A New Method To Determine Large Scale Structure From The Luminosity Distance

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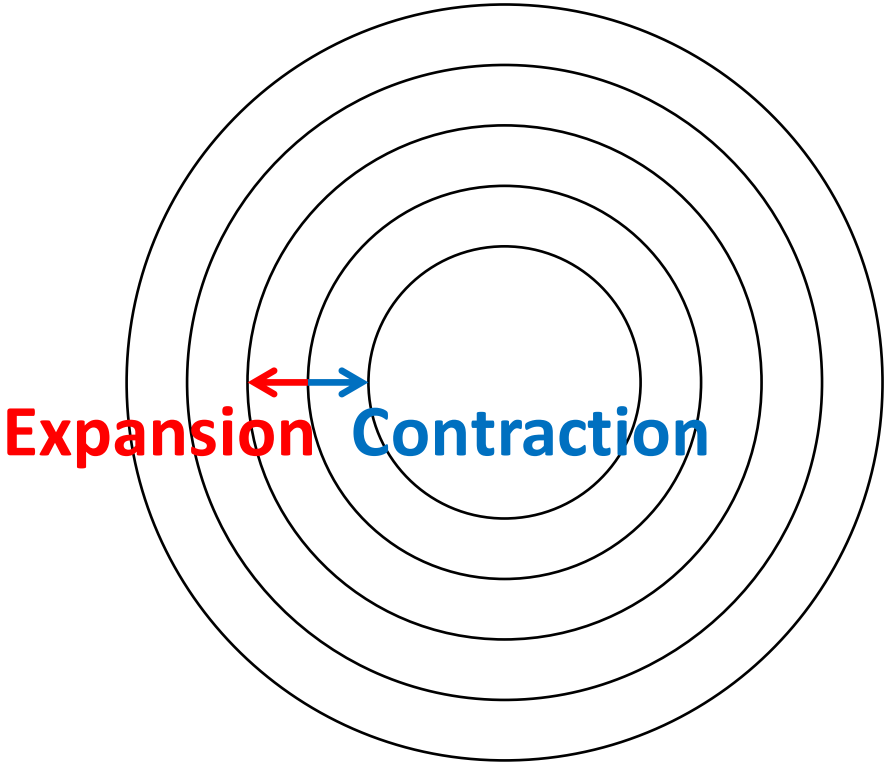
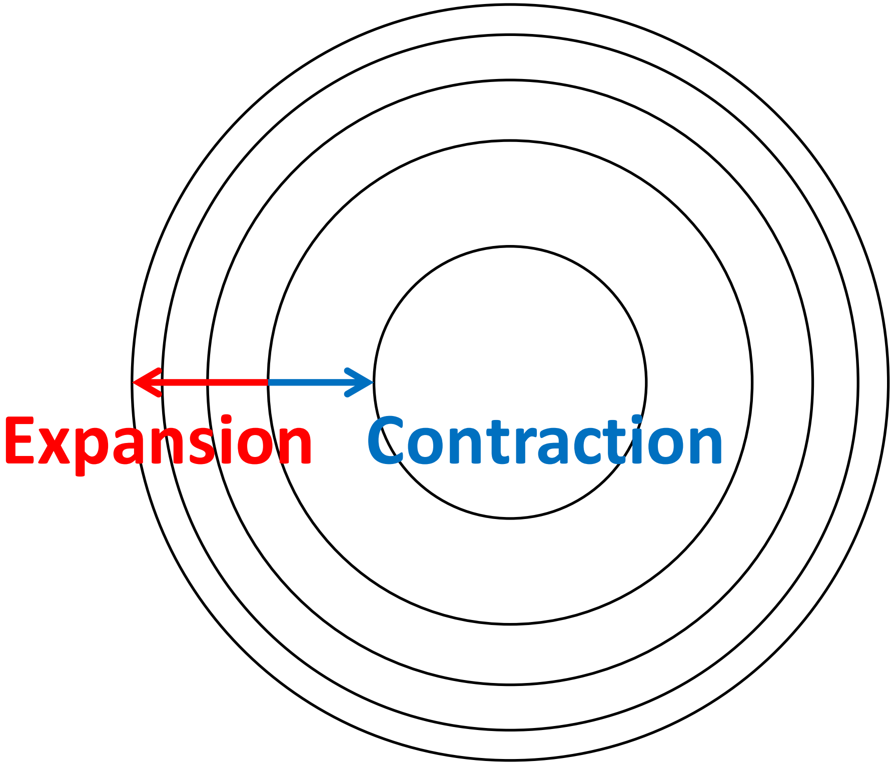
Accelerating Expansion from Large Scale Inhomogeneity

- A homogeneous universe (FRW model)

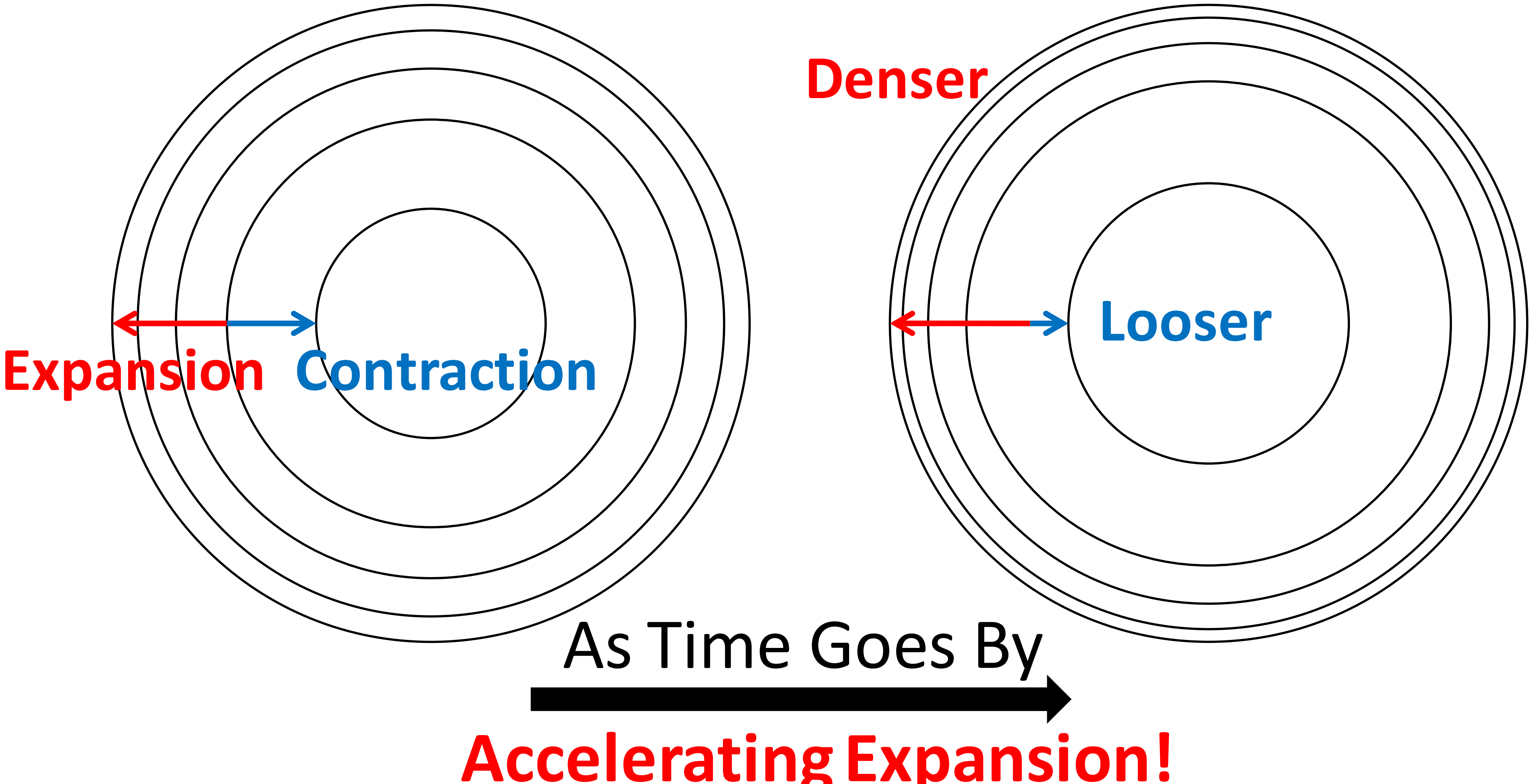


- An inhomogeneous universe (Void model)



Accelerating Expansion from Large Scale Inhomogeneity

- Early time:
- Now:



Accelerating Expansion from Large Scale Inhomogeneity

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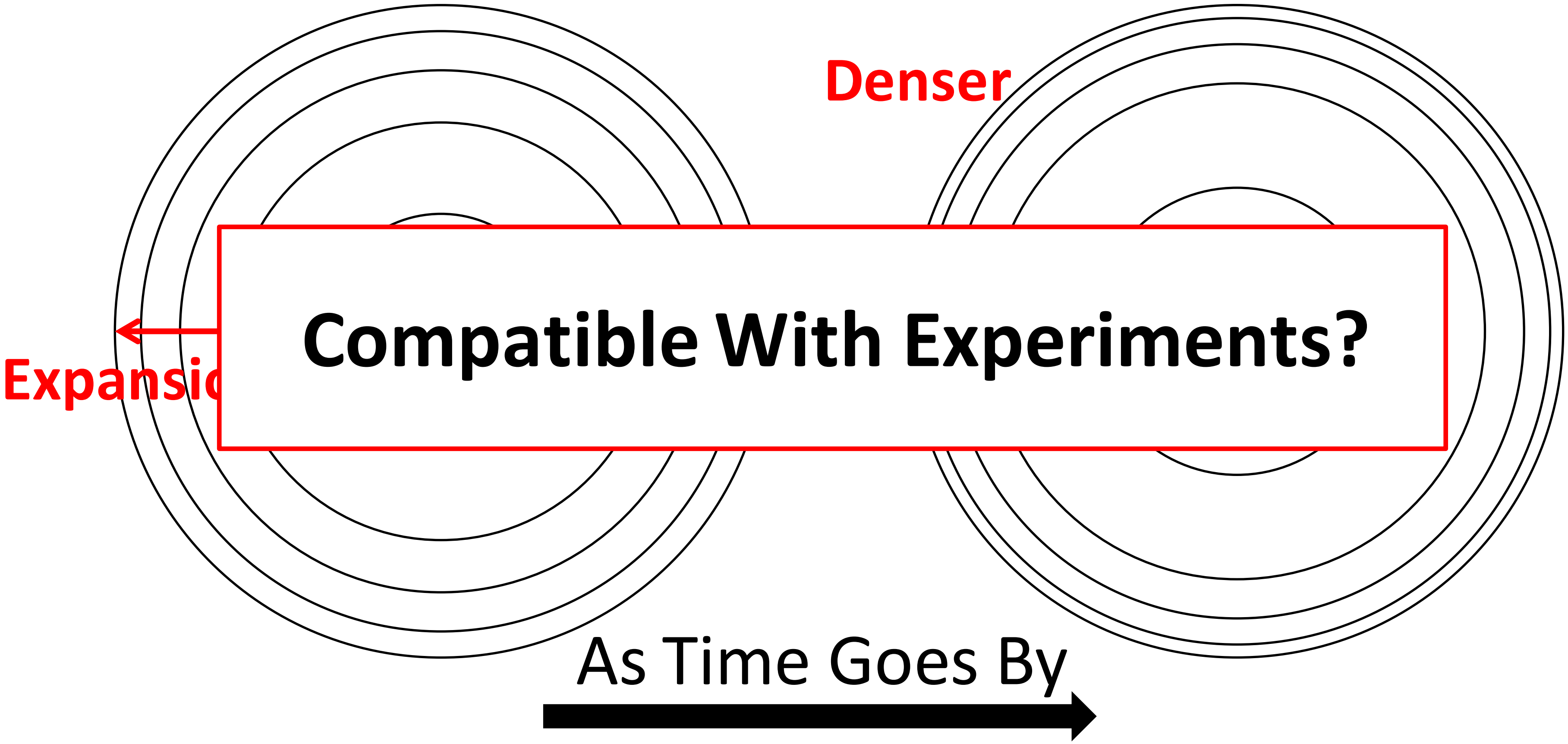


Accelerating Expansion!

Outline

- Accelerating Expansion from Large Scale Inhomogeneity: LTB Model
- Mimicking ΛCDM Model: Central Spatial Curvature as “Free Parameter”
- Climbing over Apparent Horizon: An Unique Solution

(R = Areal Radius)

Modeling Large Scale Inhomogeneity

- Assuming spherical symmetry (LTB metric):

$$ds^2 = -dt^2 + a(t,r)^2 \left[\left(1 + \frac{r\partial_r a(t,r)}{a(t,r)} \right)^2 \frac{dr^2}{1-k(r)r^2} + r^2 d\Omega^2 \right],$$

$$\rho = \frac{2\partial_r M}{a^2 r^2 (r\partial_r a + a)}, \quad H^2 = \left(\frac{\partial_t a}{a} \right)^2 = \frac{-k(r)}{a^2} + \frac{2M(r)}{a^3 r^3}$$

- Fix gauge freedom of r by setting $M(r) = \frac{1}{6}\rho_0 r^3$
- Conformal time $\eta = \int_0^t \frac{d\tau}{a(\tau,r)} + t_b(r)$

$$a(\eta,r) = \frac{\rho_0}{6k(r)} \left[1 - \cos\left(\sqrt{k(r)}\eta \right) \right]$$

From now on we set $t_b=0$

$$t(\eta,r) = \frac{\rho_0}{6k(r)} \left[\eta - \sqrt{k(r)}^{-1} \sin\left(\sqrt{k(r)}\eta \right) \right] + t_b(r)$$

(R = Areal Radius)

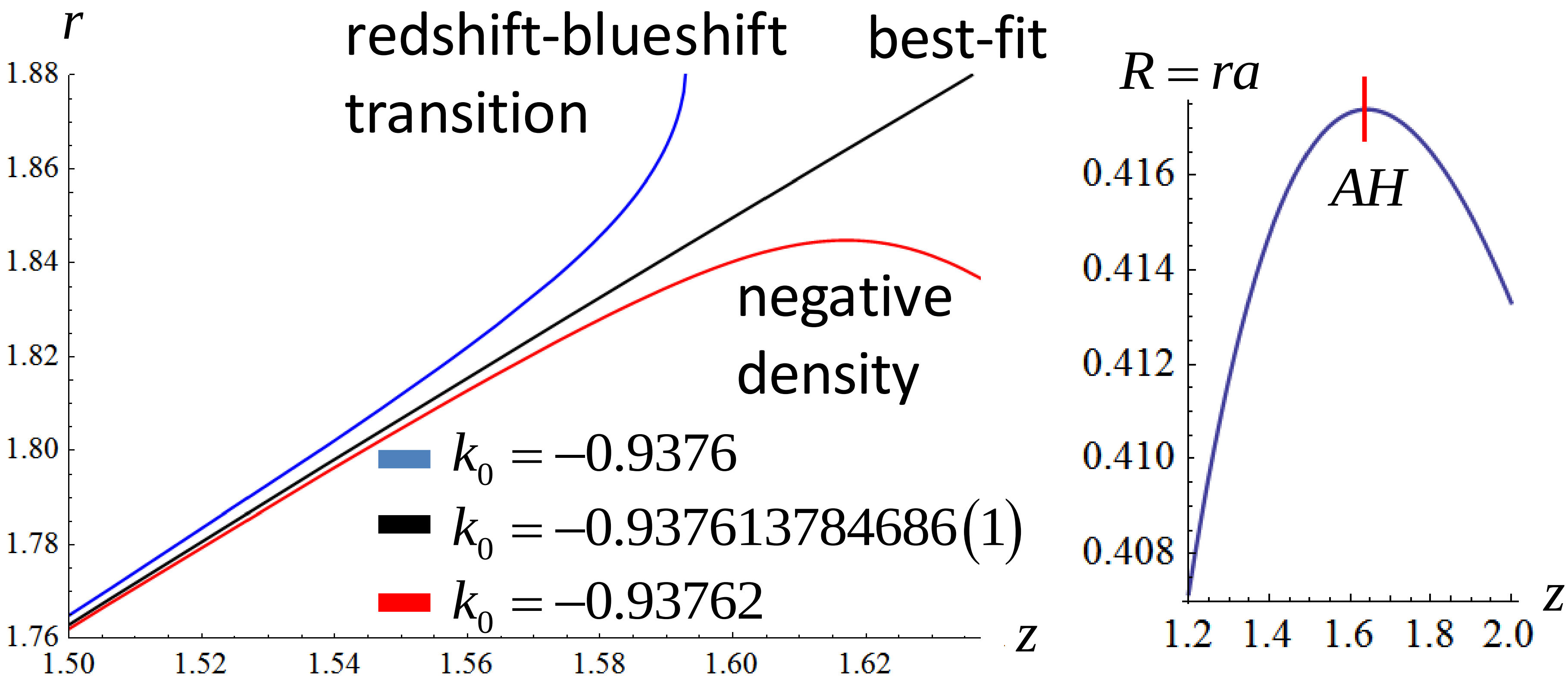
Fixing Initial Condition

- Assuming a central observer, we have $r\left(z=0\right)=0,\;\; a\left(z=0\right)=a_0,\;\; \eta\left(z=0\right)=\eta_0,\;\; k\left(z=0\right)=k_0$ and $H^{LTB}\left(z=0\right)=H_0$,where $H^{LTB}=\frac{\partial \ln a\left(t,r\right)}{\partial t}$.
- Since $a\left(\eta=\eta_0, r=0\right)=a_0$ and $H^{LTB}\left(\eta=\eta_0, r=0\right)=H_0$ are given, η_0 , k_0 and ρ_0 can be determined up to k_0.
- Central spatial curvature as “free parameter”
- Luminosity distance on past null geodesic $D_L^{obs}\left(z\right)=\left(1+z\right)^2 a\left(t\left(z\right), r\left(z\right)\right) r = D_L^{FRW}\left(z\right)$ is the input.

(R = Areal Radius)

Numerical Results

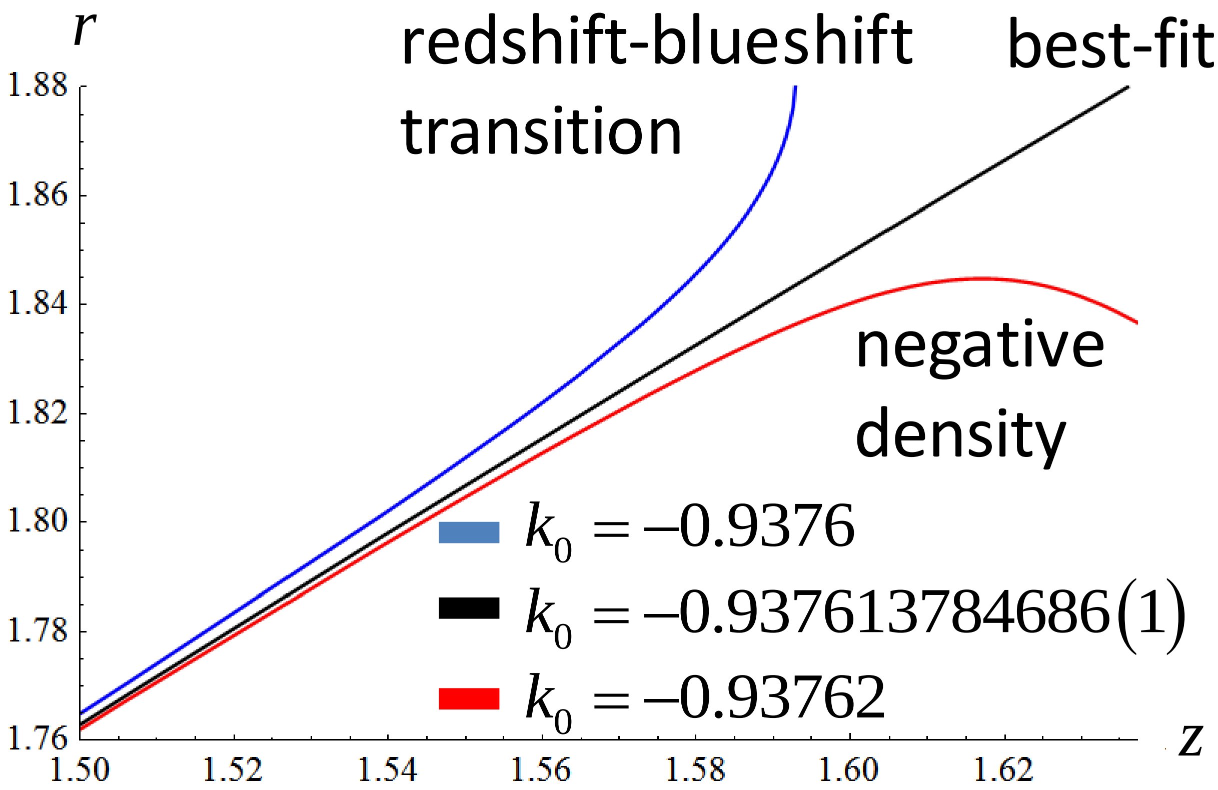
- Not so stable around apparent horizon at $z \sim 1.6$
- Overcome AH through extrapolation?!

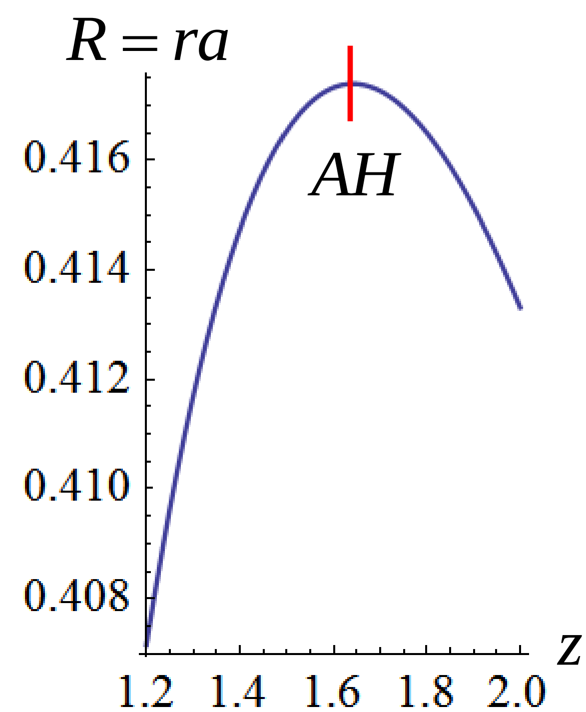


(R = Areal Radius)

Numerical Results

Only One Valid k_0, Why?





Expansion Around Apparent Horizon

- Staring at the geodesic equations, we found a common denominator $r\sqrt{k\left(1-s^2\right)}-s\sqrt{1-kr^2}$, where $s=\sqrt{H_0 ka\left(1+k_0\right)^{-1}}$.

Unstable when $R\approx\left(1+k_0\right)r^3/H_0$ or $s\approx\sqrt{kr^2}$

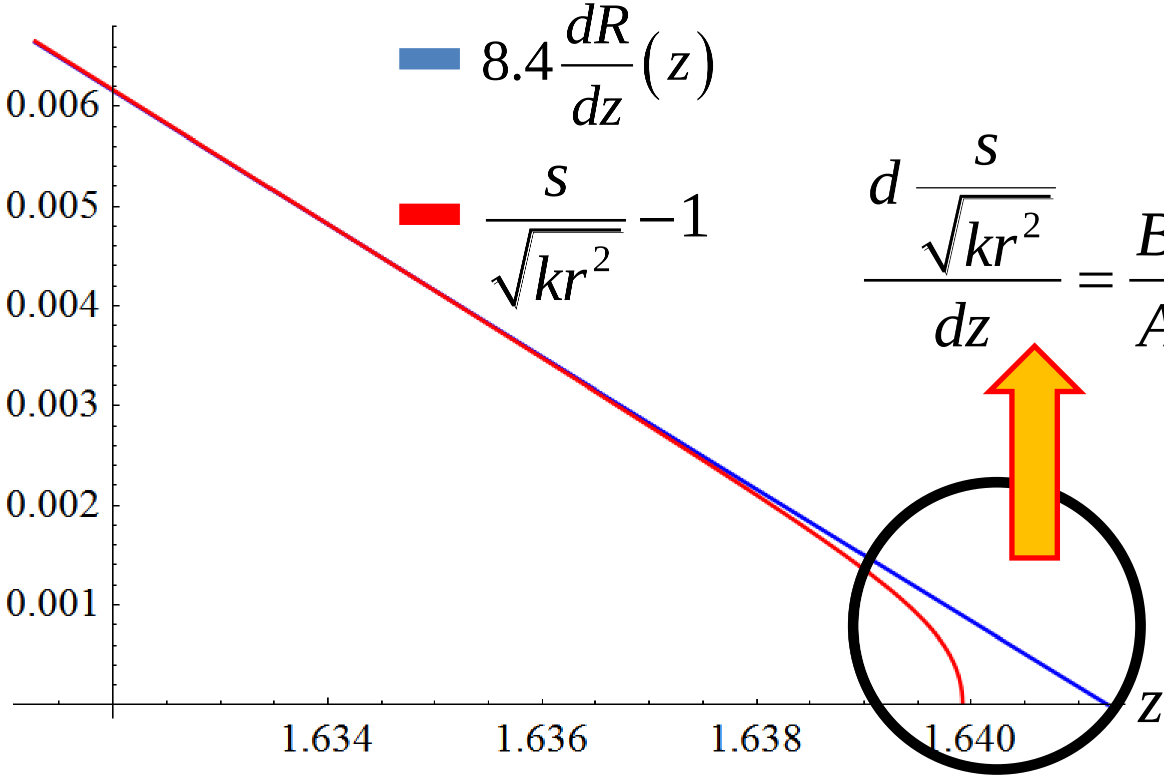
- Expand the numerator around $R=\left(1+k_0\right)r^3/H_0$

$$\frac{dk}{dz}=\frac{B_k}{A_k}+\frac{C_k\frac{dR}{dz}(z)}{A_k\left(s-\sqrt{kr^2}\right)},\quad \frac{dr}{dz}=\frac{B_r}{A_r}+\frac{C_r\frac{dR}{dz}(z)}{A_r\left(s-\sqrt{kr^2}\right)},\quad \frac{d\eta}{dz}=\frac{B_\eta}{A_\eta}+\frac{C_\eta\frac{dR}{dz}(z)}{A_\eta\left(s-\sqrt{kr^2}\right)}$$

- Needs $s=\sqrt{kr^2}$ and $\frac{dR}{dz}(z)=0$ at same spot

Expansion Around Apparent Horizon

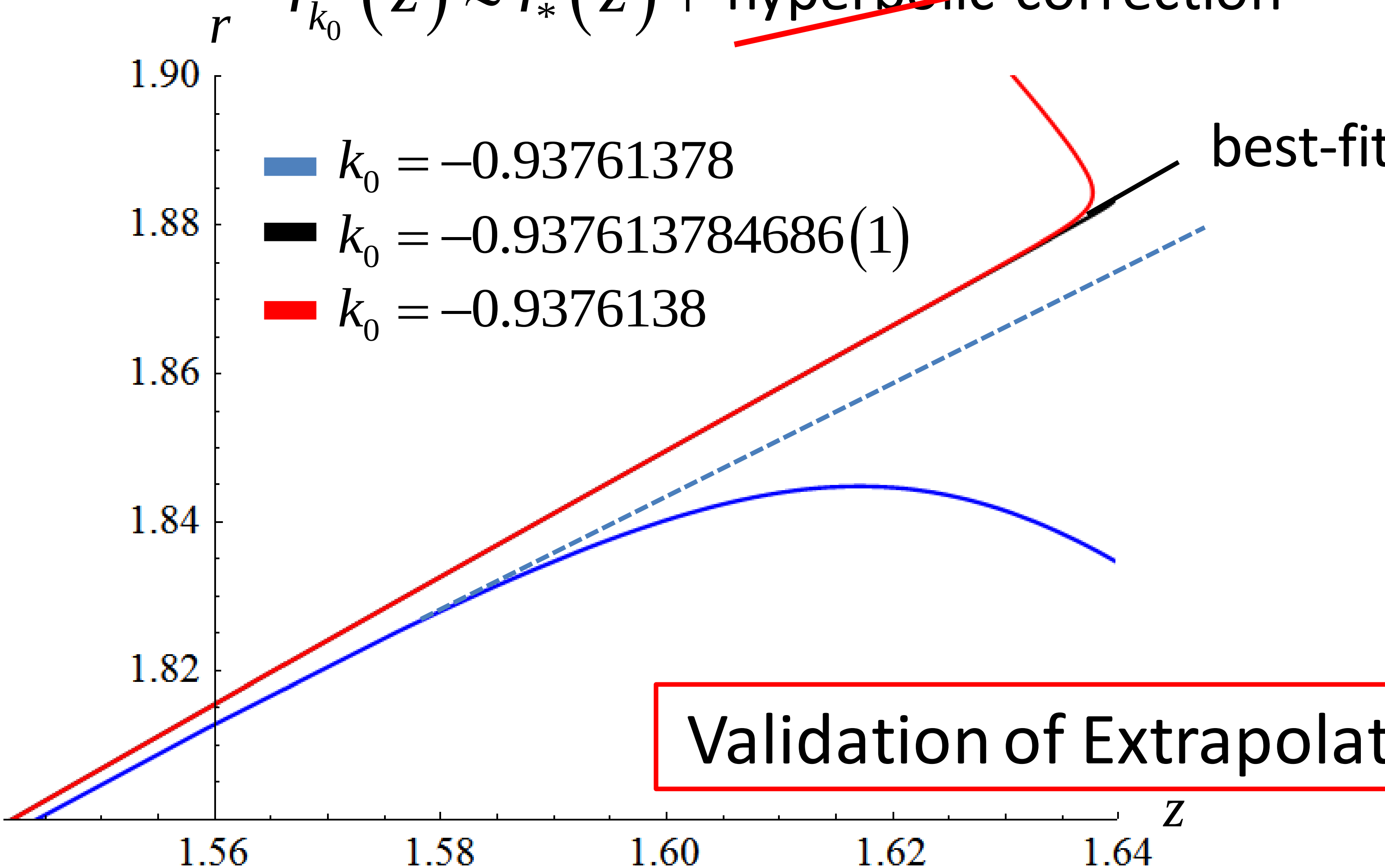
- Needs $s = \sqrt{kr^2}$ and $\frac{dR}{dz}(z) = 0$ at same spot
- Also $s / \sqrt{kr^2} - 1 \propto (z - z_{AH})$



$$\frac{d\frac{s}{\sqrt{kr^2}}}{dz} = \frac{B_s}{A_s} + \frac{C_s \frac{dR}{dz}(z)}{A_s\left(s - \sqrt{kr^2}\right)}$$

Expansion Around Apparent Horizon

$$r_{k_0}(z) \approx r_*(z) + \text{hyperbolic correction}$$



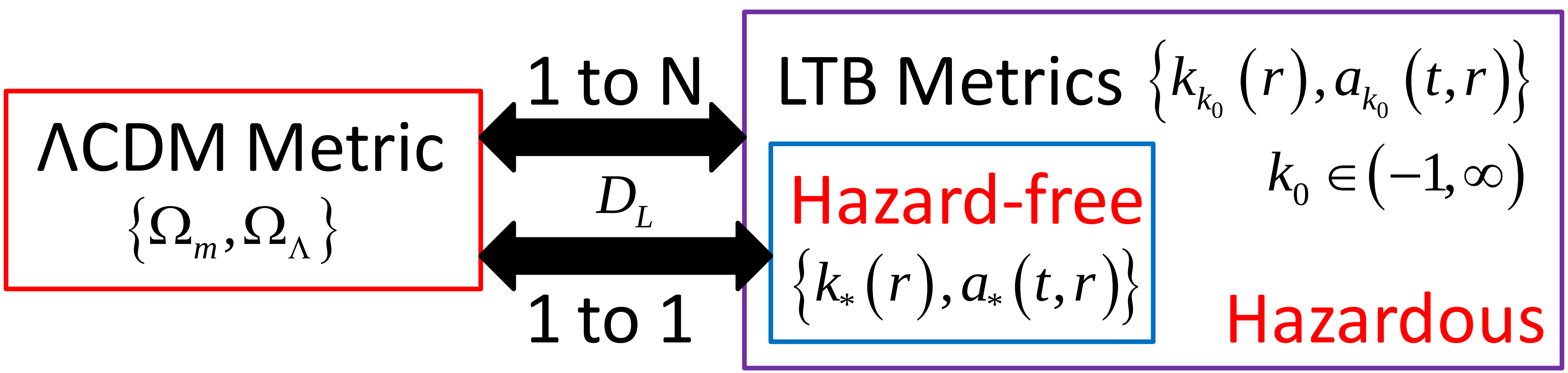
Validation of Extrapolation

Uniqueness of k_0

- Sudden jump happens only at $R_{AH} \approx (1+k_0) {r_{z_{AH}}}^3 / H_0$
- The existence of solution extended beyond AH is indicated by transit of cause of stop of integrator between $\frac{dr}{dz} = 0$ and $\frac{dz}{dr} = 0$.
- We scanned over parameter space $k_0 \in (-1, \infty)$ and found 1 solution.
- Not a rigorous proof yet

Conclusion

- There exists 1 to N correspondence between ΛCDM metric with certain parameter, and LTB metrics with specific setups that mimic the luminosity distance of that ΛCDM metric.
- But only 1 LTB metric can go beyond apparent horizon without hazards like negative density.



Conclusion

- The error of extrapolation used to overcome apparent horizon is marginal as long as k_0 used in simulation is close to the best fit k_0 value.
- Best extrapolation method is 1^{st} order Taylor expansion.

The End


Diolch
Grazie
Gracias
Arigatō
Tack
Tak
DEICING INNOVATIONS
Shukrān
Dík
Danke
Merci
THANK YOU
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Dank u
Дякую
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Köszi
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