

# Non-Gaussianity from Radiation and/or CDM

2. Sep. 2008

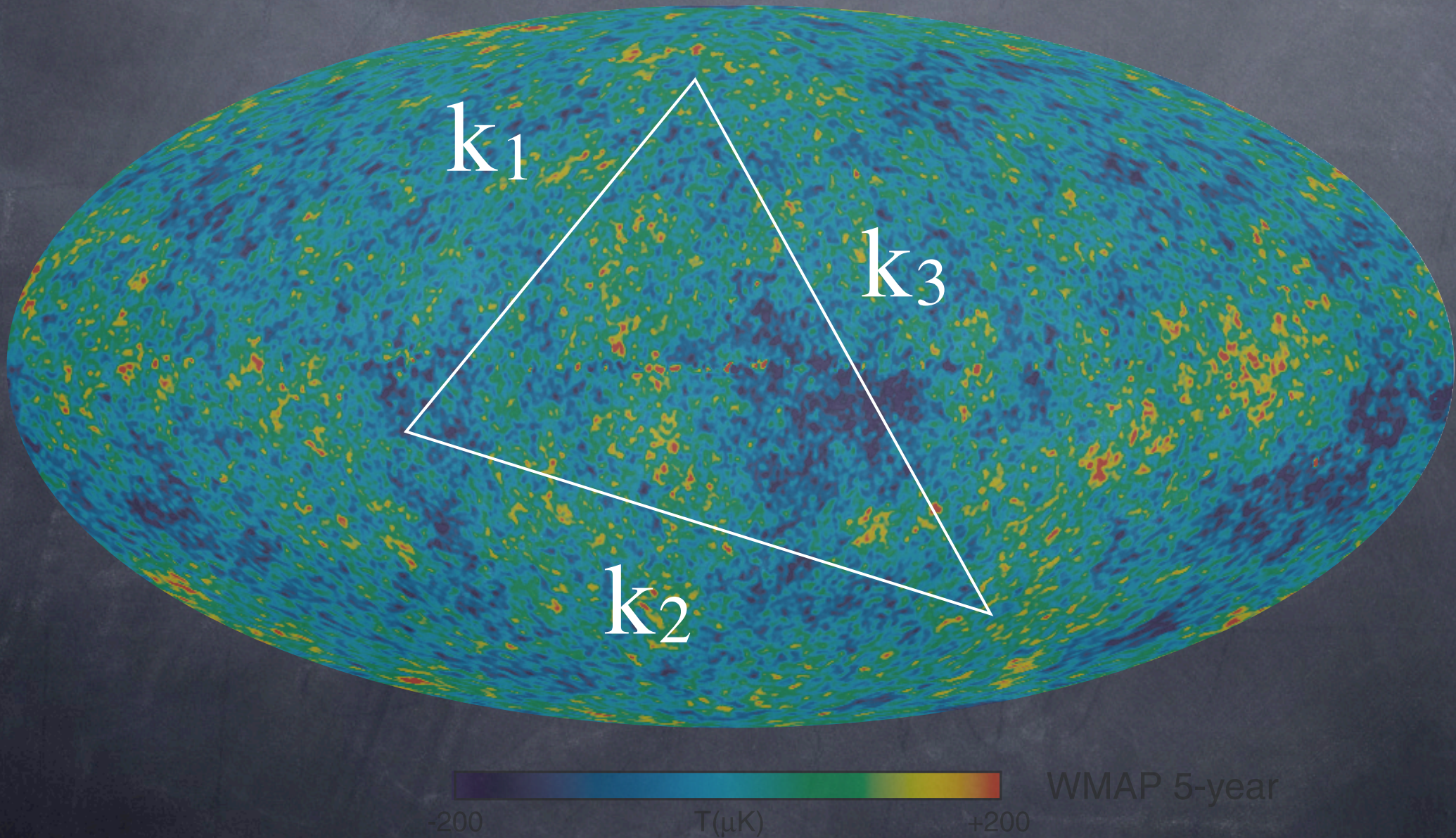
@ Asamushi-onsen, RESCEU/DENET summer  
school "Dark Energy in the Universe"

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(IPMU, Univ. of Tokyo)

arXiv:0804.0425 (to appear in JCAP), T. Suyama and F.T.

arXiv:0808.0009, Kawasaki, Nakayama, Sekiguchi, Suyama, and F.T.

# 1. Introduction



**Non-Gaussianity**, if detected, will give us a clue on the origin of the density perturbations.

Except for NG, the std. slow-roll inflation seems to be perfectly consistent with the observations.

Does detection of NG mean that we are not on the right track?

Maybe one needs some radical modifications:  
e.g.) DBI, ghost inflation, ekpyrotic etc..

**Conservative modification is to add light scalar(s)!**

# Then, why does NG exist?

NG, although large, does not seem to play any important role in cosmology.

Non-Gaussianity may be simply **common** in theoretical space.

# Symmetry

Guiding principle in modern physics.

In string theory, gauge and discrete symmetries are ubiquitous.

## Scalar fields



# Symmetry

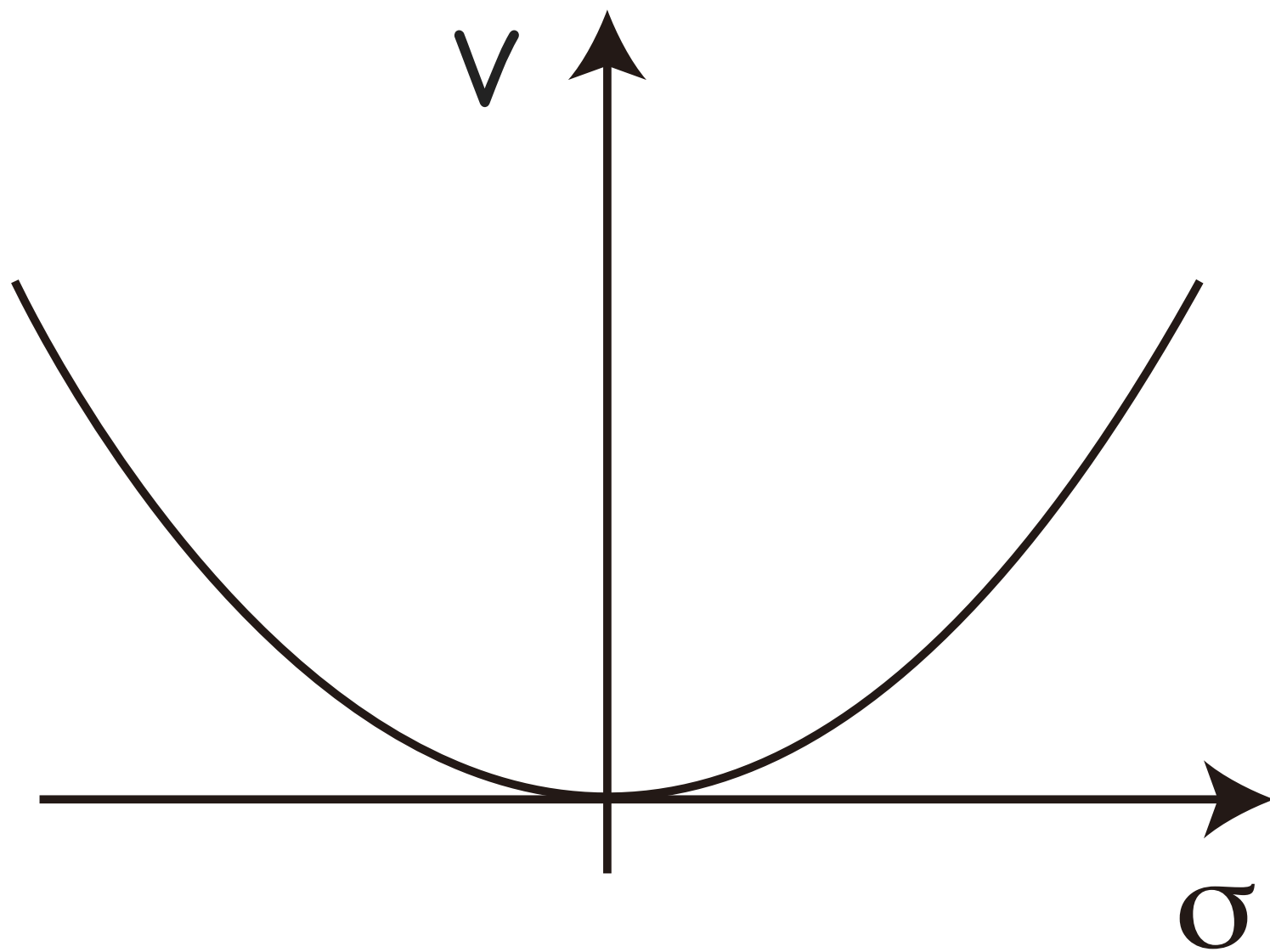
Guiding principle in modern physics.

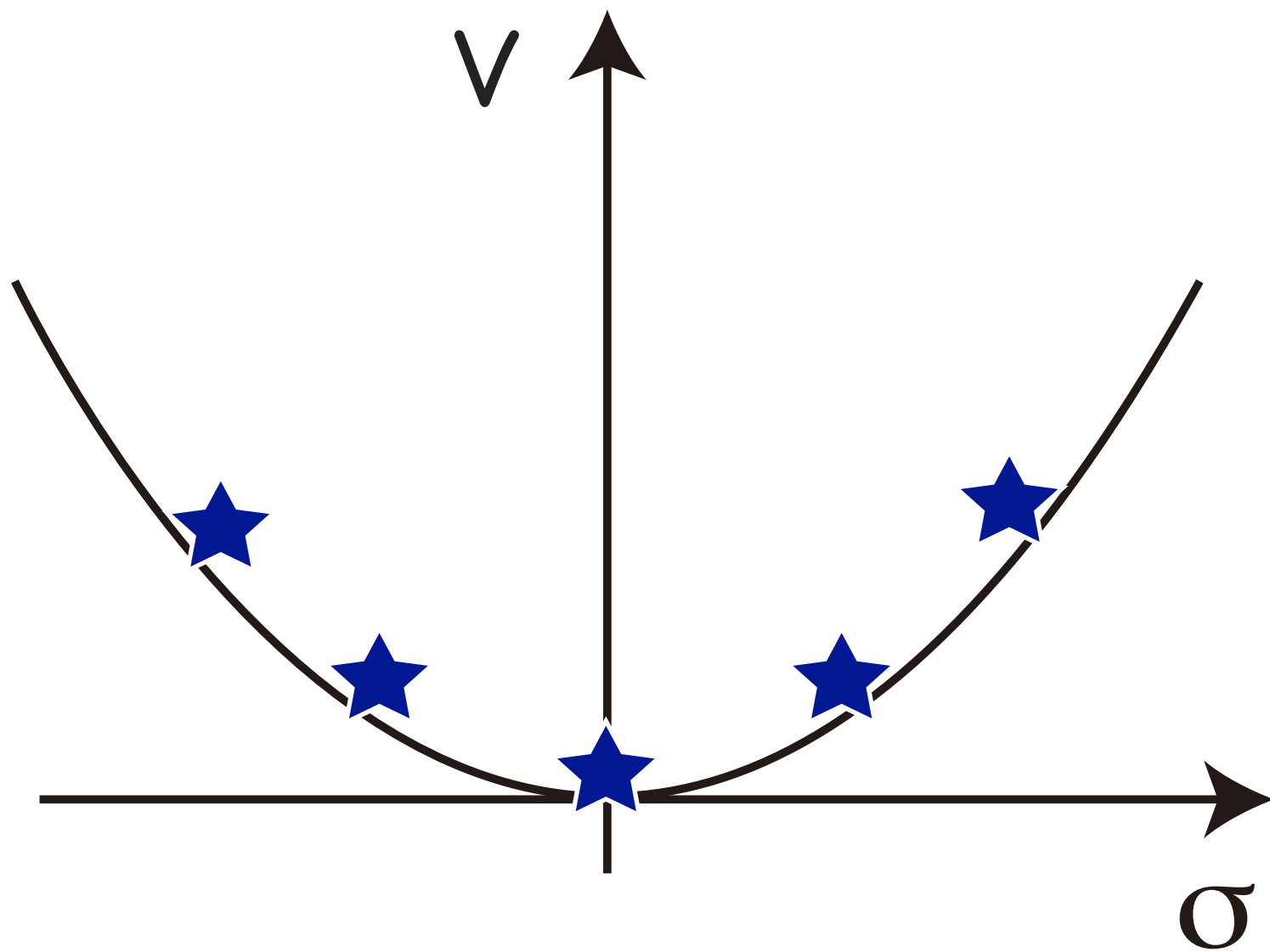
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## Scalar fields

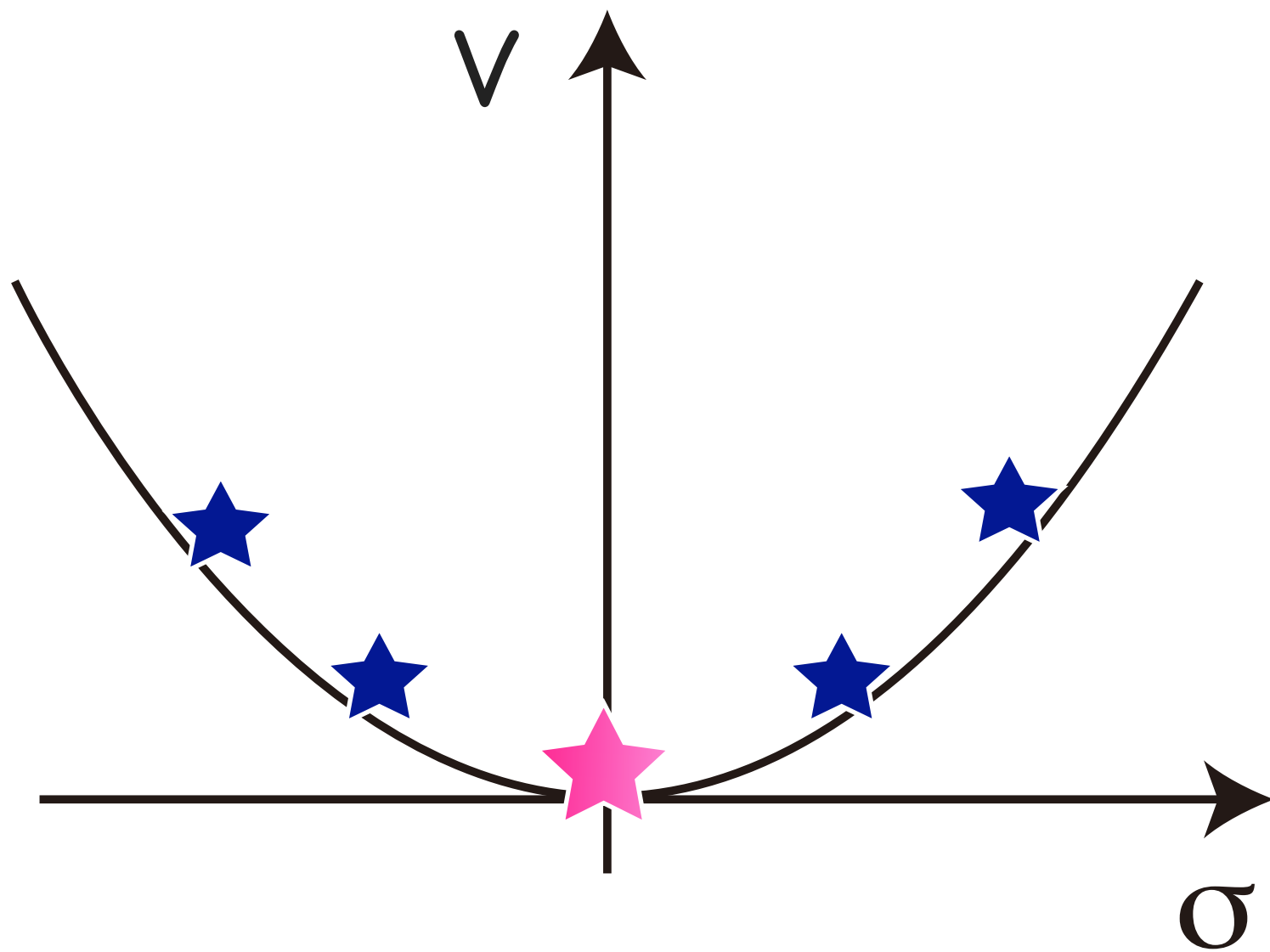


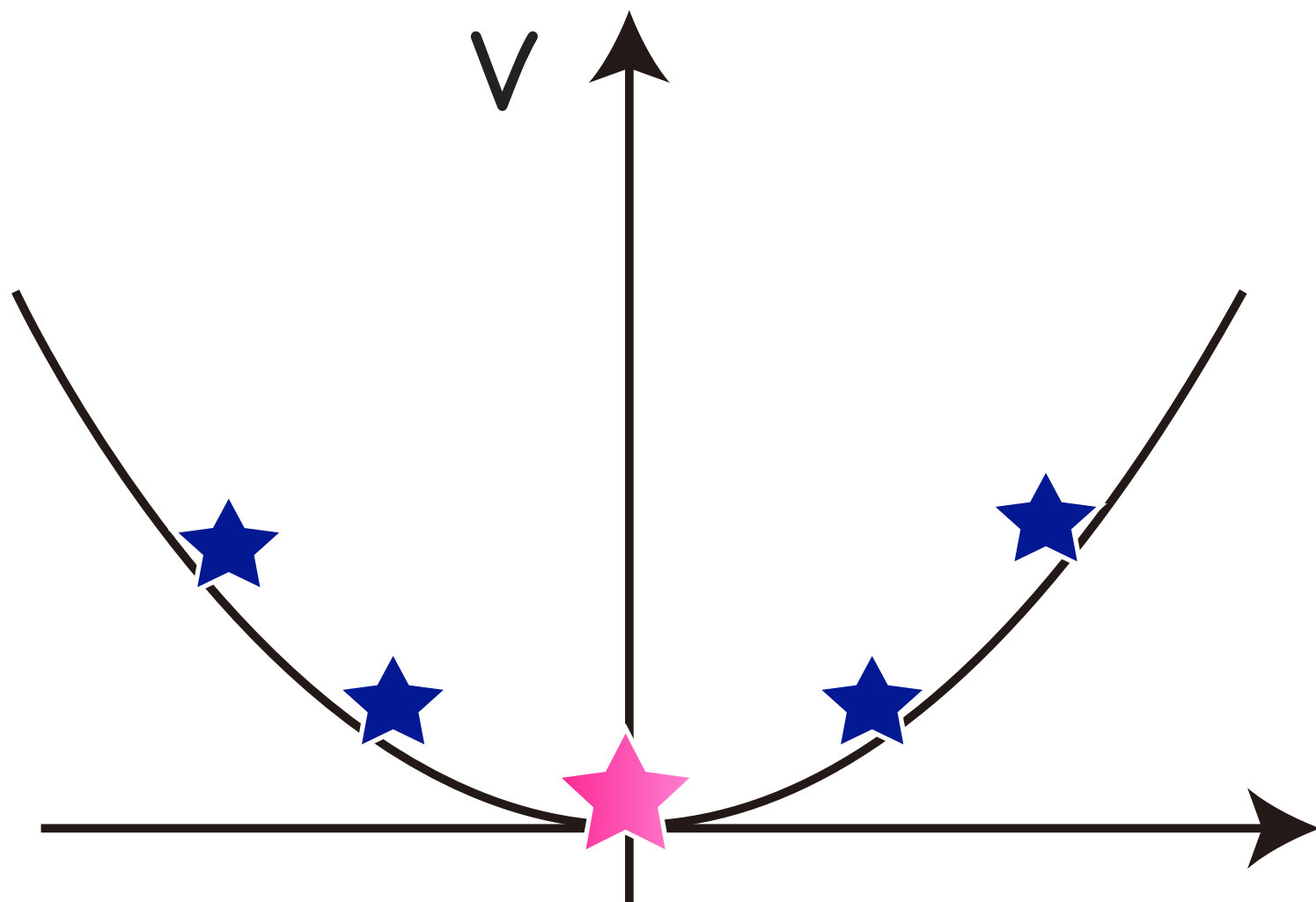
There are perhaps many scalars in nature, charged under some symmetries.





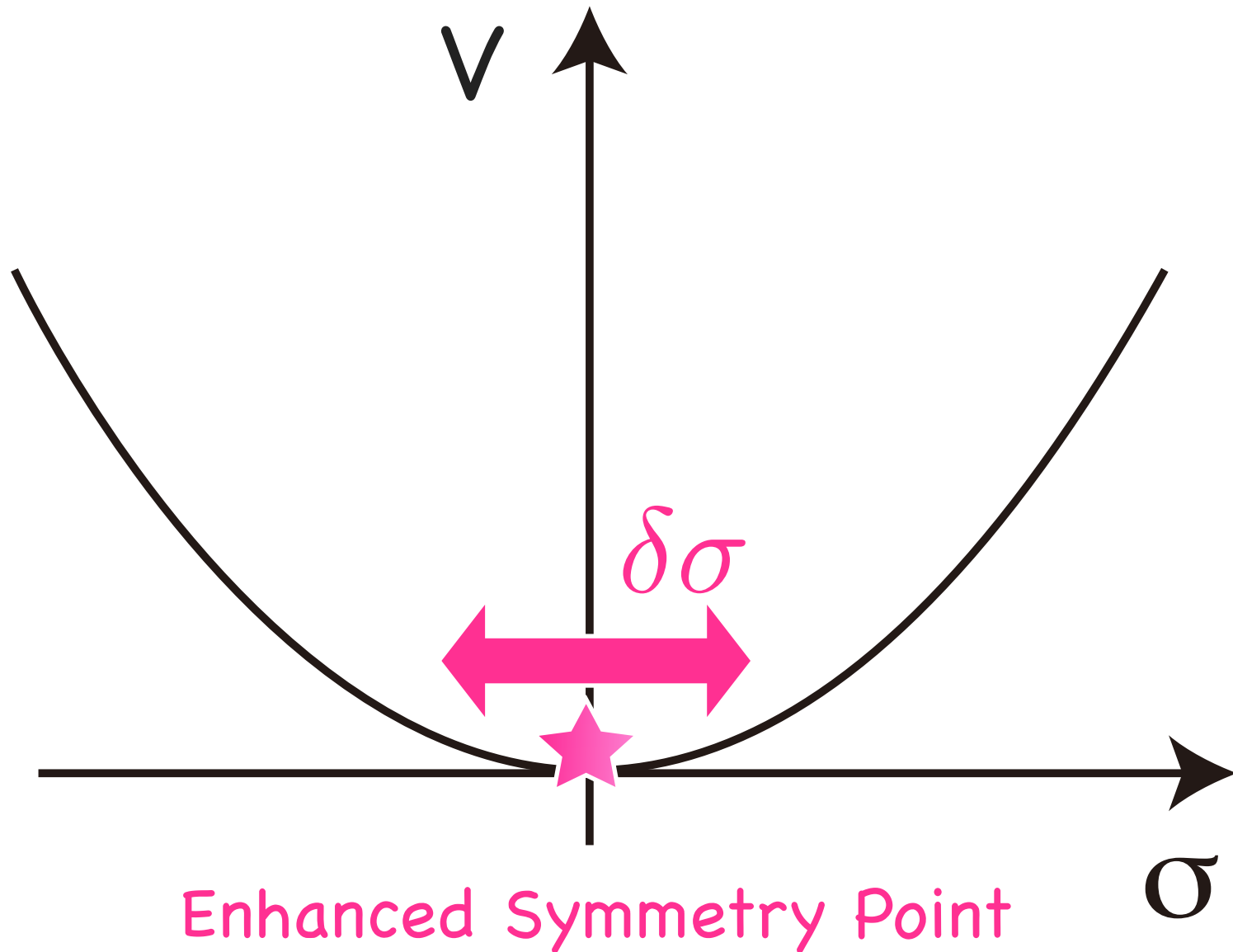




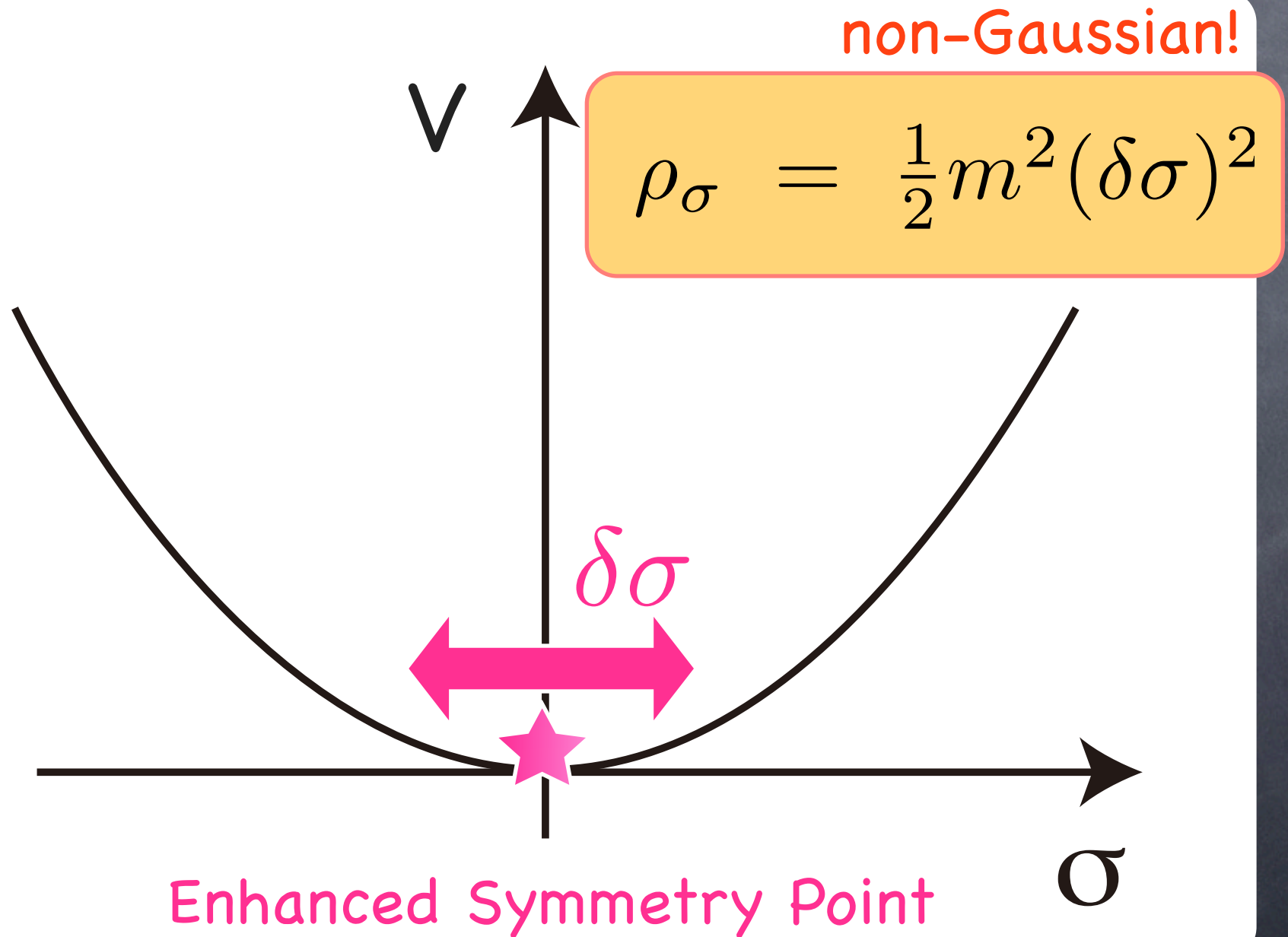


Enhanced Symmetry Point

At the quantum level, a light scalar fluctuates during inflation.



At the quantum level, a light scalar fluctuates during inflation.



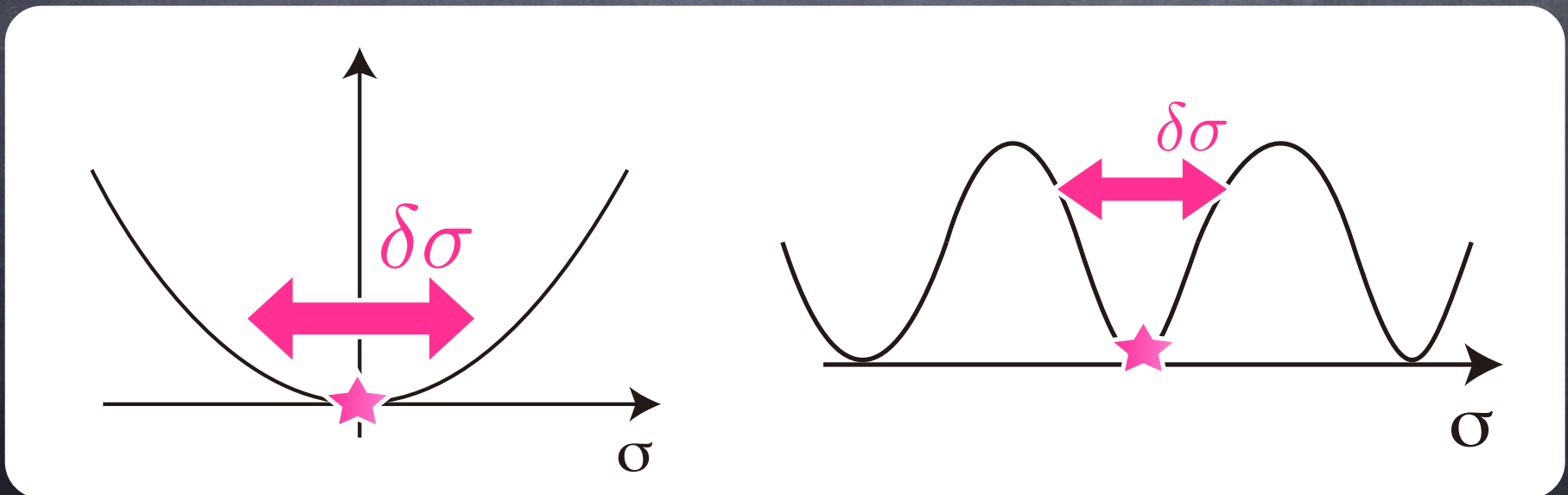
We name such a scalar “ungaussiton”.

## Ungaussiton:

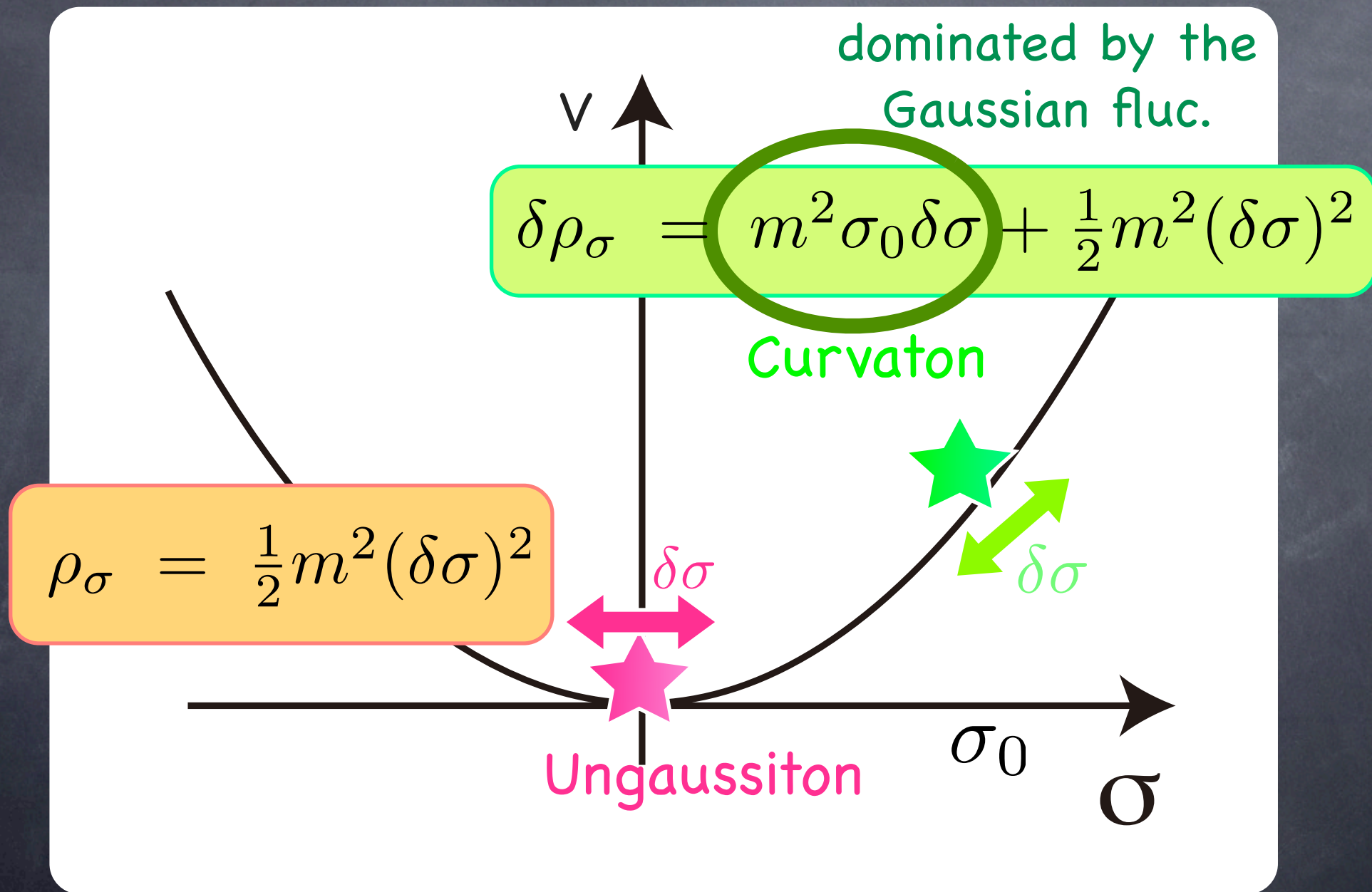
A scalar field **dominantly** produced by the quantum fluctuations, generating sizable non-Gaussianity in the density fluctuations.

e.g.)

The dominant Gaussian density perturbations are assumed to come from the inflaton.



# Ungaussiton or Curvaton?



## Related works in the past:

Linde, Mukhanov '97, '06, "Curvaton web"

Durrer, Gasperini, Sakellariadou and Veneziano '98,  
Gasperini and Veneziano '98, "Seed mechanism"

Lyth '05, "The Bunch-Davies case"

# What we did:

We have calculated the bi- and tri-spectrum from the ungaussiton, and derived a consistency relation between the two.

$$\tau_{\text{NL}} \sim 10^3 f_{\text{NL}}^{4/3}$$

This can be extended to higher corr. func. of any order.

We have derived constraints on the inflationary scale and the reheating/decay temperatures. In particular, one needs

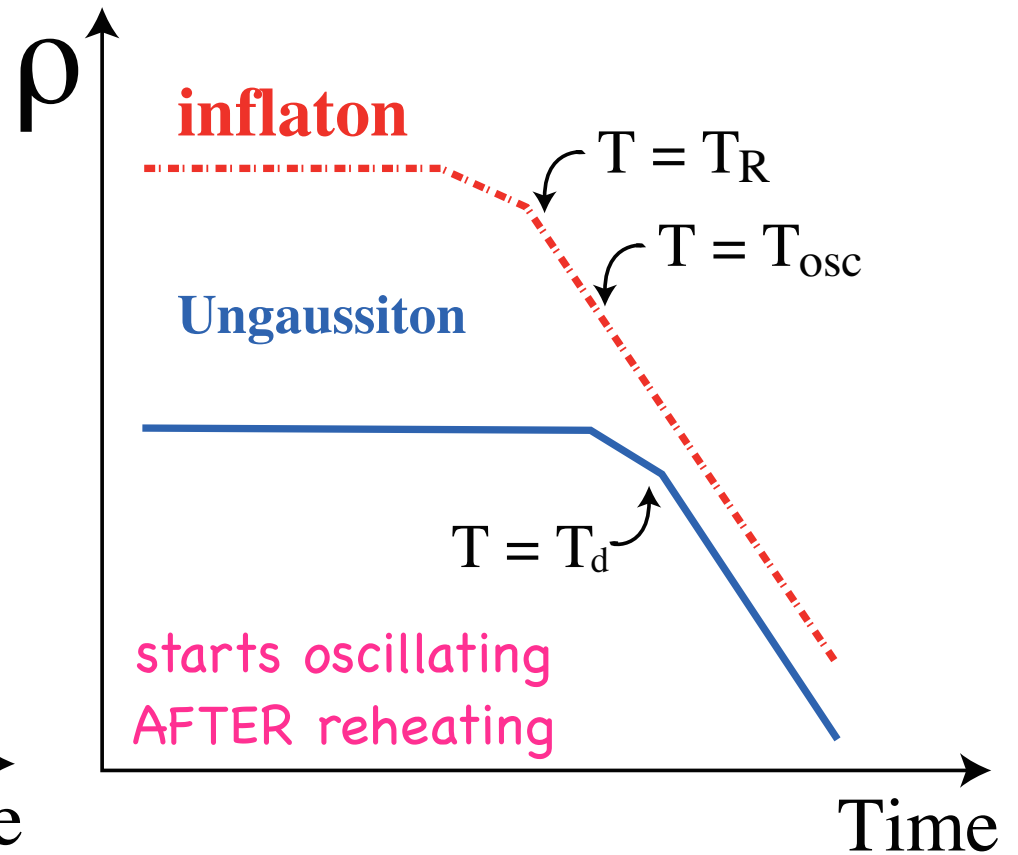
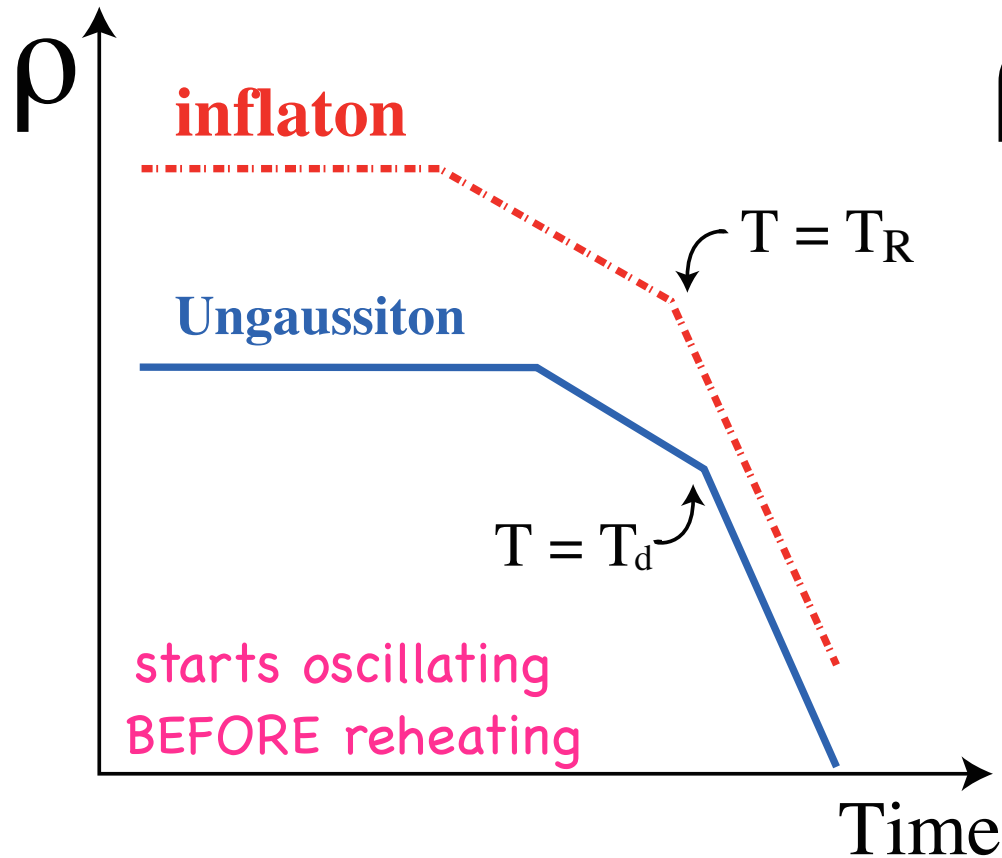
$$H_{\text{inf}} \gtrsim 10^{10} \text{ GeV}$$

for sizable NG.



# 2. Ungaussiton Mechanism

The thermal history



$$\delta\rho_\sigma = \frac{1}{2}m^2(\delta\sigma)^2$$

$$\delta\sigma = \frac{H_{inf}}{2\pi}$$

# Power spectrum, bispectrum, trispectrum:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle_c = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1),$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle_c = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta(k_1, k_2, k_3),$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle_c = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4),$$

$$B_\zeta \neq 0 \quad \text{and/or} \quad T_\zeta \gg P_\zeta^2$$



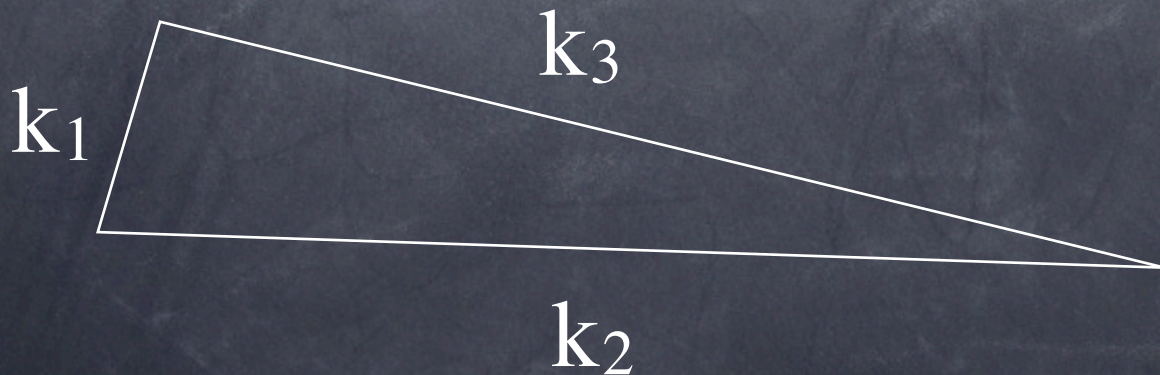
Non-Gaussianity!

# $f_{\text{NL}}$ and $\tau_{\text{NL}}$

$$B_{\zeta}(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}(P_{\zeta}(k_1)P_{\zeta}(k_2) + 2 \text{ perms.}),$$

$$T_{\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = \tau_{\text{NL}}(P_{\zeta}(k_{13})P_{\zeta}(k_3)P_{\zeta}(k_4) + 11 \text{ perms.})$$

In the squeezed config., we can approximately express B and T in terms of  $f_{\text{NL}}$  and  $\tau_{\text{NL}}$ .



# $\delta N$ formalism

Starobinsky '85, Sasaki, Stewart, '96  
Sasaki, Tanaka, '98 Lyth, Malik, Sasaki '04

On sufficiently large scales, the curvature perturbation on the uniform density surface is equal to the prtbn. in the e-folding number between the u.d.s. and the initial flat slicing.

$$\zeta \approx N_\phi \delta\phi_* + N_\sigma \delta\sigma_* + \frac{1}{2} N_{\sigma\sigma} \delta\sigma_*^2 + \frac{1}{6} N_{\sigma\sigma\sigma} \delta\sigma_*^3 + \dots,$$

$$N_\phi = \frac{\partial N}{\partial \phi} \quad \delta\phi_* \sim \delta\sigma_* \sim \frac{H_I}{2\pi} \quad \begin{array}{l} \phi : \text{inflaton} \\ \sigma : \text{ungaussiton} \end{array}$$

The coefficients are determined by  
the background evolution!

## Bispectrum

$$\frac{6}{5} f_{\text{NL}} \simeq \frac{1}{N_{\phi}^4} \left( N_{\sigma}^2 N_{\sigma\sigma} + N_{\sigma\sigma}^3 \mathcal{P}_{\sigma} \log(k_b L) \right),$$

## Trispectrum

$$\tau_{\text{NL}} \simeq \frac{1}{N_{\phi}^6} \left( N_{\sigma}^2 N_{\sigma\sigma}^2 + N_{\sigma\sigma}^4 \mathcal{P}_{\sigma} \log(k_t L) \right),$$

After some calculations,

## Bispectrum

$$\frac{6}{5}f_{\text{NL}} = \frac{\alpha^6}{216} \left(\frac{\pi}{2}\right)^{3/2} \mathcal{P}_\zeta \epsilon^3 \left(\frac{\Gamma_\sigma}{m_\sigma}\right)^{-3/2} \left\{ \left(\frac{\bar{\sigma}}{H_I/2\pi}\right)^2 + \log(k_b L) \right\},$$

## Trispectrum

$$\tau_{\text{NL}} = \frac{\pi^2 \alpha^8}{5184} \mathcal{P}_\zeta \epsilon^4 \left(\frac{\Gamma_\sigma}{m_\sigma}\right)^{-2} \left\{ \left(\frac{\bar{\sigma}}{H_I/2\pi}\right)^2 + \log(k_t L) \right\},$$

# Consistency relation

$$\tau_{\text{NL}} \approx 1 \times 10^3 C f_{\text{NL}}^{4/3};$$

$$C \equiv \frac{\frac{\bar{\sigma}^2}{(H_I/2\pi)^2} + \log k_t L}{\left(\frac{\bar{\sigma}^2}{(H_I/2\pi)^2} + \log k_b L\right)^{4/3}}.$$

# Intuitive derivation of the consistency relation.

$\zeta$  contains  $N_{\sigma\sigma}(\delta\sigma)^2$

$$\langle \zeta\zeta\zeta \rangle \sim N_{\sigma\sigma}^3 \langle (\delta\sigma)^2 \rangle^3$$

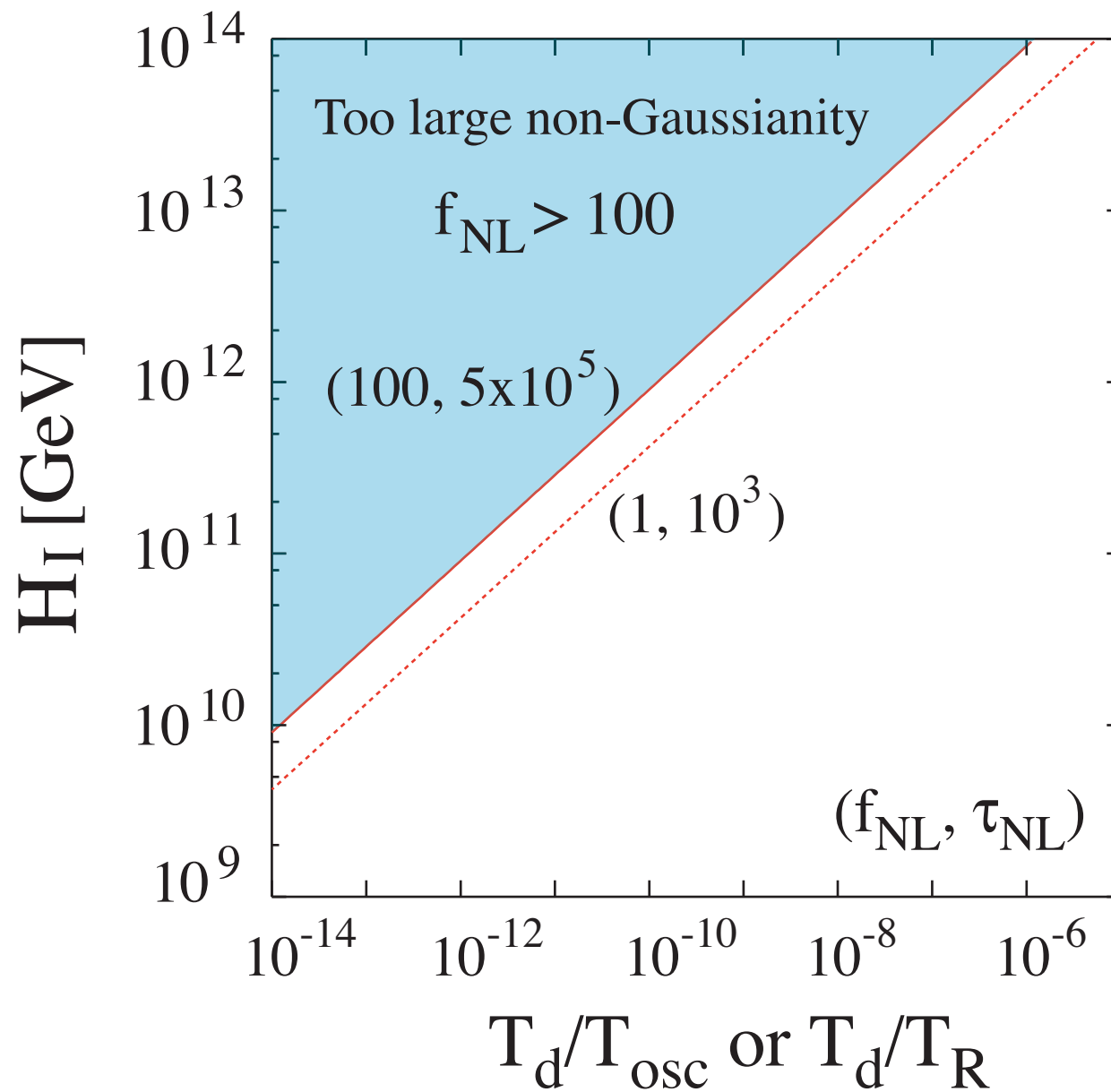
$$\langle \zeta\zeta\zeta\zeta \rangle \sim N_{\sigma\sigma}^4 \langle (\delta\sigma)^2 \rangle^4$$

Noting  $\langle \zeta\zeta\zeta \rangle \sim f_{\text{NL}} P_\zeta^2$  and  $\langle \zeta\zeta\zeta\zeta \rangle \sim \tau_{\text{NL}} P_\zeta^3$

$$\tau_{\text{NL}} \sim P_\zeta^{-\frac{1}{3}} f_{\text{NL}}^{\frac{4}{3}} \sim 10^3 f_{\text{NL}}^{\frac{4}{3}}$$



# Cosmological constraint:



## Candidates for the ungaussiton:

Moduli with enhanced symmetry points,

Right-handed sneutrino,

MSSM flat directions w/ Q-balls

etc.

# Conclusion

- A detection of NG suggests that the source of the NG may be common in nature. A light scalar fluctuating around the origin can naturally generate large NG.
- If the ungaussiton is responsible for large NG, there is a consistency relation:

$$\tau_{\text{NL}} \sim 10^3 f_{\text{NL}}^{4/3}$$

- In addition, the inflation scale should be large enough:

$$H_{\text{inf}} \gtrsim 10^{10} \text{ GeV}$$

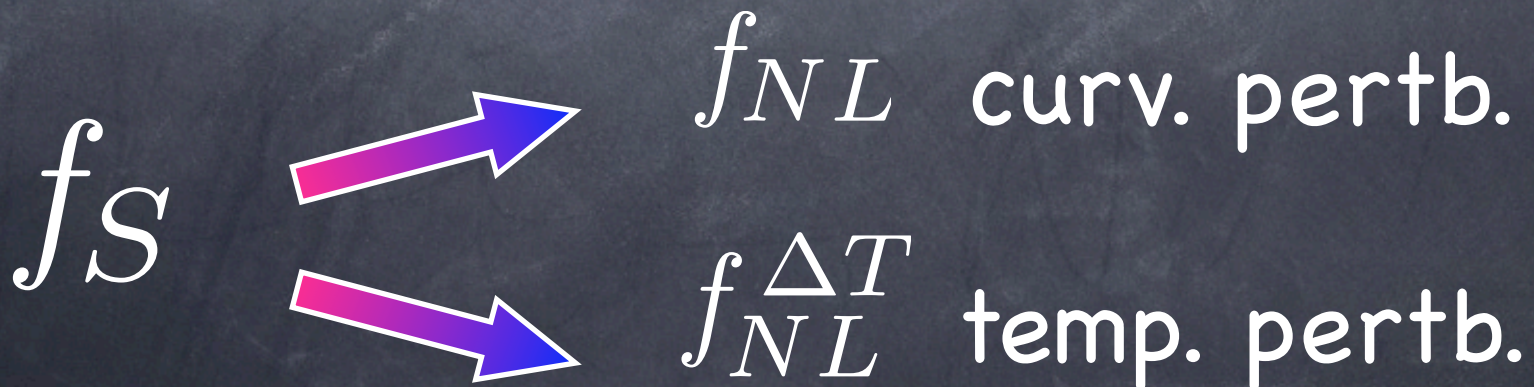
# Non-Gaussianity from CDM isocurvature fluctuations

Kawasaki, Nakayama, Sekiguchi, Suyama, FT

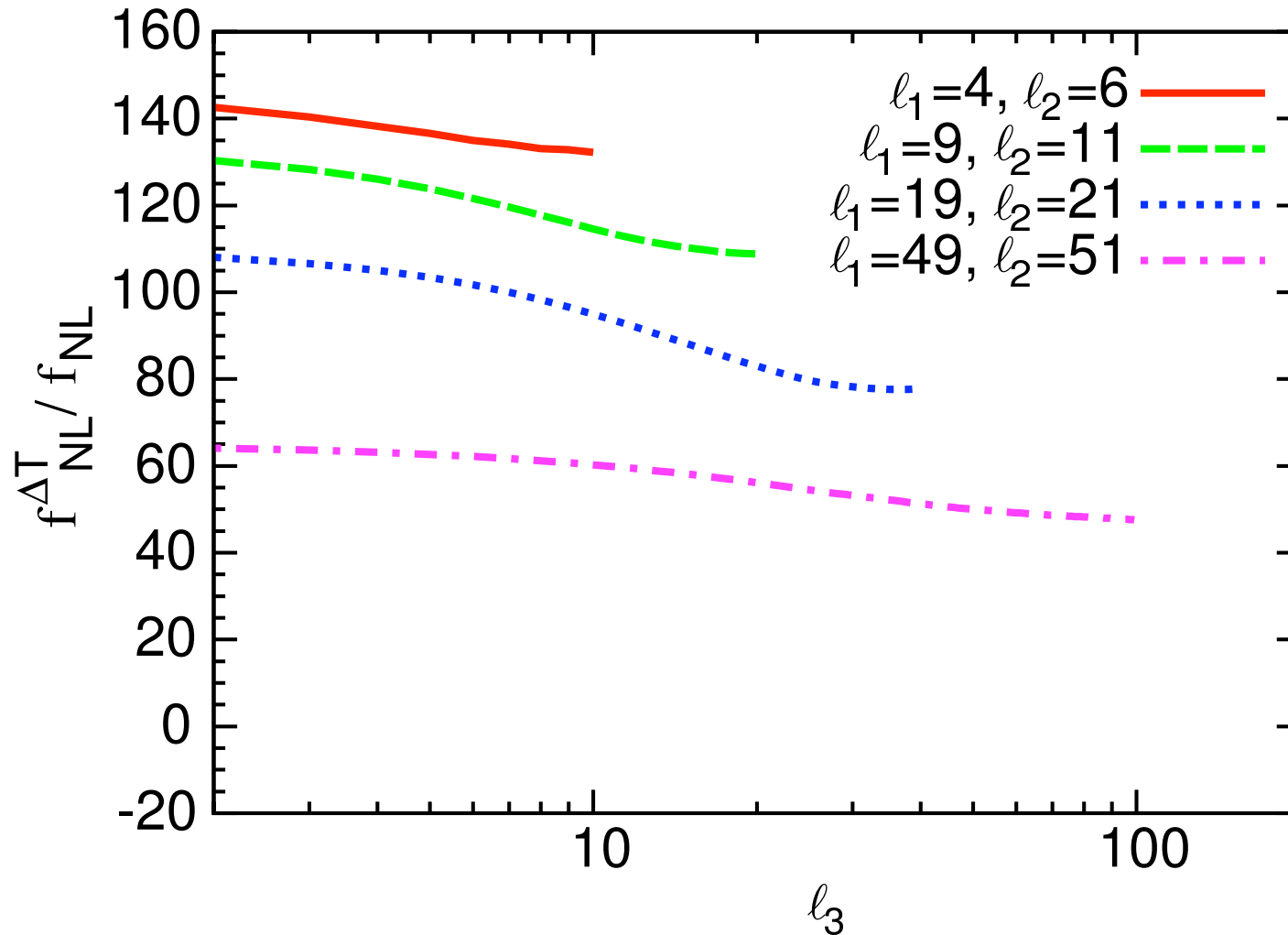
$$\langle S_{ij\vec{k}_1} S_{ij\vec{k}_2} \rangle \equiv (2\pi)^3 P_{Sij}(k_1) \delta(\vec{k}_1 + \vec{k}_2),$$

$$\langle S_{ij\vec{k}_1} S_{ij\vec{k}_2} S_{ij\vec{k}_3} \rangle \equiv (2\pi)^3 B_{Sij}(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$$

$$B_S(k_1, k_2, k_3) \equiv f_S [P_S(k_1)P_S(k_2) + 2 \text{ perms.}].$$



Due to the nature of the isocurvature perturbations, non-Gaussianity in the CMB is enhanced at large scales.



Non-Gaussianity may come  
from the QCD axion?

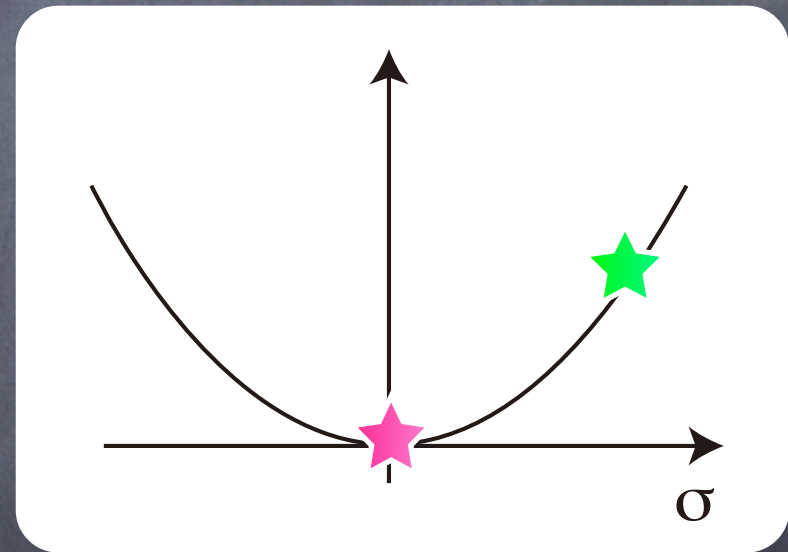
We will see...

Back-up slides

## How can a light scalar be trapped around the origin?

If the inflation lasts long enough, the distribution approaches the Bunch-Davies expression:

$$\langle \sigma^2 \rangle \sim \frac{H^2}{m}$$



If the 'homogeneous' part does not vanish, it is like a curvaton. If it vanishes, it is a ungaussiton.



1. The last stage of the inflation may have lasted only for  $N = 50-60$ . Before the last inflation, the scalar may have settled down at the origin due to some dynamics (thermal mass, preheating, Hubble-induced mass, etc.)

2. The mass of the scalar may not be much lighter than the Hubble parameter (cf. eta problem in sugra). Then it is possible that the scalar happens to be around the origin in our universe.

3. Or, the origin may be chosen by the anthropic arguments.