Modified gravity as an alternative to dark energy

Lecture 2.

Theory of modified gravity models

Theory of modified gravity

- Requirements
  - Must explain the late time acceleration
  - Must recover GR on small scales
  - Must be free from pathologies

- Two examples to see how difficult it is to satisfy these conditions!

  f(R) gravity, DGP braneworld model

Example: f(R) gravity

- The modification should act at low energies

$$S = \int d^4x \sqrt{-g}\, R \quad \Rightarrow \quad S = \int d^4x \sqrt{-g}\, F(R)$$

Ricci curvature is smaller at low energies

example $$F(R) = R - \frac{\mu^4}{R}$$

μ must be fine-tuned $\mu \sim H_0$

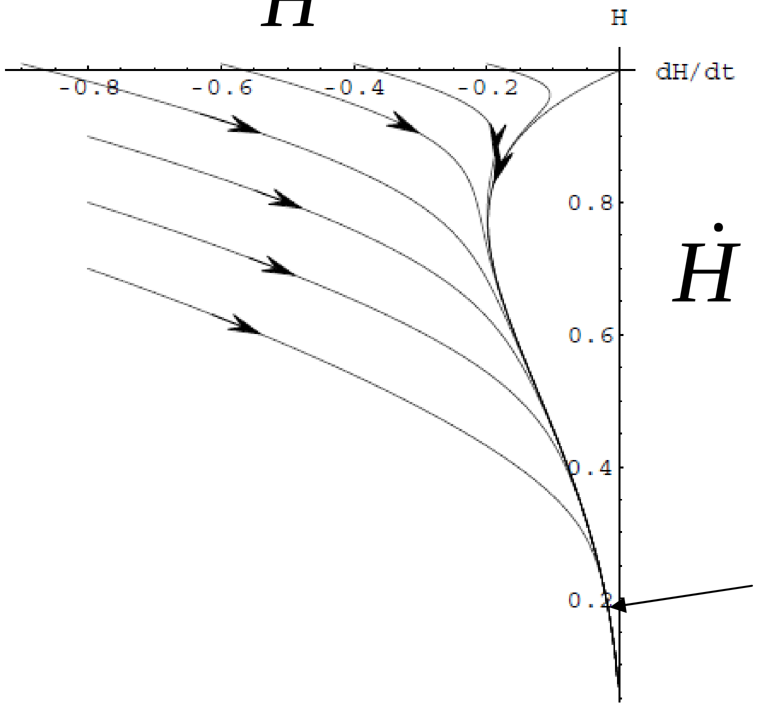
cf. high energy corrections

$$F(R) = R + \alpha R^2$$

- Late time acceleration

  Friedman equation now becomes 4th order differential equation

H



acceleration $a(t) \propto t^2$

this analysis does not include matter/radiation

Problem

- f(R) theory is equivalent to BD theory

Legendre transformation $F''(R) \neq 0$

$$S = \int d^4x \sqrt{-g}\, F(R)$$

$$S = \int d^4x \sqrt{-g}\left(F(\phi) + (R-\phi)F'(\phi)\right) \qquad R = \phi$$

$$\Rightarrow \quad S = \int d^4x \left(\psi R - V(\psi)\right)$$

This is BD theory with $\omega_{BD} = 0$ (Wands, Chiba)

The potential is of order $V(\Psi) \sim \mu^4$ for $F(R) = R - \frac{\mu^4}{R}$

- Contradicts to solar system constraints

  the potential is of order H_0^4 and can be neglected

- Cosmology

  inclusion of radiation/ matter

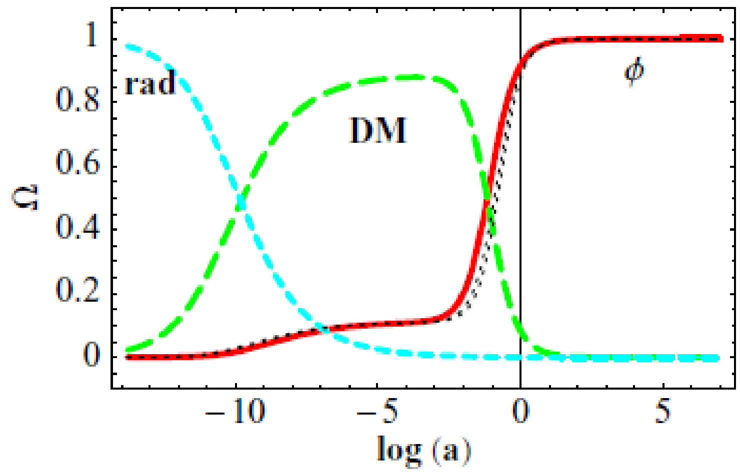
  Nariai 1969

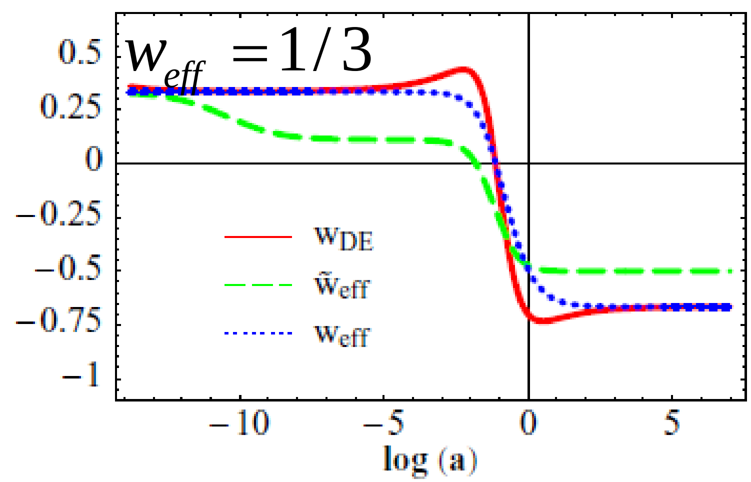
  $\omega_{BD} = 0$ BD theory yields

  $a(t) \propto t^{1/2}$ for any kind of matter

  MD era does not exist !

  We expected to recover GR at early times where the correction is tiny...





(Amendola et.al astro-ph/0603703)

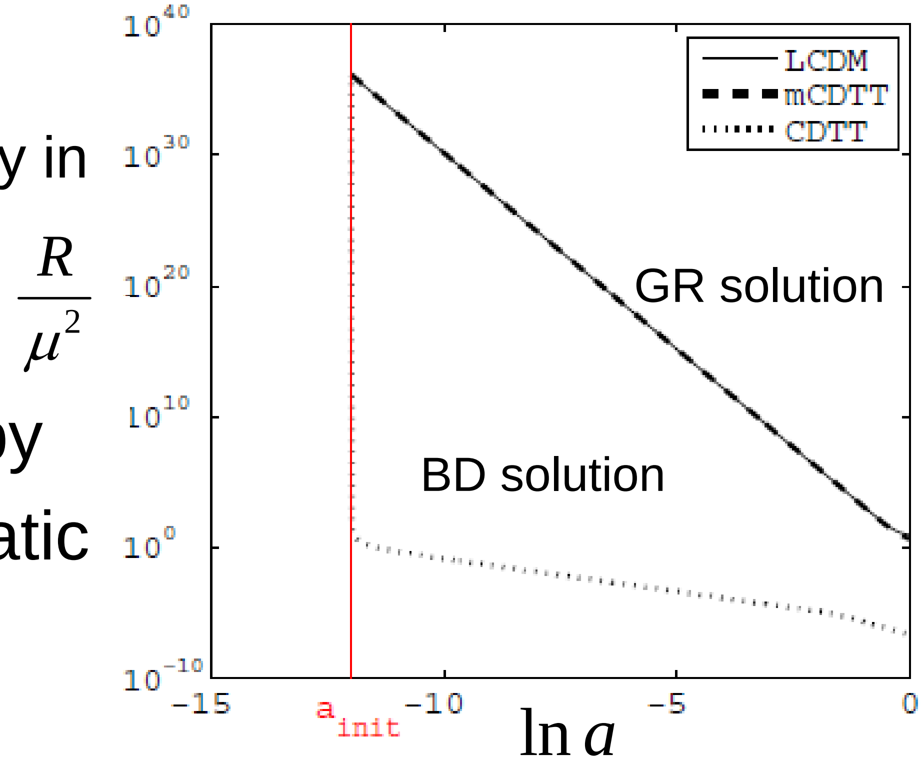
- Instability with matter

$$F(R) = R \mp \frac{\mu^4}{R}$$

CDTT

mCDTT

(mCDTT model has a singularity in the late accelerating phase)



(Hu and Sawicki astro-ph/0702278)

This instability was known by Dolgov & Kawasaki for a static source

The condition for the instability

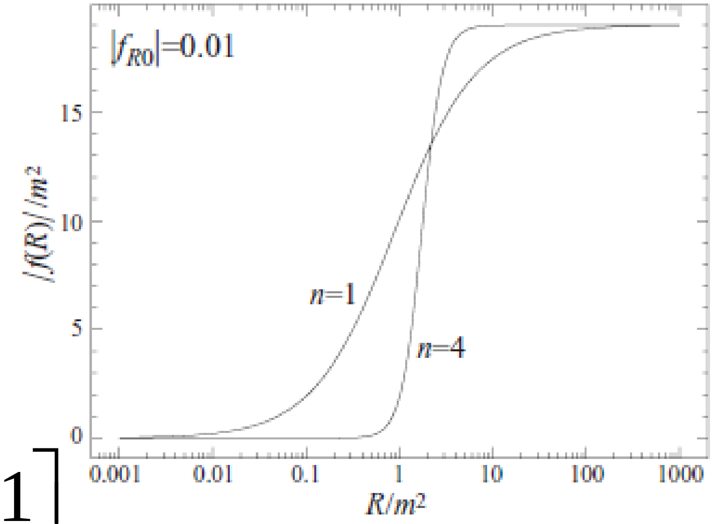
$$B = \frac{F_{RR}}{F_R} R' \frac{H}{H'} \qquad B < 0 \quad \text{unstable}$$

Engineering f(R) models

- Can we avoid solar system constraints?

Hu and Sawicki

$$F(R) = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}$$



Starobinsky

$$F(R) = R + \lambda\left[(1+(R/m)^2)^{-n} - 1\right]$$

$$\lim_{m^2/R \to 0} f(R) = R - C + DR^{-2n}$$

$$\lim_{R \to 0} f(R) = 0$$

no ‘cosmological constant’

Quasi-static perturbations

Linear perturbations

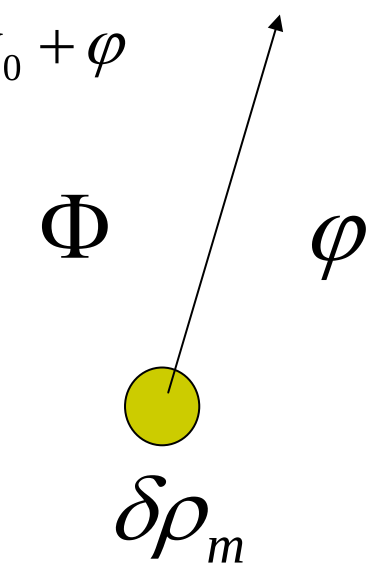
$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1+2\Phi)d\vec{x}^2$$

Equations of motion in BD theory $\quad \psi = \psi_0 + \varphi$

$$(3+2\omega_{BD})\nabla^2\varphi = a^2\delta R - 8\pi G a^2 \delta\rho_m$$

$$\nabla^2\Phi = -4\pi G a^2 \delta\rho_m - \frac{1}{2}\nabla^2\varphi$$

$$\Phi + \Psi = -\varphi$$



- GR and BD limit

$$(3+2\omega_{BD})\nabla^2\varphi = a^2\delta R - 8\pi G a^2 \delta\rho_m$$

$$\nabla^2\Phi = -4\pi G a^2 \delta\rho_m - \frac{1}{2}\nabla^2\varphi$$

$$\Phi + \Psi = -\varphi$$

$\varphi \to 0$

$$\delta R = 8\pi G \delta\rho_m$$

$$\nabla^2\Phi = -4\pi G a^2 \delta\rho_m$$

$$\Phi + \Psi = 0$$

$\delta R \to 0$

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G a^2 \delta\rho_m$$

$$\nabla^2\Phi = -4\pi G a^2 \left(\frac{2(1+\omega_{BD})}{3+2\omega_{BD}}\right)\delta\rho_m$$

$$\Psi = \frac{2+\omega_{BD}}{1+\omega_{BD}}\Phi \equiv \gamma^{-1}\Phi$$

- f(R) gravity $\omega_{BD} = 0, \psi = F_R(R)$

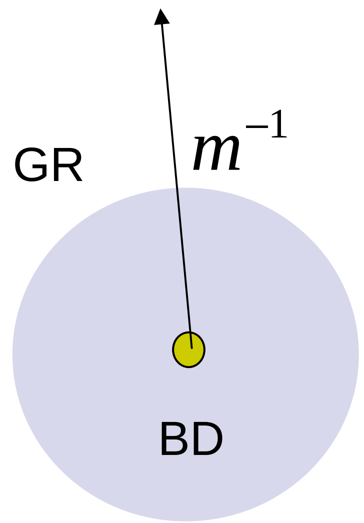
linearisation $\delta R = \left(\frac{dR}{dF_R} \right) \delta F_R \equiv 3m^2 \varphi$

$$3\nabla^2 \varphi = -3m^2 a^2 \varphi - 8\pi G a^2 \delta\rho_m$$

The inverse of mass determines the length scales where the scalar propagates

on large scales $L > m^{-1}$ we recover GR

on small scales $L < m^{-1}$ we recover $\omega_{BD} = 0$



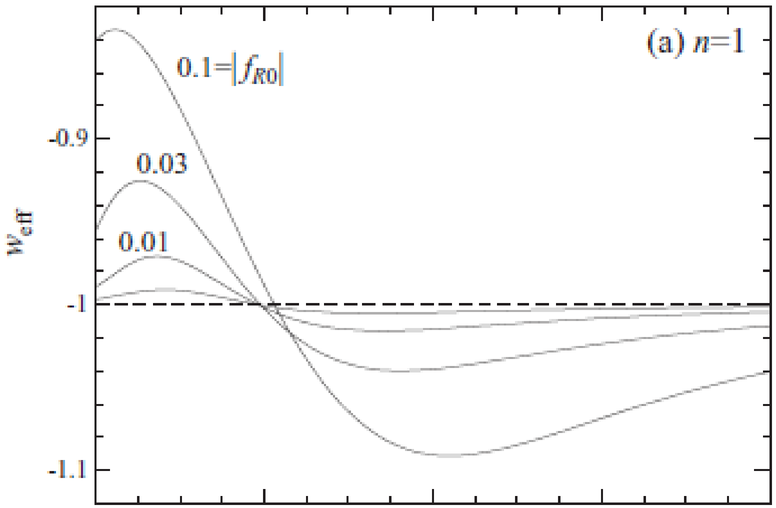
$$\gamma \equiv -\frac{\Phi}{\Psi} = \frac{1}{2}$$

enhances Newtonian potential and growth rate of structure formation

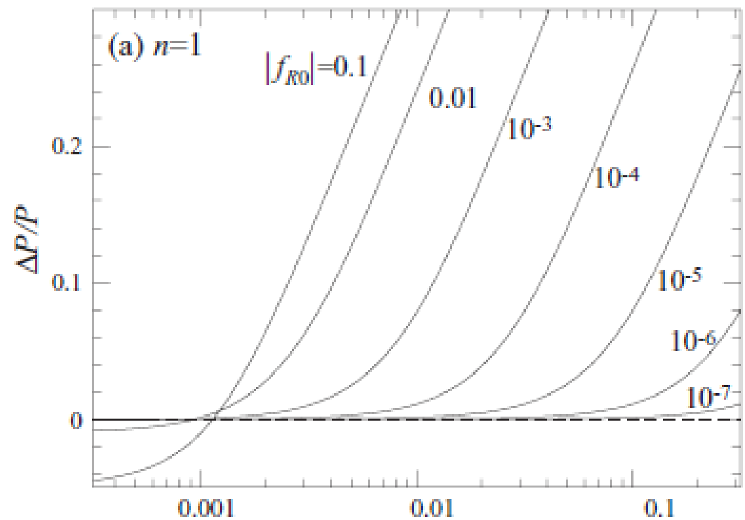
- Example (Hu and Sawicki 0705.1158)

$$f(R) = F(R) - R \propto \frac{R}{AR+1} \rightarrow C - f_{R0}\frac{R_0^2}{R} \quad m^{-1} = \sqrt{6 f_{R0}\frac{R_0^2}{R^3}}$$

In the cosmological background $m_0^{-1} \approx 3.2\sqrt{\frac{f_{R0}}{10^{-6}}}\,\mathrm{Mpc}$



z



k

f_{R0} controls the deviation from LCDM

Fractional difference of linear growth rate compared to LCDM

- On small scales, we get $\omega_{BD} = 0$

this contradicts with solar system constraints?

m^{-1} depends on curvature and becomes smaller for dense region $m^{-1} = \sqrt{6 f_{R0} \frac{R_0^2}{R^3}}$

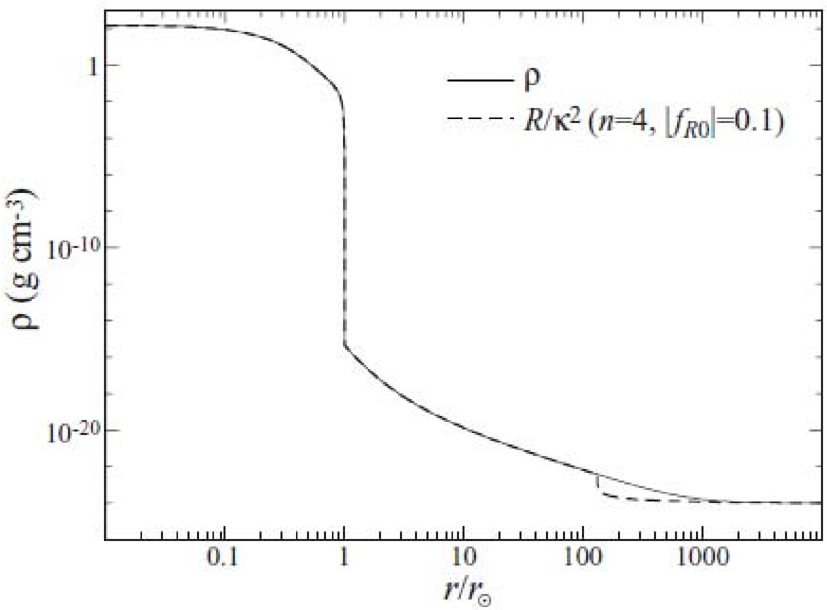
Chameleon mechanism $\rho_0 \approx 10^{-30} g / cm^3, \quad \rho_{galaxy} \approx 10^{-24} g / cm^3$

mass of the filed becomes large for dense region and hides the scalar degree of freedom

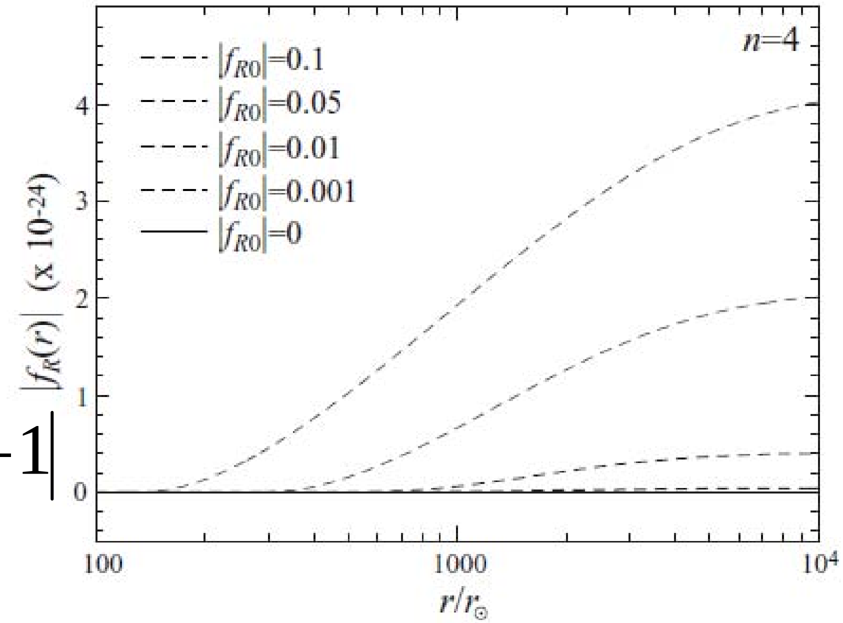
In general linearization $\delta R = R - R_0$ breaks down and need to solve the non-linear equation for $\psi = F_R$

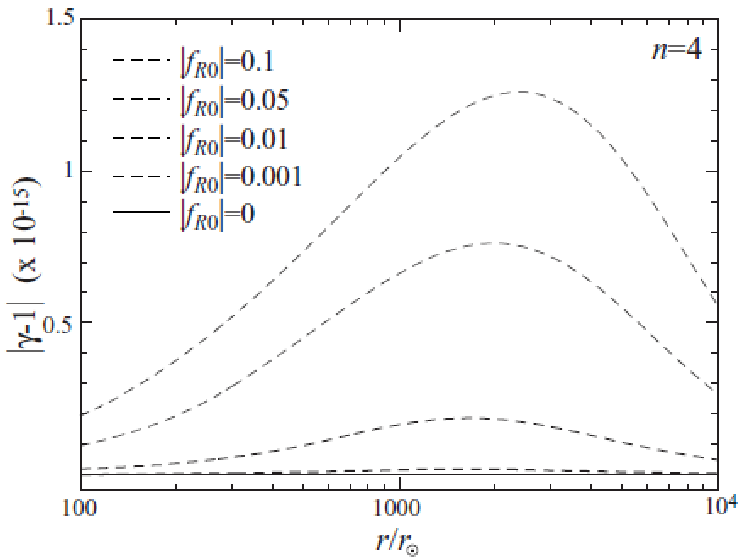
$$-\frac{\nabla^2}{a^2}\psi = -\frac{8\pi G}{3}\delta\rho_m + \frac{2\psi^3}{3}\frac{dV}{d\psi}$$

- # Example of solutions (Hu and Sawicki 0705.1158)



$$|\psi - 1|$$



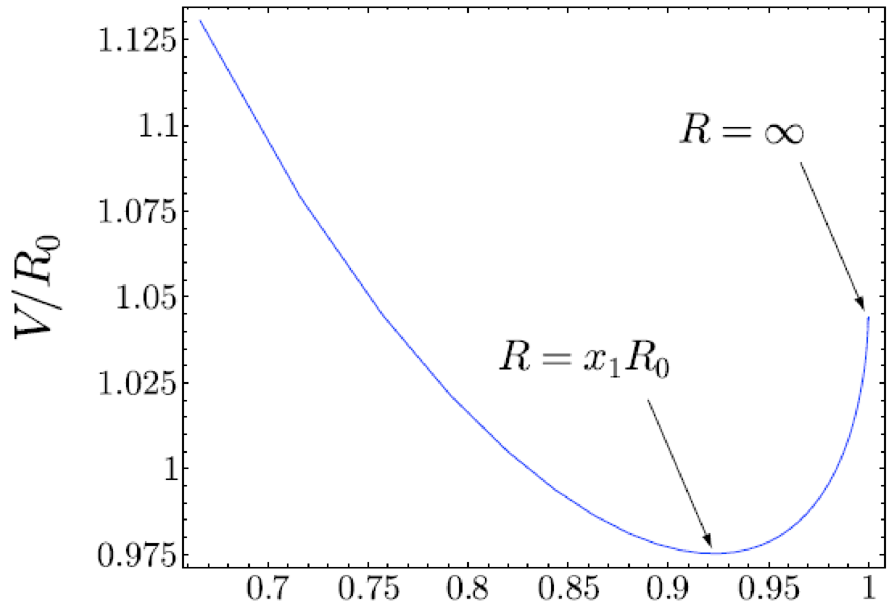


$$|\gamma - 1| < 10^{-5}$$

- Singularity problem (Frolov, Kobayashi and Maeda)

BD scalar $\psi = 1 + f_{R0}\left(\frac{R_0}{R}\right)^2$

potential



$\psi = 1$ corresponds to curvature singularity

this can be reached in strong gravity

f(R) gravity summary

- Naïve models do not work

  low curvature modification in action changes GR even in high curvature regime

- Contrive models using Chameleon mechanism can give acceptable cosmology & weak gravity
  - O(1) modification of GR on cosmological scales
  - but additional mechanisms would be needed for

    strong gravity
  - Complicated version of quintessence?

Example 2. braneworld model

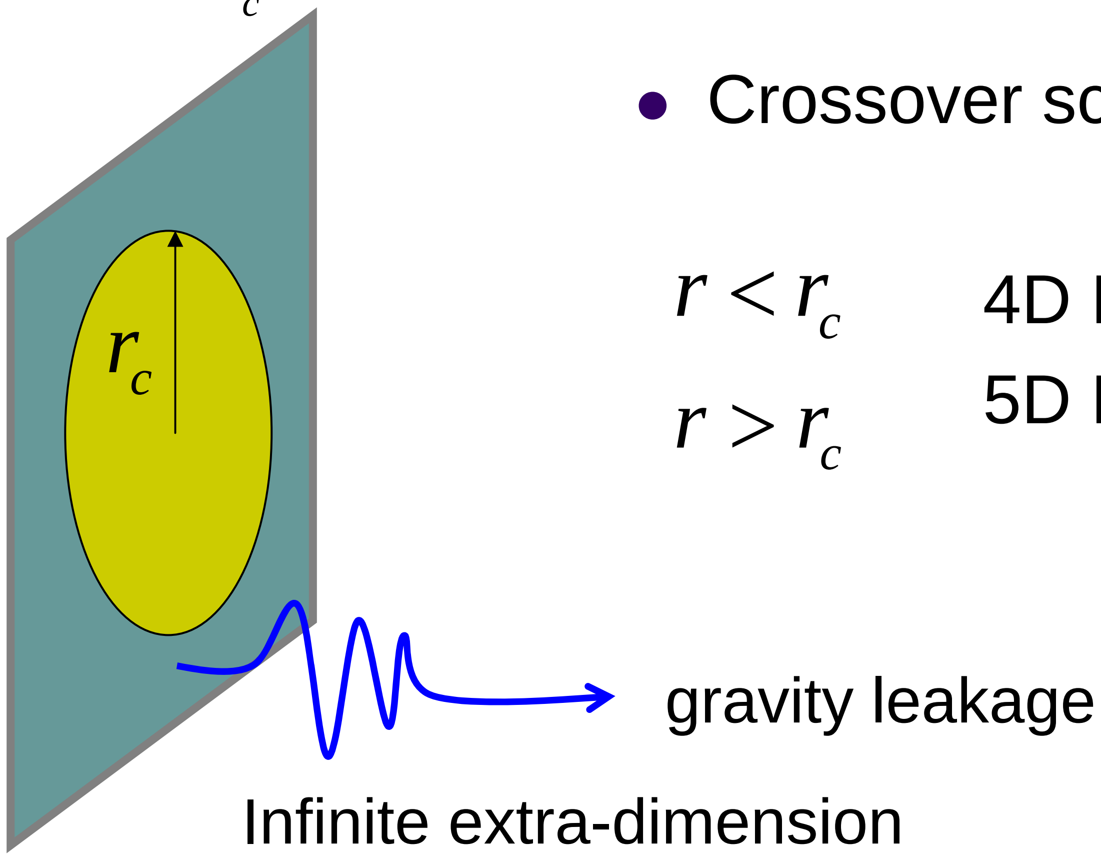
(Dvali, Gabadadze, Porrati)

$$S = \frac{1}{32\pi G r_c}\int d^5x\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

- Crossover scale r_c

$r < r_c$ 4D Newtonian gravity

$r > r_c$ 5D Newtonian gravity



Cosmology in DGP model

- Friedmann equation (Deffayet)

$$\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$

early times $Hr_c >> 1$ 4D Friedmann

late times $\rho \to 0$ $H \to \frac{1}{r_c}$

As simple as LCDM model

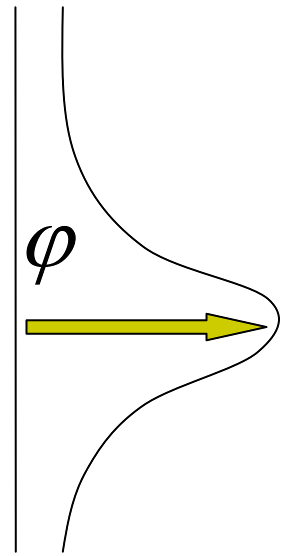
(and as fine-tuned as LCDM $r_c \approx H_0$)

Quasi-static perturbations

Silva and KK hep-th/0702169

- Non-linearity of brane bending mode

$$ds^2 = -N^2\left(1+2\Psi\right)dt^2 + A^2\left(1+2\Phi\right)d\vec{x}^2 + (1+2G)dy^2$$
$$+2\,r_c\varphi_{,i}\,dydx^i$$



Solving bulk perturbations

imposing regularity condition in the bulk

junction conditions on a brane

$$-\nabla^2\Phi = 4\pi G a^2\rho\delta + \frac{1}{2}\nabla^2\varphi, \qquad \Phi+\Psi = -\varphi \qquad \beta = 1-2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$

$$3\beta(t)\nabla^2\varphi + r_c^2\left\{\partial_j\left(\partial^j\varphi\,\nabla^2\varphi\right) - \partial_j\left(\partial^i\varphi\,\partial_i\partial^j\varphi\right)\right\} = 8\pi G a^2\rho\delta$$

Linear theory

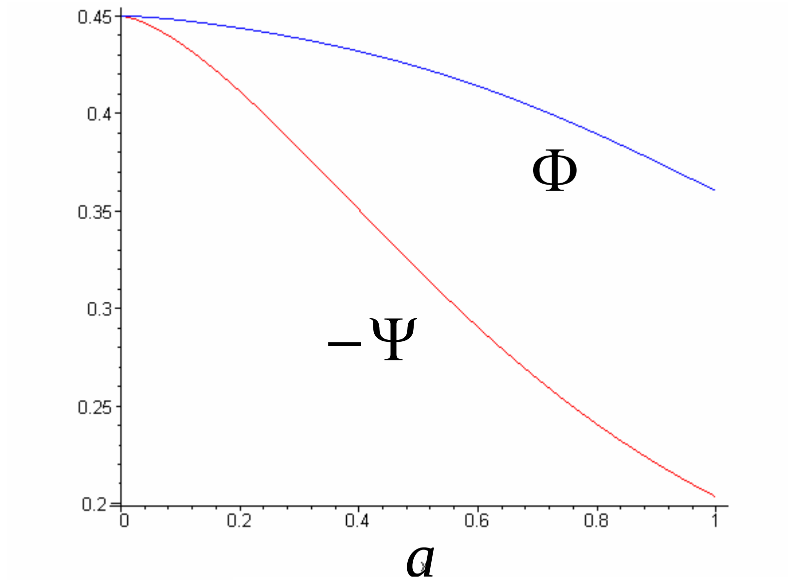
Lue et.al, KK and Maartens

- Solutions for metric perturbations

$$ds^2 = -\left(1+2\Psi\right)dt^2 + a(t)^2\left(1+2\Phi\right)d\vec{x}^2$$

$$\frac{k^2}{a^2}\Phi = 4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta,$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta,$$



$$\omega_{BD} = \frac{3}{2}(\beta-1) \sim O(1)$$

$$\beta = 1 - 2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$

Non-linear evolution

$$-\nabla^2\Phi = 4\pi G a^2 \rho\delta + \frac{1}{2}\nabla^2\varphi, \qquad \Phi + \Psi = -\varphi$$

$$3\beta(t)\nabla^2\varphi + r_c^2\left\{\partial_j\left(\partial^j\varphi\,\nabla^2\varphi\right) - \partial_j\left(\partial^i\varphi\,\partial_i\partial^j\varphi\right)\right\} = 8\pi G a^2\rho\delta$$

- Non-linearity of brane bending becomes important

  when $\beta^2 (H r_c)^{-2} \sim O(1) < \delta$

$$\beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

  then $\nabla^2\varphi \sim \dfrac{H}{r_c}\sqrt{\delta} \sim \nabla^2\Phi \sim H^2\delta$

GR is recovered on non-linear scales

Spherically symmetric solution

(Gruzinov; Middleton, Siopsis; Tanaka; Lue, Sccoccimarro, Starkman)

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3\beta}\left(\frac{r}{r_*}\right)^3\left(\sqrt{1+\left(\frac{r_*}{r}\right)^3} - 1\right)$$

Vainstein radius

$$r_* = \left(\frac{8r_c^2 r_g}{9\beta^2}\right)^{\frac{1}{3}}, \quad r_g = 2G_4M$$

Solar system constraints are avoided if $r_c > H_0^{-1}$

4D Einstein ($r < r_*$)	4D BD ($r_* < r < r_c$)	5D ($r > r_c$)
$\Phi = \frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}}$,	$\Phi = \frac{r_g}{2r}\left(1 - \frac{1}{3\beta}\right)$,	
$\Psi = -\frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}}$	$\Psi = -\frac{r_g}{2r}\left(1 + \frac{1}{3\beta}\right)$	

Problem

- Ghost

Negative BD parameter

$$\omega_{BD} = \frac{3}{2}(\beta - 1) \qquad \beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

In Einstein frame, kinetic term for the scalar $-\frac{3}{2}\beta$
if $\beta < 0$ the scalar becomes a ghost

ghost mediates anti-gravity and suppressed the growth of structure

- Strong coupling problem

  Covariant effective theory

  Minkowski background

$$S = \frac{1}{2}\left(\partial_\mu \varphi\right)^2 + \frac{1}{\Lambda}\left(\partial_\mu \varphi\right)^2 \left(\Box \varphi\right), \quad \Lambda = \left(\frac{M_4}{r_c^2}\right)^{\frac{1}{3}}$$

We need quantum gravity below $\Lambda^{-1} = 1000\,\mathrm{km}$!

This is due to the fact φ disappears as $r_c \to \infty$

- Strong gravity

  no known BH solution

DGP gravity summary

- Classically, the model works very well

  there is only one parameter and we do not need to introduce additional mechanism to recover solar system constraints

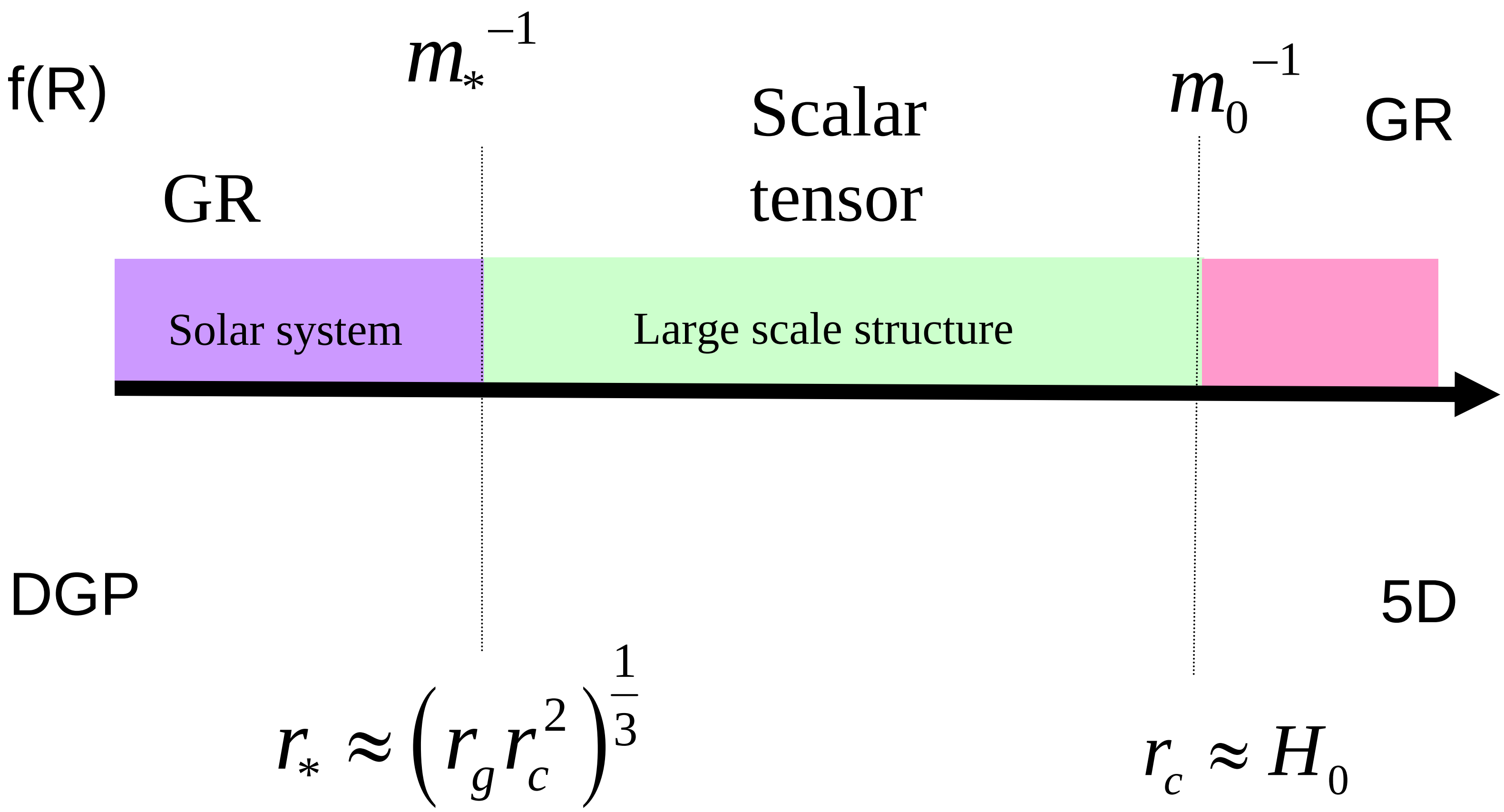
- Theory shows pathologies at quantum level

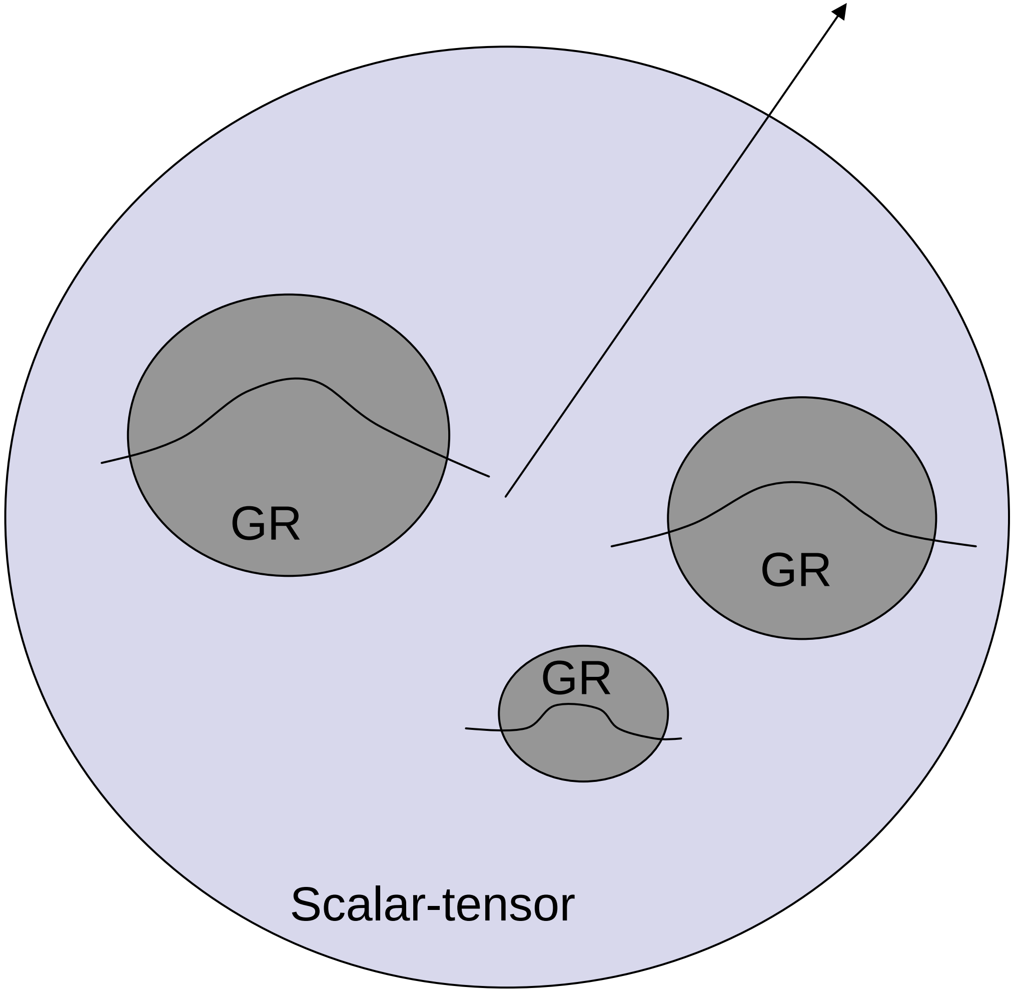
  there are debates on whether they are fatal or not

  KK, Class.Quant.Grav.24:R231-R253,2007.arXiv:0709.2399 [hep-th]

Common features in f(R) and DGP

- 3 regime of gravity




GR
GR
GR
Scalar-tensor