RESCEU-DENET joint meeting & summer school

Stability of Freund-Rubin compactification

Shunichiro Kinoshita

(University of Tokyo)

with S. Mukohyama and T. Kobayashi

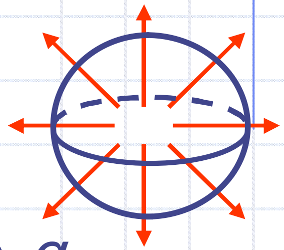
Introduction

- De Sitter spacetime
  - It well describes our accelerating Universe in the early epoch and in the present epoch
- Realization the Universe in higher-dimensional theories
  - We need to stabilize the compact extra-dimensions
  - Without stabilization, it is impossible to obtain effective 4D theory

We want successful embeddings of 4D de Sitter space in higher-dimensions

Freund-Rubin compactification

Freund and Rubin 1980



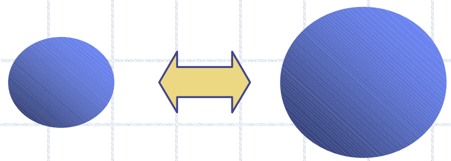
- Simple model with compactification and stabilization mechanism by flux
  - There is a *q*-form flux field for stabilizing the *q*-dimensional compact space, considering (*p*+*q*)-dimensional spacetime
  - Introducing bulk cosmological constant allows an external de Sitter space and an internal manifold with positive curvature
- We obtain a (*p*+*q*)-D product spacetime dS_p and S^q ,Freund-Rubin solution:

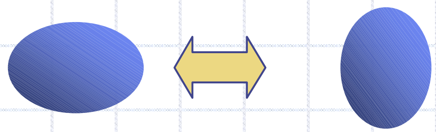
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2ht}\mathrm{d}\vec{x}^2_{p-1} + \rho^2 \mathrm{d}\Omega^2_q \qquad F_{(q)} = b\epsilon_{(q)}$$

Dynamical stability of Freund-Rubin compactifibations

R. Bousso, O. DeWolfe and R. C. Myers 2003

- The analysis of linear perturbations show that there are two channels of instabilities in the scalar sector
  - Homogeneous excitation ($l=0$ mode)
  - Inhomogeneous excitations with higher multi-pole moments ($l \geqq 2$ mode)





Homogeneous mode (l=0)

- It is so-called volume modulus or radion, which corresponds to the change of the radius of the extra-dimensions
- It becomes unstable as the Hubble parameter h of dS_p becomes very large (the flux density b small)
  - Stability condition:

$$h^2 \le \frac{2\Lambda(p-2)}{(p-1)^2(p+q-2)} \quad \text{or} \quad b^2 \ge \frac{2\Lambda}{(p-1)(q-1)}.$$

  - This seems to be a generic feature of de Sitter compactifications for other models

Higher multi-pole modes ($l \geqq 2$)

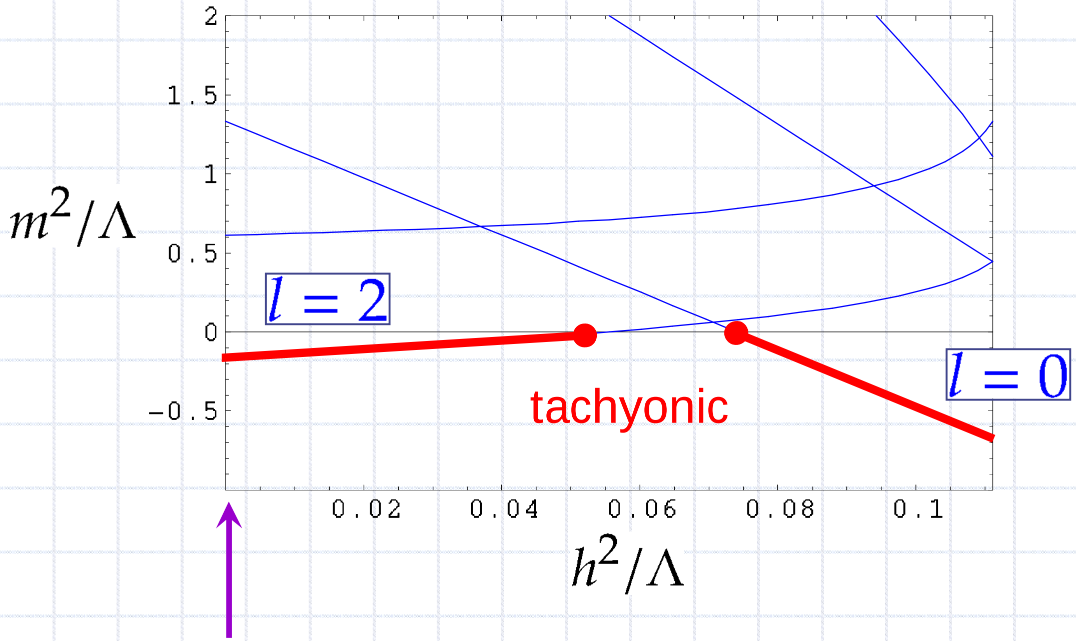
- It corresponds to deformation of the shape of the internal compact space
- It becomes unstable as the flux density *b* becomes very large (the Hubble parameter *h* small, including Minkowski)
  - Stability condition:

$$h^2 \geq \frac{2\Lambda[2+q-3pq+(p-1)q^2]}{q(q-3)(p-1)^2(p+q-2)} \quad \text{or} \quad b^2 \leq \frac{4\Lambda}{q(q-3)(p-1)}.$$

  - These tachyonic modes appear as the number of the extra-dimensions is large, $q \geqq 4$

KK mass spectrum of Freund-Rubin solution

- The case p=4 and q=4



When external spacetime is Minkowski, this configuration is unstable

How can we interpret physically?

Thermodynamic argument

S. K., Y. Sendouda and S. Mukohyama 2007

- The instability arising from l=0 mode can be interpreted based on thermodynamic argument using de Sitter entropy
  - Entropy: $S = \frac{\mathcal{A}}{4} = \frac{\Omega_{p-2}}{4h^{p-2}}\Omega_q \rho^q$, Total flux: $\Phi = \oint F_{(q)}$
  - Relation between entropy and total flux

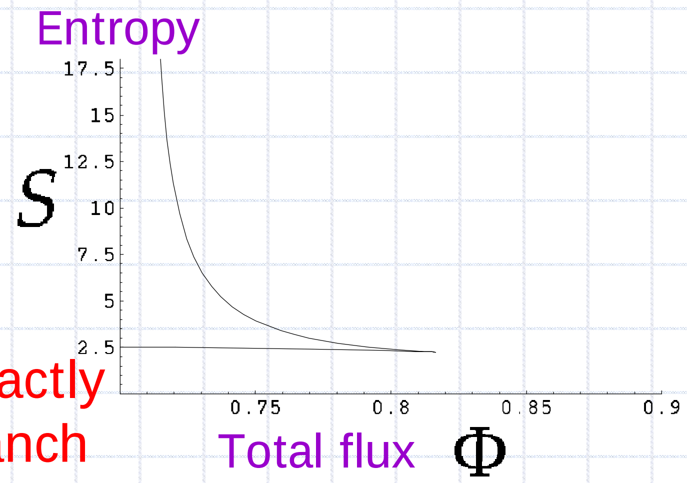
$$dS = -\frac{\Omega_{p-2} b}{4(p-1)h^p} d\Phi$$

The entropy is a function of the total flux

Correlation between dynamical and thermodynamic stability

- The entropy is a double valued function of the total flux
  - The family of solutions is divided into two distinct branches: high-entropy and low entropy branches

The high-entropy branch must be preferred thermodynamically



Dynamically stable solutions for l=0 mode exactly correspond to those of the high-entropy branch

Deformed solutions

S.K. 2007

- The higher multi-pole instabilities suggests the existence of a new branch of solutions
  - Solutions with a deformed internal space
- We explore warped product solutions such that the *p*-D external space has a de Sitter symmetry with a warp factor and the *q*-D internal space is a deformed sphere

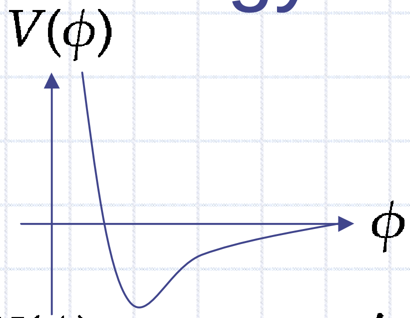
$$ds^2 = e^{2\phi(r)}[-dt^2 + e^{2ht}d\vec{x}^2_{p-1}] + e^{-\frac{2p}{q-2}\phi(r)}[dr^2 + a^2(r)d\Omega^2_{q-1}],$$

ansatz

$$F_{(q)} = b e^{-\frac{2p(q-1)}{q-2}\phi} a^{q-1} dr \wedge d\Omega_{q-1},$$

Equations of motion

- The system is similar to FRW cosmology with scalar field



EOM:
$$\begin{cases} \phi'' = -(q-1)\mathcal{H}\phi' - \dfrac{q-2}{p(p+q-2)}V_\phi(\phi), \\ \mathcal{H}' = -\mathcal{H}^2 - \dfrac{p(p+q-2)}{(q-1)(q-2)}\phi'^2 + \dfrac{2}{(q-1)(q-2)}V(\phi), \end{cases}$$

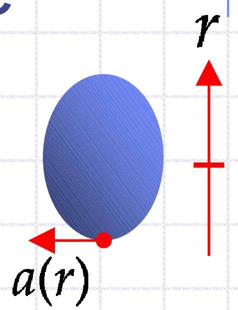
$$\mathcal{H} \equiv \frac{a'}{a}$$

Potential: $$V(\phi) \equiv \frac{p(p-1)}{2}h^2 e^{-\frac{2(p+q-2)}{q-2}\phi} + \frac{b^2}{2}e^{-\frac{2p(q-1)}{q-2}\phi} - \Lambda e^{-\frac{2p}{q-2}\phi}$$

Trivial FR solution corresponds to $\phi(r) = \text{const.}$ at the bottom of $V(\phi)$

Numerical solutions for p=4 and q=4

- We integrate the ODEs to obtain the desired solutions satisfying the boundary conditions



  - boundary conditions:

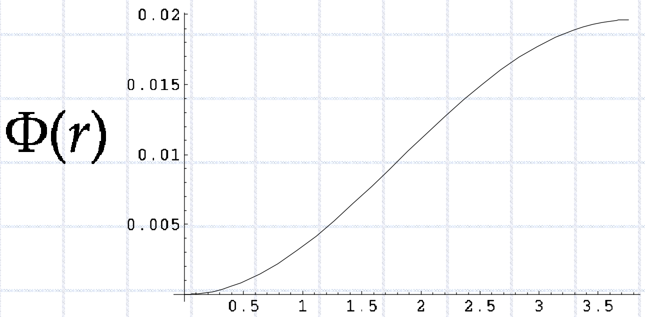
$$\Phi'(0) = 0, a'(0) = 0$$

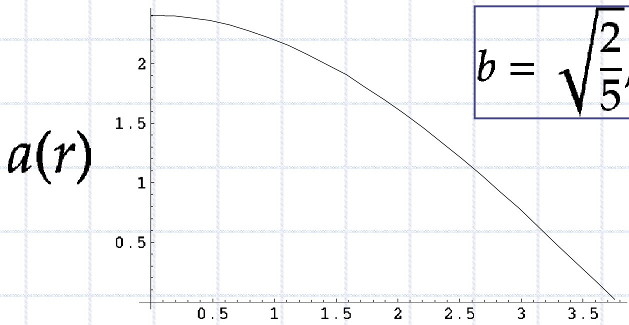
Symmetric at the equatorial plane

$$\Phi'(r_0) = 0, a(r_0) = 0, a'(r_0) = 1$$

regular at the pole

example

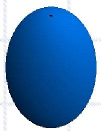


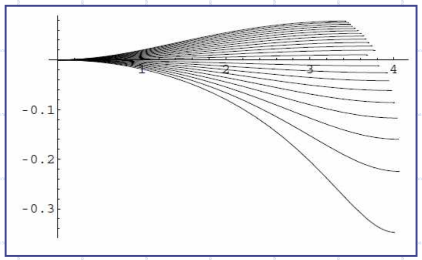


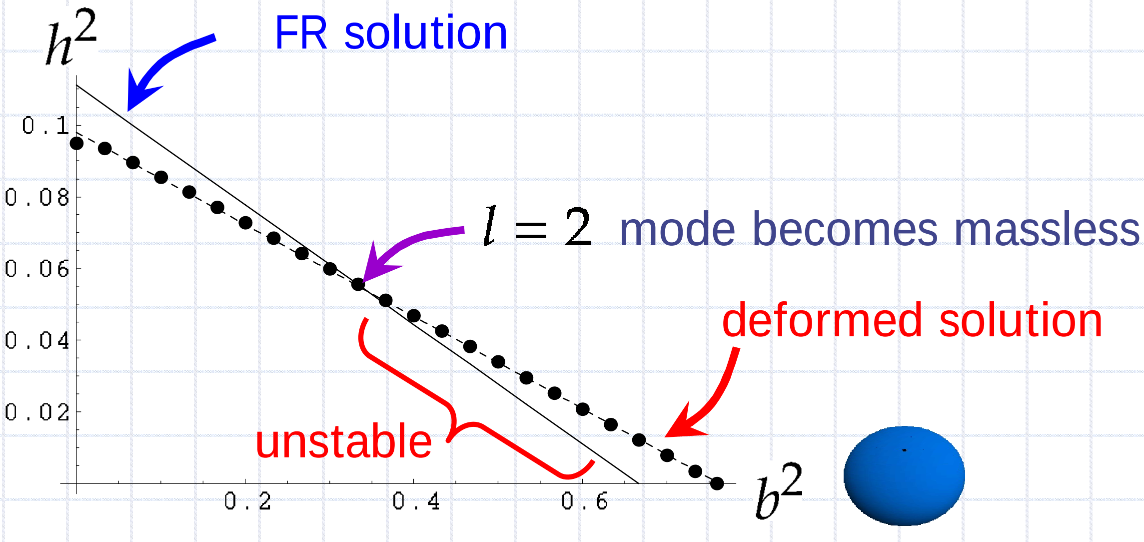
$$b = \sqrt{\frac{2}{5}}, h = 0.21656$$

Two families of solutions

- The branch of trivial solutions and that of deformed solutions intersect at one point







The new branch emanates from the marginally stable FR solution

Dynamical stability of new branch of solutions

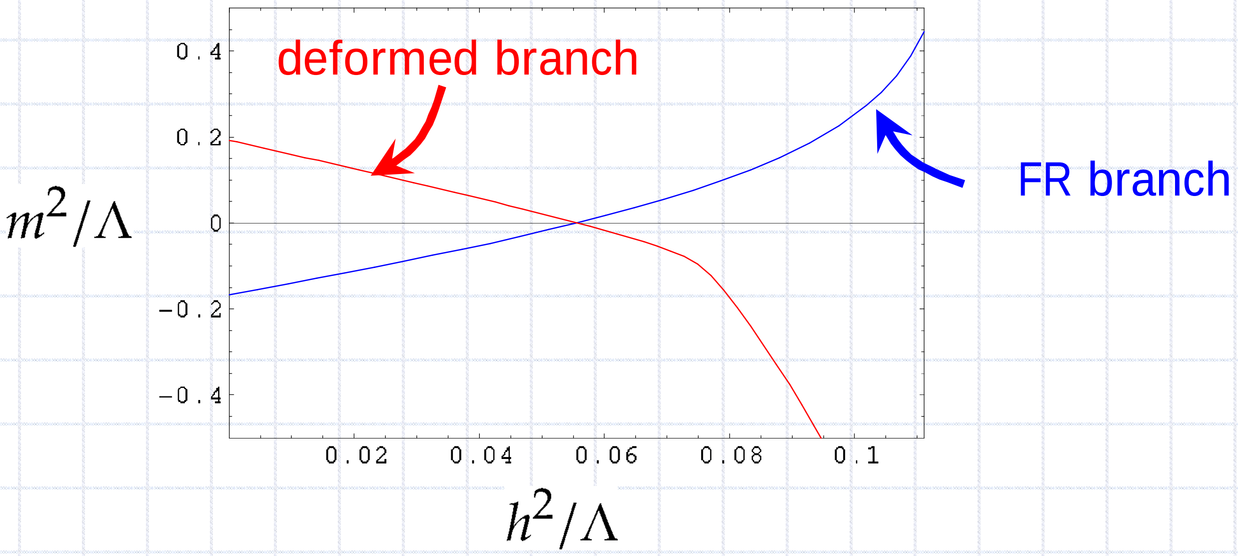
- We consider scalar perturbations around the deformed solutions
  - For simplicity, we assume perturbations have *SO(q)* symmetry

$$ds^2 = (1+\Pi)A^2(\rho)g_{\mu\nu}dx^\mu dx^\nu + (1+\Pi-\Omega)d\rho^2 + (1+\pi)B^2(\rho)q_{ij}dy^i dy^j$$

Perturbation equations

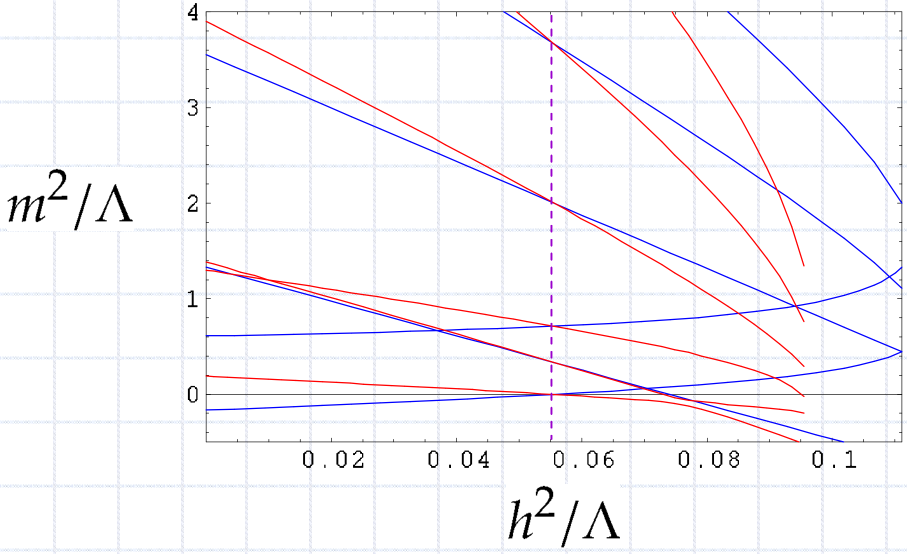
$$\begin{cases} (p+q-2)\Pi'' + (q-2)\Omega'' + (p+q-2)\left[n\frac{A'}{A} - (q-1)\frac{B'}{B}\right]\Pi' + (q-2)\left[n\frac{A'}{A} + (q-1)\frac{B'}{B}\right]\Omega' \\ \quad + \left[\frac{m^2}{A^2} + \frac{2(q-2)}{B^2}\right][(p+q-2)\Pi - q\Omega] = 0, \\ \Omega'' + \left[(3p-2)\frac{A'}{A} + 3(q-1)\frac{B'}{B}\right]\Omega' + \left[\frac{m^2 + 2h^2(p-1)^2}{A^2} + \frac{2q(q-2)}{B^2} - 4\Lambda\right]\Omega \\ \quad - \left[\frac{2(p+q-2)(q-2)}{B^2} - 4\Lambda\right]\Pi = 0, \end{cases}$$

- The KK mass for *l*=2 mode



The *l*=2 mode for deformed solutions is stable where trivial FR solutions are unstable

- KK mass spectrum（l=0,1,2）

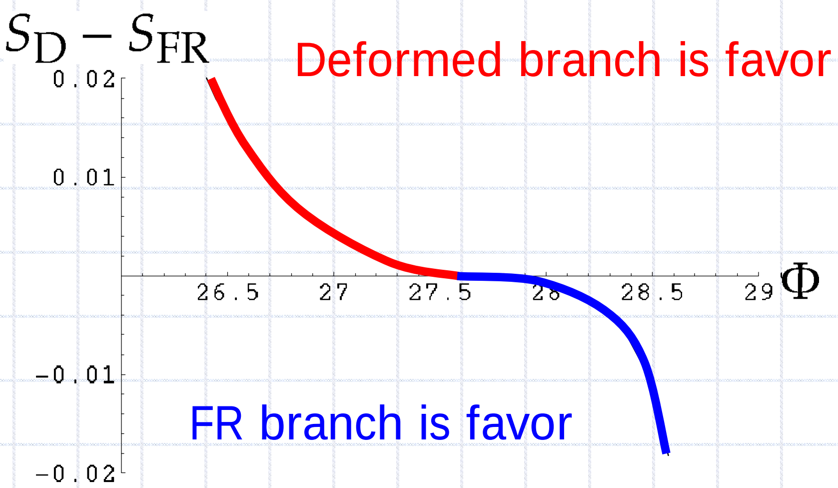


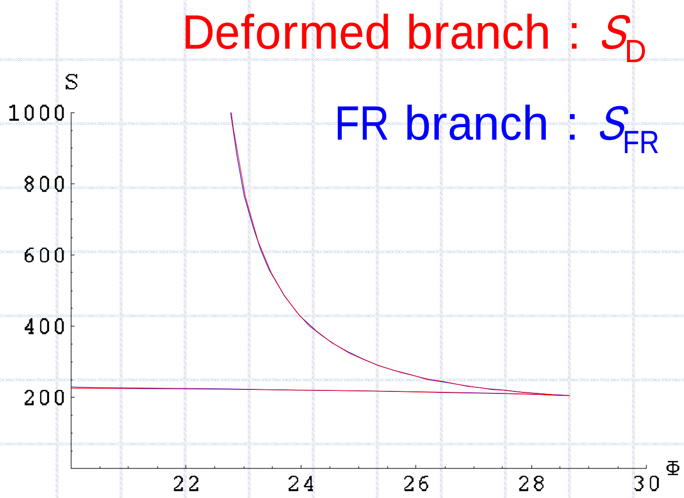
m^2 tends to lift up at the side of lower h^2 solutions

Deformation will stabilize the internal space for the low Hubble region

Thermodynamic stability

- Entropy argument





Correlation between thermodynamic and dynamical stability exists for deformed solutions

Summary

- We have studied new solutions in Freund-Rubin compactification
  - We found warped solutions with a deformed sphere of the internal space in the case p=4 and q=4
  - New branch emanates from the marginally stable solutions for the l=2 mode
- We have analyzed linear perturbations of deformed Freund-Rubin solution
  - The deformed solutions are stable for the region where Freund-Rubin solutions are unstable for l=2 mode
- Correlation between thermodynamic stability and dynamical stability exists for FR solutions and deformed solutions