Modified gravity as an alternative to dark energy

Lecture 3.

Observational tests of MG models

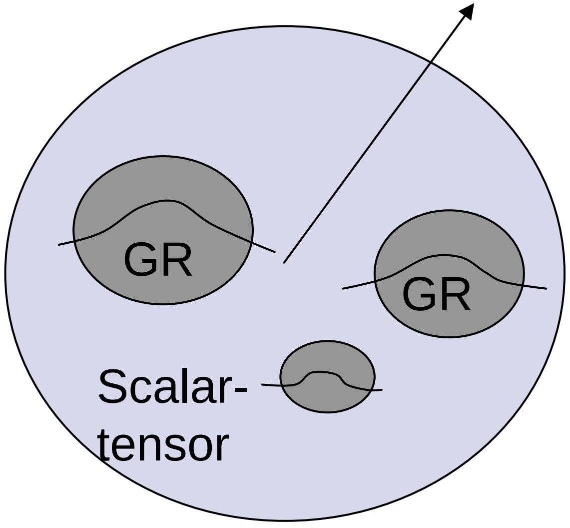
Observational tests

- Assume that we manage to construct a model

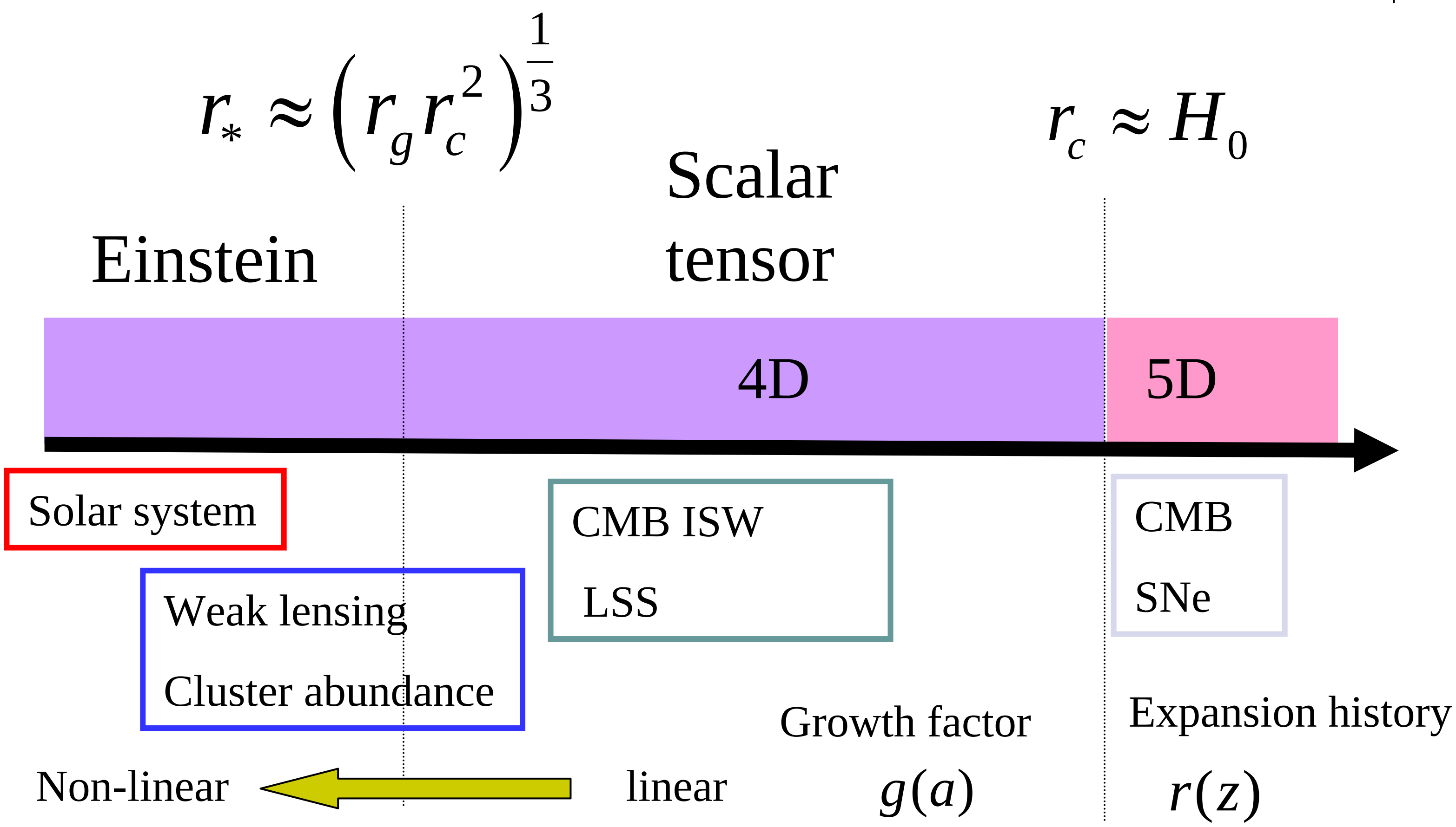
How well can we test the model and distinguish it from LCDM model?

There have been a lot of activities

Here I focus on DGP as this is one-parameter theory



Three regime of gravity -DGP



Expansion history

- Friedmann equation

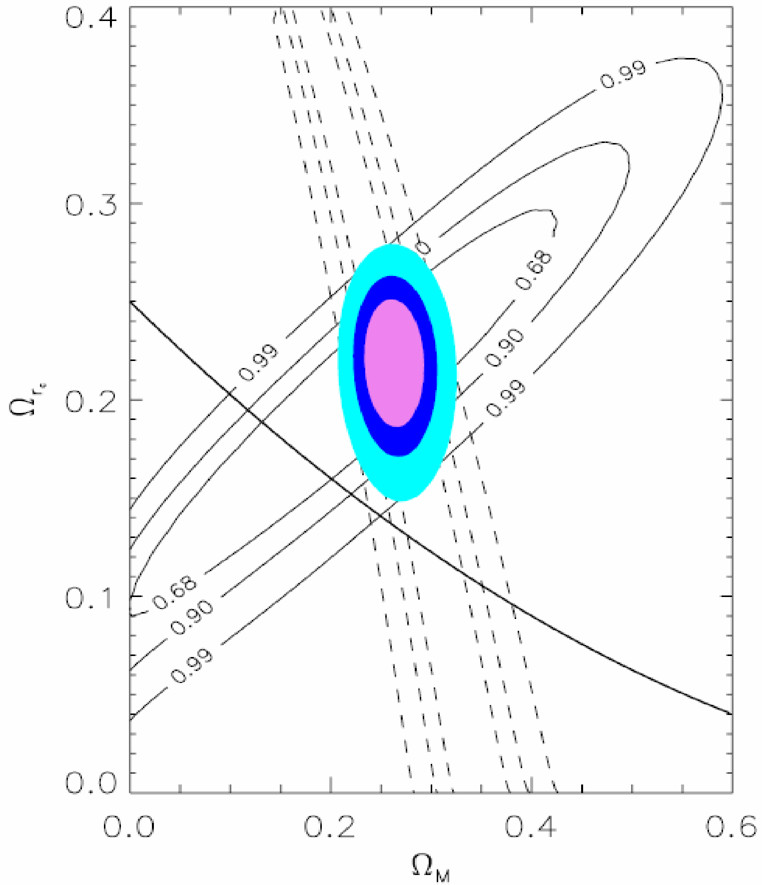
$$H^2 = \left(\frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{8\pi G}{3}\rho_m} \right)^2 + \frac{K}{a^2}$$

$$1 = \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m} \right)^2 + \Omega_K , \quad \Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}$$

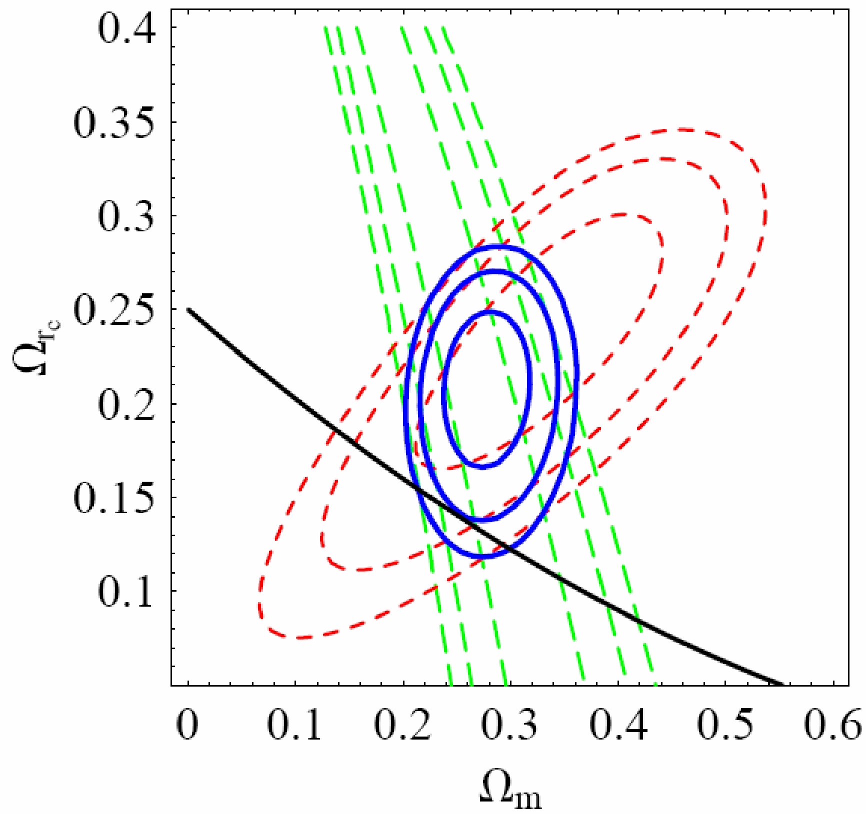
cf. LCDM

$$1 = \Omega_m + \Omega_\Lambda + \Omega_K$$

SNe + baryon oscillation



SNLS + SDSS



'Gold' set + SDSS

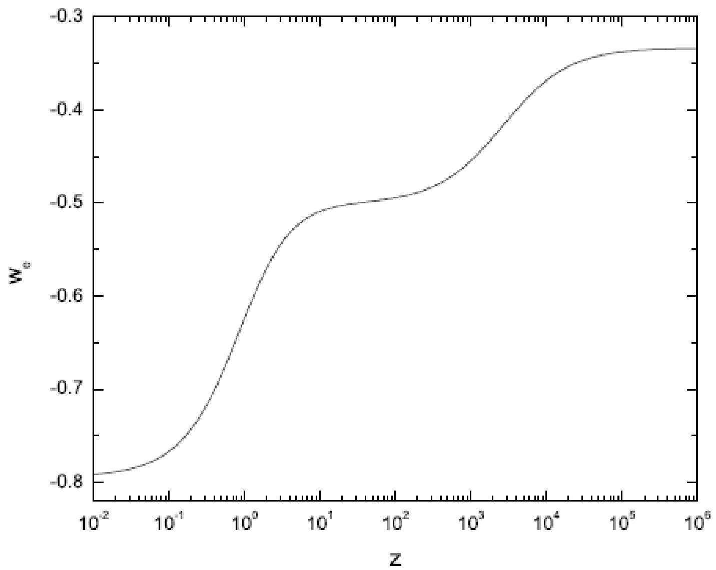
(Fairbairn and Goobar astro-ph/0511029)

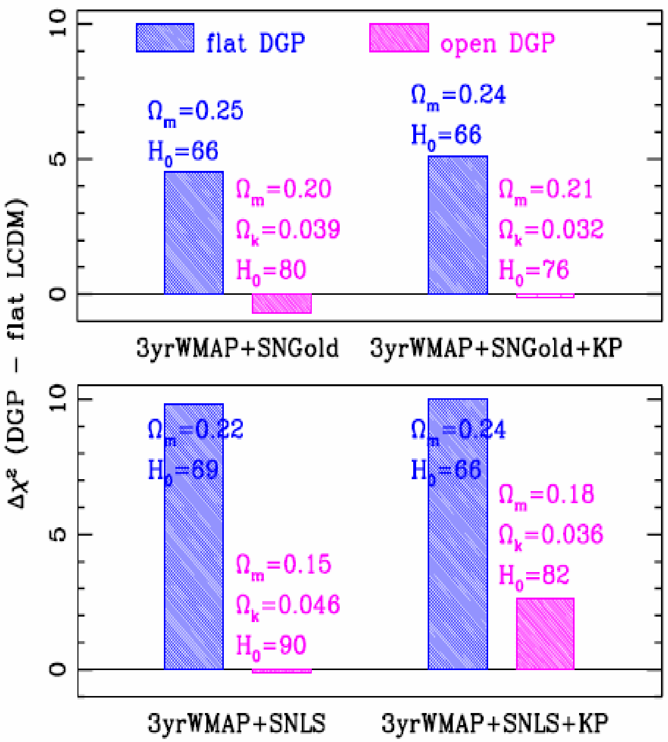
(Maartens and Majerotto astro-ph/0603353)
(cf. Alam and Sahni, astro-ph/0511473)

- flat model conflicts with data

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{H}{r_c}$$

$$\rho_{DE} \equiv \frac{H}{r_c} \qquad w_{DE} = -\frac{1}{1+\Omega_m(a)}$$



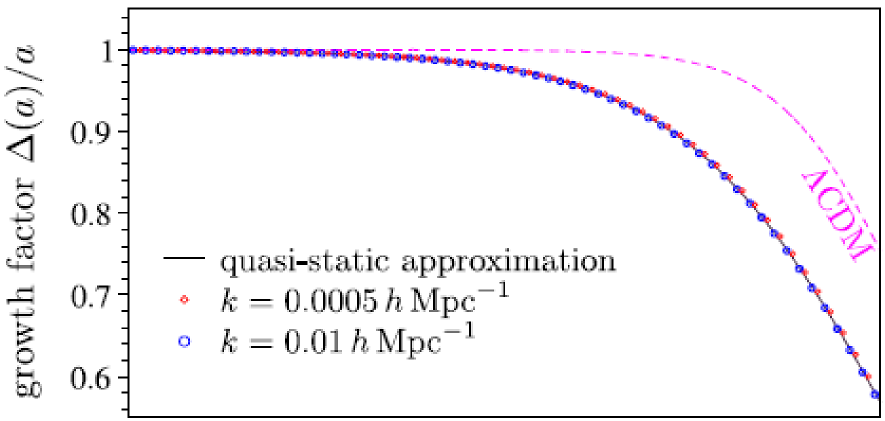


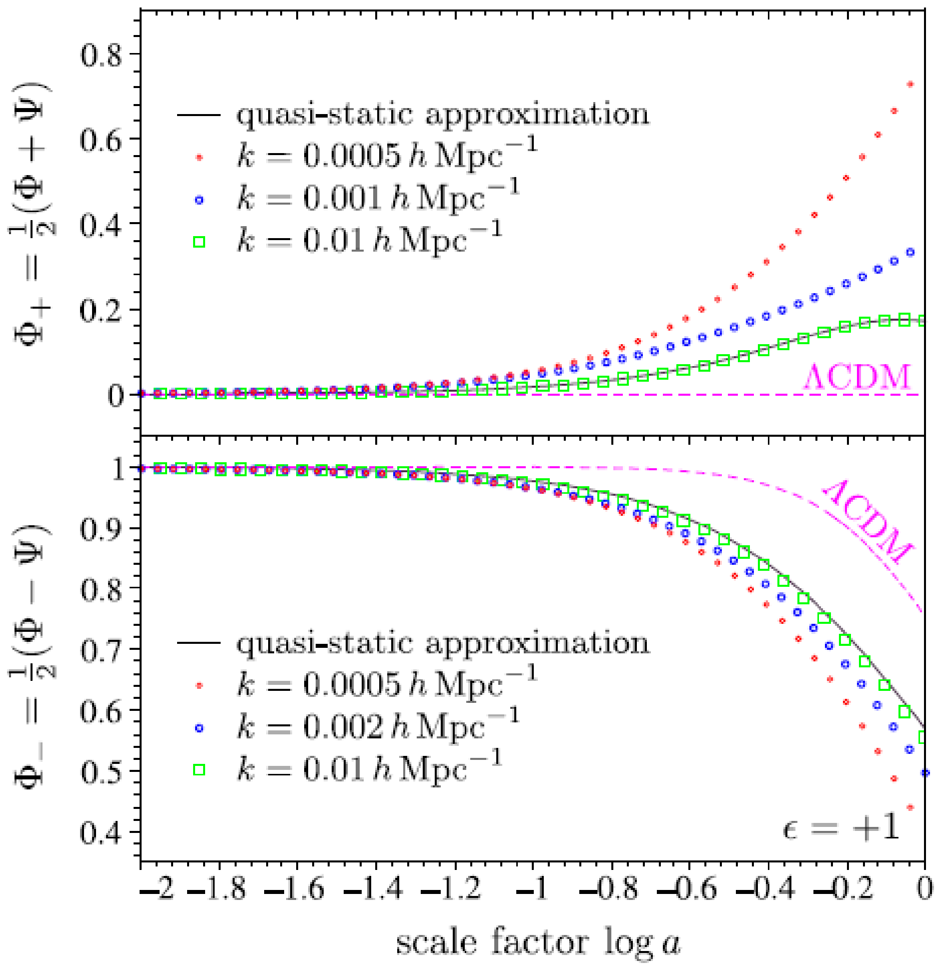
(Song, Hu and Sawicki)

inclusion of curvature improves a fit

Linear perturbations (Cardoso et.al.)

$$ds_b^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)\delta_{ij}dx^i dx^j$$

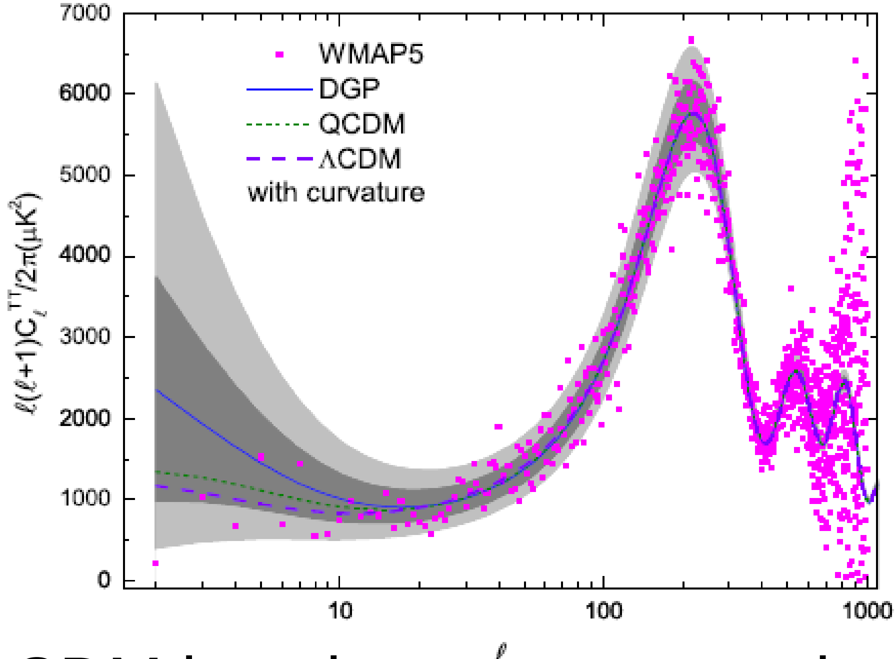




$$\Phi_+ = \frac{1}{2}(\Phi + \Psi) = 0$$ GR

$$\Phi_- = \frac{1}{2}(\Phi - \Psi)$$ CMB ISW

- Enhancement of low multipoles



parameters	DGP	QCDM	ΛCDM
$100\Omega_b h^2$	2.38	2.36	2.27
$\Omega_c h^2$	0.0937	0.0960	0.107
$100\theta_s$	1.04	1.04	1.04
τ	0.0887	0.0914	0.0884
Ω_K	0.0189	0.0268	-0.00553
n_s	0.996	0.992	0.959
$\ln[10^{10} A_s]$	3.02	3.05	3.18
H_0	73.8	78.3	69.8
Ω_m	0.216	0.195	0.266
Ω_{r_c}	0.149	..	..
$-2\ln L$	2800.8	2787.2	2777.5

QCDM has the same expansion history as DGP

DGP is a poorer fit than LCDM at 5.3 σ level

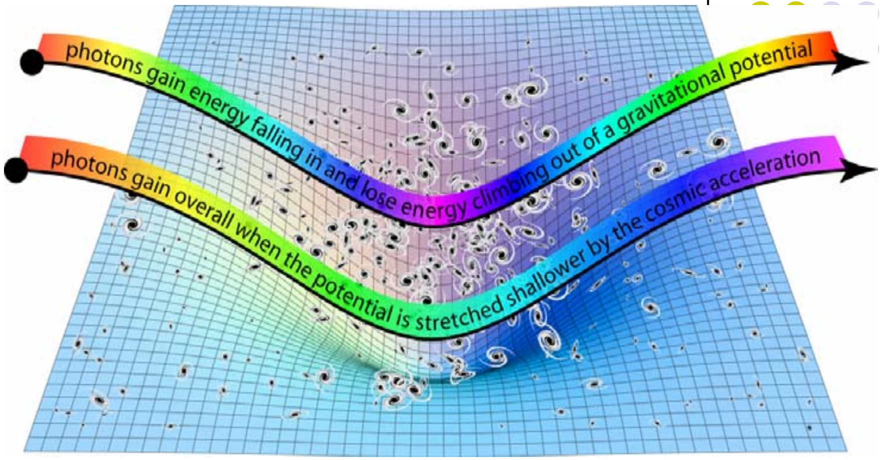
inclusion of large scale CMB has 30% contribution to this conclusion

(Fang et.al. 0808.2208)

Integrated Sachs-Wolfe effects

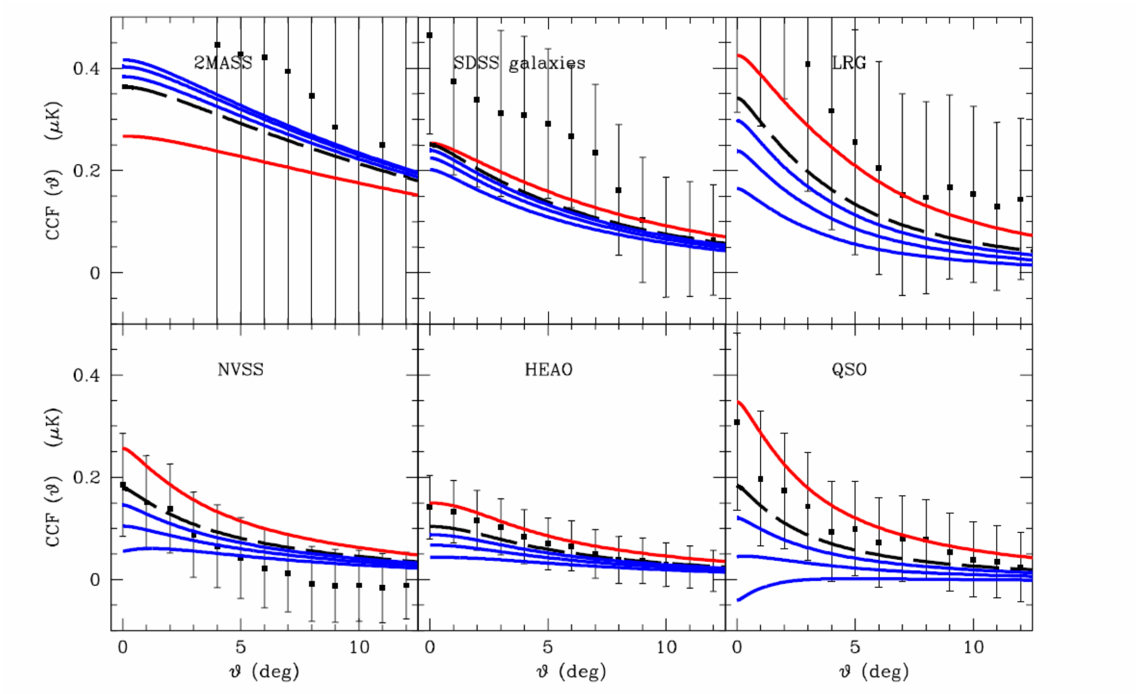
- ISW effects

Sensitive to time variation of growth rate / MG

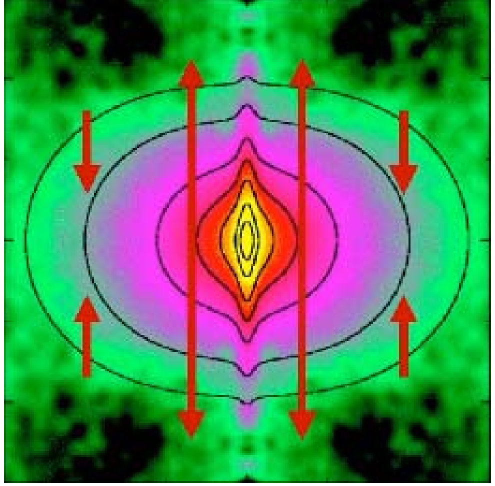


Cross correlation to matter perturbations

(Giannantonio et.al.)



Peculiar velocity



- Redshift distortion

due to peculiar velocities of galaxies, red-shift space power spectrum of galaxies becomes anisotropic

$$P_g^s = \left(P_g(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}\right) F\left(\frac{k^2\mu^2\sigma_v^2}{H^2(z)}\right)$$

Non-linear effects

μ cos of angle between the line of sight and wave number

$\theta = -a\dot{\delta}$ is divergence of peculiar velocities

multi-pole moment expansion

$$P(k,\mu) = \sum_{\ell=0,2,} P_\ell(k) L_\ell(\mu)(2\ell+1)$$

- Quadrupole spectrum

linear theory

$$\frac{P_2(k)}{P_0(k)}=\frac{1}{5}\frac{4\beta/3+4\beta^2/7}{(1+2\beta/3+\beta^2/5)},\ \beta=f/b,\ f=\frac{d\ln\delta}{d\ln a}$$

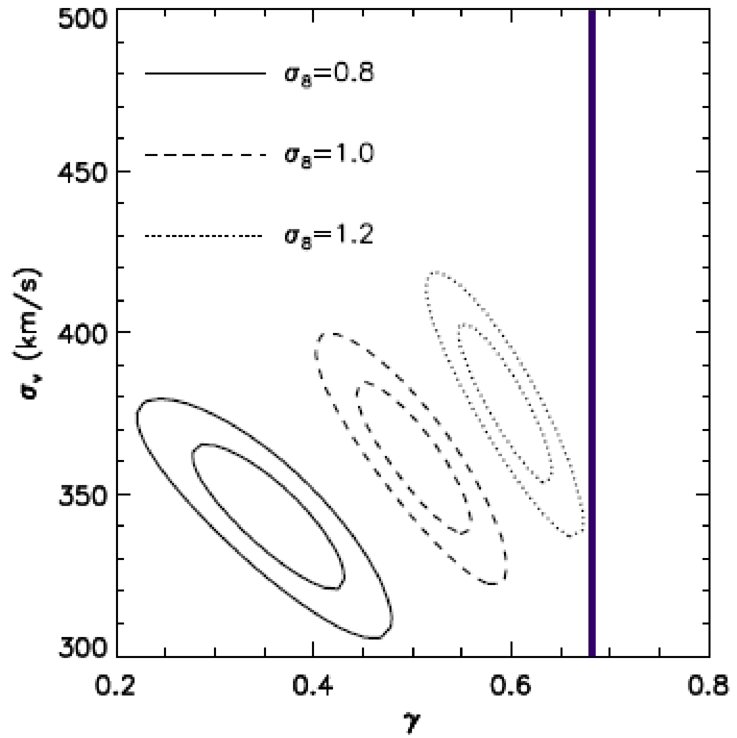
b: galaxy bias

- SDSS LRGs (Yamamoto et.al.)

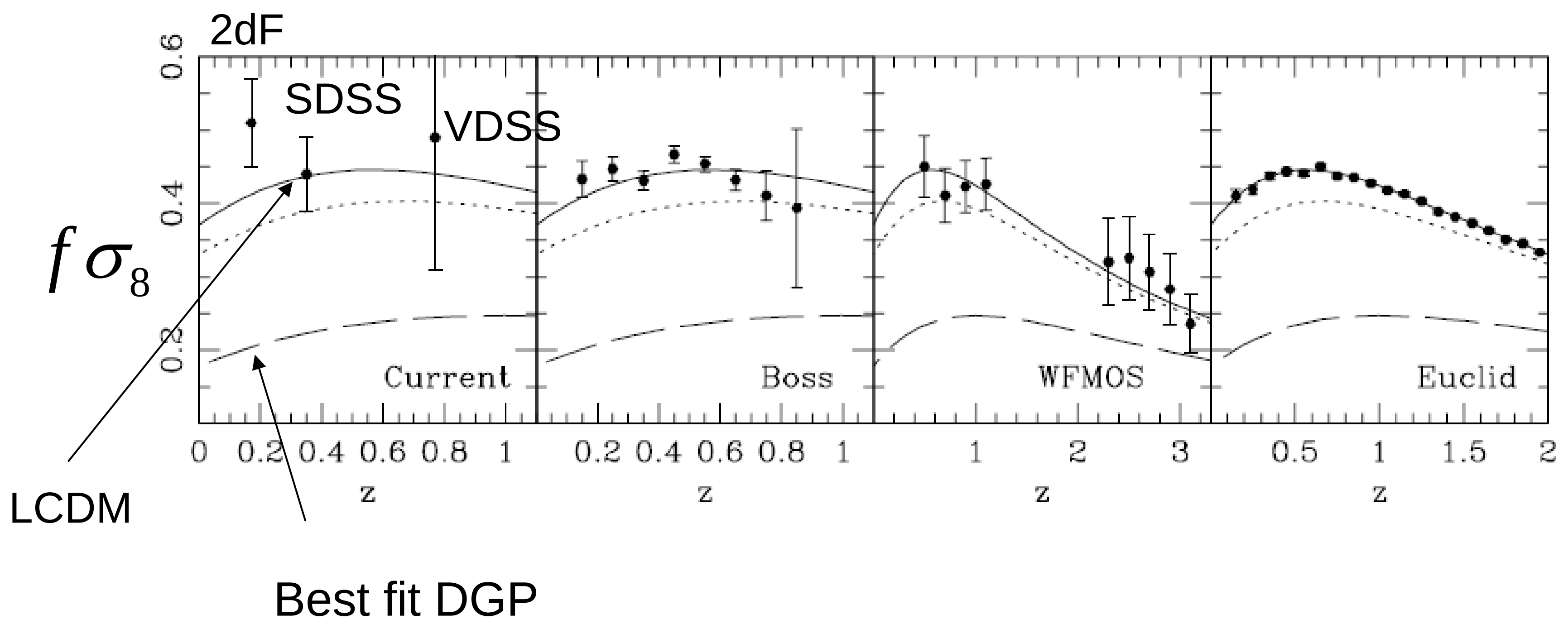
significantly high σ_8 is required

Combining to the CMB will exclude the model significantly

$f=\Omega_m^\gamma,\quad \gamma=0.68$ (DGP)



- # Future forecast (Song and Percival)



peculiar velocity can give bias-free measurements of

$\sigma_{\theta 8} = f\sigma_8$ from $P_{\theta\theta}$

Weak lensing

- Sensitive to growth rate $g(a) = \delta / a$

Linder's parametrisation

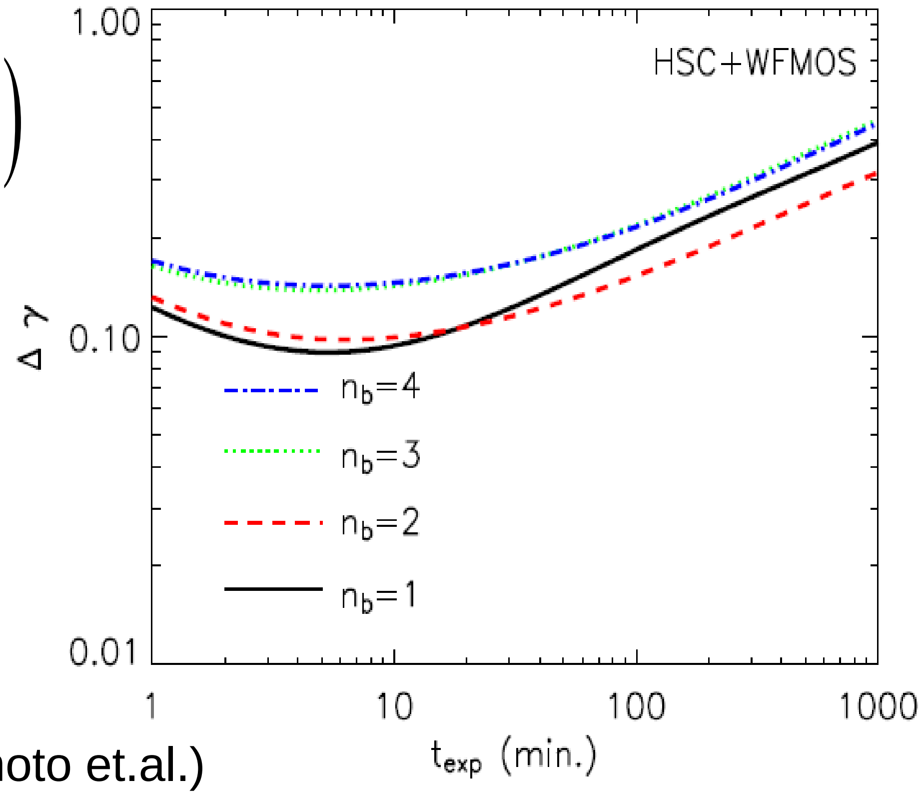
$$g(a) = \exp\left(\int_0^a d\ln a\left(\Omega_m(a)^\gamma - 1\right)\right)$$

dark energy

$$\gamma = 0.55 + 0.05(1 + w(z=1))$$

DGP

$$\gamma = 0.68$$



WFMOS $\Delta\gamma = 0.1$ (Yamamoto et.al.)

Non-linear power spectrum

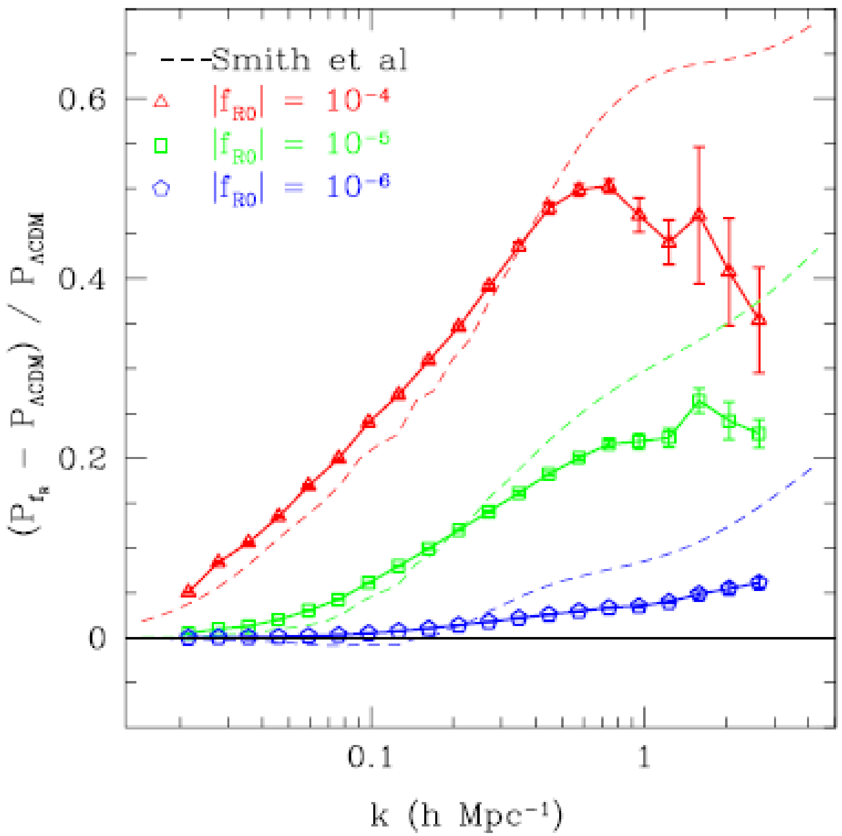
- So far, GR mapping formula is used

  this neglects the subtlety of non-linear recovery of GR on non-linear scales

  example from f(R)

  mapping formula fails

  (Oyaizu et.al. 0807.2462)



- Need to understand

  non-linear physics in MG

  - N-body
  - perturbation theory

  (Hiramatsu, Taruya, KK)

Summary

- We have enough observations!

  current data has an ability to exclude DGP at 5 sigma level against LCDM

- Structure formation test can give significant contributions

  large scale CMB anisotropies

  peculiar velocities

  weak lensing

Can we distinguish between MG and DE?

- Can we surely prove that the acceleration is driven by MG not by DE

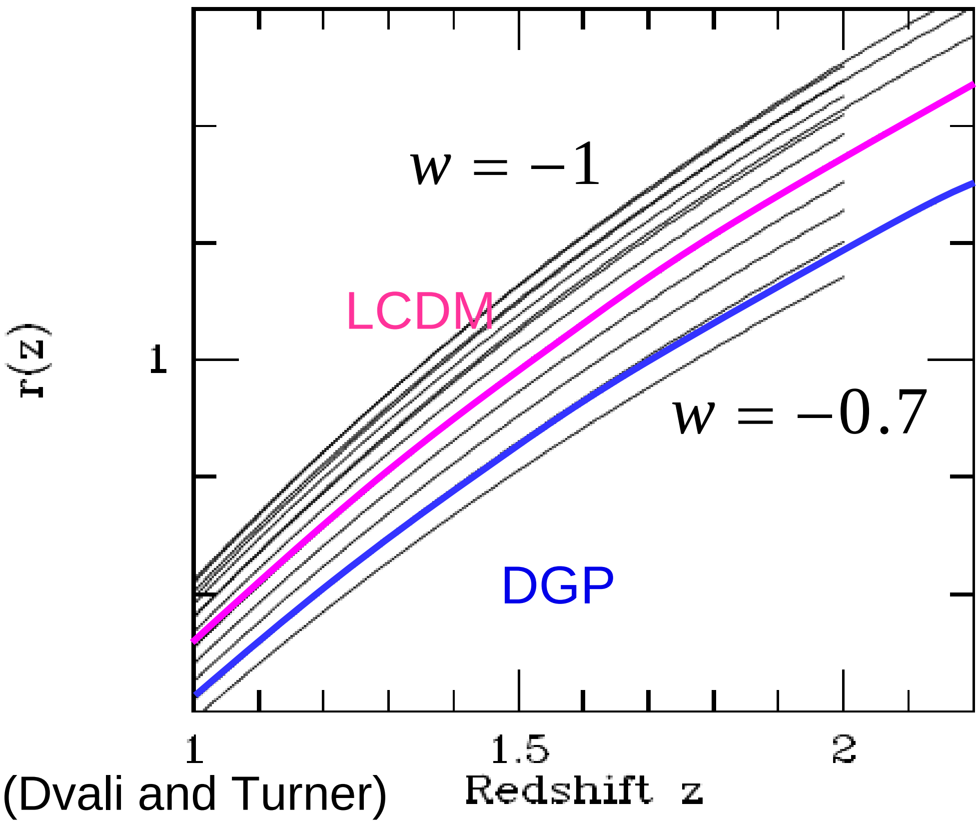
This is clearly impossible in the background

structure formation test is essential

(+ sensible assumption for dark energy perturbations)

Dark energy vs DGP

- Can we distinguish between dark energy in GR and DGP ?



(Dvali and Turner)

$$r(z) = \int_0^z dz H(z)^{-1}$$

DGP model is fitted by

$$w(a) = w_0 + w_a(1-a),$$
$$w_0 = -0.78, w_a = 0.32$$

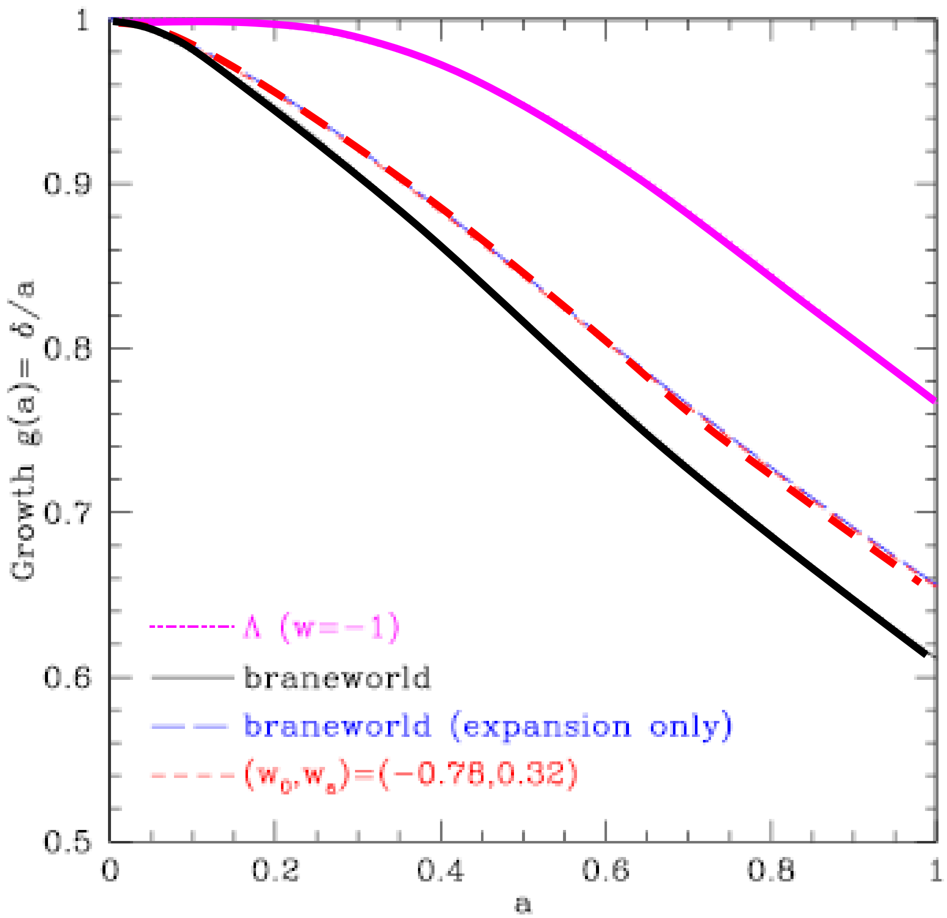
(Linder)

Expansion history vs growth rate

(Lue.et.al, Koyama & Maartens, Koyama)

- Growth rate resolves the degeneracy

$$g(a) = \frac{\delta}{a}$$

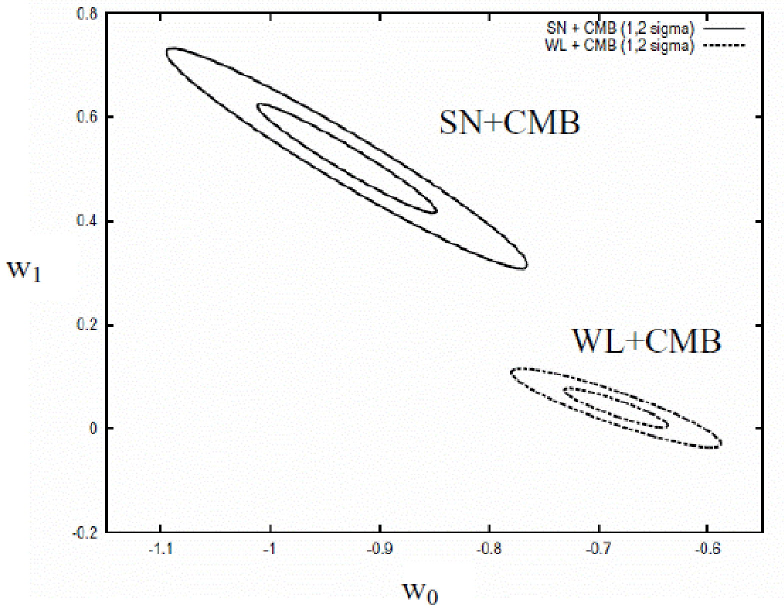


Experiments

(Ishak,Upadhye and Spergel, astro-ph/0507184)

- ASSUME our universe is DGP braneworld

but you do not want to believe this,so fit the data using dark energy model



m(z):
apparent magnitude

R:
CMB shift parameter

G(a):
Growth rate

OR

SNe+CMB

SNe+weak lensing

Inconsistent!

Observables

- Density perturbations $\delta = \frac{\delta\rho}{\rho}$

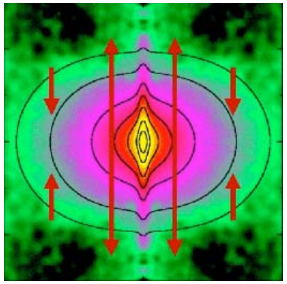
  galaxy clustering

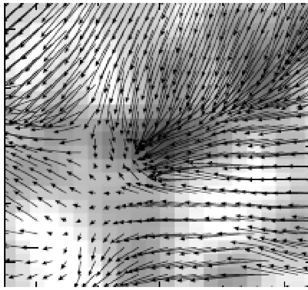
  mass function of clusters

- Peculiar velocity $\theta = \partial_i v^i$

  red-shift distortions

  internal dynamics of clusters/galaxies

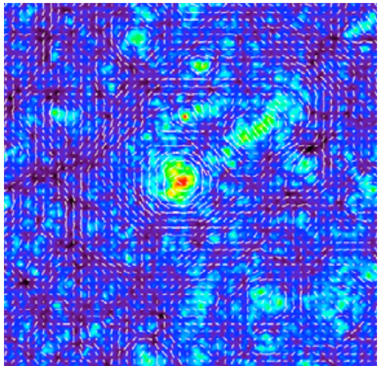




- Lensing potential $\Phi - \Psi$

  weak lensing

  ISW of CMB



Equations under horizon

- Gravitational equations (GR)

$$H^2 = \frac{8\pi G}{3}\rho_T, \quad \rho_T = \sum_i \rho_i$$

$$\frac{k^2}{a^2}\Phi = 4\pi G a^2 \rho_T \delta_T, \quad \rho_T \delta_T = \sum_i \rho_i \delta_i$$

- Equation of motion for matter (no interaction)

$$\frac{k^2}{a}\Psi = \dot{\theta}_m + H\theta_m$$

$$\dot{\delta}_m = -\frac{\theta_m}{a}$$

Consistency condition

- Eliminate Newton constant from Friedmann eq. and Possion eq.

$$\alpha(k,t) = \frac{2k^2}{3a^2H^2}\frac{(\Phi-\Psi)+\Psi}{\delta_T}$$

Peculiar velocity

$$k^2\Psi = \frac{d(a\theta_m)}{dt}$$

background

Weak lensing

Galaxy distribution

$$\delta_g = b_T\delta_T$$

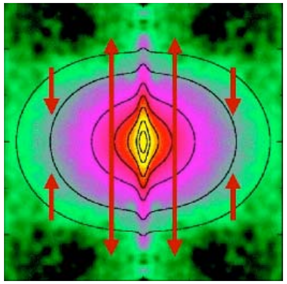
In GR, $\alpha(k,t)=1$

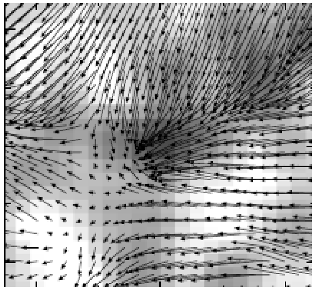
This is written only by observables

Different observables measure different physical quantities and they are all valuable if we just extend our theoretical prior to include MG

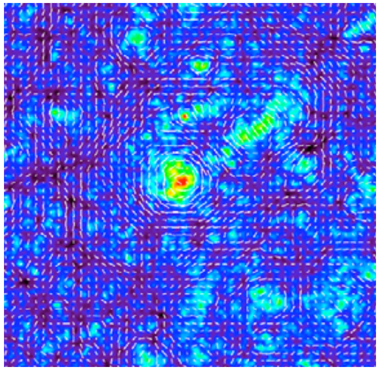
$$\delta$$

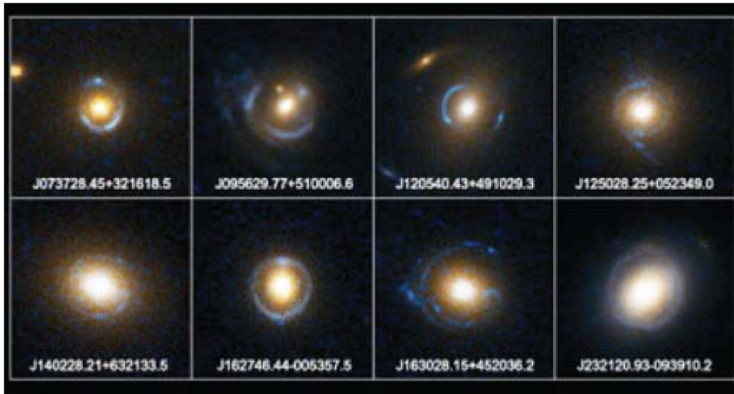
$$\Psi$$





$$\Phi - \Psi$$





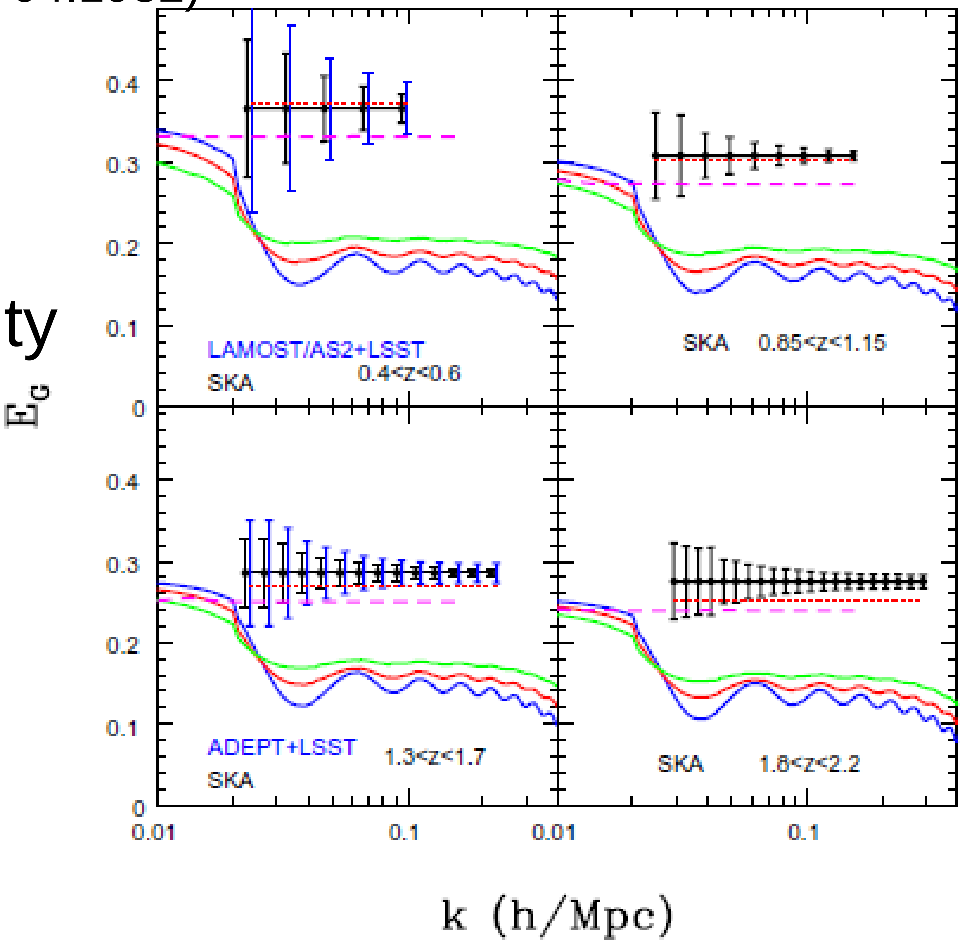
How to combine various observations?

- Estimator (Zhang et.al. 0704.1932)

$$\langle E_G \rangle = \frac{\nabla^2(\Psi - \Phi)}{-3H_0^2 a^{-1}\theta}$$

galaxy-lensing and -velocity cross correlation

$$E_g \approx \frac{\Omega_m}{f(\Omega_m)}, \quad f = \frac{d\ln\delta(a)}{d\ln a}$$



Current status

- SDSS LRGs

  (Reyes et.al)

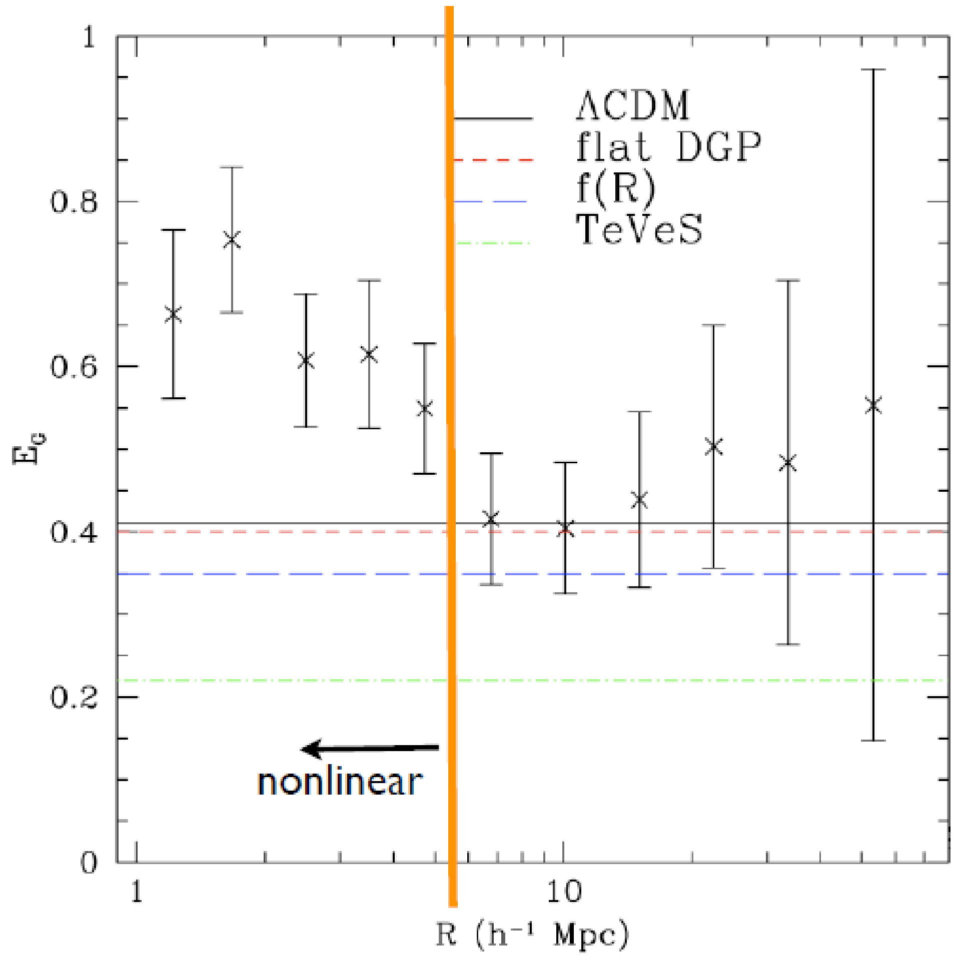
It is free from bias

but also eliminates

Information on modified

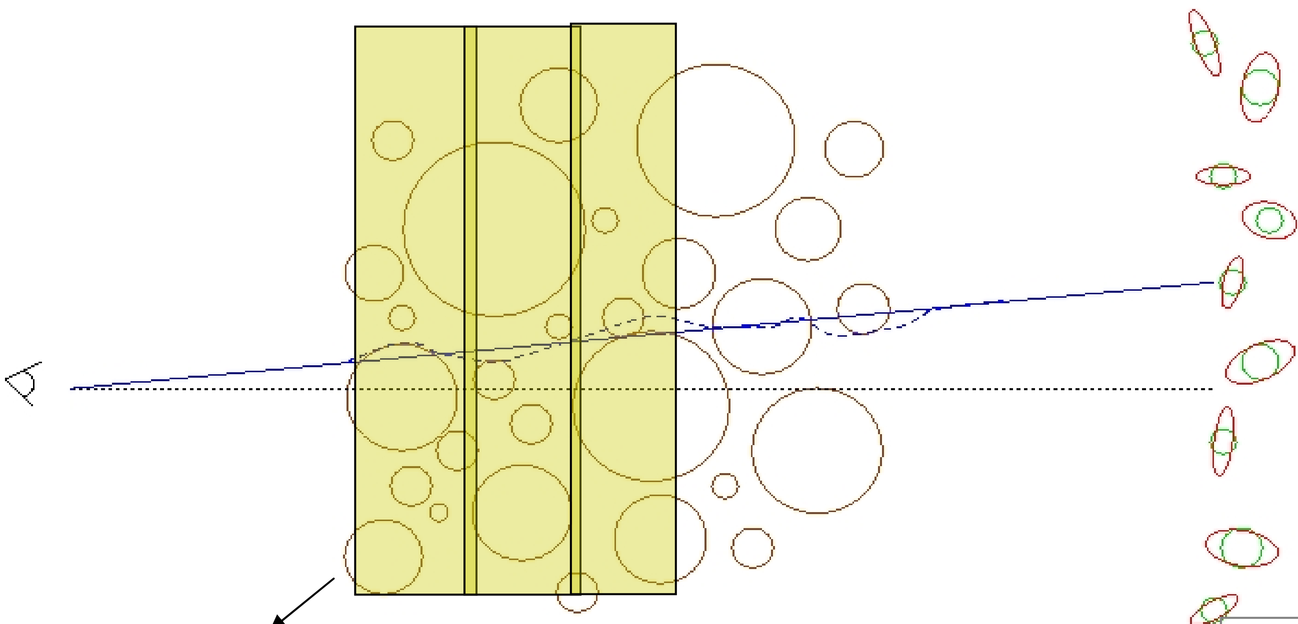
Newton constant

$$\nabla^2\Phi = -4\pi\left(G_{eff}/b\right)\delta_g$$



Consistency test (Song and KK)

- Reconstruction of gravitational lensing from density distribution and peculiar velocity at each red-shift bin



$$k^2\Phi = 4\pi G a^2 \rho_T^{GR}\delta_T^{GR} \;\Rightarrow\; \phi_i(\ell/\chi_i, z_i) = \Phi - \Psi \;\Rightarrow\; \text{Reconstructed gravitational lensing}$$

$$k^2\Psi = \frac{d(a\theta_m)}{dt}$$

GR is assumed

Consistency test

Reconstructed gravitational lensing

Observed gravitational lensing

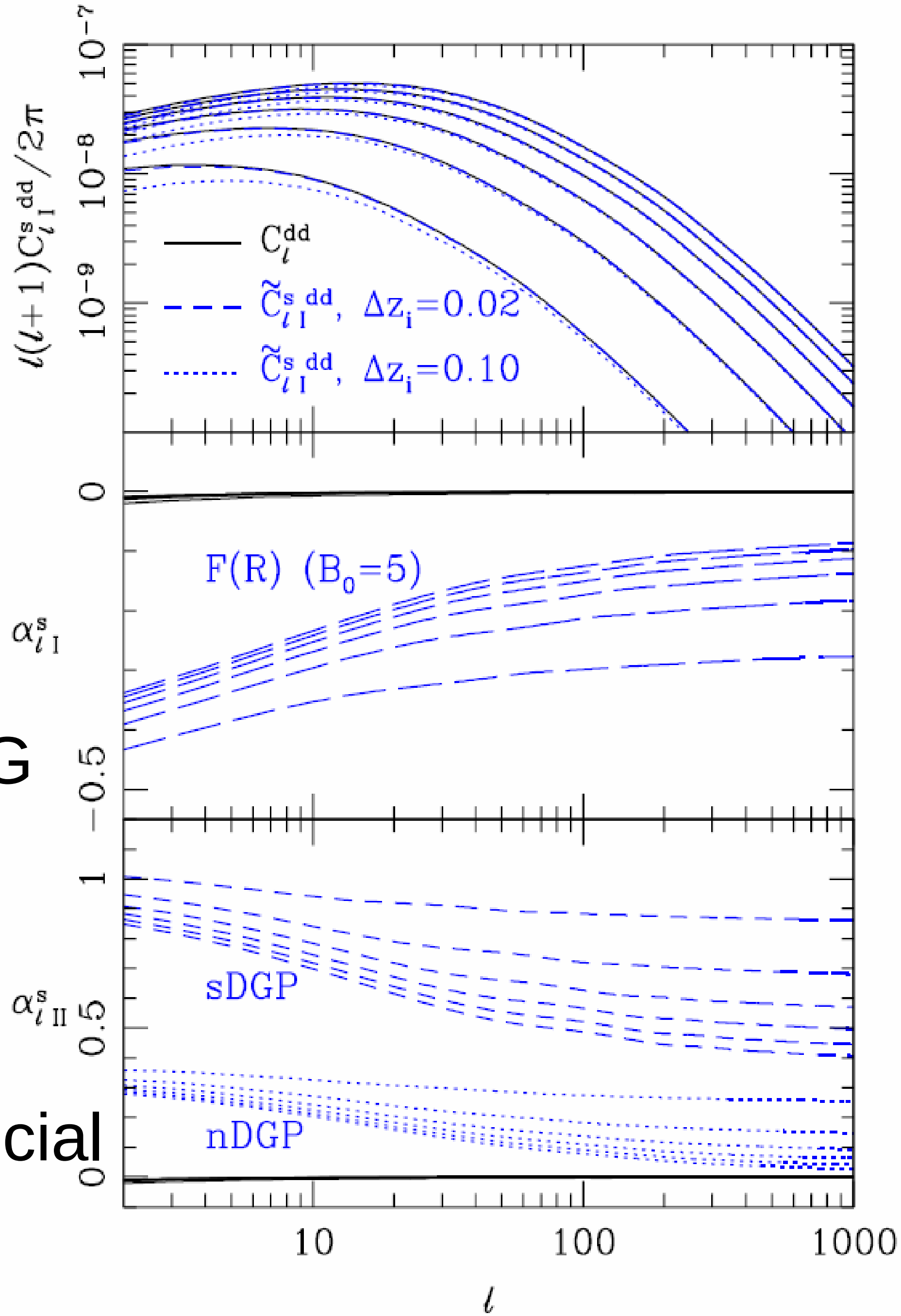
- 2D power spectrum of lensing potential

test of reconstruction in GR

- Ratio of reconstructed and true spectrum in MG

$$\alpha = \log_{10}\left(\frac{C_l^{\,recon}}{C_l}\right)$$

- Information of bias is crucial



Summary

- Observational test on MG models

  geometrical test + structure formation

  theoretical models are required

  understanding of non-linear clustering is necessary

- Model independent test of GR

  peculiar velocity + WL + galaxy distribution

  need to find the best estimator

  understanding of systematic in observation is necessary

Objective

Seek solutions to the question of dark energy
By challenging conventional GR

- construct consistent theoretical models building on rapid progress in understanding the law of gravity beyond GR
- develop efficient ways to combine observational data sets to distinguish modified gravity models from dark energy models based on GR
- provide tests of GR on largest scales