Modified gravity as an alternative to dark energy

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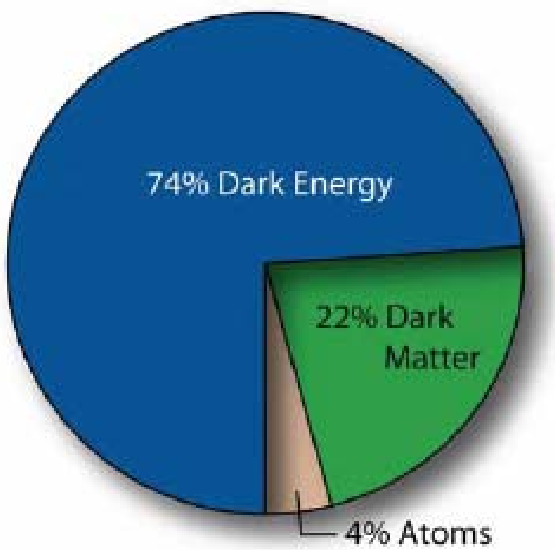
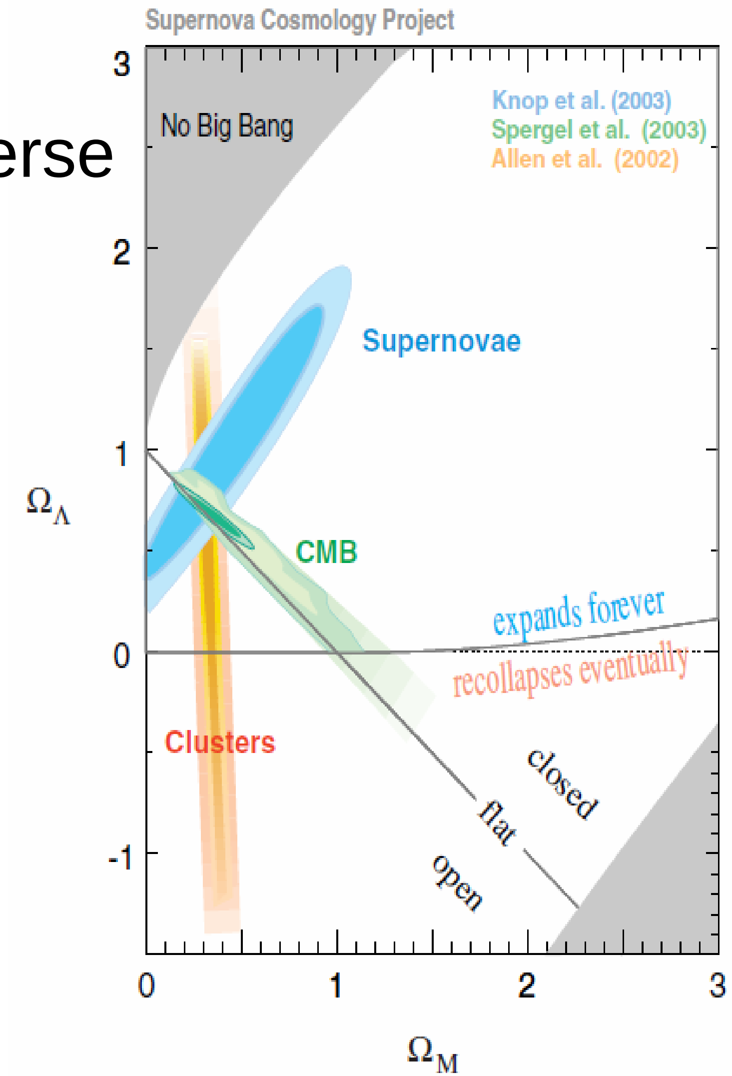
Cosmic acceleration

- Many independent data sets indicate expansion of the Universe is accelerating

- Standard cosmology requires 74% of unknown 'dark' energy





Dark energy v Dark gravity

- Cosmological constant is the simplest candidate

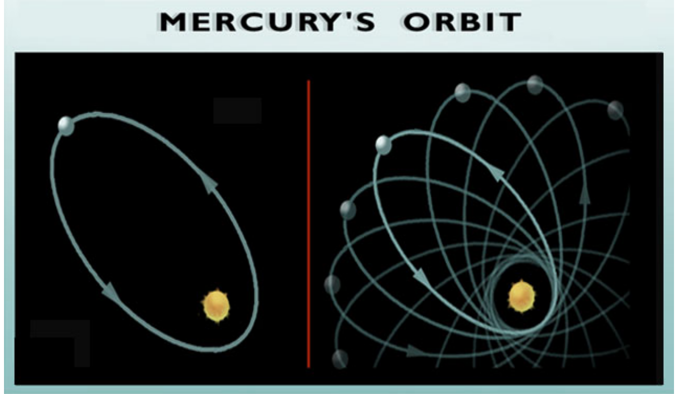
  but the theoretical prediction is more than 50 orders of magnitude larger than the observed values

  - ***Most embarrassing observation in physics***

- Standard model of cosmology is based on GR but we have never tested GR on cosmological scales

  cf. precession of perihelion

  dark planet v GR



Objective

Seek solutions to the question of dark energy by challenging conventional GR

- construct consistent theoretical models building on rapid progress in understanding the law of gravity beyond GR
- develop efficient ways to combine observational data sets to distinguish modified gravity models from dark energy models based on GR
- provide tests of GR on largest scales

Plan of lectures

- Lecture 1 Motivation for modified gravity
- Lecture 2 Theory of modified gravity
- Lecture 3 Observational constraints

Modified gravity as an alternative to dark energy

Lecture.1 Motivation

Cosmological constant

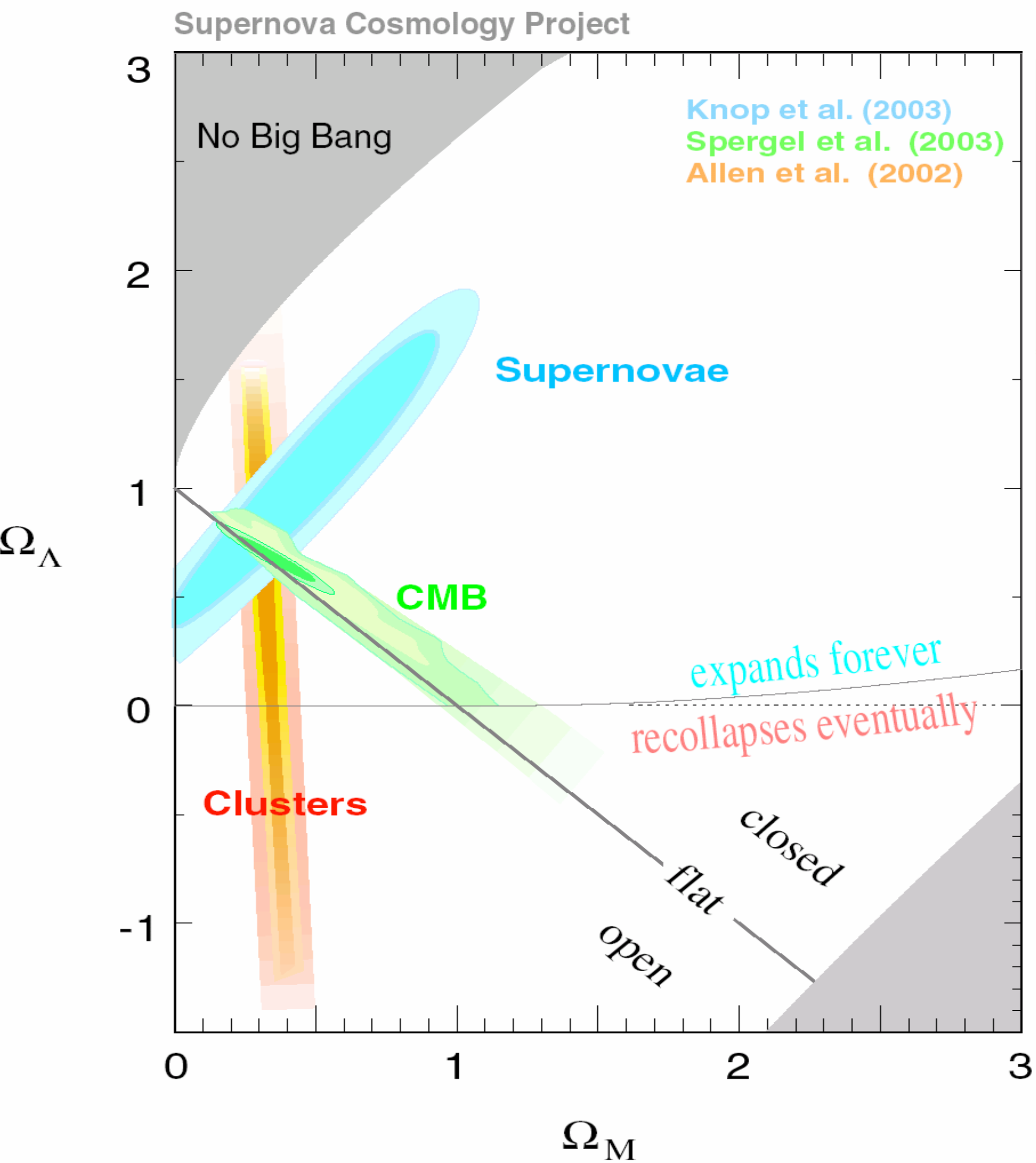
- Prior from theory

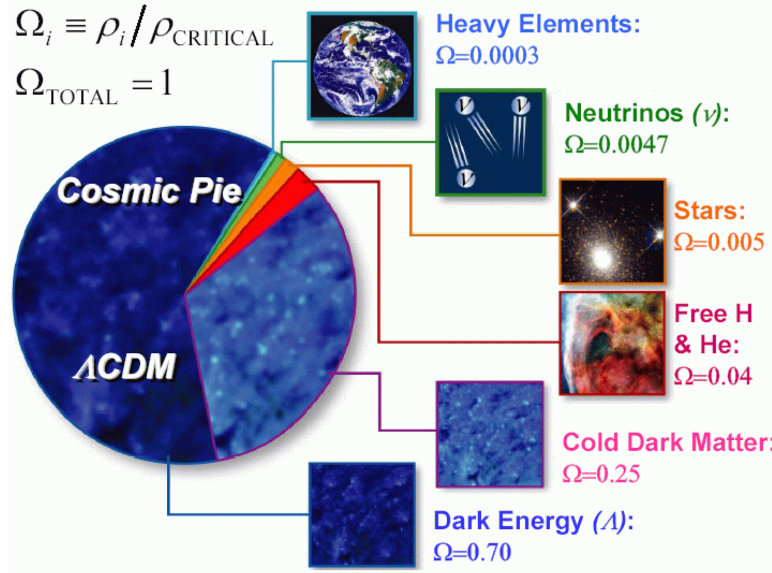
cosmological principle + general relativity

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{K}{a^2} + \frac{\Lambda}{3}$$

$$1 = \Omega_{m0} + \Omega_{K0} + \Omega_{\Lambda 0}$$

- Fit to observational data





$$\Omega_\Lambda = 0.7, \quad \Omega_m = 0.3$$

If this is true...

- There is a cosmological constant

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \sim H_0^2 M_{pl}^2 = (10^{-33}\mathrm{eV})^2 (10^{19}\mathrm{GeV})^2 = \left(10^{-3}\mathrm{eV}\right)^4$$

- Then we have an incredible fine-tuning

$$\Lambda_{tot} = \Lambda_{classical} + \Lambda_{quantum}$$

$\Lambda_{classical}$:determined by boundary condition of the Universe

$\Lambda_{quantum}$: determined by UV (high energy) cut-off of QFT

standard model $\rho_{\Lambda_{SM}} \sim (\mathrm{TeV})^4$

$\Rightarrow$ $\rho_{classical} = (10^{-3}\mathrm{eV})^4 - (10^9\mathrm{eV})^4$ fine-tuning!!

(old) cosmological constant problem

Why there is (almost) no cosmological constant?

- Supersymmetry $\Lambda_{quantum} = \Lambda_{boson} + \Lambda_{fermion} = 0$

  does not help as it must be broken at TeV

- Anthropic principle/Landscape

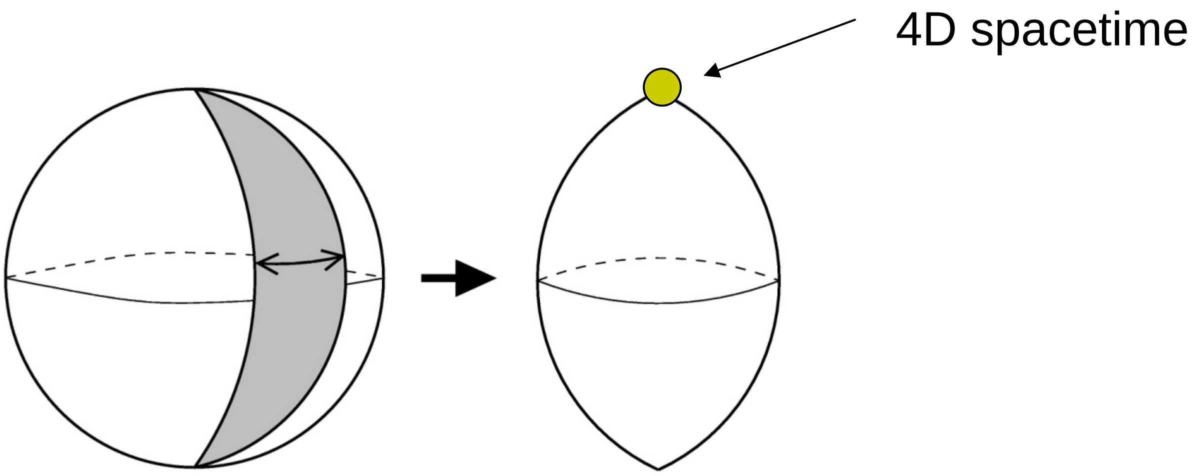
- Modified gravity

  Even if there is a vacuum energy, it may be possible that it does not curve our 4D spacetime

- 6D models

2-dimensional extra-dimensions

cosmological constant in 4D spacetime only change a geometry of 2-extra-dimensions and 4D spacetime remains flat



Still no fully successful model is obtained

(new) cosmological constant problem

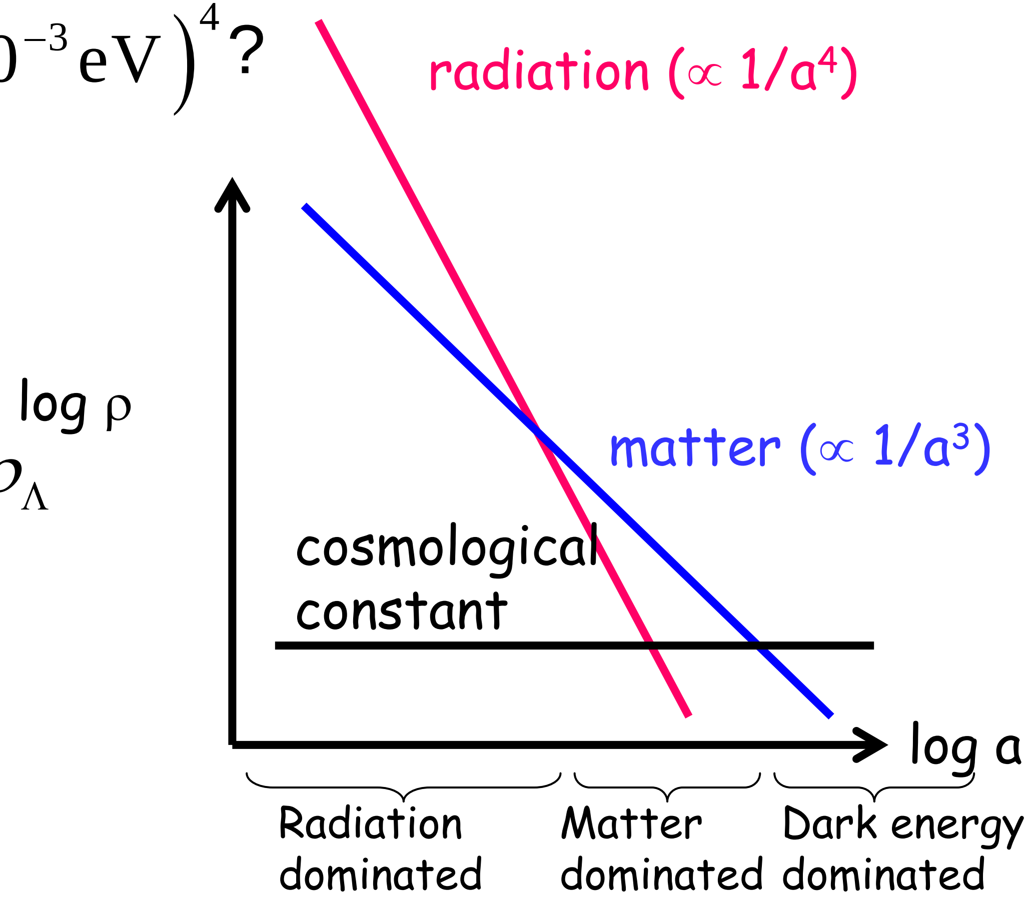
Assuming $\Lambda_{tot} \simeq 0$ is realized somehow

how to explain $\rho_\Lambda = \left(10^{-3}\,\mathrm{eV}\right)^4$?

‘Why now’ problem

at present $\rho_m \simeq \rho_\Lambda$

Implies in the past $\rho_m \gg \rho_\Lambda$



Anthoropic principle?

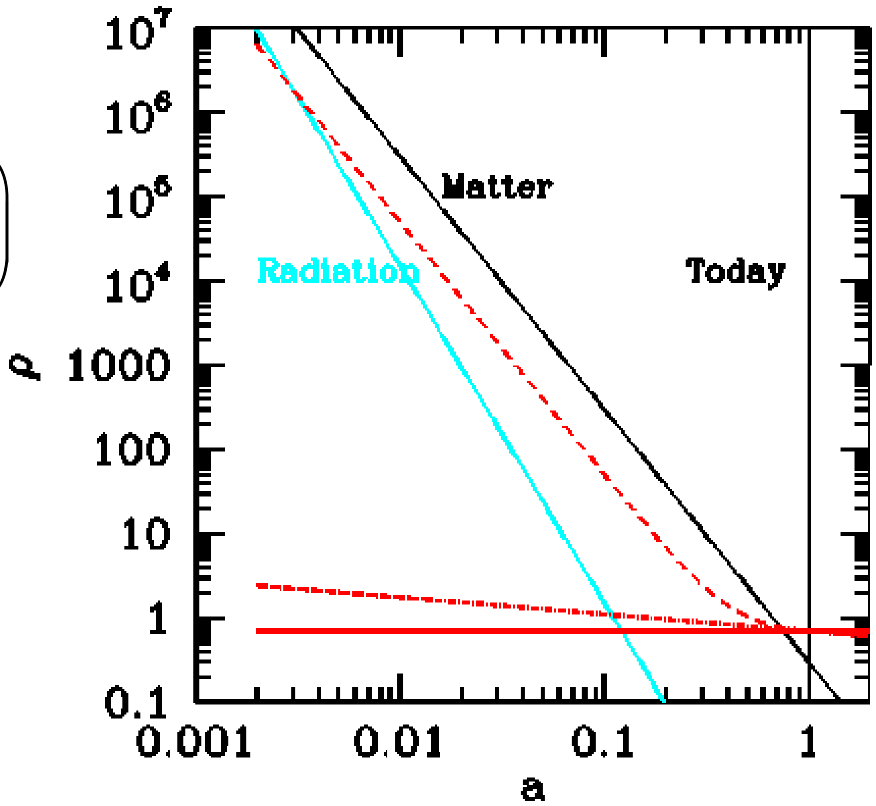
Quintessence

- Scalar field

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

- Tracking solution

$$\rho_\phi \approx \rho_r , \rho_m$$



All model so far introduces other tiny scales $V_0 , V_0 ', \ldots$
and need fine-tuning to get the acceleration

Dark energy

- Dark energy

let us call unknown matter that drives acceleration dark energy

$$P_{DE} = w_{DE}\rho_{DE}$$

it can be anything

Still no one knows what it is and there is no true alternative to cosmological constant

Alternatives?

If $\rho_{DE} = 0$, then what can account for the acceleration?

Basic assumption

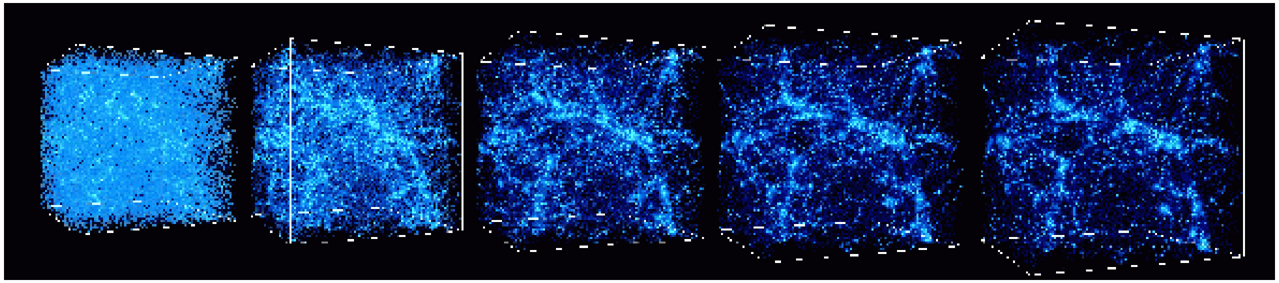
cosmological principle + general relativity

⇨ inhomogeneous universe

modification of gravitational theory

Inhomogeneous universe

- Back-reaction



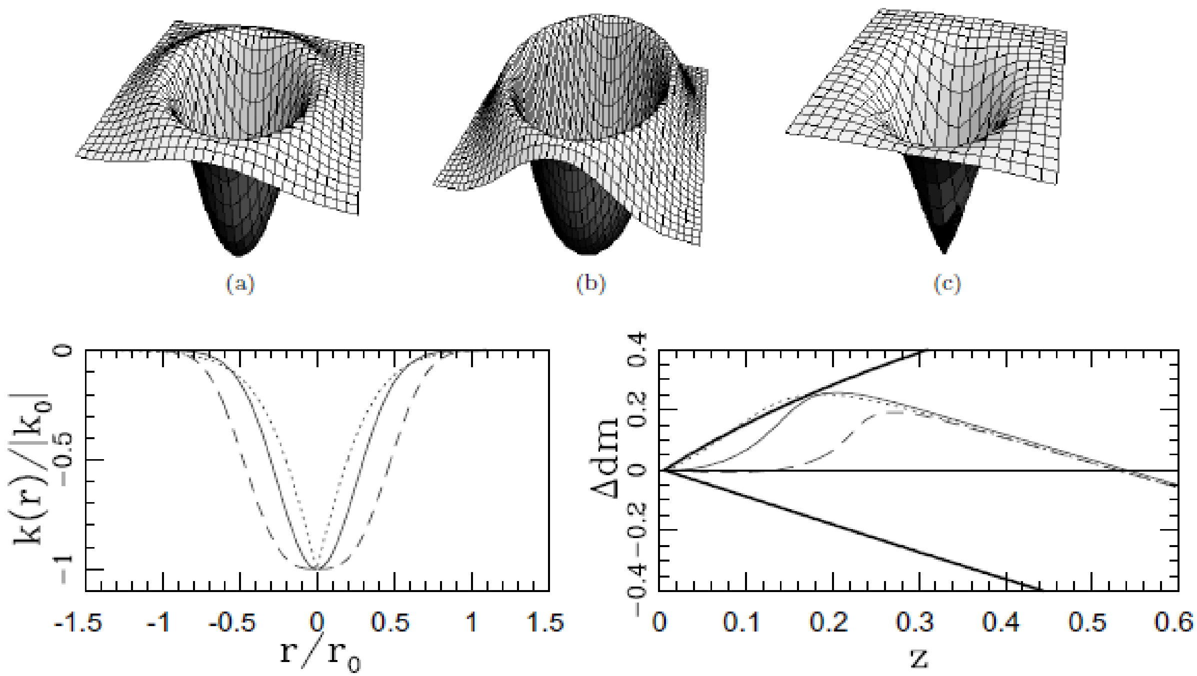
As structure grows, the Universe becomes inhomogeneous which can cause the acceleration?

- It is a tough problem in GR how to define observables in an inhomogeneous universe
- Back-reaction from structure formation must be small

$$\Phi < 1$$

- Give up Copernican principle

  we are living in a large void (Tomita 2000, ...)



(Clifton et.al. arXiv:0807.1443)

It is not clear how to calculate other observations such as CMB, BAO etc.

Modified gravity

- Modification of GR on cosmological scales

  dark energy dominates at late times

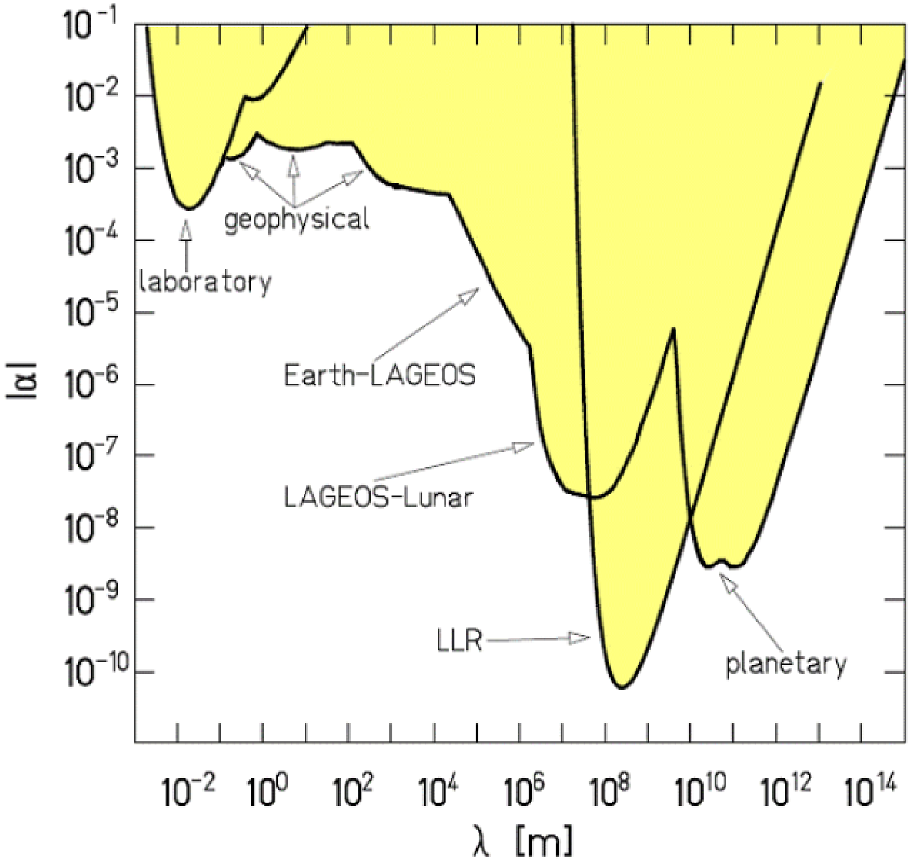
  in order to mimic dark energy, gravity must be modified at large distance/ low energies

- Local / solar system test of gravity

  Any modified gravity models should pass the existing constraints on the deviation from GR

Constraints on gravity

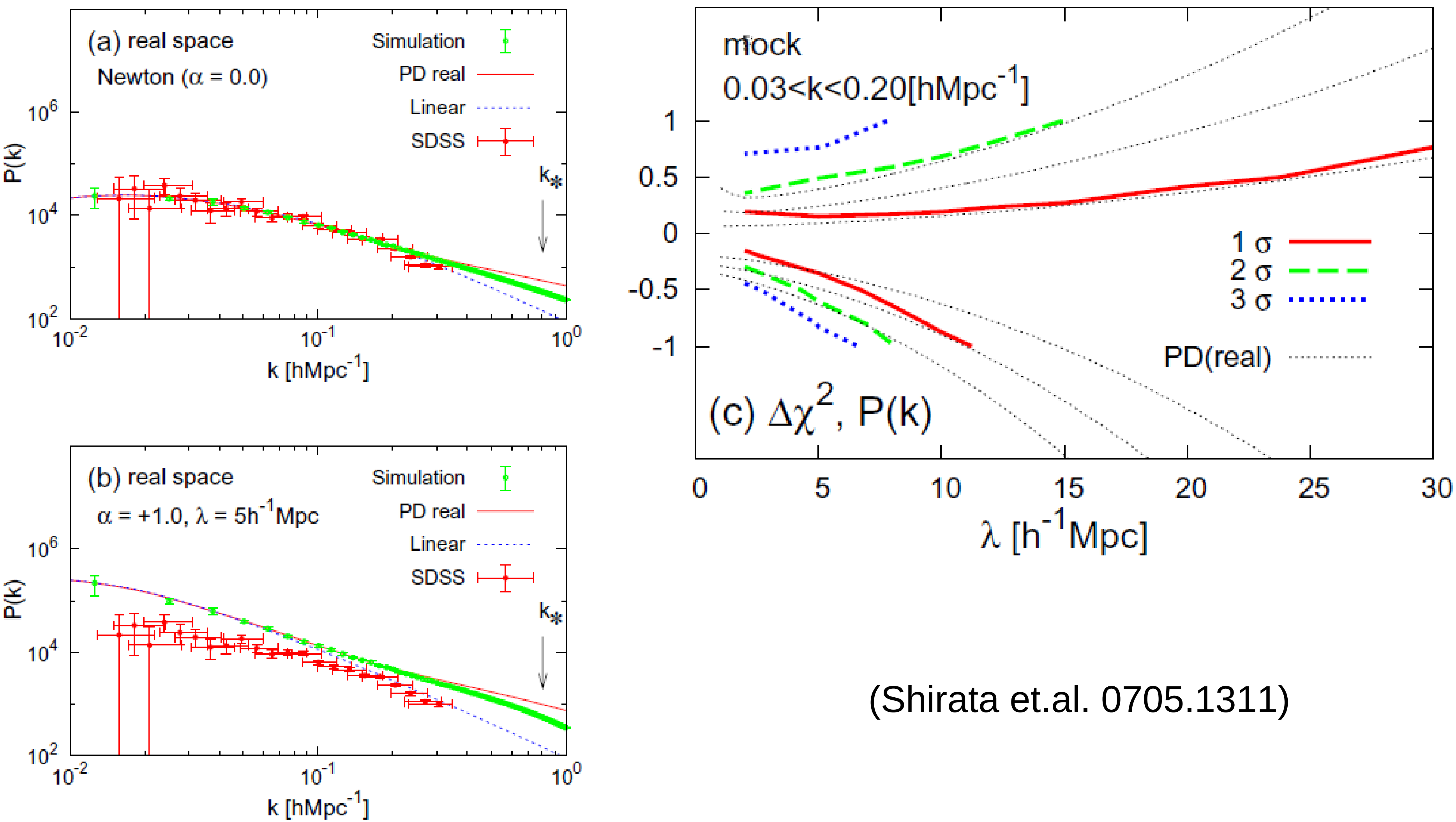
- Constraints on Newton's law of gravity



$$V(r) = -G\frac{m_1 m_2}{r}\left[1+\alpha\exp\left(-\frac{r}{\lambda}\right)\right]$$

(Adelberger et.al. hep-ph/0307284)

- On cosmological scales
  shape of power spectrum



(Shirata et.al. 0705.1311)

Constraint on GR

- PPN formalism

$$ds^2 = -(1-2U)dt^2 + (1+2\gamma U)\delta_{ij}dx^i dx^j \quad U = \int d^3x' \frac{\rho(x')}{|x-x'|}$$

$$|\gamma - 1| < 2.3\times 10^{-5}$$ time delay (Cassini)

ex) Brans-Dicke gravity

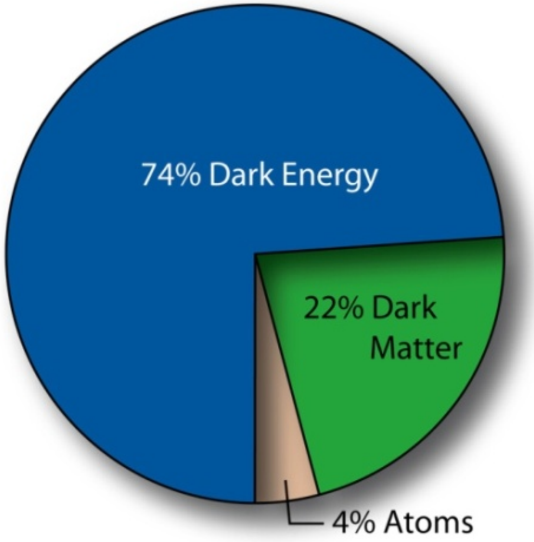
$$S = \int d^4x \left(\Psi R - \frac{\omega_{BD}}{\Psi} (\nabla\Psi)^2 \right)$$

$$\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}} \qquad \omega_{BD} > 40000$$

- Cosmological scale

  test of GR is available only in cosmology

success of cosmology may indicate that GR is valid on cosmological scales *given that we know dark energy*



or GR is badly broken on cosmological scales

Decoupling theorem

- High energy corrections

Quantum gravity is important at high energies at low energies, $E < M_{pl}$ it's effect is negligible

- Low energy corrections

Even if we try to modify gravity at low energies, there could be modifications at high energies $E > M_{MG}$

this can spoil the success of GR at small scales

Challenge for Modified gravity

- It should explain cosmic acceleration without Λ

  modification of gravity does not necessarily leads to the acceleration

- It should not spoil the success of GR on small scales

  cf Brans-Dicke theory $\omega_{BD} > 40000$

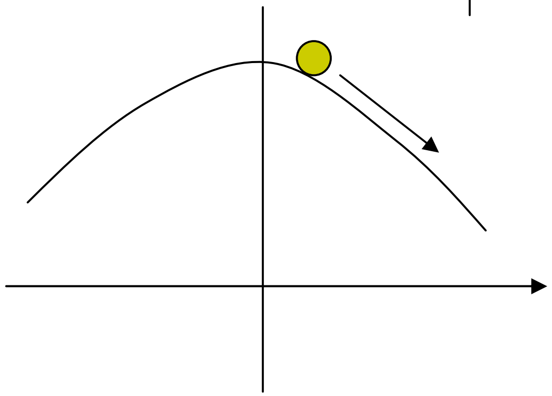
  In order to mimic DE, we need O(1) modification to GR

- It should not have pathologies
  - Tachyon

$$S=\int d^4x\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right)$$

there is a time scale of instability

$$t \sim \sqrt{-m^2}$$

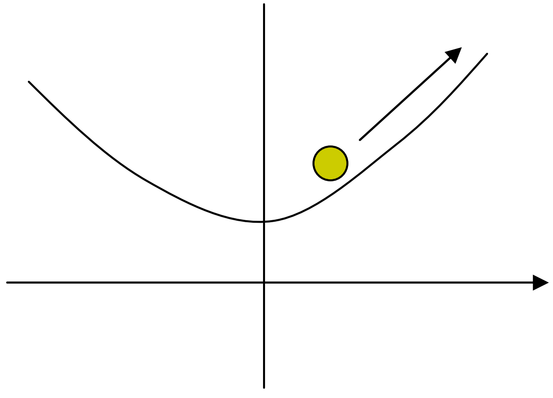


  - Ghost

$$S=\int d^4x\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right)$$

wrong sign for its kinetic term

$$\rho=-\frac{1}{2}\dot{\phi}^2+V$$

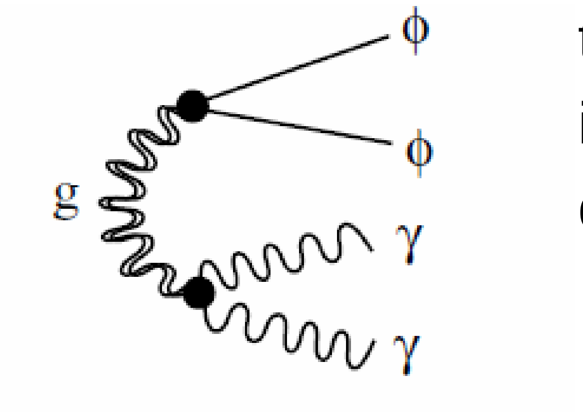


Problem of ghosts

ghost carries negative energy density

quantum mechanically, particles can be created from vacuum without costing any energy

⇨ instability of vacuum



there is no time scale for instability in Lorentz invariant theory

decay is instantaneous

- Strong coupling problem

  quantum loops can introduce higher order terms in action suppressed by a scale M

$$S = \int d^4x \left(\frac{1}{2} Z(\phi)\partial_\mu \phi \partial^\mu \phi + O\left(\frac{(\partial_\mu \phi)^2 \Box \phi}{M^3} \right) \right)$$

In terms of canonically normalized filed, the UV scale is $\tilde{M} = \sqrt{Z} M$

When $Z \to 0$ all the higher terms becomes important and the theory looses its predictive power

cf GR $\tilde{M} \sim M_{pl}$

It turns out that it is really hard to find even a toy model that satisfies these conditions!

We need to go further to ask

can we evade the fine-tuning problem?

The small number will appear in a different way

Is it stable against radiative corrections?

Is there something like see-saw mechanism?

Unfortunately we are not in a position to answer these questions yet...

- Several attempts achieved partial success in the first step and they teach us valuable lessons

  f(R) gravity model

  DGP braneworld model

- There are other attempts to unify dark energy and dark matter from modified gravity using Lorentz invariance violation

  ghost condensation $S = \int d^4x \sqrt{-g} P(X), X = \frac{1}{2}(\partial\phi)^2, \phi_0 = Mt$

  (generalized) Einstein-Aether $A^\mu = (1, 0, 0, 0)$

These fields change gravity even at linearized level (but not changing the theory of gravitation)