < 10^{-8}$ is a constraint in the future by PIXIE observation
- If $r = 0.2$ (BICEP2 suggestion), PIXIE can rule out $n_T > 1.0$



Conclusions

1. We found a new generation mechanism of spectrum distortions by Thomson isotropic nature
2. We obtained constraints on tensor-to-scalar ratio and tensor spectrum index through CMB μ-distortion alone
3. For $r = 0.1$ and $n_T = 0$, expected μ distortion is the order of 10^{-14}
4. Blue shifted spectrum will be ruled out by the PIXIE observation (e.g. $r = 0.2$ and $n_T = 1$ makes comparable μ from adiabatic scalar perturbation)

Mixing of Blackbodies under energy conservation law

1. Let's take two Blackbodies with $T+\delta T$ and $T-\delta T$, then mix them:
   - $\rho_{\text{initial}} = \frac{\alpha}{2}[(T+\delta T)^4 + (T-\delta T)^4] = \alpha T^4 \left[1 + \frac{3}{2}\left(\frac{\delta T}{T}\right)^2 + \cdots\right]^4$
   - $n_{\text{initial}} = \frac{\beta}{2}[(T+\delta T)^3 + (T-\delta T)^3] = \beta T^3 \left[1 + \left(\frac{\delta T}{T}\right)^2 + \cdots\right]^3$
2. Energy conservation: $\rho_{\text{initial}} = \rho_{\text{final}}$
   - $T_{\text{final}} = T\left[1 + \frac{3}{2}\left(\frac{\delta T}{T}\right)^2 + \cdots\right]$
   - $n_{\text{final}} = \beta T^3 \left[1 + \frac{9}{2}\left(\frac{\delta T}{T}\right)^2 + \cdots\right] \neq n_{\text{initial}}$

New system is not a blackbody under number conservation

Realized distribution function is Bose distribution with nonzero chemical potential!

- Thermalization under the photon number conserving process makes μ-distortion

Mixing of Black bodies to a Bose distribution

T_{BE} is temperature of Bose distribution

1. Energy density and Number density of Boson:
   - $\rho = \alpha T_{\mathrm{BE}}^4 \left(1 - \frac{90\zeta(3)}{\pi^4}\mu\right)$
   - $n = \beta T_{\mathrm{BE}}^3 \left(1 - \frac{\pi^2}{6\zeta(3)}\mu\right)$
2. Again imposing number conservation,
   - $T_{\mathrm{BE}} = T\left[1 + \left(\frac{\delta T}{T}\right)^2 + \frac{\pi^2}{18\zeta(3)}\mu + \cdots\right]$
   - $\rho_{\mathrm{final}} = \alpha T^4 \left[1 + 4\left(\frac{\delta T}{T}\right)^2 + \left(\frac{2\pi^2}{9\zeta(3)} - \frac{90\zeta(3)}{\pi^4}\right)\mu\right]$

Initial energy perturbation $6\delta T^2/T^2$ goes into Two parts

- 1/3 makes distortions
- 2/3 raise average temperature

The distortion and temperature shift are the **second order of temperature perturbation**

Locally thermodynamic equilibrium

In cosmological perturbation theory,

- Distribution of photons is inhomogeneous
- But, locally thermodynamic equilibrium

Each spectra are locally thermodynamic equilibrium

$$f(x,\omega) = \frac{1}{e^{\frac{\omega}{T_{\mathrm{BE}}(x)}+\mu(x)} - 1}$$

We can write down energy and number density to the first order of μ

- $\rho(x) = \alpha T_{\mathrm{BE}}^4(x)\left(1 - \frac{90\zeta(3)}{\pi^4}\mu(x)\right)$
- $n(x) = \beta T_{\mathrm{BE}}^3(x)\left(1 - \frac{\pi^2}{6\zeta(3)}\mu(x)\right)$

Bose temperature and Planck temperature

We expand local Bose temperature T_{BE} as follows:

$$\begin{aligned} T_{\mathsf{BE}}(x) &= T_{\mathsf{pl}}(x) + t_{\mathsf{BE}}(x) \\ &= \langle T_{\mathsf{pl}} \rangle + \delta T(x) + t_{\mathsf{BE}}(x) \\ &= T_{\mathsf{rf}} + \Delta T + \delta T(x) + t_{\mathsf{BE}}(x) \end{aligned}$$

Notice that $\langle T_{\mathsf{pl}} \rangle \neq T_{\mathsf{rf}} = (\beta^{-1} \langle n \rangle)^{1/3}$

- t_{BE} is difference between $T_{\mathsf{BE}}(x)$ and $T_{\mathsf{pl}}(x)$
- $\delta T(x)$ is inhomogeneous part of local Planck temperature
- ΔT is homogeneous temperature variation
- T_{rf} is 'reference temperature' which scales as a^{-1}

Expand $n(x)$ and $\rho(x)$ around $T_{\rm rf}$

We define dimensionless perturbations as follows:

- $T_{\rm rf} + \delta T(x) + \Delta T + t_{\rm BE}(x) = T_{\rm rf}(1 + \Theta(x) + \Delta + t(x))$
- ΔT and $t_{\rm BE}(x)$ are the 2nd order of $\Theta(x)$

Then we get,

- $n(x) = \beta T_{\rm rf}^3 \left(1 + 3\Theta(x) + 3\Delta + 3t(x) + 3\Theta^2(x) - \frac{\pi^2}{6\zeta(3)}\mu(x)\right)$
- $\rho(x) = \alpha T_{\rm rf}^4 \left(1 + 4\Theta(x) + 4\Delta + 4t(x) + 6\Theta^2(x) - \frac{90\zeta(3)}{\pi^4}\mu(x)\right)$

Impose number conservation law: $d(a^3\langle n\rangle)/d\eta = 0$

- $\Delta + \langle t\rangle = \frac{\pi^2}{18\zeta(3)}\langle\mu\rangle - \langle\Theta^2\rangle$

Substitute this into $\rho(x)$, we obtain

- $\langle\rho\rangle = \alpha T_{\rm rf}^4 \left[1 + 2\langle\Theta^2\rangle + \left(\frac{2\pi^2}{9\zeta(3)} - \frac{90\zeta(3)}{\pi^4}\right)\langle\mu\rangle\right]$