Massive spin-2 ghost in de Sitter space

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K.I., K. Koyama and T. Tanaka JHEP 0704:053,2007
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K. I. and T. Tanaka in preparation

1, introduction

The present-day accelerated expansion of the Universe is one of the most hot topic in cosmology.

approaches to this problem

- Modification of scalar (spin-0) sector → ・cosmological constant ・quintessence :

example

- <u>Modification of spin-2 sector</u>
  ⇒ Modification of the gravity theory → ・massive gravity theory ・DGP braneworld model :

・Massive gravity theory (Fierz, Pauli, Proc. Roy. Soc., 173A,211 (1939))

$$S = m_{pl}^2 \int d^4x \sqrt{-g}\,(R + \frac{m^2}{4}(h^2 - h^{\mu\nu}h_{\mu\nu}))$$

To explain the accelerated expansion of the universe, $m = H$.
However,
a spin-2 graviton with mass in the range $0 < m^2 < 2H^2$
in the de Sitter background has the ghost excitation
in its helicity-0 component. (Higuchi Nucl. Phy. **B282**, 397 1986)

Since the models which have the quadratic term
in the metric perturbation can be regarded as
the massive gravity theory,
most of the modified gravity models fall in to this category.

Question

① **Can we build a ghost-free model in which modification of the spin-2 sector explains the accelerated cosmic expansion?**

➡ Section 2

② **Is this ghost really harmful?**

➡ Section 3

2-1, Dvali-Gabadadze-Porrati braneworld model

- action

(Phys. Lett. **B485**, 208 (2000))

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,R + \int d^4x\sqrt{-g^{(4)}}\left(\frac{1}{2\kappa_4^2}R^{(4)} + \frac{1}{\kappa^2}K + L_m\right)$$

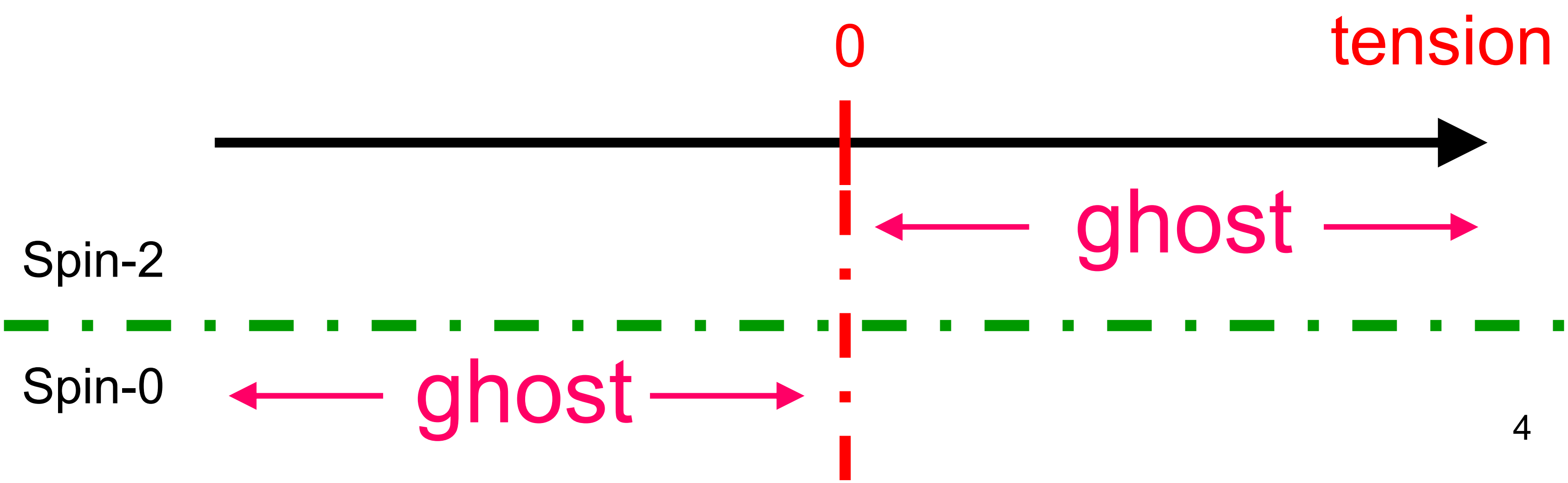
- ghost in self-acceleration branch

(Koyama, Phys. Rev. **D72**, 123511 (2005))
(Gorbunov, Koyama and Sibiryakov, Phys. Rev. **D73,** 044016 (2006))

For positive tension, the lowest mass of KK graviton satisfies $m^2 < 2H^2$.
A helicity-0 excitation of graviton is a ghost.

For negative tension, a spin-0 mode is a ghost
which is a brane bending mode.

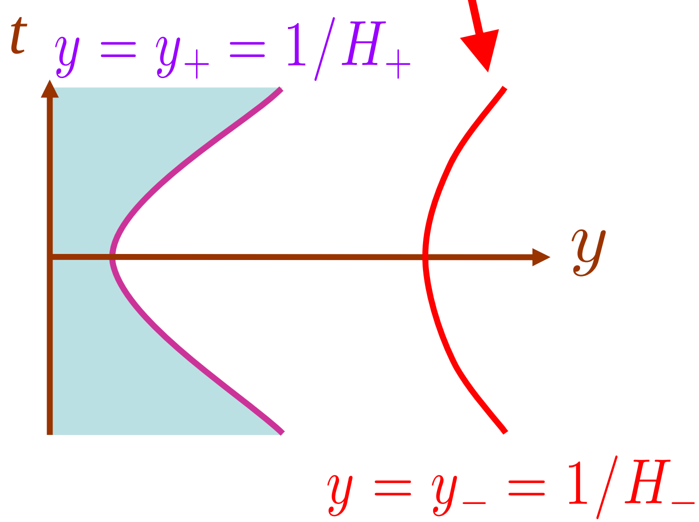
For tensionless, the lowest mass of the KK gravitons conforms to $m^2 = 2H^2$
Though this is marginal case, there is ghost excitation.



2-2, Two-branes model

(K.I., K. Koyama and T. Tanaka 2007)

We put another brane in the bulk in order to make the KK mass increase.



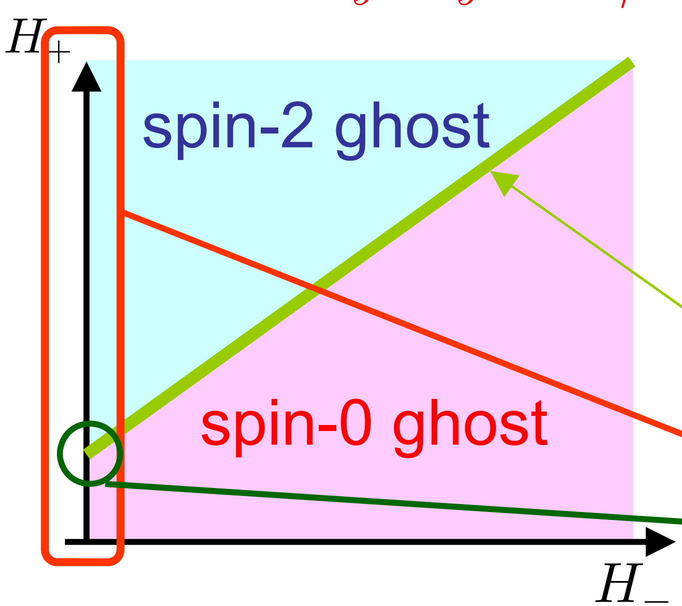
Bulk is Rindler wedge of Minkowski space

$$ds^2 = dy^2 + y^2 ds^2_{4D-deSitter}$$

$$H_\pm = \pm \frac{1}{2r_c} + \sqrt{\frac{\kappa^2}{3}\sigma_\pm + \left(\frac{1}{2r_c}\right)^2}$$

$\sigma_\pm$ is tension of $\pm$- branch brane

We can make all masses of spin-2 satisfy $m^2 > 2H^2$. But then the spin-0 mode, which is radion mode, becomes a ghost.



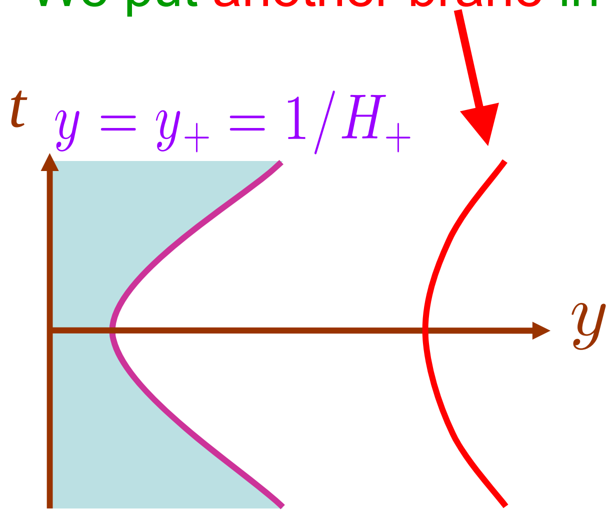
$H_+ = H_- + r_c^{-1} \quad (\sigma_+ = \sigma_-)$

original DGP braneworld model limit (one brane limit)

self-acceleration branch without tension

2-3,Two-branes model with stabilization

We put another brane in the bulk in order to make the KK mass increase.



In order to stabilize this radion mode,
we introduce the bulk scalar field.

Stabilization of brane distance
by a bulk scalar field .
(Goldberger-Wise mechanism)

When spin-2 mode has no ghost excitation,
spin-0 mode, which is derived from bulk scalar field, becomes ghost.

Ghost cannot be removed in this mechanism.

It is difficult ,or maybe impossible, to remove the ghost
by continuous transformation of DGP braneworld model.

2-4, Why the ghost property transfer?

Usually spin-2 mode and spin-0 mode are completely decoupled.

However, when a scalar function f satisfies $(\Box^{(4)} + 4H^2)f = 0$,
a tensor which is constructed by f ; $(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)})f$;
satisfies a transverse-traceless condition.

and

$$(\Box^{(4)} - (\mu^2 + 4H^2))(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)})f = (\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)})(\Box^{(4)} - (\mu^2 - 4H^2))f$$

$= m^2_{s=2} + 2H^2$

$= m^2_{S=0}$

$$m^2_{S=2} = 2H^2$$ degenerate $$m^2_{S=0} = -4H^2$$

At the critical mass, which corresponds to the mass of the boundary of the ghost condition, spin-0 mode and spin-2 mode degenerate.

3-1, Is ghost harmful?

Spontaneous pair production of ghost and usual particles

➡ Vacuum becomes unstable.

In flat background, since, due to the Lorentz symmetry,
the coupling is the same value in any flame
instantaneously the particle production is infinity.

However, in de Sitter background of the massive gravity theory
coupling of helicity-0 mode of graviton depend on the 3D momentum.

Let’s estimate the particle production in the de Sitter background
of the massive gravity theory.

3-2, Simple model: conformal coupling scalar field

Action for spin 2, helicity-0 mode $S = \sum_k \int \frac{M_{4pl}^2 m^2(m^2-2H^2)}{k^4\eta^2} h_k^{(0)} \left(\Box_{flat} + \frac{2}{\eta} - \frac{m^2}{(H\eta)^2}\right) h_k^{(0)}\, d\eta$

For $m^2 < 2H^2$, the signature of the action flips.

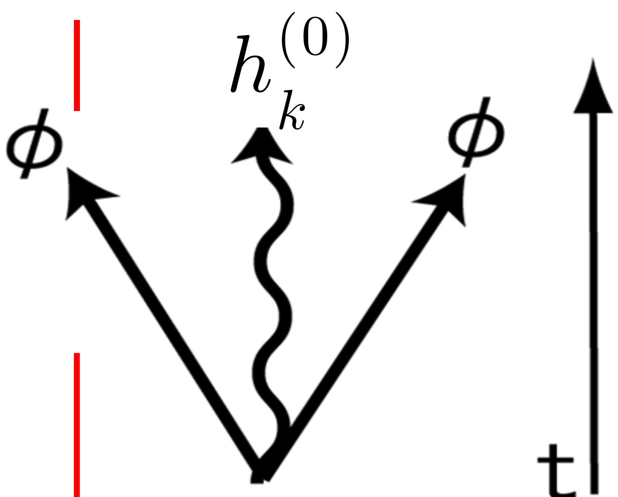
For large *k*, $< s_k^2 >$ becomes large ⇒ strong coupling

Action for a conformally coupled scalar field and interaction term

$$S_\phi = \sum_k \int \frac{\phi_k}{2\eta} \Box_{flat} \frac{\phi_k}{\eta} d\eta \quad S_{int} = -\frac{1}{2}\int d^4x \sqrt{-g}\, h^{\mu\nu} T_{\mu\nu}$$

Particle creation in de Sitter space

(K. I. and T. Tanaka in preparation)



total energy density of created scalar particles

$$\rho \approx \frac{H^6\eta_f^3}{M_{4pl}^2}\Lambda^3$$

Λ : 3D cutoff scale

Result is divergent, but

if we set $\Lambda = \frac{M_{4pl}}{H\eta_f}$ ⇒ $\rho \approx H^3 M_{4pl} \ll \rho_{crit}$

Strong coupling scale $\Lambda = (M_{4pl}m^n)^{1/n+1}/H\eta_f$ is much lower.

If we set cutoff at this energy scale,

particle creation is extremely suppressed.

3-3, de-Sitter symmetry breaking in vacuum

(K. I. and T. Tanaka 2007)

Simple example of symmetry breaking

$$S=\int dt(\partial_t\phi_1)^2-(\partial_t\psi_1)^2 \quad\rightarrow\quad S=\int dt(\partial_t\phi_2)^2-(\partial_t\psi_2)^2$$

Ghost?? Ghost??

$$\phi_1\rightarrow\phi_2=\phi_1\cosh\theta+\psi_1\sinh\theta$$
$$\psi_1\rightarrow\psi_2=\psi_1\cosh\theta+\phi_1\sinh\theta$$

$$\phi_1=\int d\omega(a_{\phi,1}\exp(-i\omega t)+a^\dagger_{\phi,1}\exp(i\omega t))$$
$$\psi_1=\int d\omega(a_{\psi,1}\exp(i\omega t)+a^\dagger_{\psi,1}\exp(-i\omega t))$$

$$\phi_2=\int d\omega(a_{\phi,2}\exp(-i\omega t)+a^\dagger_{\phi,2}\exp(i\omega t))$$
$$\psi_2=\int d\omega(a_{\psi,2}\exp(i\omega t)+a^\dagger_{\psi,2}\exp(-i\omega t))$$

"1"-vacuum state

$$a_{\phi,1}\mid 0>_1=0$$
$$a_{\psi,1}\mid 0>_1=0$$

$$a_{\phi,2}=a_{\phi,1}\cosh\theta+a^\dagger_{\psi,1}\sinh\theta$$
$$\downarrow$$
$$a_{\phi,2}|0\rangle_1=a^\dagger_{\psi,1}\sinh(\theta)\,|0\rangle_1\neq 0$$

Symmetry breaking !!

In the de-Sitter background of the massive gravity theory, helicity-0 mode of graviton is ghost.
Since the different helicities can be mixed by the rotation of the de Sitter group, the helicity-0 is different from the one of the different frame.

There are no de-Sitter symmetry vacuum.

3-4, Spin-0 ghost

- Two-branes model

Spin-2 ⟷ Spin-0

No de Sitter inv. vacuum

$m^2 = -4H^2$

When $m^2 \leq 0$,
there is no de Sitter inv. vacuum
(Allen and Folacci (1987))

- Equivalence theorem

In high energy limit,
Helicity-0 of Spin-2 → Spin-0 ??

⬇

This theorem is not applied
because the meaning of ”high energy limit” is that
the relative momentum to other external line is large.

4, Summary

①To built the ghost-free theory in which spin-2 sector accelerate the expansion of universe, we improve the DGP braneworld model

Two-branes model

If $H_+ > H_- + r_c^{-1}$, helicity-0 of spin-2 becomes ghost.
If $H_+ < H_- + r_c^{-1}$, spin-0 becomes ghost.

Stabilization

If the mass of spin-2 mode approaches the critical mass the mass of spin-0 goes to the critical mass.
So the property of the ghost can transfer from spin-2 to spin-0

②We investigate if ghost is harmful or not

Pair creation

We calculate the particle production from de Sitter vacuum in the massive gravity theory with conformal coupling scalar field.
Result is UV divergent, but if we set that the 3D cutoff equals Planck mass, energy density is less than critical density of universe.

de-Sitter breaking vacuum

Since we must select ghost mode, which is helicity-0 mode of graviton, de-Sitter symmetry is broken.