Dark Energy Models in F(R) Gravity

[Part I: The Story So Far...]

Andrei Frolov

Department of Physics
Simon Fraser University

RESCEU Summer School on Dark Energy

Asamushi Onsen, Aomori, Japan

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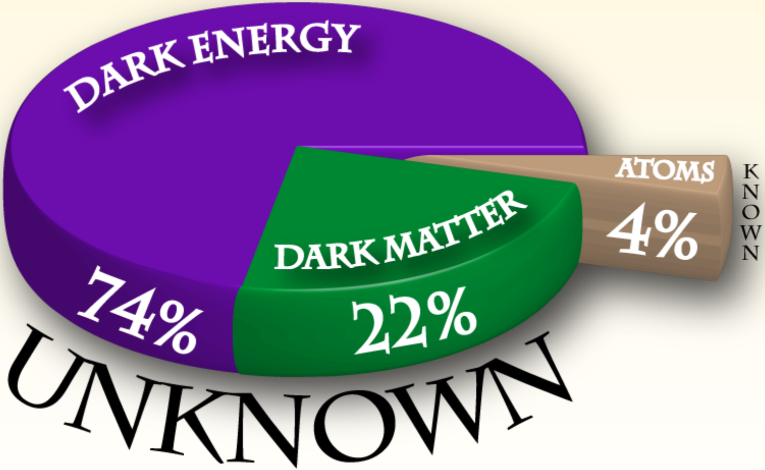
What's the Matter with Cosmology?



Maybe it's Gravity we Don't Understand...

What if instead of curvature in Einstein-Hilbert action we had

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_{\mathrm{m}} \right\} \sqrt{-g}\, d^4x$$

UV modification:

$$f(R) = R + \frac{R^2}{M^2}$$

Starobinsky (1980)

IR modification:

$$f(R) = R - \frac{\mu^4}{R}$$

Capozziello et. al. [astro-ph/0303041]
Carroll et. al. [astro-ph/0306438]

For F(R) theory to make sense we need:

- $f' > 0$ – otherwise gravity is a ghost
- $f'' > 0$ – otherwise gravity is a tachyon

Field Equations in F(R) Gravity

- Vary the action with respect to the metric:

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_{\mathrm{m}} \right\} \sqrt{-g}\, d^4x$$

- Einstein equations turn into a fourth-order equation:

$$f' R_{\mu\nu} - f'_{;\mu\nu} + \left(\Box f' - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- A new scalar degree of freedom $\phi \equiv f' - 1$ appears:

$$\Box f' = \frac{1}{3}(2f - f'R) + \frac{8\pi G}{3}\, T$$

- Can rewrite fourth-order field equation as two second order ones!

A New Scalar Degree of Freedom

- Equation for $\phi \equiv f' - 1$ is just a scalar wave equation:

$$\Box\phi = V'(\phi) - \mathscr{F}$$

- Matter directly drives the field ϕ by a force term:

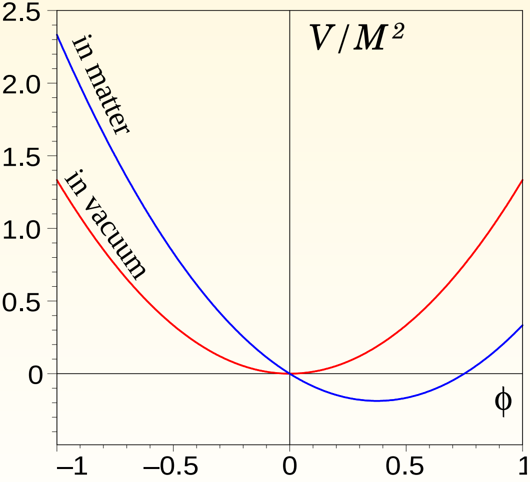
$$\mathscr{F} = \frac{8\pi G}{3}(\rho - 3p)$$

- Effective potential can be found by integrating

$$V'(\phi) \equiv \frac{dV}{d\phi} = \frac{1}{3}(2f - f'R)$$

- In practice, easier to obtain in parametric form:

$$\frac{dV}{dR} \equiv \frac{dV}{d\phi}\frac{d\phi}{dR} = \frac{1}{3}(2f - f'R)f''$$



$$f(R) = R + \frac{R^2}{M^2}$$

$$\phi = \frac{2R}{M^2}$$

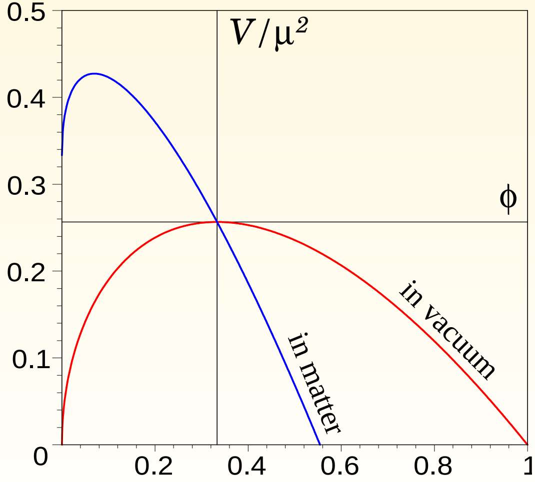
$$V = \frac{1}{3}\frac{R^2}{M^2} = \frac{M^2}{12}\phi^2$$

massive scalar field!

scalar degree of freedom ϕ is heavy and hard to excite

$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

Example II: (Failed) IR Modification



$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

$$f(R) = R - \frac{\mu^4}{R}$$

$$\phi = \frac{\mu^4}{R^2}$$

$$V = \frac{2}{3}\left(\frac{\mu^4}{R} - \frac{\mu^8}{R^3}\right) = \frac{2}{3}\mu^2\left(\phi^{\frac{1}{2}} - \phi^{\frac{3}{2}}\right)$$

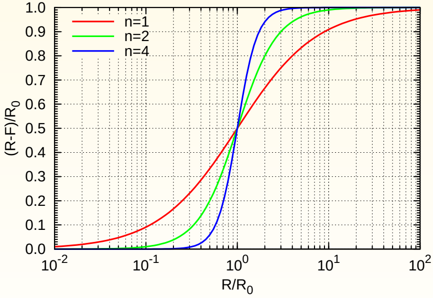
field ϕ is unstable!

Dolgov & Kawasaki
(astro-ph/0307285)

Can We Come Up With Something Better?..

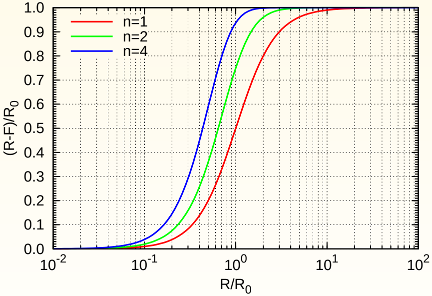
Hu and Sawicki [0705.1158]

$$f(R) = R - \frac{\alpha (R/R_0)^n}{1+\beta (R/R_0)^n} R_0$$



Starobinsky [0706.2041]

$$f(R) = R + \lambda \left[\frac{1}{\left(1+(R/R_0)^2\right)^n} - 1 \right] R_0$$



... and many other models ...

Cosmology in F(R) Gravity

- Homogeneous flat cosmology is described by FRW metric:

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$

- Scalar degree of freedom looks as usual (albeit with a force term):

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \mathscr{F}$$

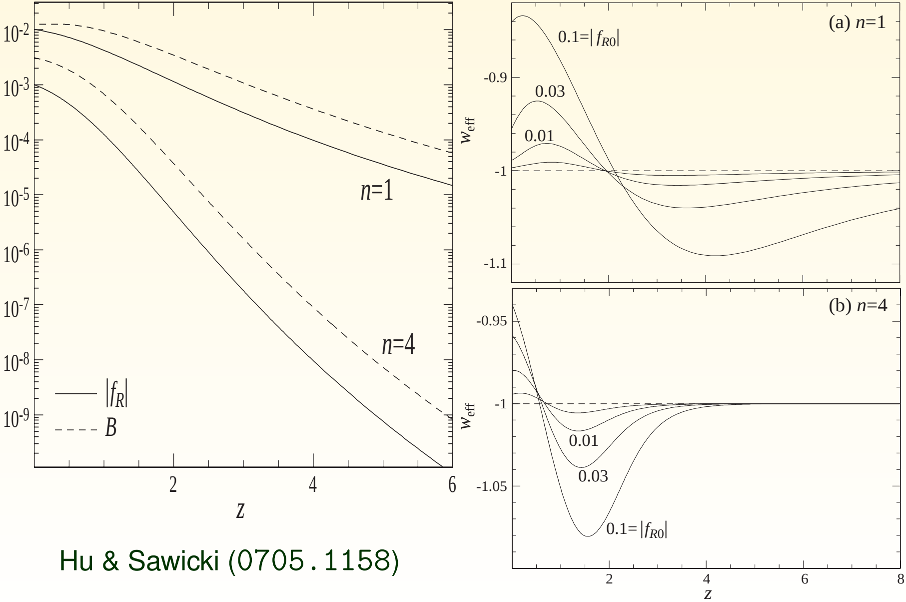
- What about Friedman equation? It looks strange...

$$3H(f')^{\cdot} - 3\frac{\ddot{a}}{a} f' + \frac{1}{2} f = 8\pi G \rho$$

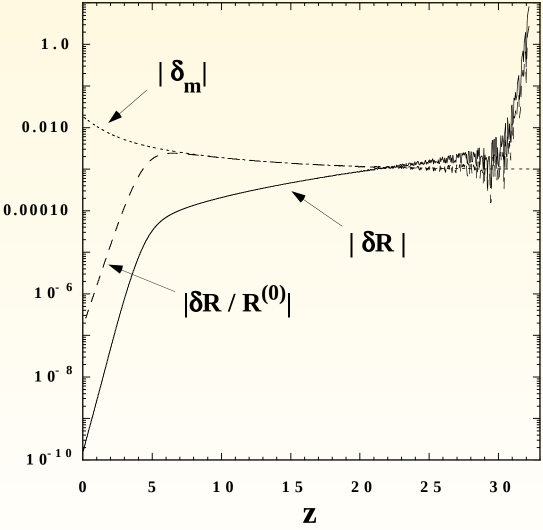
- ... but it isn't! Eliminating $\ddot{a}$ in favor of $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ does the trick:

$$H^2 + (\ln f')^{\cdot} H + \frac{1}{6}\frac{f - f'R}{f'} = \frac{8\pi G}{3f'}\rho$$

Deviation from ΛCDM Cosmology



Hu & Sawicki (0705.1158)



Tsujikawa [0709.1391]

Local Tests of F(R) Dark Energy

- Spherically symmetric static metric:

$$ds^2 = -e^Q \left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

- After same trick we used for Friedmann equation:

$$\left(1 + \phi + \tfrac{1}{2} r \phi'\right) M' = 4\pi G r^2 \rho + O(\phi)$$

$$O(\phi) \equiv \frac{r^2}{2}\left(1 - \frac{2M}{r}\right)\phi'' + r\left(1 - \frac{3M}{2r}\right)\phi' + \frac{r^2}{4}\left(R + \phi R - f\right)$$

$$\left(1 + \phi + \tfrac{1}{2} r \phi'\right) Q' = 8\pi G r \frac{\rho + p}{1 - \frac{2M}{r}} + r\phi''$$

- Once again, dynamics are governed by equation for scalar DOF:

$$\Delta\phi = -\frac{8\pi}{3} G(\rho - 3p) + V'(\phi)$$

Local Tests of F(R) Dark Energy (continued)

We need to solve a non-linear differential equation:

$$\Delta\phi = -\frac{8\pi}{3} G(\rho - 3p) + V'(\phi)$$

How do we understand its solutions?

"Equilibrium" regime:

$$V'(\phi) = \frac{8\pi}{3} G(\rho - 3p)$$

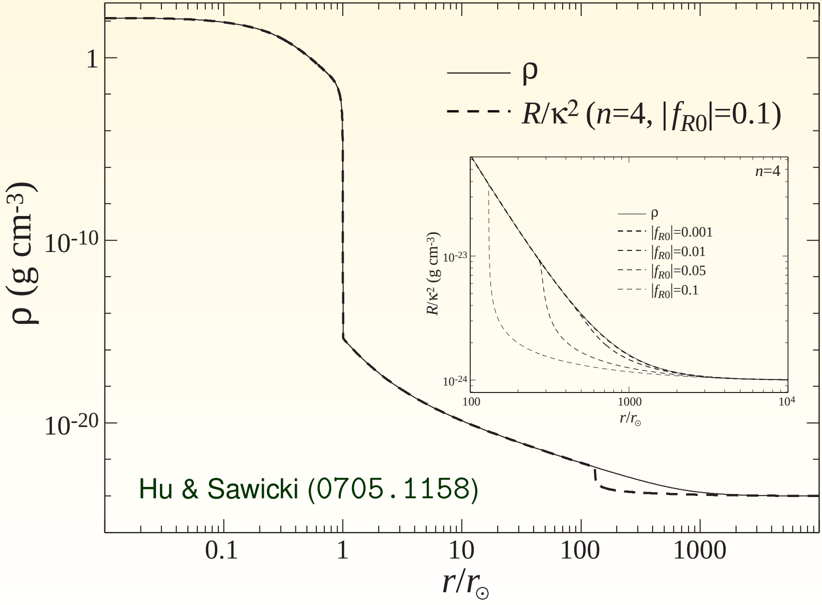
chameleon mechanism

"Ballistic" Regime:

$$\Delta\phi = -\frac{8\pi}{3} G(\rho - 3p)$$

which one is realized depends on environment!

Solar System Curvature Profile

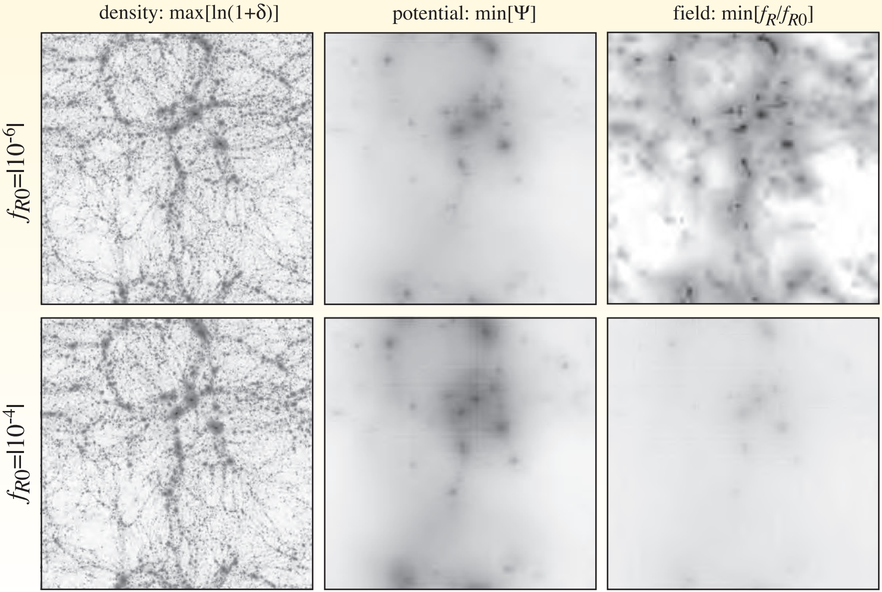


So Far, So Good...

Looks like we can
avoid local constraints...

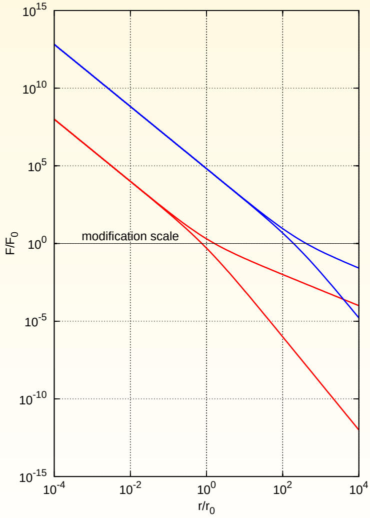
Let's see if there is
any effect on cosmology!

N-Body Simulations with F(R) Dark Energy



Oyaizu, Lima & Hu (0807.2462)

Large Scale Structure Growth is Tricky...



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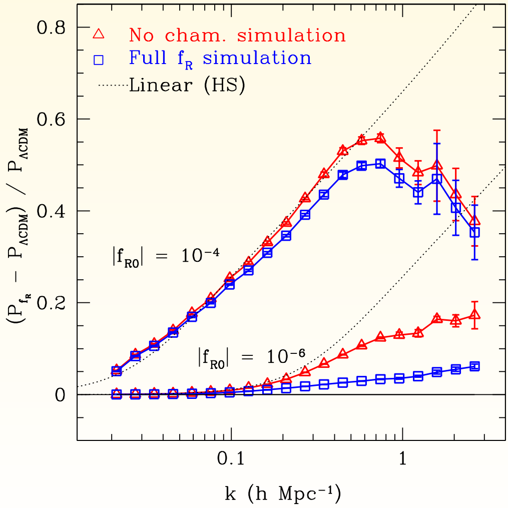

no superposition!

Next Time... Problems with F(R) Dark Energy

F(R) Dark Energy is sort of "geometrical quintessence"

But it does not work!

Surprizingly, IR-modification breaks the strong gravity...