RESCEU-DENET summer school 08
@ Asamushi Onsen, Aomori

2008. 8. 31

Non-gaussianity from the bispectrum in general multiple field inflation

Shuntaro Mizuno (RESCEU, Tokyo)

With Frederico Arroja (Portsmouth)
Kazuya Koyama (Portsmouth)

JCAP08(2008)015

Introduction

Observations on non-Gaussianities

deviation from Gaussian distribution

- conventional parametrisation Komatsu and Sperbel 2001

$$\zeta = \zeta_L - \frac{3}{5} f_{NL} \zeta_L^2$$

Curvature perturbations

ζ_L obeys Gaussian statistics

$f_{NL} \sim 0$ for almost free theories like standard inflation

- Constraints on f_{NL} from WMAP 5-year

$$-9 < f_{NL} < 111$$

Komatsu et al. 2008

favoring relatively large non-Gaussianity

⇒ Need to consider the early universe scenarios other than the standard inflationary sceario?

DBI inflation: model

Silverstein and Tong 2004

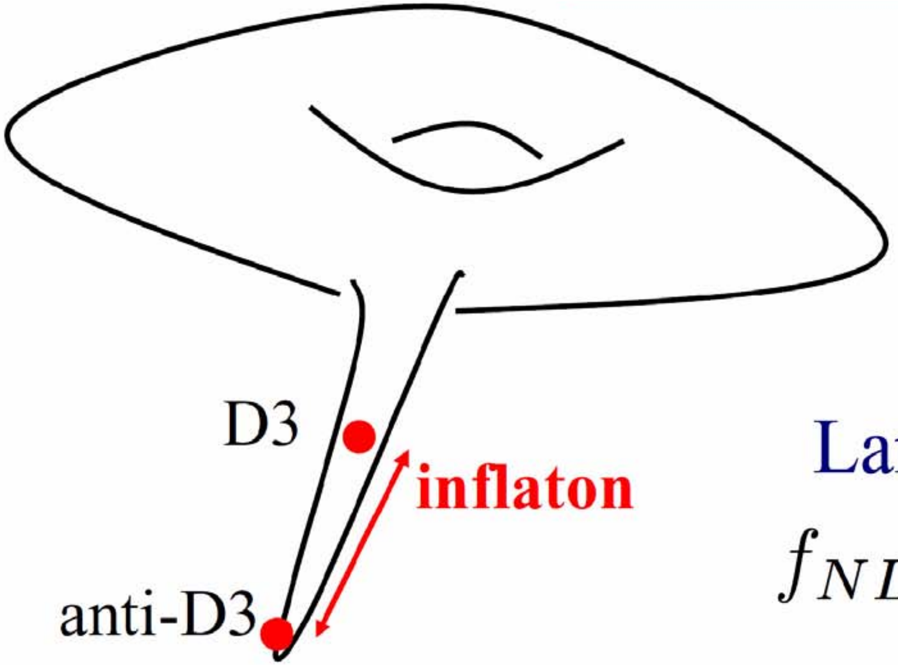
- Set-up

Inflation is driven by a mobile D3-brane with relativistic speed $\gamma = \frac{1}{\sqrt{1-\dot{\phi}^2/T(\phi)}}$

- Action

$$S = \int d^4\xi \sqrt{-g^{(-4)}}\left[-T(\phi)\sqrt{1+\partial^\mu\phi\partial_\mu\phi/T(\phi)} + T(\phi) - V(\phi)\right]$$

DBI part



$d\phi = T_3^{1/2} d\rho \quad T(\phi) = T_3^{1/2} h^4$

ρ : radial position of the brane

Large non-gaussianity is possible

$$f_{NL} \simeq \frac{1}{3}\gamma^2 \simeq 1/3c_s^2$$

sound speed

DBI inflation: present status

Baumann and Mcllister 2006, Lidsey and Huston 2007

• DBI inflation with large non-gaussianity seems inconsistent with WMAP data

• It can be consistent only in the limit when it goes back to a standard slow-roll inflation

Consistency relation $r \geq 4(1-n_s)/\sqrt{1+3f_{NL}}$

large but not too large $|f_{NL}|$ ⇒ large r

⇒ large $\Delta\phi/M_{pl}$

i) large $\Delta\phi$ ⇒ long throat ⇒ large V_6

ii) not large M_{pl} ⇒ not large V_6

contradiction!!

• Can the situation be better for multi-field models?

d.o.f corresponding to the angular directions

Multi-field K-inflation

- Action

Langlois and Renaux-Petel (2008)

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + P(X,\phi^I)\right)$$

with $I = 1, \cdots, N$

$$X = -\frac{1}{2}G_{IJ}(\phi)\nabla_\mu\phi^I\nabla^\mu\phi^J$$

field space metric

- Energy-momentum tensor

$$T^{\mu\nu} = Pg^{\mu\nu} + P_{,X}G_{IJ}\nabla^\mu\phi^I\nabla^\nu\phi^J$$

⇒ $\rho = 2XP_{,X} - P, \quad P = P$ Flat FRW

⇒ Sound speed $c_s^2 \equiv \frac{P_{,X}}{\rho_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$

⇓

- Propagation speed of modes

c_s for adiabatic mode

1 for entropy modes

⇒ Effects of entropy perturbations are suppressed for $c_s^2 \ll 1$

Multi-field DBI inflation

Langlois, Renaux-Petel, Steer and Tanaka (2008)

They carefully observed DBI action

- kinetic function

$$\tilde{P}(\tilde{X},\phi^I) = -\frac{1}{f(\phi^I)}\left(\sqrt{1-2f(\phi^I)\tilde{X}}-1\right) - V(\phi^I)$$

with $\tilde{X} = \frac{1-\mathcal{D}}{2f}, \quad \mathcal{D} = \det(\delta^I_J + f\partial^\mu\phi^I\partial_\mu\phi_J)$

X and $\tilde{X}$ differ for the perturbed values in multi-field models

⟺ Multi-field DBI inflation does not belong to K-inflation

⇓

- Propagation speed of modes

All perturbations propagete with the same speed c_s

⇒ no suppression for the entropy modes

General multi-field inflation

SM with Arroja, Koyama (2008)

Need to analyze more general classes of multi-field models including DBI inflation

- Action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + P(X^{IJ}, \phi^I)\right) \qquad X^{IJ} \equiv -\frac{1}{2}\partial^\mu\phi^I\partial_\mu\phi^J$$

ex.) $$\mathcal{D} = 1 - 2fX + 4f^2X_I^{[I}X_J^{J]} - 8f^3X_I^{[I}X_J^JX_K^{K]} + 16f^4X_I^{[I}X_J^JX_K^KX_L^{L]}$$

$$P(X^{IJ}, \phi^I) = \tilde{P}(X, \phi^I)$$ ⟶ K-inflation

$$P(X^{IJ}, \phi^I) = \tilde{P}(\tilde{X}, \phi^I) = -\frac{1}{f}\left(\sqrt{1 - 2f\tilde{X}} - 1\right) - V(\phi^I), \qquad \tilde{X} = \frac{1-\mathcal{D}}{2f}$$

⟶ DBI inflation

Linear perturbations

- Scalar fields on the flat hypersurface

$$\phi^I = \phi_0^I + \underline{Q^I}$$ linear perturbation

- Second order action for Q

$$S_{(2)} = \frac{1}{2}\int dt d^3x a^3 \Big[\underline{(P_{,X^{IJ}} + P_{,X^{IK}X^{JL}}\dot{\phi}_0^K\dot{\phi}_0^L)\dot{Q}^I\dot{Q}^J}$$
$$\underline{-\frac{1}{a^2}P_{,X^{IJ}}\partial_i Q^I \partial^i Q^J} - \mathcal{M}_{IJ}Q^I Q^J + \mathcal{N}_{IJ}Q^I\dot{Q}^J\Big]$$

To go further, we impose the following assumtion

- Kinetic function

$$P(X^{IJ}, \phi^I) = \tilde{P}(Y, \phi^I)\,, \quad Y = G_{IJ}(\phi)X^{IJ} + \frac{b(\phi)}{2}(X^2 - X_I^J X_J^I)$$

$b = 0$ ⟶ K-inflation

$b = -2f$ ⟶ DBI inflation

Adiabatic and entropy perturbations

- Decomposition of the perturbations

$$Q^I = Q_n e^I_n$$

new basis

$$\begin{cases} e^I_1 \equiv \dfrac{\dot{\phi}^I}{\sqrt{P_{,X^{JK}}\dot{\phi}^J\dot{\phi}^K}} & \text{adiabatic vector} \\ P_{,X^{IJ}} e^I_m e^J_n = \delta_{mn} & \text{orthonormality condition} \end{cases}$$

- Second order action

adiabatic and entropy fields

$$S_{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[K_{mn} D_t Q^m D_t Q^n - \frac{1}{a^2}\delta_{mn}\partial_i Q^m \partial^i Q^n + \cdots\right]$$

$$K_{mn} = \begin{cases} \dfrac{1}{c_{ad}^2} & (m = n = 1) \qquad c_{ad}^2 = \dfrac{\tilde{P}_{,Y}}{\tilde{P}_{,Y} + 2X\tilde{P}_{,YY}} \\ \dfrac{1}{c_{en}^2} & (m = n \neq 1) \qquad c_{en}^2 = 1 + bX \end{cases}$$

In general, propagation speeds are determined independently !!

Leading order three-point function

- Assumptions

i) Coupling between adiabatic mode and entropy mode is negligible (usual quantization is possible)

ii) The following parameters are small (slow-roll)

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}, \quad \chi_{ad} \equiv \frac{\dot{c}_{ad}}{c_{ad} H}, \quad \chi_{en} \equiv \frac{\dot{c}_{en}}{c_{en} H}, \quad \ell \equiv \frac{\dot{\lambda}}{\lambda H}$$

$$(\lambda \equiv 2/3 X^3 \tilde{P}_{,YYY} + X^2 \tilde{P}_{,YY})$$

- Third order action adiabatic and entropy fields

$$S_{(3)} = \int dx^3 dt a^3 \left[\frac{1}{2} \Xi_{nml} \dot{Q}_n \dot{Q}_m \dot{Q}_l - \frac{1}{2a^2} \Xi_{nml} \dot{Q}_n (\partial_i Q_m)(\partial^i Q_l) \right]$$

$$\begin{cases} \Xi_{nml} = (2X\tilde{P}_{,Y})^{-\frac{1}{2}} \left[\frac{(1-c_{ad}^2)}{c_{ad}^2 c_{en}^2} \delta_{1(n}\delta_{ml)} + \left(\frac{4}{3} \frac{X^2 \tilde{P}_{,YYY}}{\tilde{P}_{,Y}} - \frac{(1-c_{ad}^2)(1-c_{en}^2)}{c_{ad}^2 c_{en}^2} \right) \delta_{n1}\delta_{m1}\delta_{l1} \right] \\ \Upsilon_{nml} = (2X\tilde{P}_{,Y})^{-\frac{1}{2}} \left(\frac{1-c_{ad}^2}{c_{ad}^2} \delta_{n1}\delta_{ml} - \frac{2(1-c_{en}^2)}{c_{en}^2} \left(\delta_{n1}\delta_{ml} - \delta_{n(m}\delta_{l)1} \right) \right) \end{cases}$$

Three-point function of the fields

- Definition $\langle\Omega|Q_l(t,k_1)Q_m(t,k_2)Q_n(t,k_3)|\Omega\rangle$ Maldacena (2003)

$$= -i\int_{t_0}^{t} d\tilde{t}\langle 0|\left[Q_l(t,k_1)Q_m(t,k_2)Q_n(t,k_3), H_I(\tilde{t})\right]|0\rangle$$

- mixed component

$\langle\Omega|Q_\sigma(0,k_1)Q_s(0,k_2)Q_s(0,k_3)|\Omega\rangle$ $\tilde{K} = c_{ad}k_1 + c_{en}(k_2+k_3)$

$$= (2\pi)^3\delta^{(3)}(k_1+k_2+k_3)\frac{H^5}{8c_{ad}c_{en}^2}\frac{1}{\Pi_{i=1}^3 k_i^3}\frac{1}{\tilde{K}}\times$$

$$\left[C_2 c_{en}^2 k_3^2 k_1\cdot k_2\left(1+\frac{c_{ad}k_1+c_{en}k_2}{\tilde{K}}+\frac{2c_{ad}c_{en}k_1k_2}{\tilde{K}^2}\right)+(k_2\leftrightarrow k_3)\right.$$

$$\left.+4C_3 c_{ad}^2 c_{en}^4\frac{k_1^2k_2^2k_3^2}{\tilde{K}^2} - 2(C_1+C_2)c_{ad}^2 k_1^2 k_2\cdot k_3\left(1+c_{en}\frac{k_2+k_3}{\tilde{K}}+2c_{en}^2\frac{k_2k_3}{\tilde{K}^2}\right)\right]$$

For DBI inflation, this momentum dependence is same as purely adiabatic component

Non-gaussianities in multi-field DBI inflation

cf.) Langlois, Renax-Petel, Steer and Tanaka (2008)

- curvature perturbation

Wands, Bartolo, Matarrese and Riotto (2002)

$$\mathcal{R} = \mathcal{A}_\sigma Q_{\sigma *} + \mathcal{A}_s Q_{s*}$$

$$\mathcal{A}_\sigma = \left(\frac{H\sqrt{c_s}}{\dot{\sigma}}\right)_*, \quad \mathcal{A}_s = \underline{\mathcal{T}_{\mathcal{RS}}}\left(\frac{H\sqrt{c_s}}{\dot{\sigma}}\right)_*$$

transfer function

$$\Rightarrow \begin{cases} \mathcal{P}_\mathcal{R} = (1+T^2_{\mathcal{RS}})\mathcal{P}_{\mathcal{R}*} \\ \langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3)\rangle = \mathcal{A}^3_\sigma \langle Q_\sigma(k_1)Q_\sigma(k_2)Q_\sigma(k_3)\rangle(1+T^2_{\mathcal{RS}}) \end{cases}$$

- non-linear parameter

$$f_{NL} \propto \frac{1}{c_s^2}\underline{\frac{1}{1+T^2_{\mathcal{RS}}}}$$

good for stringy inflation

Conclusion

- Perturbations in general multi-field inflation model

$$P = P(X^{IJ}, \phi^I) \qquad X^{IJ} \equiv -\frac{1}{2}\partial^\mu\phi^I\partial_\mu\phi^J$$

including multi-field K-inflation, multi-field DBI inflation as special cases

- Generally, the sound speeds for the adiabatic and entropy perturbations are different

- Generally, the momentum dependence of the three point function from the adiabatic and entropy modes are different

- multi-field effect could help to ease the constraints on the stringy DBI-inflation models

$$f_{NL} \propto \frac{1}{c_s^2}\frac{1}{1+T_{\mathcal{RS}}^2}$$

Discussion

- Is there some deep reason for the coincidence in the multi-field DBI inflation models?

  ref) For the four-point function, the momentum dependence from the entropy mode is different from the adiabatic mode

- Reheating mechanism in DBI-inflation

$$T_{\mathcal{RS}}$$

- Relaxing the assumptions

  slow-roll approximation

  coupling between the adiabatic and entropy mode