Relativistic stars in $f(R)$ dark energy models, and absence thereof

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Based on work with 前田惠一,
arXiv:0807.2503, PRD accepted

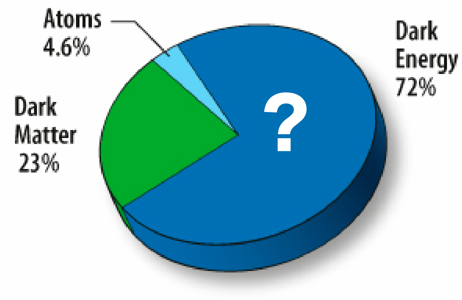
Alternative explanation for dark energy

- General relativity (GR) is the most successful theory of gravity.

$$G_{\mu\nu} = T_{\mu\nu}$$

- However, the mystery of the accelerating universe remains unsolved.
  - Observational evidences: CMB, SNe Ia, …
  - Unknown energy-momentum components?

$$G_{\mu\nu} = T_{\mu\nu} \boxed{- \Lambda g_{\mu\nu}} \ ? \qquad G_{\mu\nu} = T_{\mu\nu} \boxed{+ T^{(\phi)}_{\mu\nu}} \ ?$$



- GR may be modified at long distances (~ H_0^{-1} ~ 3Gpc).

$$? \quad \boxed{G_{\mu\nu} + \cdots} = T_{\mu\nu}$$

Motivation

- $f(R)$重力: 長距離で一般相対論を変更

Carroll et al. (2003)

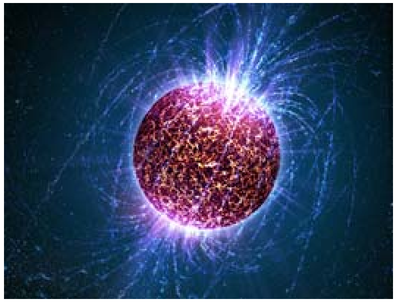
Einsitein-Hilbert action R ⇨ $f(R)$

例: $f(R) = R - \frac{\mu^4}{R}$

- 要請:
  - 確立されている宇宙の歴史を再現すべし
    - 元素合成、輻射優勢→物質優勢→加速膨張、…
  - 地上や太陽系での重力実験に抵触しない (弱重力場テスト)
    - ニュートン則、水星の近星点移動、光の曲がり (ポストニュートニアン)、…

- これらすべての制限をパスし、生き残っているモデル
  - Hu and Sawicki (2007); Starobinsky (2007); Appleby and Battye (2007)

強重力場の性質？　中性子星の解？
➔ ここで綻びが現れるかもしれない



c.f. Frolov's singularity problem (2008)

$f(R)$ gravity

- Action:

$$S = \int d^4x \sqrt{-g} \left[\frac{f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- EOM:

$$f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + \left(\Box f_R - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi G T_{\mu\nu},$$ where $f_R := df/dR$

metricの4階微分まで含む

- Energy conservation:

$$\nabla_\mu T^{\mu\nu} = 0$$

$f(R)$ gravity as a scalar-tensor theory

Maeda (1989)

- Trace of the EOM:

$$\Box f_R = \frac{8\pi G}{3}T + \frac{1}{3}(2f - f_R R)$$

Ricci はダイナミカルに決まる
c.f. 一般相対論では代数的に決まる

$$R = -T$$

$$\chi := f_R \qquad R = Q(\chi)$$

- Equivalence with a scalar-tensor theory

Jordan frame equations

$$\begin{cases} \chi G_\mu{}^\nu = 8\pi G T_\mu{}^\nu + \left(\nabla_\mu \nabla^\nu - \delta_\mu{}^\nu \Box\right)\chi - \chi^2 V(\chi)\delta_\mu{}^\nu, \\ \Box \chi = \frac{8\pi G}{3}T + \frac{2\chi^3}{3}\frac{dV}{d\chi}, \end{cases}$$

where $V(\chi) := \frac{1}{2\chi^2}\left[\chi Q(\chi) - f(Q(\chi))\right]$

- 追加のスカラー自由度
- 2階微分までしか含まない

物質場がスカラー場のダイナミクスに影響を与える

The model

- Starobinsky's *f*(*R*):

$$f(R) = R + \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right]$$

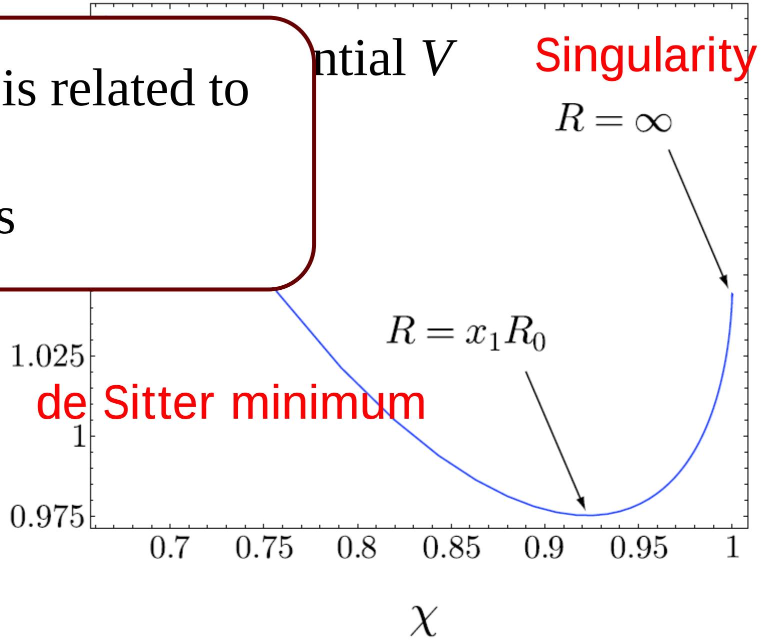
Starobinsky (2007)

very similar to Hu and Sawicki's *f*(*R*) (2007)

- High curvature regime:

- The effective scalar degree of freedom is related to the Ricci scalar in a nontrivial way.
- Its dynamics is affected by matter fields

$$R = R_1 = \text{constant} = x_1 R_0$$



- Scalar field written in terms of *R*

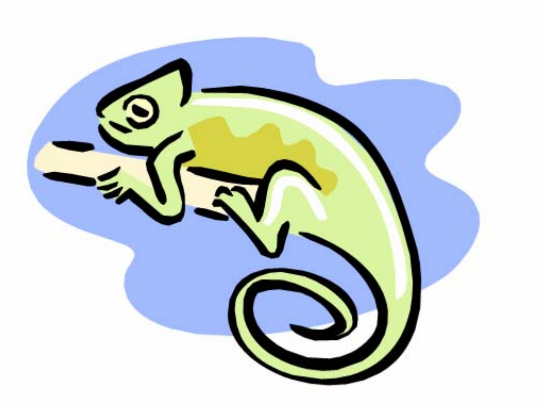
$$\chi = 1 - 2n\lambda \frac{R}{R_0} \left(1 + \frac{R^2}{R_0^2} \right)^{-n-1}$$

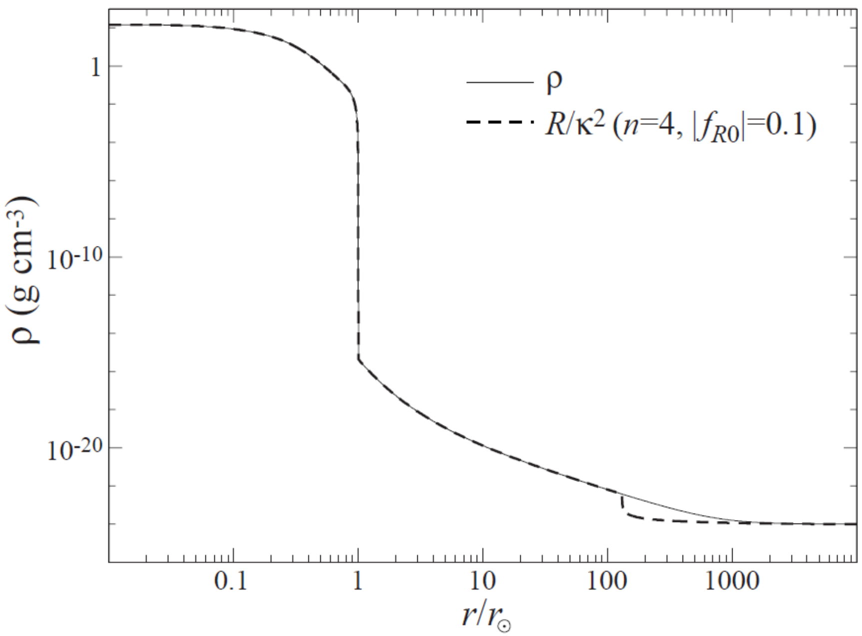
"Chameleon" mechanism

- スカラー場の effective mass は物質のエネルギー密度に依存
  - 太陽系の密度では重い → 「第5の力」は短距離力、実験をパス
  - 宇宙論的な密度では軽い → 加速膨張を引き起こす(~ quintessence)

"Chameleon" field

Khoury and Weltman (2003)





Density profile in the solar interior and vicinity

Hu and Sawicki (2007)

Spherically symmetric stars: Basic equations

- Metric:

$$ds^2 = -N(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

- Matter:

$$T_\mu^{\ \nu} = \mathrm{diag}\left(-\rho, p, p, p\right)$$

- Energy-momentum conservation:

$$p' + \frac{N'}{2N}(\rho + p) = 0$$

- Field equations:

(tt) $$\frac{\chi}{r^2}(-1 + B + rB') = -8\pi G\rho - \chi^2 V - B\left[\chi'' + \left(\frac{2}{r} + \frac{B'}{2B}\right)\chi'\right],$$

(rr) $$\frac{\chi}{r^2}\left(-1 + B + rB\frac{N'}{N}\right) = 8\pi G p - \chi^2 V - B\left(\frac{2}{r} + \frac{N'}{2N}\right)\chi'.$$

- Scalar field EOM:

$$B\left[\chi'' + \left(\frac{2}{r} + \frac{N'}{2N} + \frac{B'}{2B}\right)\chi'\right] = \frac{8\pi G}{3}(-\rho + 3p) + \frac{2\chi^3}{3}\frac{dV}{d\chi}$$

Spherically symmetric stars: Boundary conditions

- Regular at the center:

$$N(r) = 1 + N_2 r^2 + \cdots, \quad B(r) = 1 + B_2 r^2 + \cdots$$
$$\chi(r) = \chi_c \left(1 + \frac{C_2}{2} r^2 + \cdots\right),$$
$$\rho(r) = \rho_c + \frac{\rho_2}{2} r^2 \cdots, \quad p(r) = p_c + \frac{p_2}{2} r^2 + \cdots,$$

- Surface of the star:

$$r = \mathcal{R}, \quad p(\mathcal{R}) = 0$$

- Asymptotically de Sitter:

$$r \to \infty, \quad \chi(r) \to \chi_1$$

Consider constant density stars

$$\rho = \rho_0$$

Parameters of a solution

$$\rho_0, \ p_c, \text{ and } \chi_c \text{ (or, equivalently, } R_c)$$

Classical mechanics picture

- 星の解を求める ⇔ 古典力学における質点の運動の問題
- Scalar field EOM:

$$\Box\chi = \frac{8\pi G}{3}T + \frac{2\chi^3}{3}\frac{dV}{d\chi}$$

(neglecting the effect of the metric…)

$$\frac{d^2}{dr^2}\chi + \frac{2}{r}\frac{d}{dr}\chi = -\frac{dU}{d\chi} + \mathcal{F},$$

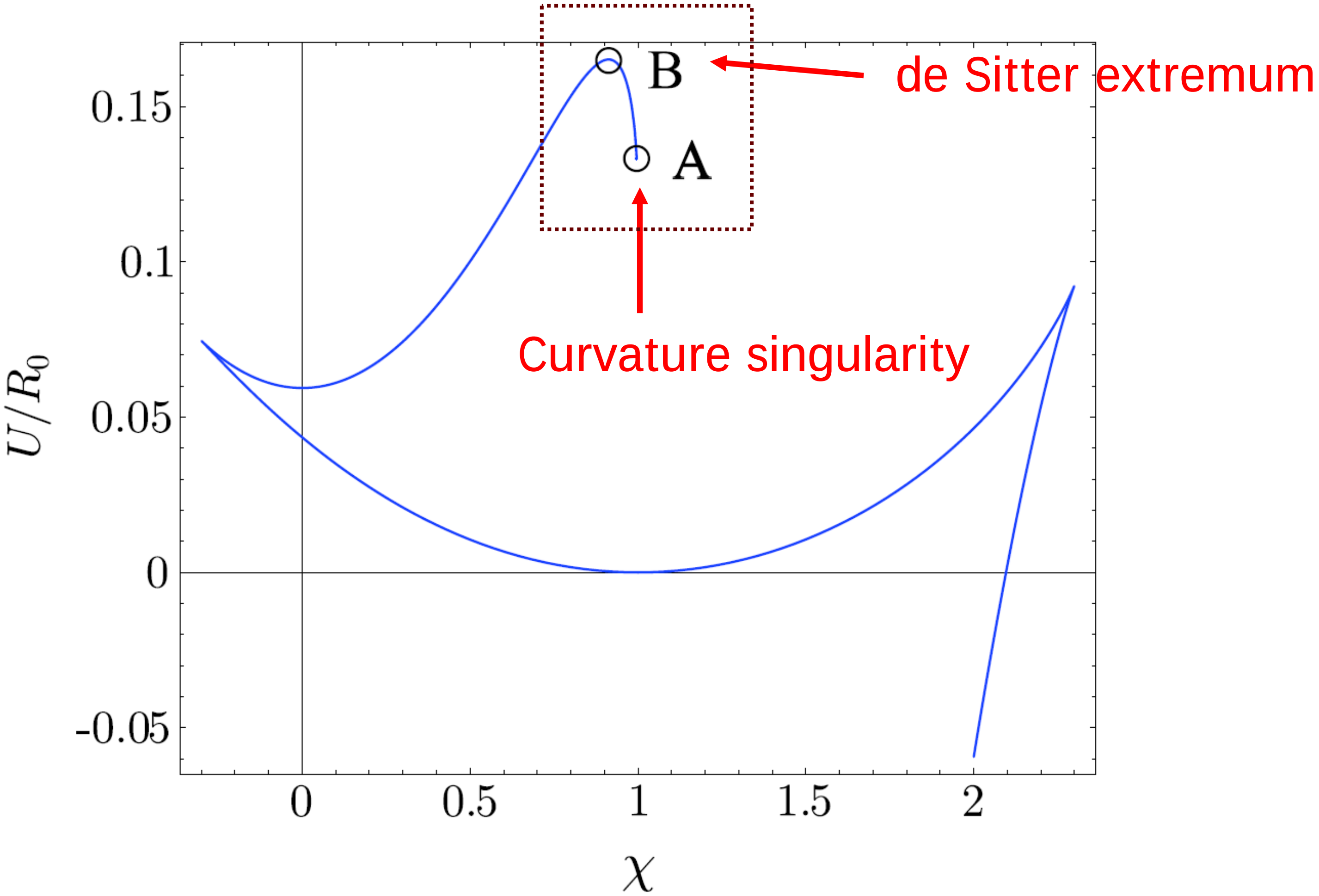
where

$$\frac{dU}{d\chi} := \frac{1}{3}\left[f_R Q - 2f\right]$$

$$\mathcal{F} := -\frac{8\pi G}{3}(\rho - 3p)$$

- r を時間座標と見なす
- ポテンシャル U の中で
  力 F を受けて運動する質点の問題
- (摩擦項)

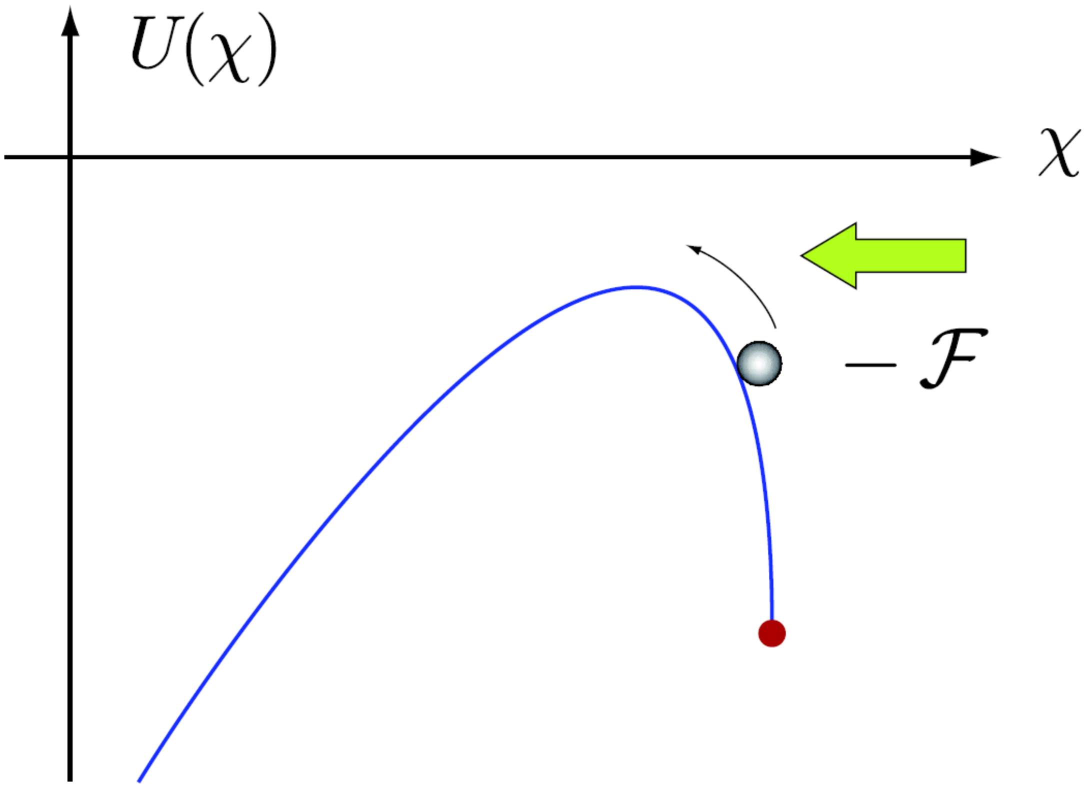
Inverted potential



Classical mechanics picture

- The particle starts at rest, since $\chi'|_{r=0} = 0$
- $\rho_0 , \, p_c$ → initial strength of the force
- χ_c → initial position of the particle

$$\mathcal{F} := -\frac{8\pi G}{3}(\rho - 3p)$$



- The force term vanishes for $r > \mathcal{R}$

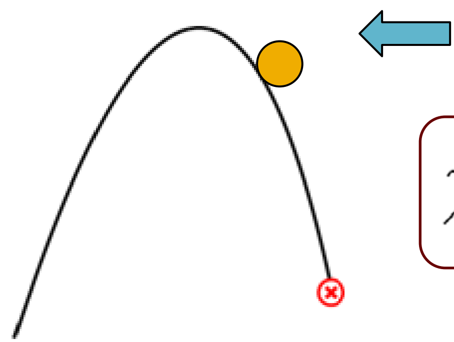
Motion of a particle 1

- Fall into the singularity

$$\chi_c > \chi_s$$

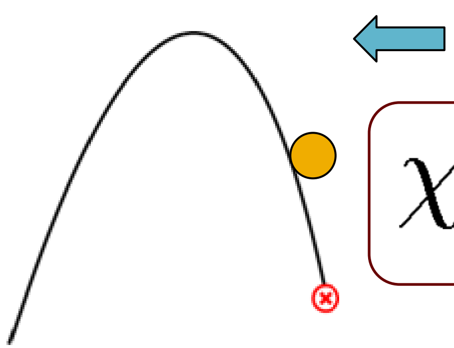
$$[\mathcal{F} - dU/d\chi]_{\chi_s} = 0$$

- Overshoot



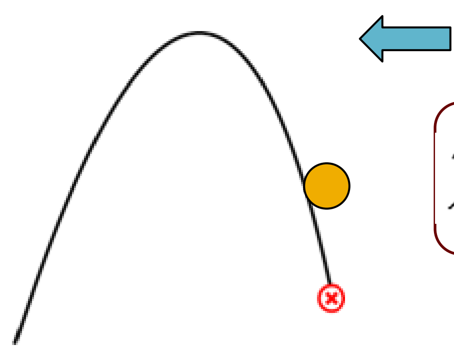
$$\chi_c < \chi_{\mathrm{crit}}$$

- Stop at the top



$$\chi_c = \chi_{\mathrm{crit}}$$

- Turn around



$$\chi_c > \chi_{\mathrm{crit}}$$

Motion of a particle 2

- もし $\chi_s > \chi_{\rm crit}$ となると、山を越える か 直で特異点に落ちる かのどちらかしかない
  - これは重力ポテンシャルが大きいと起こり得る
  - なぜなら、 重力ポテンシャル大 $(\rho_0\mathcal{R}^3)/\mathcal{R} = \rho_0\mathcal{R}^2$

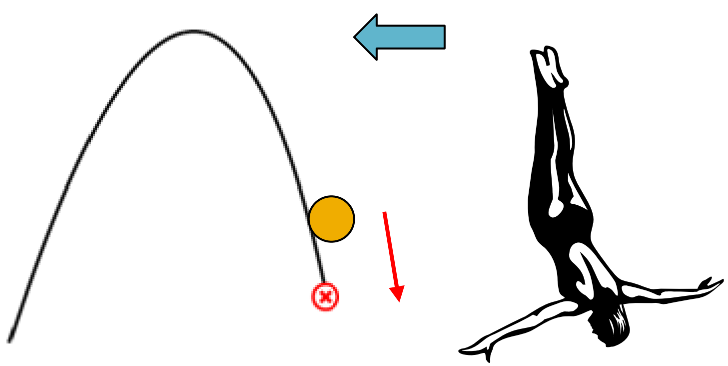
→ 密度 and/or 半径 大

→ 力の大きさ and/or 継続時間 大

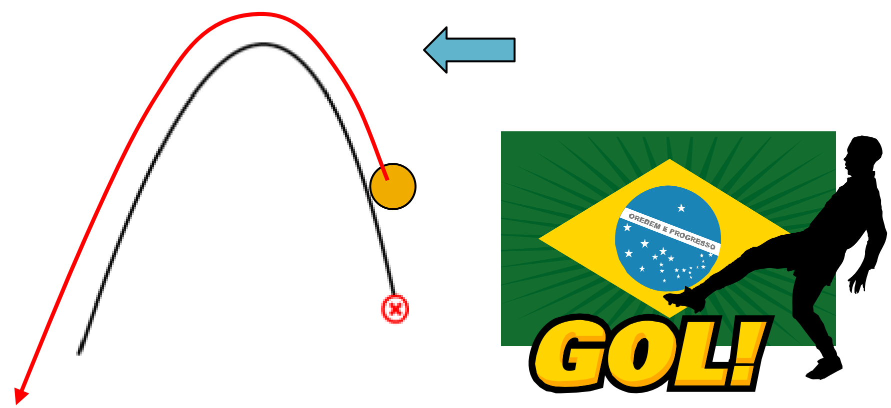
→ 頂上で止めるための初期位置はより右に

物理的な星の解の重力ポテンシャルには上限が存在する

- Fall into the singularity



- Overshoot



So close to danger!

Frolov (2008)

- Simple estimate
  - Poisson eq.:

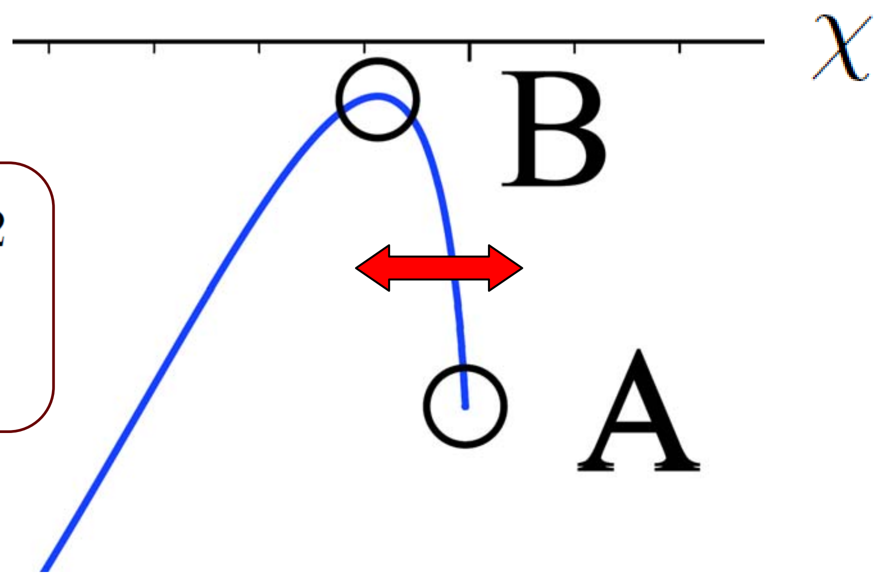
$$\nabla^2 \Phi \sim G\rho$$

Excitation around matter distribution:

$$\delta\chi \sim \mathcal{O}(\Phi)$$

  - Scalar field EOM:

$$\nabla^2 \chi \sim G\rho$$



Typical model with $n = 1\,,\, \lambda = 2$

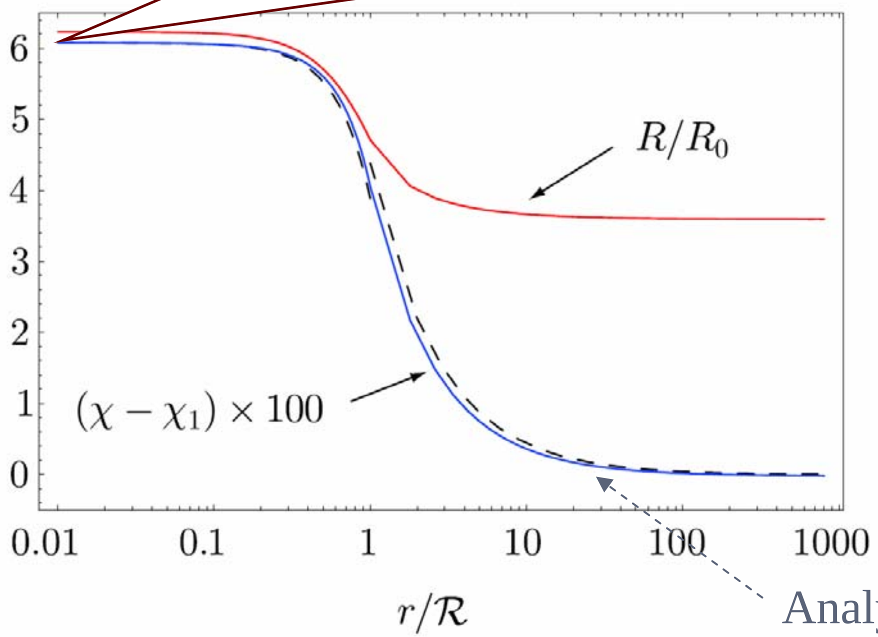
→ ⟷ **~ 0.077 !**

Numerical results 1: Nonrelativistic stars

- Model parameters: $n = 1\,, \lambda = 2.088$
- Look for nonrelativistic stellar configurations: $\begin{cases} 4\pi G\rho_0 = 10^6 \Lambda_{\rm eff} \\ p_c = 5 \times 10^{-2} \rho_0 \end{cases}$

Fine-tuned initial condition: $\chi_c = 0.9836$

$\dfrac{\hat{G}M}{\mathcal{R}} \simeq 0.06687$



Analytic approximation

Asymptotically de Sitter solution

$R \to x_1 R_0$ and $\chi \to \chi_1$

as $r \to \infty$

Numerical results 1: Nonrelativistic stars

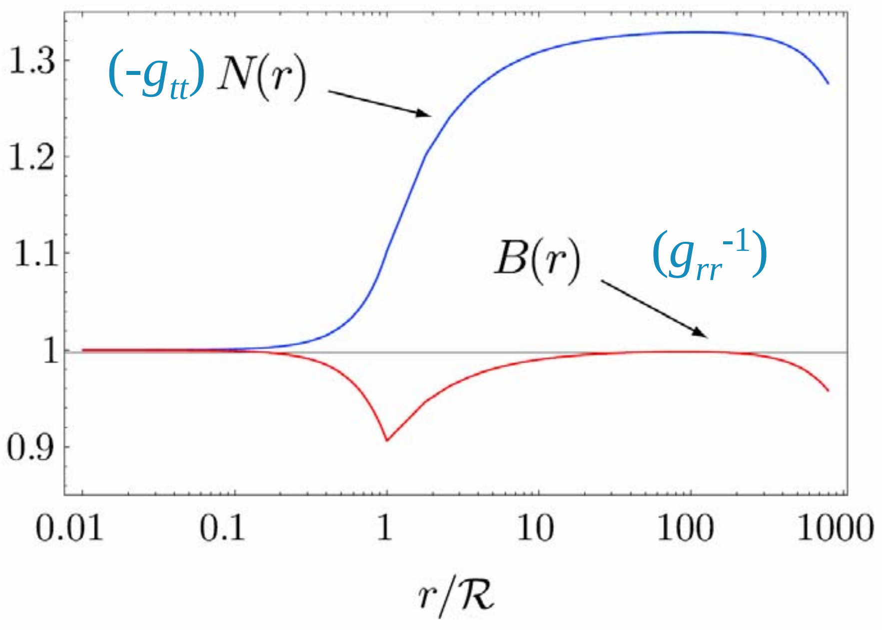
- Metric
  - Numerical fitting:

$$N \simeq N_\infty \left(1 - 2c_1 \frac{\mathcal{R}}{r} - \frac{c_2}{3}\Lambda_{\rm eff} r^2\right),$$

$$B \simeq 1 - 2c_3 \frac{\mathcal{R}}{r} - \frac{c_4}{3}\Lambda_{\rm eff} r^2,$$

with

$$N_\infty = 1.332,\ c_1 = 0.08716,\ c_2 = 0.9973,\ c_3 = 0.04747,\ c_4 = 0.9993.$$



Numerical results 2: (Absence of) Relativistic stars

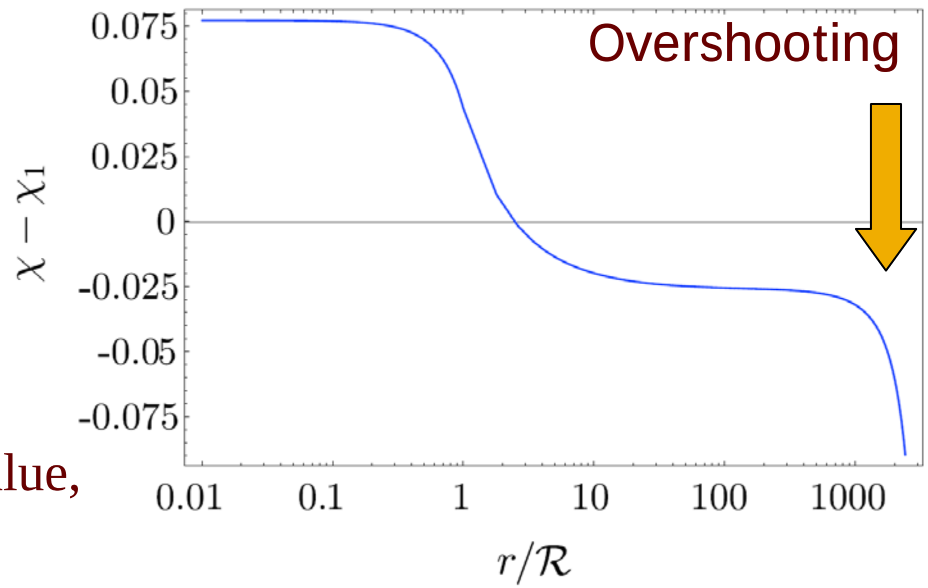
- Try to construct relativistic stellar configurations:

$$4\pi G\rho_0 = 10^6 \Lambda_{\rm eff} \qquad p_c = 0.1 \times \rho_0$$

- "Rolling-down" solution for $R_c = 0.7000 \times 8\pi G\rho_0$
  - regular inside the star

$$\frac{\hat{G}M}{\mathcal{R}} \simeq 0.1203$$

  - but unphysical exterior



- The Ricci scalar rapidly diverges inside the star for a slightly larger value,

$$R_c = 0.7001 \times 8\pi G\rho_0$$

Conclusions

- *f*(*R*) 重力 – 宇宙の加速膨張を説明可能
  - 宇宙論的制限、弱重力場テスト(太陽系 etc.)のすべてをパスするモデル
  - 強重力場(中性子星の解) ……

- 重力ポテンシャルに最大値が存在 (~ 0.1) ➔ 中性子星が存在できない

f(*R*)理論は強重力場の性質により否定される

- 現在生き残っているすべての(長距離で重力を変更する)*f*(*R*)モデルに普遍的な問題
  - 有効ポテンシャルの形で判断可能
- High energy correctionによってこの問題はどのような影響を受けるか？