Constraint on neutrino masses with nonlinear galaxy power spectrum

Phys. Rev. Lett 100, 191301 (2008)
full paper in preparation

The 8-th RESCEU-DENET joint meeting

August 30 2008

Shun Saito

The University of Tokyo, Theoretical Astrophysics Group

collaborate with M. Takada (IPMU) & A. Taruya (RESCEU)

浅虫温泉について

＊ 876年　慈覚大使（円仁）により発見

布を織るために麻を蒸したことから、麻蒸と呼ばれる

＊ 1190年 円光大使（法然）が、

青鹿が傷を癒すのにつかっているのを見て、
村人に入浴をすすめる

＊現在の「浅虫」は、

火事が多いことから、火にゆかりのある文字を嫌って転じた

じゃあ、「浅」は？

古文書には、「朝蒸」「浅蒸」なども登場するらしい
-麻でできた衣服の色が浅いから？
-麻は丈が伸びるのが早く、朝起きると伸びているから？

Neutrino mass from cosmology

Cosmology provides stringent constraints on total neutrino masses

1. Distance test

WMAP5 only	$\sum m_\nu \lesssim 1.3\ \mathrm{eV}$	Komatsu et al (2008)
WMAP5 + BAO + SN	$\sum m_\nu \lesssim 0.6\ \mathrm{eV}$	

This is trustworthy bound, but cannot expect more strict constraint.

CMB is important to determine the other cosmological parameters.

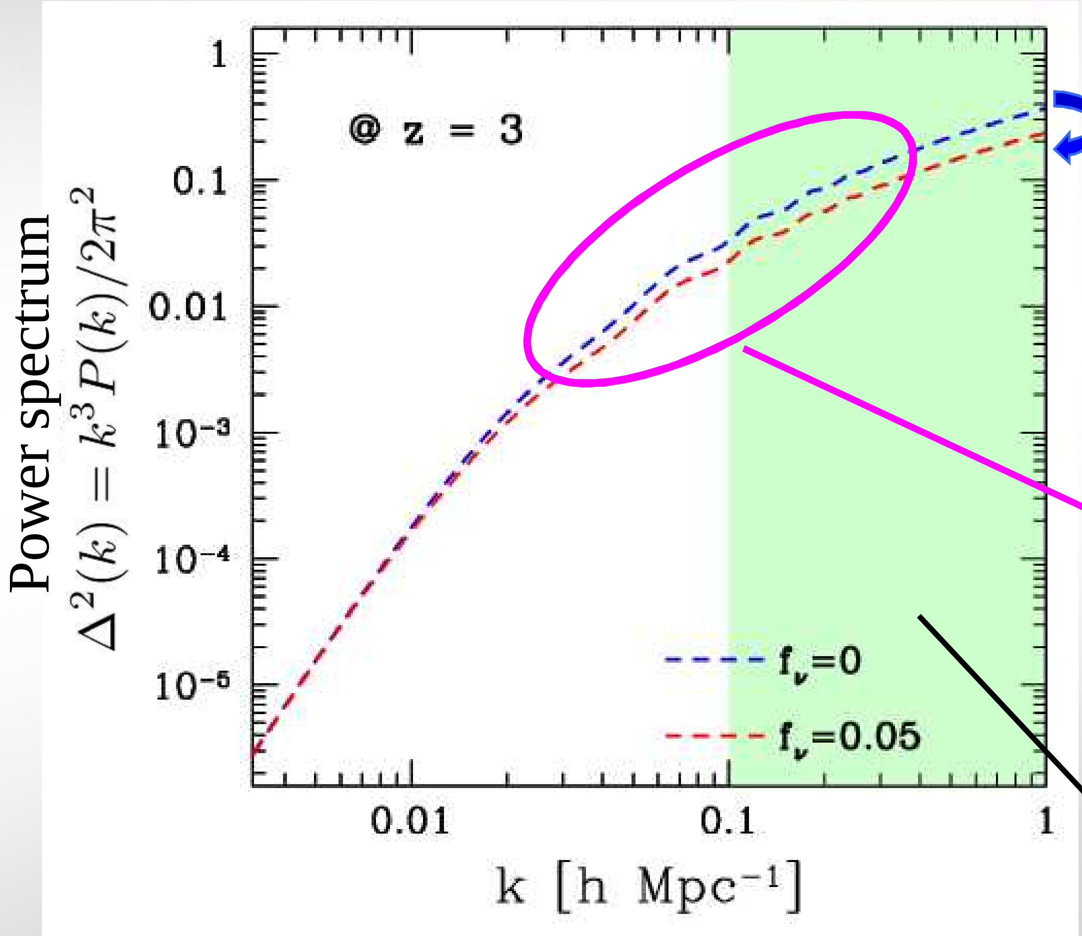
2. Suppression of growth

WMAP3 + SDSS LRG	$\sum m_\nu \lesssim 0.6\ \mathrm{eV}$	Tegmark et al (2006)
WMAP3 + SDSS Lyα	$\sum m_\nu \lesssim 0.2\ \mathrm{eV}$	Seljak et al (2006)

Precision cosmology like CMB will come for next-generation experiment.

Neutrino suppression effect

Neutrino perturbations cannot stay at smaller scale than neutrino free-streaming ➜ weaken gravitational potential



suppression !!

scale-dependent growth

$$\frac{P(k)_{f_\nu \neq 0}}{P(k)_{f_\nu = 0}} - 1 \approx -8 f_\nu \geq 4\%$$

Baryon Acoustic Oscillations (BAOs)

calculated by **Linear** ~~theory~~

Non-linear regime !!

Future galaxy redshift survey

- Many galaxy redshift surveys, e.g. **WFMOS**, HETDEX, are proposed for measuring BAOs (~100Mpc) to probe the nature of dark energy.

- BAO scale is comparable to sub-eV neutrino free-streaming scale.
  Moreover, neutrino suppression effect cannot be neglected.
  This is a good chance to constrain or *determine* the neutrino masses!

- BAO scale is in weakly nonlinear regime ($k < 0.5$ hMpc^-1).
  - Nonlinearity is being understood theoretically.
    - standard perturbation theory — **Makino et al (1992), Nishimichi et al (2007)**
    - renormalized PT — **Crocce & Scoccimarro (2006) Matsubara (2008) Taruya & Hiramatsu (2008)**
    - N-body simulation — **Jeong, Komatsu (2006), Takahashi et al (2008)**
  - All these studies are based on only CDM cosmology without neutrinos.

- Note that other probes of P(k), e.g. weak lensing & Lyα, suffer from more strong nonlinearity ($k \sim 1$ hMpc^-1).

Our Work

巣For the cosmology with CDM & massive neutrinos, we carefully develop the approach to calculate the nonlinear matter power spectrum based on cosmological perturbation theory.
- Check our approximations for simple calculation.

巣Using our refined nonlinear theory, we demonstrate how well neutrino masses are constrained for WFMOS-like survey.
- But assuming linear galaxy biasing and linear redshift distortion

巣For the consistent treatment, we include nonlinear galaxy biasing and nonlinear redshift distortion.
- Discuss the neutrino effect on biasing & redshift-distortion.

Methodology

Perturbation Theory : natural extension of linear theory

multi-fluid component of baryon + mixed dark matter (CDM + Neutrinos)

$$\delta_{\rm m} = f_{\rm cb}\delta_{\rm cb} + f_\nu \delta_\nu \quad \left[f_{\rm cb} \equiv \frac{\Omega_{\rm c}+\Omega_{\rm b}}{\Omega_{\rm m}},\ f_\nu \equiv \frac{\Omega_\nu}{\Omega_{\rm m}} = \frac{\sum m_\nu}{94.1\Omega_{\rm m}h^2} \lesssim 0.05 \right]$$

Power spectrum

$$P(k) = \langle \delta_{\rm m}\delta_{\rm m} \rangle = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

⇨ Perturbative expansion of nonlinear Continuity & Euler equations

∗ Contrasted to only CDM case, some difficulties are involved:

① Nonlinear growth functions are also scale-dependent, which complicates the calculation of nonlinear correction.

② Neutrinos cannot be treated as fluid-component.

① One-loop correction for Pcb

calculate next-to-leading order correction for Pcb(k)

From standard perturbation theory **Makino,Sasaki,Suto (1992)**

$$P_{\rm cb}^{\rm Approx}(k) = P_{\rm cb}^{L} + P_{\rm cb}^{(22)} + P_{\rm cb}^{(13)}$$

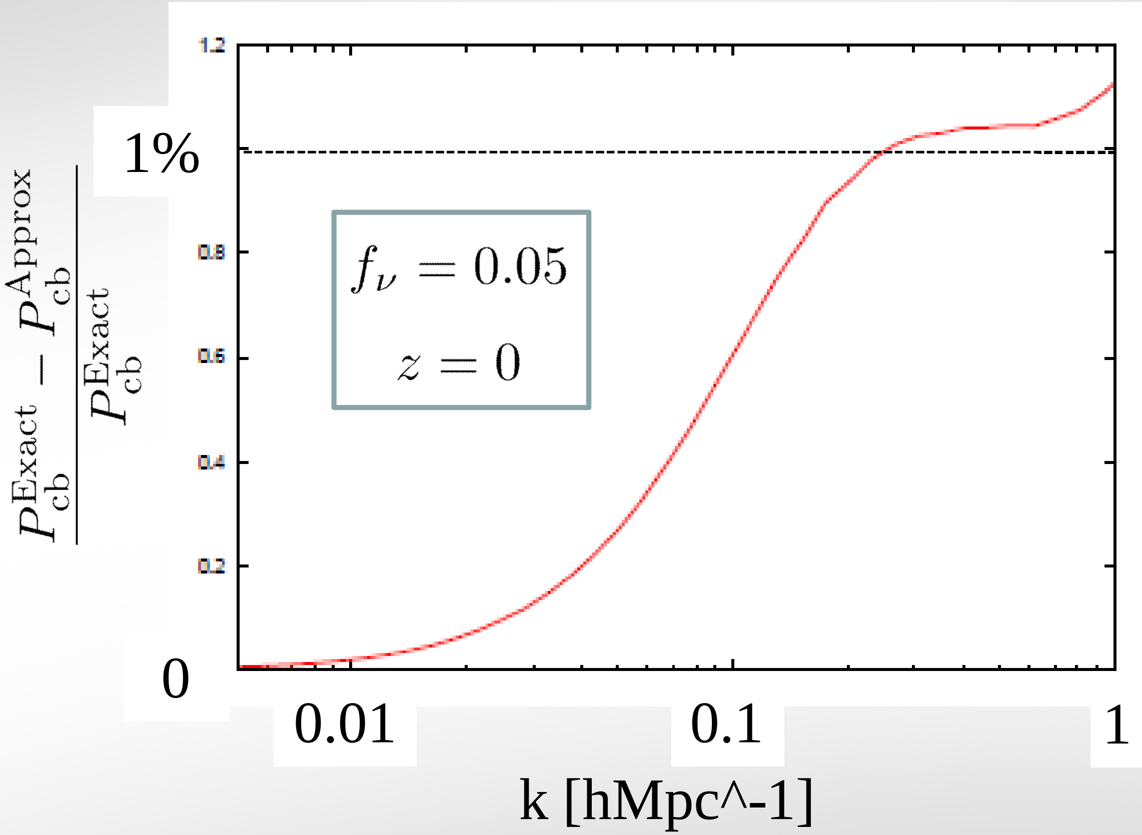
$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z)\int_{-1}^{1} d\mu P_{\rm cb}^L(k\sqrt{1+r^2-2\mu r};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}$$

$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3}{252(2\pi)^2}P_{\rm cb}^L(kr;z)\int_0^\infty dr P_{\rm cb}^L(kr;z)$$
$$\times\left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right]$$

However, this is an approximation in the sense that scale-dependency of growth functions are neglected.

① One-loop correction for Pcb

difference between exact and approximated $P_{cb}(k) = P^{L}_{cb} + P^{(22)}_{cb} + P^{(13)}_{cb}$



Numerically

solve the 2nd & 3rd order equations

⇩

integrate mode-coupling

⇩

$P^{\rm Exact}_{cb}(k)$

The fractional difference is less than ~ 1%.
Because only nearby mode-coupling contributes to one-loop integration.

② Neutrino fluctuations

- Neutrino perturbations cannot be treated as fluid.
  We have to solve the Vlasov (collisionless Boltzmann) equation.

- The dynamics of neutrino perturbations are controlled by Newton potential, which is supported by CDM + baryon.
  (c.f.) dynamics of CDM + baryon are controlled by CDM + baryon itself
  ➜ causes nonlinearity

- For smaller scales, neutrinos cannot stay due to free-streaming.
  Tiny contributions from neutrinos perturbation to total P(k) for $f_\nu \lesssim 0.05$

$$P(k) = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

We assume **neutrino perturbations stay at linear level** and add nonlinear corrections **only for Pcb** term.

② Neutrino fluctuations

巣 Is it a good approximation that Pν(k) is calculate from linear theory?

巣 linear Vlasov equations **Ma & Bertschinger (1995)**

$$\dot{\Psi}_0 = -\frac{qk}{a\epsilon}\Psi_1 + H\phi\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3a\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3aq}\phi\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_\ell = \frac{qk}{(2\ell+1)a\epsilon}[\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}] \quad (\ell \geq 2)$$

Newton potential

q : 3-momentum
ε : proper energy
Ψ : fluctuated distribution

$$f(x^i, q_j/a, t) = f_0(q)[1 + \Psi(x^i, q, n_j, t)]$$

$$\Rightarrow \quad \delta_\nu^{(1)} = \frac{4\pi}{a^4\rho_m}\int q^2 dq\, \epsilon f_0(q)\Psi_0 \quad \Rightarrow \quad P_\nu(k) = \langle \delta_\nu^{(1)}\delta_\nu^{(1)} \rangle$$

巣 Even if nonlinear CDM + baryon fluctuations are included in Newton potential, less than 0.01% change of Pm(k).

Recipe for nonlinear matter P(k)

- We can develop the theory to calculate the nonlinear P(k) with massive neutrinos having mass of ~0.1eV.

- Next-to-leading order correction, one-loop correction, is included.

Recipe to calculate the nonlinear P(k;z)

① calculate **linear** power spectra $P^L_{\rm cb}(k;z),\ P^L_{{\rm cb}\nu}(k;z),\ P^L_{\nu}(k;z)$ for redshift z from CAMB or CMBFAST

② add **one-loop correction** for CDM + baryon term

$$P^{\rm Approx}_{\rm cb}(k) = P^L_{\rm cb} + P^{(22)}_{\rm cb} + P^{(13)}_{\rm cb}$$

③ sum up all components $P(k) = f^2_{\rm cb} P_{\rm cb} + 2 f_{\rm cb} f_\nu P^L_{{\rm cb},\nu} + f^2_\nu P^L_\nu$

Notes on nonlinear P(k)

- For smaller neutrino masses, our PT results become better approximation.

- From our PT, the limitation of linear theory can be known.
  Meanwhile, the validity of our PT cannot be provided, which is derived from comparison with N-body simulations with neutrinos.

- One-loop corrections are roughly proportional to linear P(k).

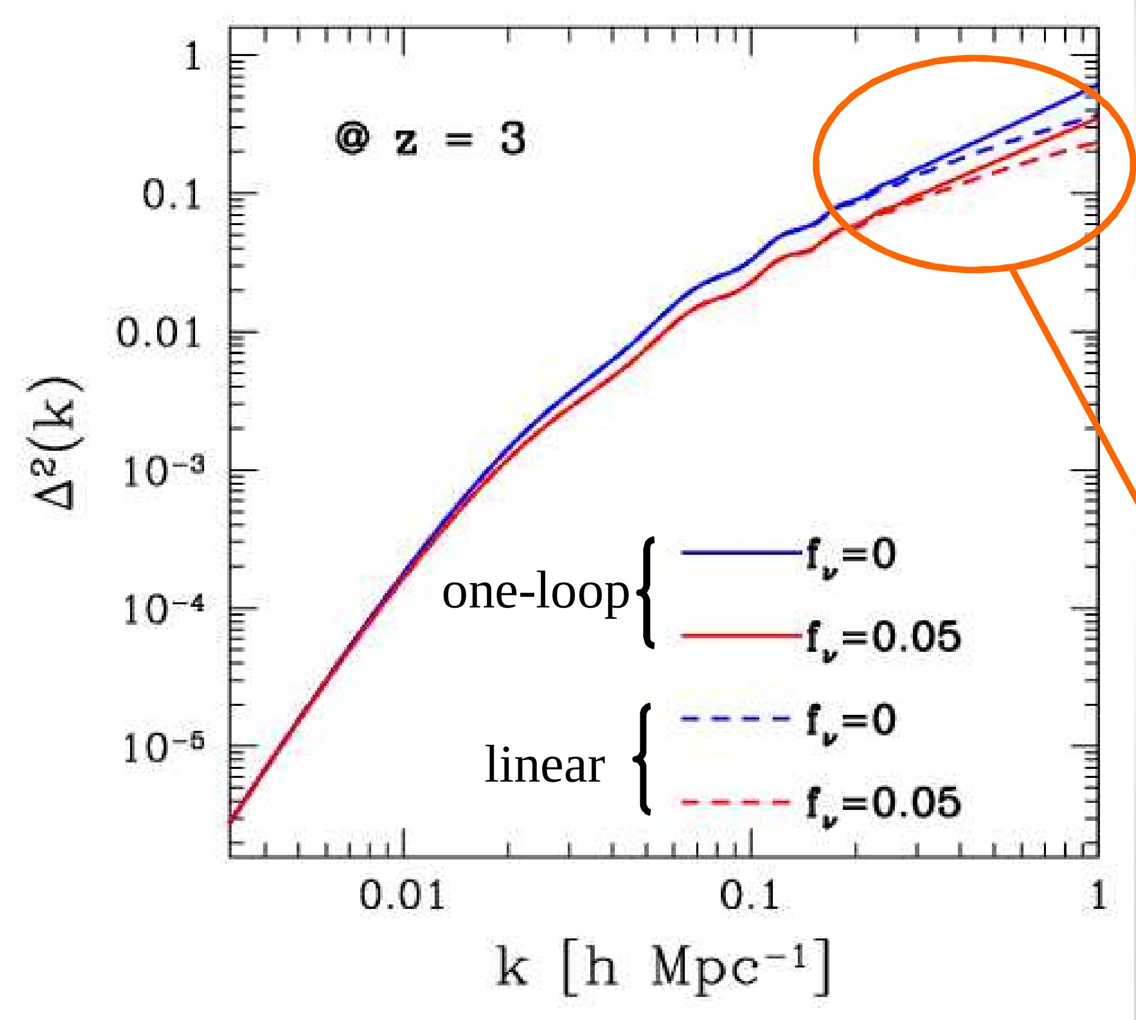
$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr\,P_{\rm cb}^L(kr;z)\int_{-1}^1 d\mu\,P_{\rm cb}^L(k\sqrt{1+r^2-2\mu r};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}$$

$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3}{252(2\pi)^2}P_{\rm cb}^L(kr;z)\int_0^\infty dr\,P_{\rm cb}^L(kr;z)\times\left[\frac{12}{r^2}-158+100r^2-42r^4+\frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right]$$

➜ neutrino suppression effect is expected to be enhanced in weakly nonlinear regime!

Nonlinear P(k)

dimensionless power $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$



fiducial cosmology

$\Omega_b h^2 = 0.0223$

$\Omega_m h^2 = 0.1277$

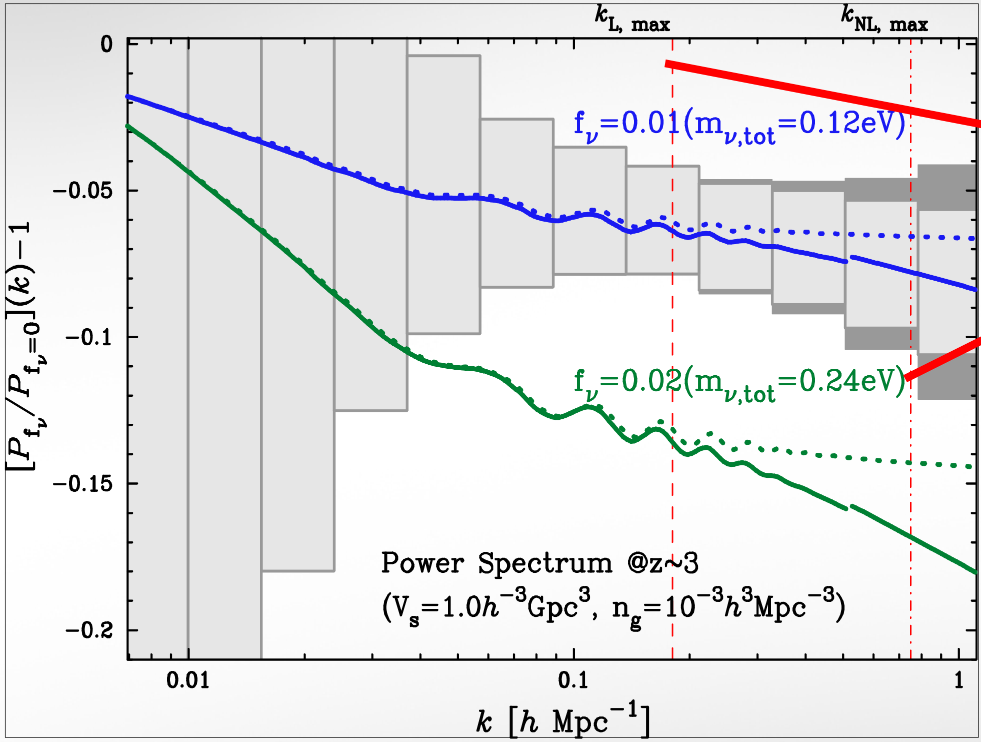
$h = 0.73, \; w_0 = -1$

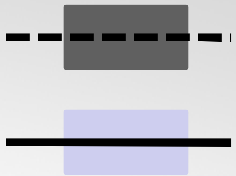
$\Delta^2_{\mathcal{R}} = 2.35 \times 10^{-9}, \; n_S = 1$

The amplitude is enhanced by non-linear gravitational evolution.

Neutrino suppression effect

$P_{f_\nu \neq 0}/P_{f_\nu = 0} - 1$





Linear

Perturbation Theory

Limitation of linear theory

applicapable range of PT？

$k_{\rm NLmax}$: 0.69 hMpc^-1

$\Delta^2 = k^3 P^{NL}_{f_\nu=0}(k)/2\pi^2 < 0.4$

Jeong,Komatsu (2006)

True criterion should be derived from simulations with neutrinos

Neutrino suppression effect is enhanced in weakly nonlinear regime.

The larger amplitude leads to less shot noise error.

Forecast

- 箂How well our PT improve the constraint on neutrino masses for future galaxy redshift survey?

- 箂Fisher information formalism

  Seo & Eisenstein (2003)
  Takada, Komatsu , Futamase (2006)

$$P_{\rm g}(k,\mu) = b_1^2[1+\beta\mu^2]^2 P_{\rm m}^{\rm NL}(k) + P_{\rm shot}$$

- assumption: linear bias, linear redshift distortion, Nν = 3

- CMB prior : Planck

- 23 free parameters (5 z slices):

$$\mathbf{p} = (\Omega_{\rm b}h^2, \Omega_{\rm c}h^2, \Omega_m, \Delta^2_{\mathcal{R}}, n_S, \alpha, w_0, f_\nu, b_1(z_i), \beta(z_i), P_{\rm shot}(z_i))$$

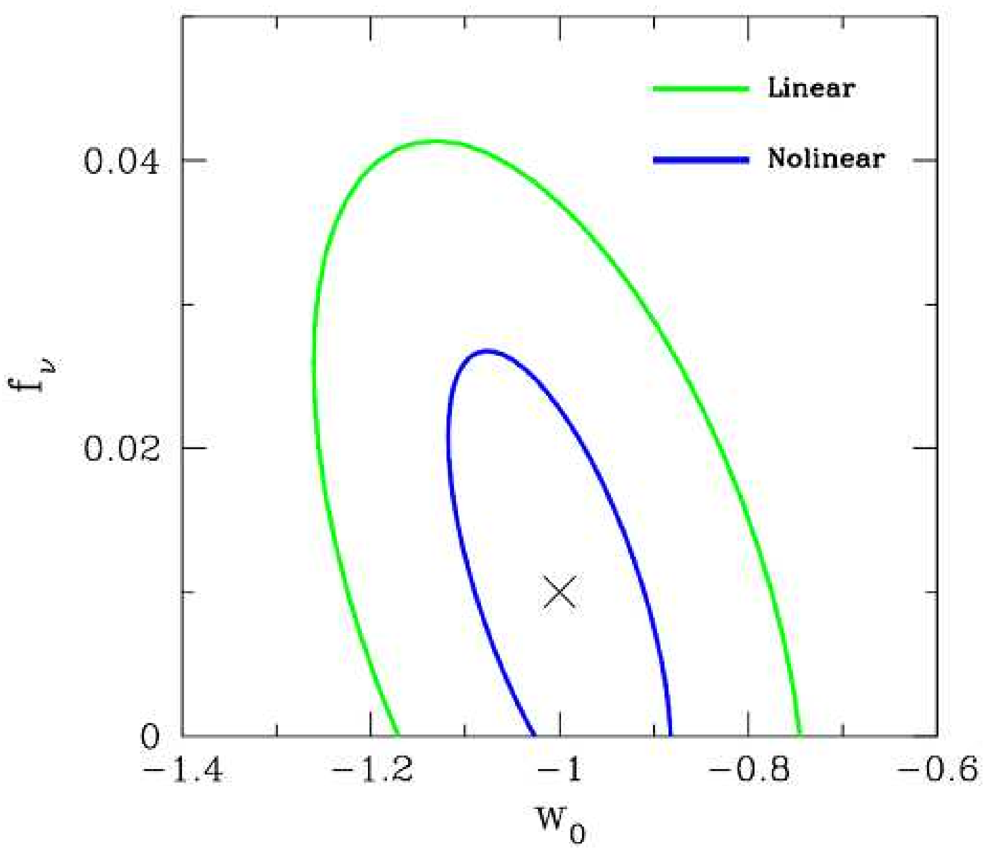
- fiducial parameter: $f_\nu = 0.01, \iff \sum m_\nu = 0.12{\rm eV}$

- survey parameters: Wide-field Fiber-fed Multi-Object Spectroscopy

  z~1 2000deg^2, z~3 300deg^2

Neutrino mass constraint

2D marginalized error between w_0 & f_nu



1σ marginalized error

Linear	$\sigma\left(\sum m_\nu\right) = 0.248\ \mathrm{eV}$
Nonlinear	$\sigma\left(\sum m_\nu\right) = 0.132\ \mathrm{eV}$

∗ potentially ~ factor of 2 improvement !!

∗ neutrinos does not shift BAOs
- Constraint on w0 is consistent with the result in Seo et al (2003)

Other Nonlinear effects

∗ Galaxy redshift survey suffers from other nonlinear effects !!

- **Galaxy biasing & redshift distortion**

∗ We should include these nonlinear effects at 1-loop PT level.

Galaxy biasing --- McDonald 2006

$$P_{\mathrm{g},\delta\delta}(k) = b_1^2 \left\{ P^{\mathrm{NL}}_{\mathrm{m},\delta\delta}(k) + b_2 P_{b2,\delta\delta}(k) + b_2^2 P_{b22}(k) \right\} + N$$

- reparametrized only by 3 bias parameters, b1, b2 & N.

Redshift distortion --- Scoccimarro 2004 + α

$$P^s_{\mathrm{m}}(k,\mu) = \frac{1}{1+f^2\sigma_v^2 k^2 \mu^2} \left\{ P^{NL}_{\mathrm{m},\delta\delta} + 2f\mu^2 P^{NL}_{\mathrm{m},\delta\theta} + f^2\mu^4 P^{NL}_{\mathrm{m},\theta\theta} \right\}$$

- FOG factor is replaced to Lorentzian type. Jeong & Komatsu 2008

Nonlinear galaxy P(k) in redshift space

∗ We find that the above 2 models can be combined consistently.

Assuming local galaxy bias and no bias of velocity fields, we obtain

$$P_{\mathrm{g},\delta\theta}(k) = b_1 \left\{ P^{\mathrm{NL}}_{\mathrm{m},\delta\theta}(k) + b_2 P_{b2,\delta\theta}(k) \right\}$$

Then, nonlinear galaxy P(k) in s-space is written as

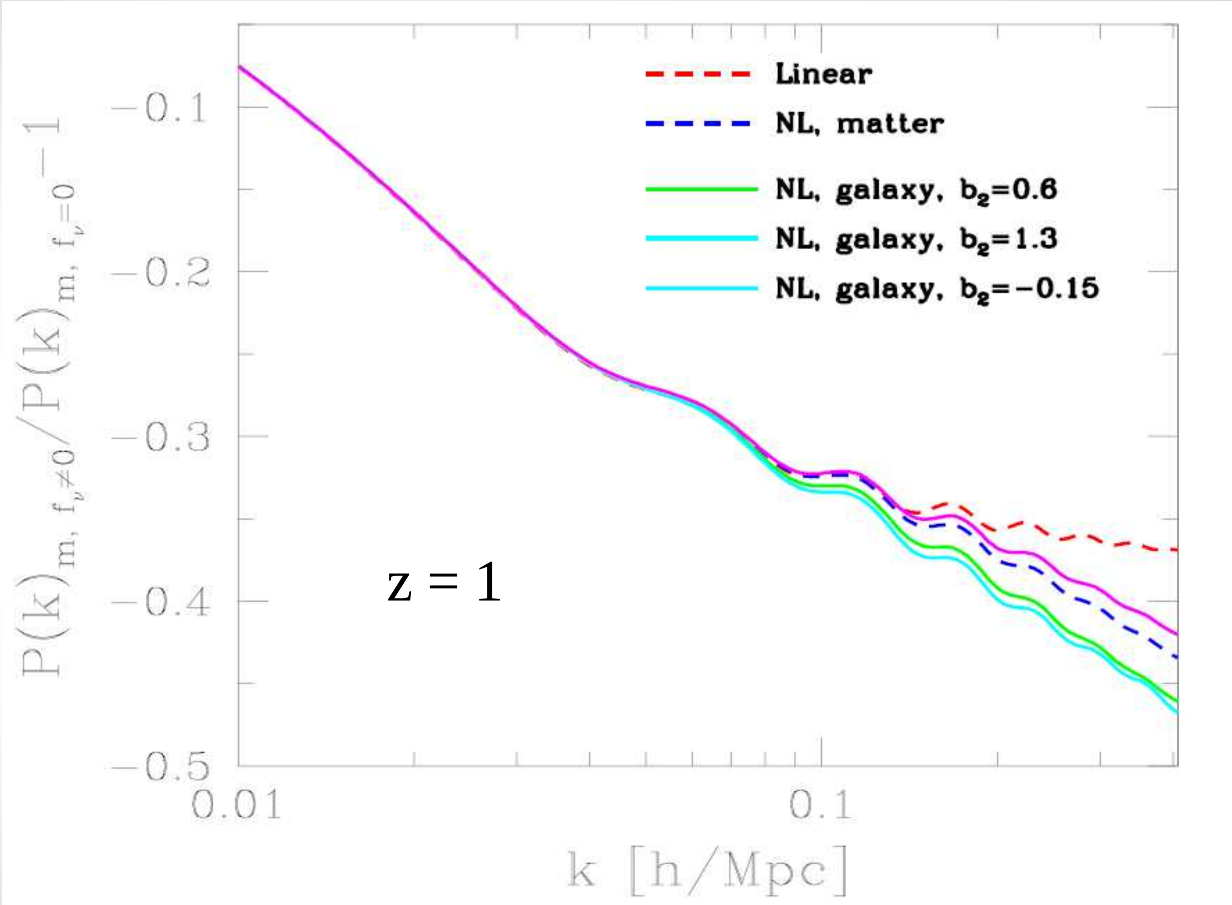
$$P^s_{\mathrm{g}}(k,\mu) = \frac{1}{1+f^2\sigma_v^2 k^2\mu^2} \left\{ P^{NL}_{\mathrm{g},\delta\delta} + 2f\mu^2 P^{NL}_{\mathrm{g},\delta\theta} + f^2\mu^4 P^{NL}_{\mathrm{m},\theta\theta} \right\}$$

- Note that Scoccimarro 2004 model agrees with N-body simulation in ~10% level. More precise model is strongly desired.

- Forecast based on this consistent treatment is in progress!!

Neutrinos vs Nonlinear biasing

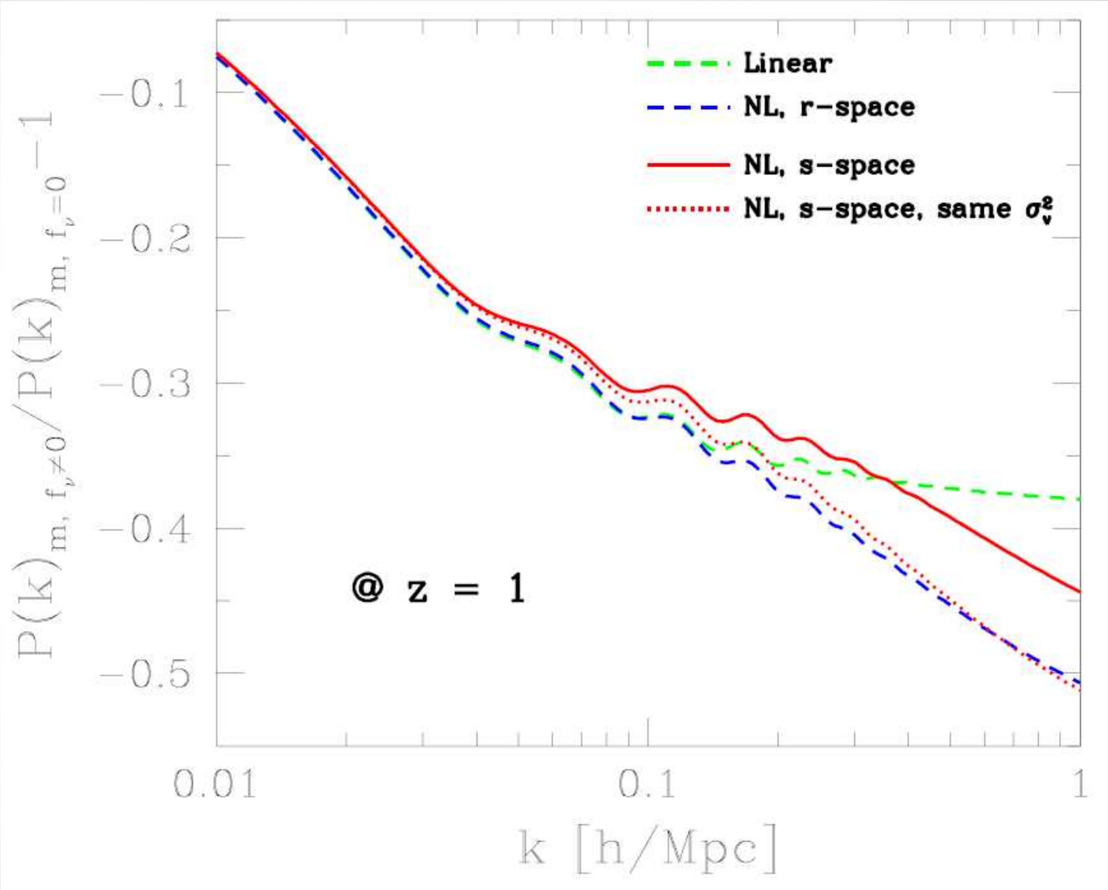
the ratio of nonlinear galaxy power spectra in r-space



∗ neutrino suppression effect strongly depends on the value of b2.

Neutrinos vs Redshift space distortion

the ratio of nonlinear matter power spectra in s-space (monopole)



∗FOG effect smears out the neutrino suppression effect.