Can F(R) Dark Energy Really Work?

Part II: Singularity Problem [arXiv:0803.2500]

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LAST TIME...

Linearly stable models have been found:

Hu and Sawicki [0705.1158]

Starobinsky [0706.2041]

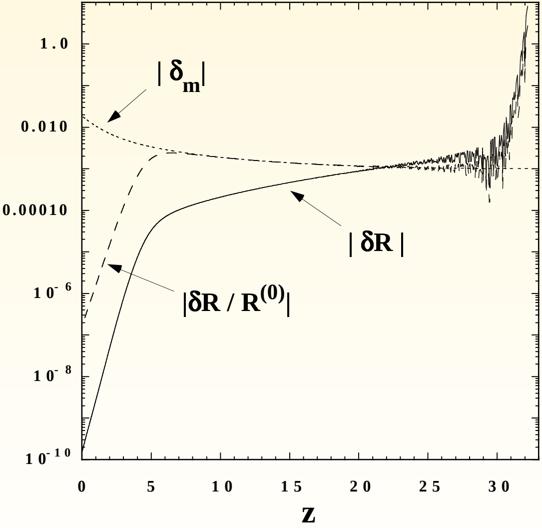
various “designer” models

LSS growth studied:

Pogosian & Silvestri [0709.0296]

Oyaizu, Lima & Hu [0807.2462]

But something is amiss...



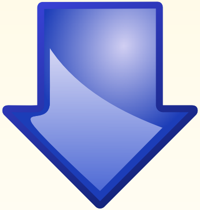
Tsujikawa [0709.1391]

Why Doesn't F(R) Dark Energy Work?

IR <- 60 orders of magnitude -> UV

IR modification

$$f(R) = R - \frac{\mu^4}{R}$$



$$\phi = \frac{\mu^4}{R^2}$$

damage to UV

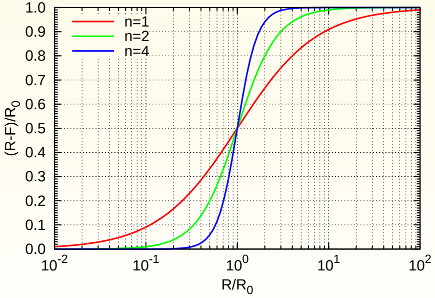
IR UV

O(1)

IR-Modified F(R) Gravity Models

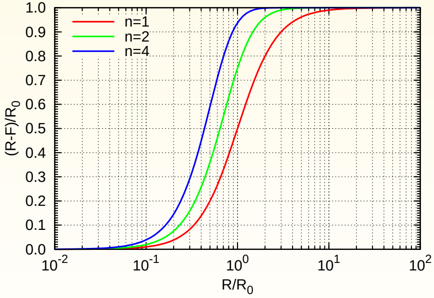
Hu and Sawicki [0705.1158]

$$f(R) = R - \frac{\alpha (R/R_0)^n}{1 + \beta (R/R_0)^n} R_0$$



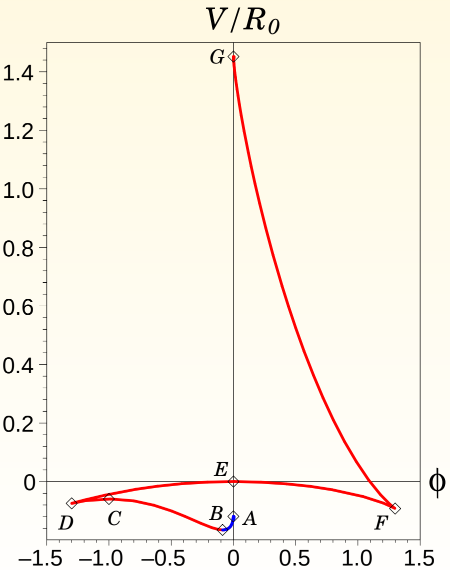
Starobinsky [0706.2041]

$$f(R) = R + \lambda \left[\frac{1}{\left(1 + (R/R_0)^2\right)^n} - 1 \right] R_0$$



... and many other models ...

Disappearing Cosmological Constant Model



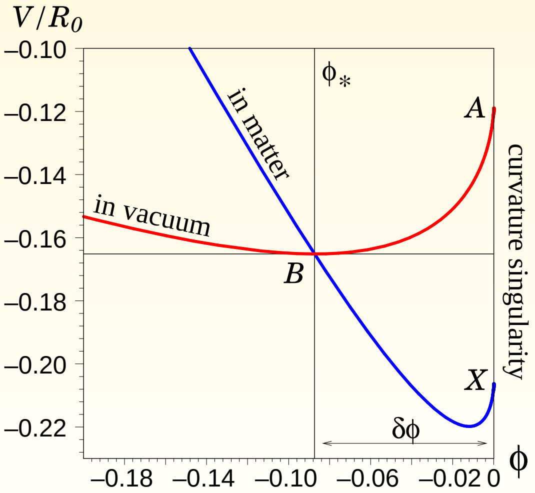
Starobinsky [0706.2041]

$$f(R) = R + \lambda\left[\left(1+R^2\right)^{-1} - 1\right]$$

$$\phi = -\frac{2\lambda R}{(1+R^2)^2}$$

- **A** singularity $(R = +\infty)$
- **B** stable dS min $(f' = 0)$
- **C** unstable dS max $(f' = 0)$
- **D** critical point $(f'' = 0)$
- **E** flat spacetime $(f' = 0)$
- **F** critical point $(f'' = 0)$
- **G** singularity $(R = -\infty)$

Singularity is Finite Distance Away!



$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

in large R limit:

$$f(R) = R + \Lambda + \frac{1}{R^\alpha} \sum_{n=0}^{\infty} \frac{\mu_n}{R^n}$$

$$\phi \equiv f' - 1 \simeq -\frac{\alpha \mu_0}{R^{\alpha+1}}$$

$$\frac{dV}{dR} \simeq \frac{R f''}{3} = \frac{\alpha(\alpha+1)\mu_0}{3 R^{\alpha+1}}$$

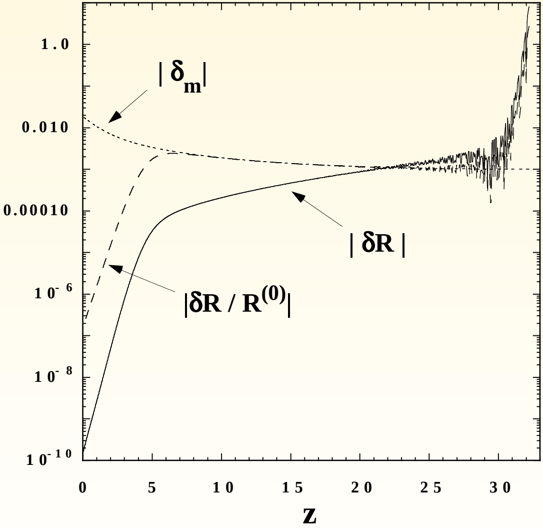
weak power-law singularity:

$$V(\phi) \simeq \text{const} - \frac{(\alpha+1)\mu_0}{3|\alpha\mu_0|^\gamma} |\phi|^\gamma$$

$$\gamma = \frac{\alpha}{\alpha+1}$$

Mechanical Analogy: a Ball in a Bowl

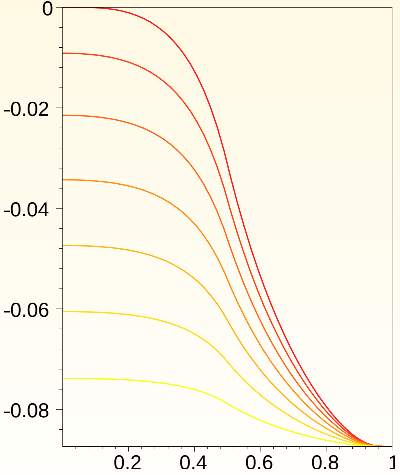
Now we Understand What's Going On Here!



Tsujikawa [0709.1391]

Bigger Problem: Singular Compact Objects!

scalar DOF ϕ



$x = r/(1+r)$

Potential well of
a compact object:

$$\Delta\phi = -\frac{8\pi}{3} G\rho + \underbrace{V'(\phi)}_{\text{negligible}}$$

$$\Delta\Phi = 4\pi G\rho$$

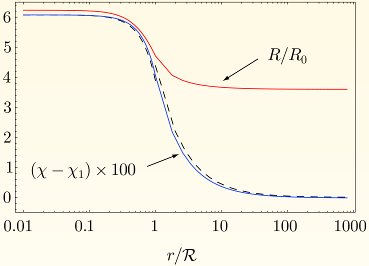
Excitations of f(R) degree of freedom ϕ and Newtonian potential Φ are related:

$$\phi \approx \phi_* - \frac{2}{3}\Phi$$

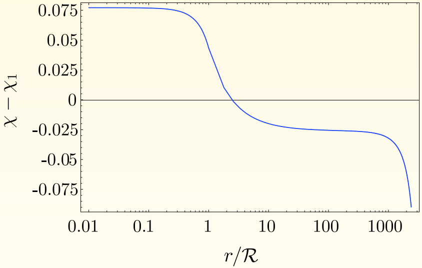
Reach singularity if $\delta\phi \lesssim \frac{1}{3}$!

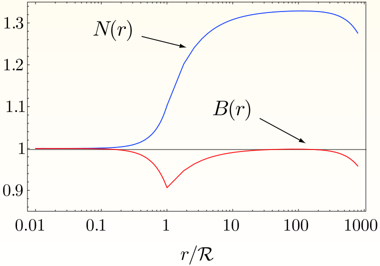
Kobayashi & Maeda (0807.2503)

shallow potential well



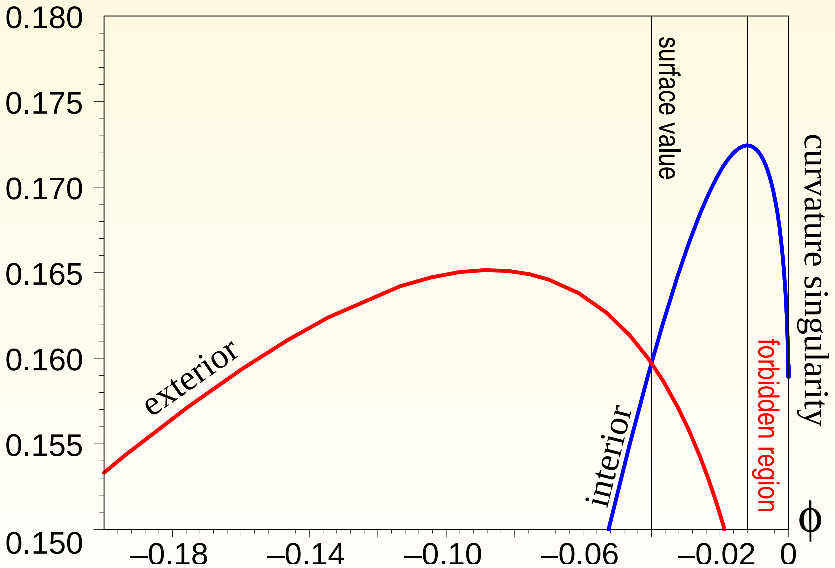
deep potential well





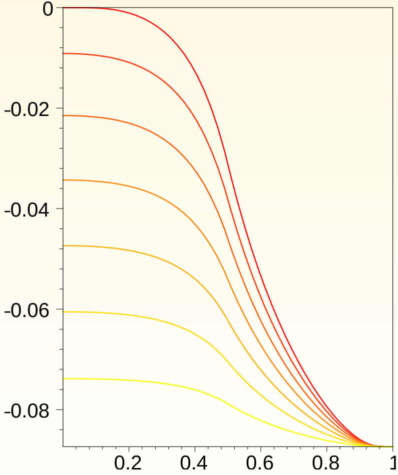
no regular static solution!

Mechanical Analogy: Shooting in the Hills



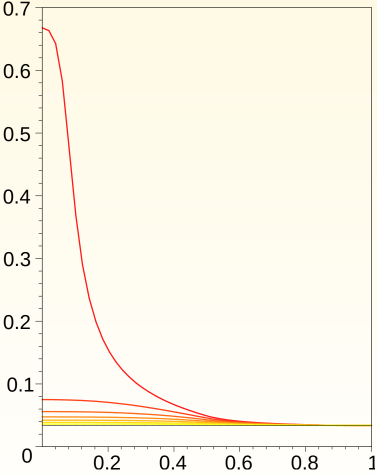
How do I Lose the Regular Solution Branch?

scalar DOF ϕ



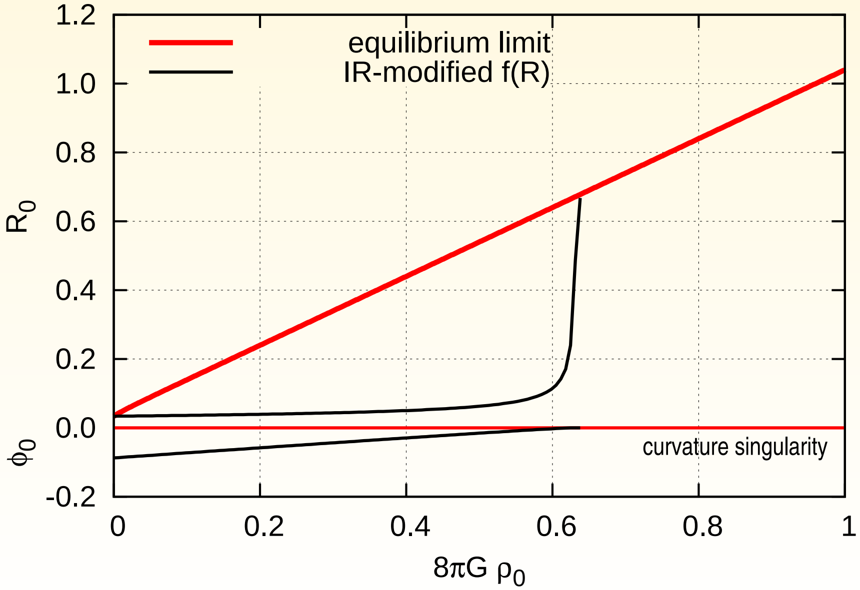
mildly non-linear

curvature R

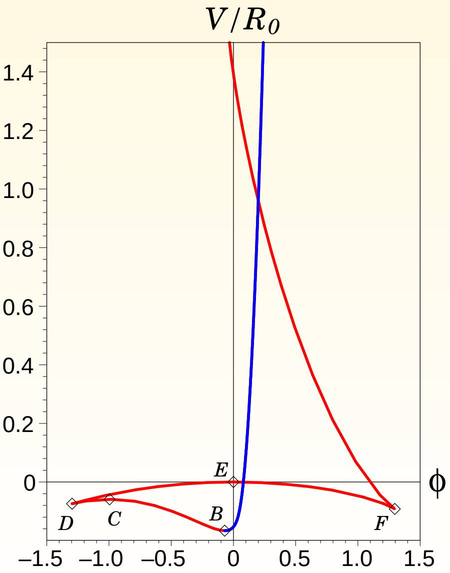


very non-linear

How do I Lose the Regular Solution Branch?



Can UV Completion Save the Day?



Starobinsky [0706.2041]

$$f(R) = R + \lambda\left[\left(1+R^2\right)^{-1} - 1\right] + \frac{R^2}{M^2}$$

$$\phi = -\frac{2\lambda R}{(1+R^2)^2} + \frac{2R}{M^2}$$

A	singularity	$(R = +\infty)$
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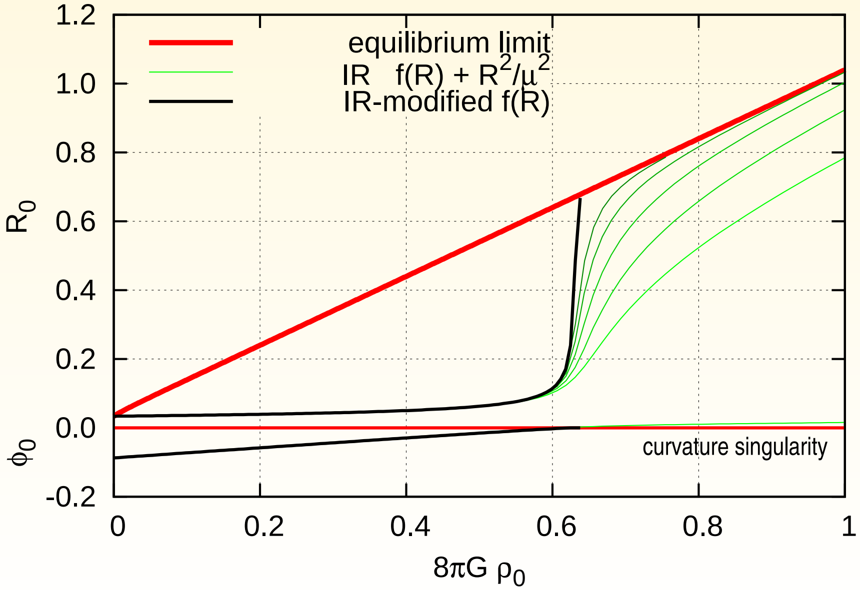

Not quite yet!

But we are forced to confront UV-completion, and even if we fix it we might not get Einstein gravity...

Need to understand how bad it really is!