Probing dark energy with weak lensing

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Outline of lectures

- Lecture 1
  - Basics of lensing
  - Thin lenses: 2D reconstruction
- Lecture 2
  - Cosmological lensing
  - 3D reconstruction
  - Cosmological Parameter Estimation
- Lecture 3
  - Observations, problems and surveys
  - Probing Dark Energy with lensing
  - Probing Gravity

Lensing by ordinary matter (stars, planets, galaxies)

Lensing by Dark Matter

Effects of Dark Energy

Effects of Modifying Gravity

The cosmology you need, in one slide

$$\text{Metric}: \quad ds^2 = c^2dt^2 - a^2(t)\left[dr^2 + S_k^2(r)(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

r = comoving distance label; *t* = cosmic time, *a(t)* = scale factor = *1/(1+z)*, and proper distances = comoving distances x *a(t)*

NO GR in this, just symmetry. Note: space may be curved, with curvature k, so

$$S_k(r) = r_0 \sinh(r/r_0),\ r,\ r_0 \sin(r/r_0) \text{ for k} = -1, 0, 1$$

GR ⇒ Hubble parameter $H(a) \equiv a^{-1}\, da/dt$ obeys the *Friedmann equation*:

$$H^2(a) - 8\pi G\rho/3 = -k/a^2 \qquad (\tfrac{1}{2}v^2 - GM(<r)/r = \text{constant})$$

Dark energy contribution to ρ is controlled by continuity equation:

$$\frac{d}{da}\left(\rho_q a^3\right) = -p_q a^2 \equiv -w(a)\rho_q c^2$$

This defines the 'equation of state' *w(a)*

$$H^2(a) = H_0^2\left[\Omega_m a^{-3} + (1-\Omega)a^{-2} + \Omega_q \exp\left\{\int_1^a \frac{da'}{a'}\left[1 + w(a')\right]\right\}\right]$$

Photons have $ds^2=0$, so for radial orbits, $dr = -dt/a(t)$, so $r = c\int_a^1 da'/[a'^2 H(a')]$

And the angular diameter distance is $D_A(r) = a(t)S_k(r)$

Growth of fluctuations

- Linear perturbation theory (GR specific):

Assuming DE is smooth,

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0$$

where $\delta = \delta\rho_m/\rho_m$.

Influence of Dark Energy

In GR, Dark Energy influences cosmology through

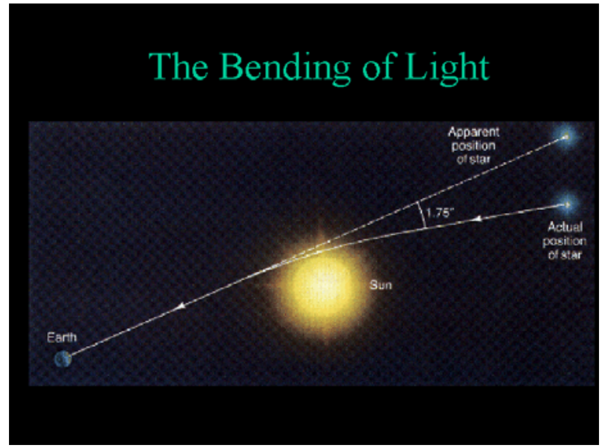
- Distance Measurements, and
- Growth Rate,
- both via *H(a)* or equivalently the expansion rate *a(t)*

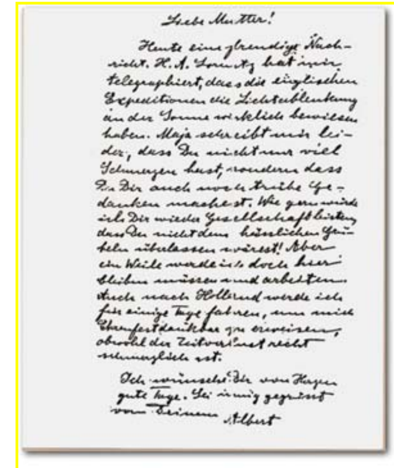
Basics of Lensing

- The bend angle:

(circular symmetry)

$$\tilde{\alpha}(R) = \frac{4GM(<R)}{Rc^2}$$





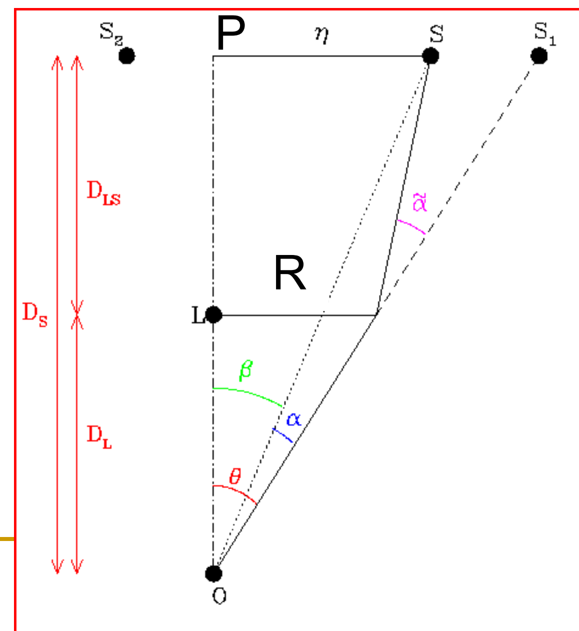
Liebe Mutter!
Heute eine freudige Nachricht. H. A. Lorentz hat mir telegraphiert, dass die englischen Expeditionen die Lichtablenkung an der Sonne wirklich bewiesen haben. Maja schreibt mir leider, dass Du nicht nur viel Schmerzen hast, sondern dass Du Dir auch noch trübe Gedanken machst. Wie gern würde ich Dir wieder Gesellschaft leisten, dass Du nicht dem hässlichen Grübeln überlassen wärest! Aber eine Weile werde ich doch hier bleiben müssen und arbeiten. Auch nach Holland werde ich für einige Tage fahren, um mich Ehrenfest dankbar zu erweisen, obwohl der Zeitverlust recht schmerzlich ist.
Ich wünsche Dir von Herzen gute Tage. Sei innig gegrüsst von Deinem Albert

- The lens equation:

$$D_S\theta = D_S\beta + D_{LS}\tilde{\alpha}$$

$$\text{Let } \alpha = \tilde{\alpha}\frac{D_{LS}}{D_S}$$

$$\beta = \theta - \alpha$$



Point mass lens

- Lens equation soluble analytically:

$$\beta = \theta - \frac{4GM}{c^2\theta}\frac{D_{LS}}{D_L D_S}$$

- Quadratic for θ:

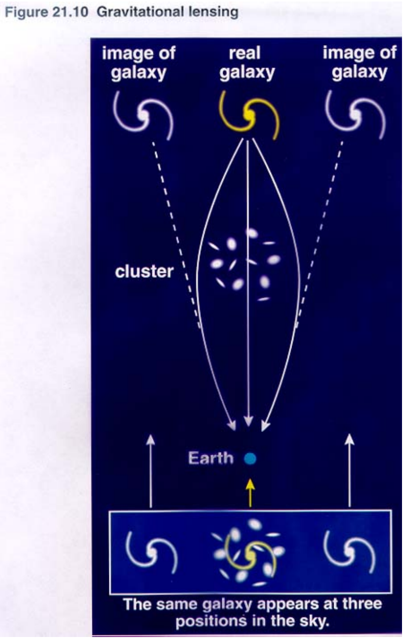
$$\theta^2 - \beta\,\theta - \theta_E^2 = 0$$

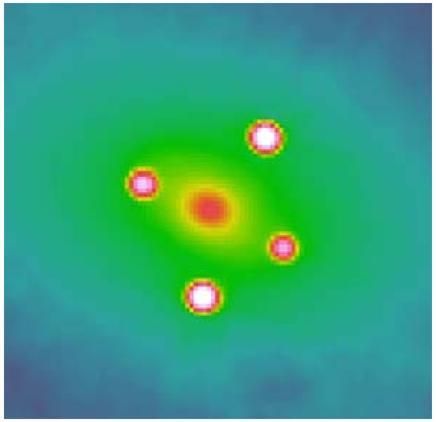
- where the *Einstein Angle* is

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

- Two images at

$$\theta_\pm = \frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \theta_E^2}$$

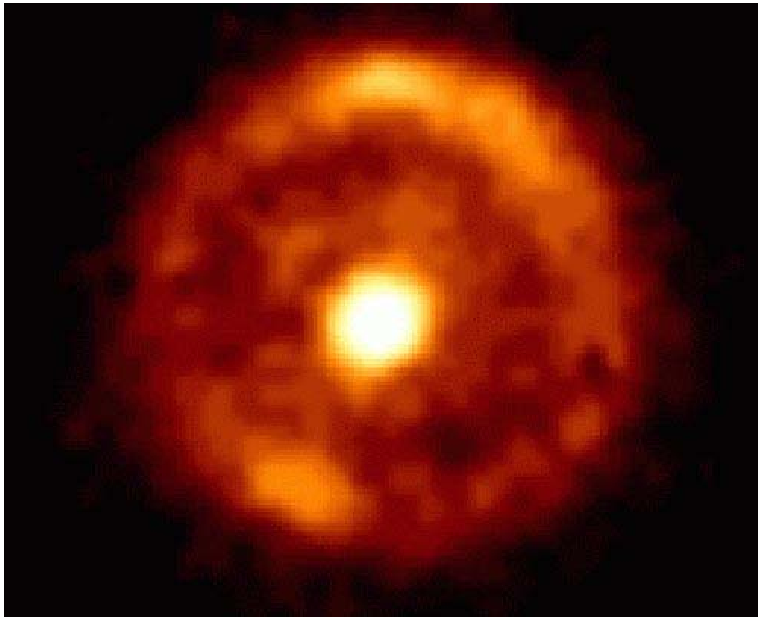


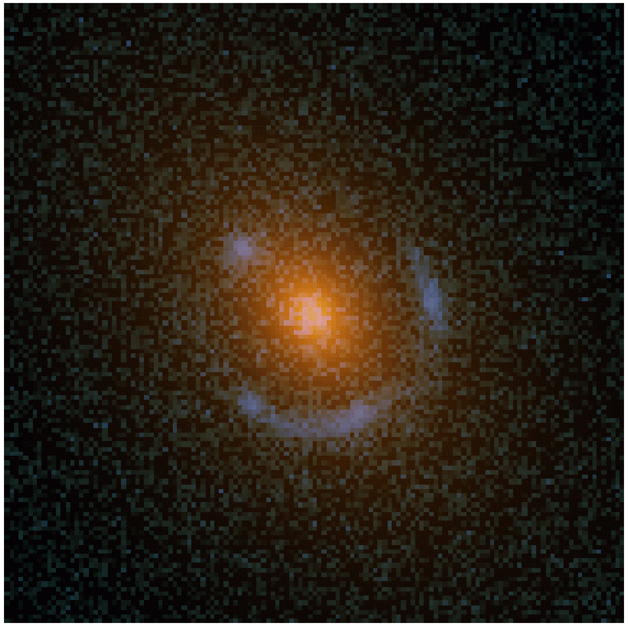


Einstein Rings

$$\theta_{\pm} = \frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \theta_E^2}$$

If $\beta = 0$ then the image is a complete ring.





Requires surface density $\Sigma \geq M/(\pi D_L^2 \theta_E^2) \equiv \Sigma_{crit}$, the *Critical Surface Density*

$$\Sigma_{crit} \equiv \frac{c^2 D_S}{4\pi G D_L D_{LS}}.$$

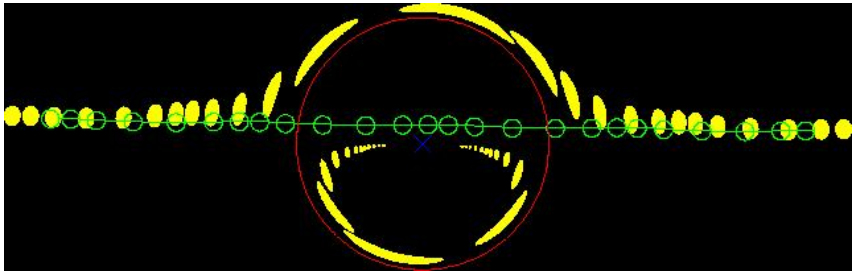
Magnification and Amplification

- Lensing preserves surface brightness → brightness proportional to solid angle:

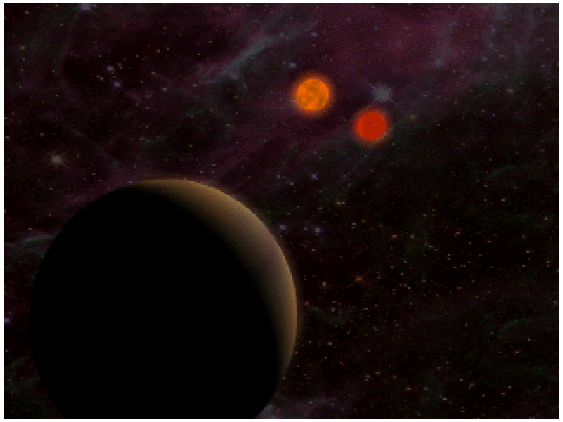
$$A = \frac{\theta}{\beta}\frac{d\theta}{d\beta}.$$

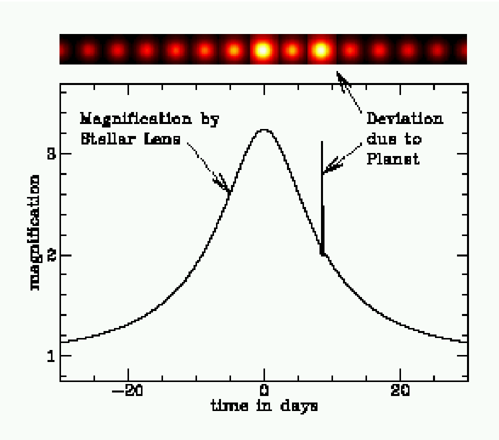
$$A_{\pm} = \frac{1}{2}\left(1 \pm \frac{\beta^2 + 2\theta_E^2}{\beta\sqrt{\beta^2 + 4\theta_E^2}}\right).$$

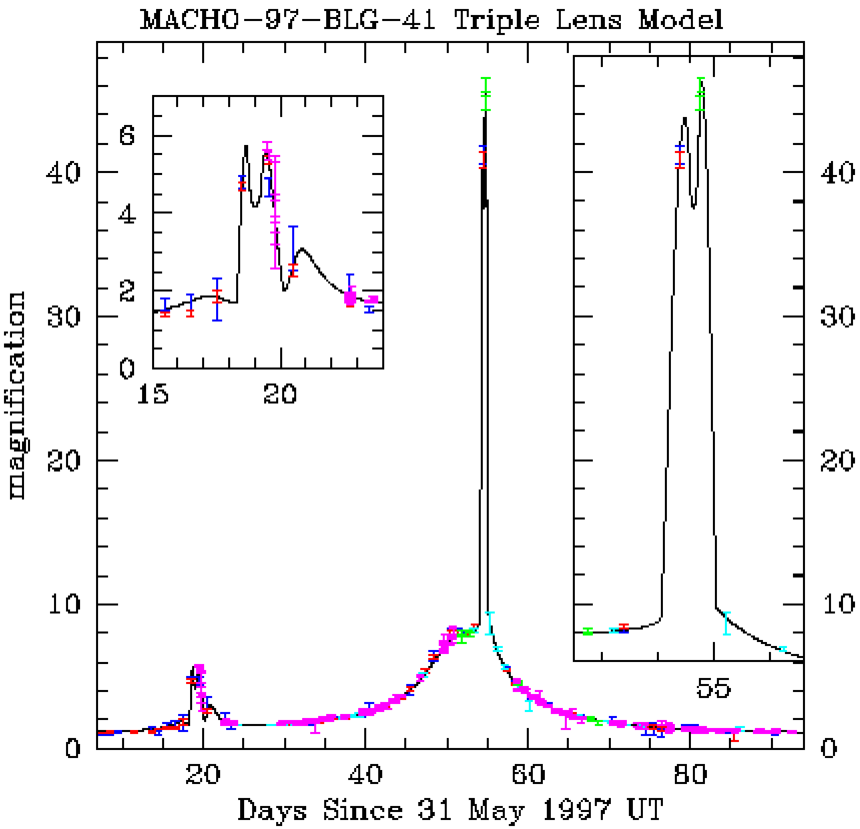
$A < 0 \;\; \rightarrow$ image reversed.



Planet detection







General thin lens

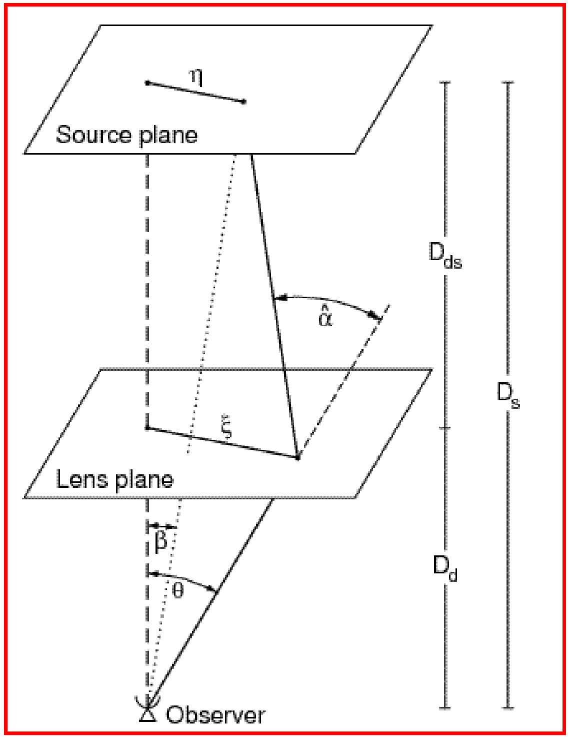
Surface mass density $\Sigma(\vec{\theta})$

- Bend angle is a 2D vector (on the sky)

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}$$

$$\vec{\alpha}(\vec{\theta}) = \frac{4GD_L D_{LS}}{D_S c^2} \int d^2\vec{\theta}\,' \frac{\Sigma(\vec{\theta}\,')(\vec{\theta} - \vec{\theta}\,')}{|\vec{\theta} - \vec{\theta}\,'|^2}$$

- Bend angle is 2D gradient of *lensing potential*



Schneider

$$\phi(\vec{\theta}) = \frac{4GD_L D_{LS}}{c^2 D_S} \int d^2\vec{\theta}\,'\Sigma(\vec{\theta}\,') \ln(|\vec{\theta} - \vec{\theta}\,'|) \qquad \text{(thin lens)}$$

$$\vec{\alpha} = \nabla\phi.$$

2D Poisson Equation

- ϕ satisfies the 2D Poisson equation:

$$\nabla^2\phi(\vec{\theta}) = 2\kappa(\vec{\theta})$$

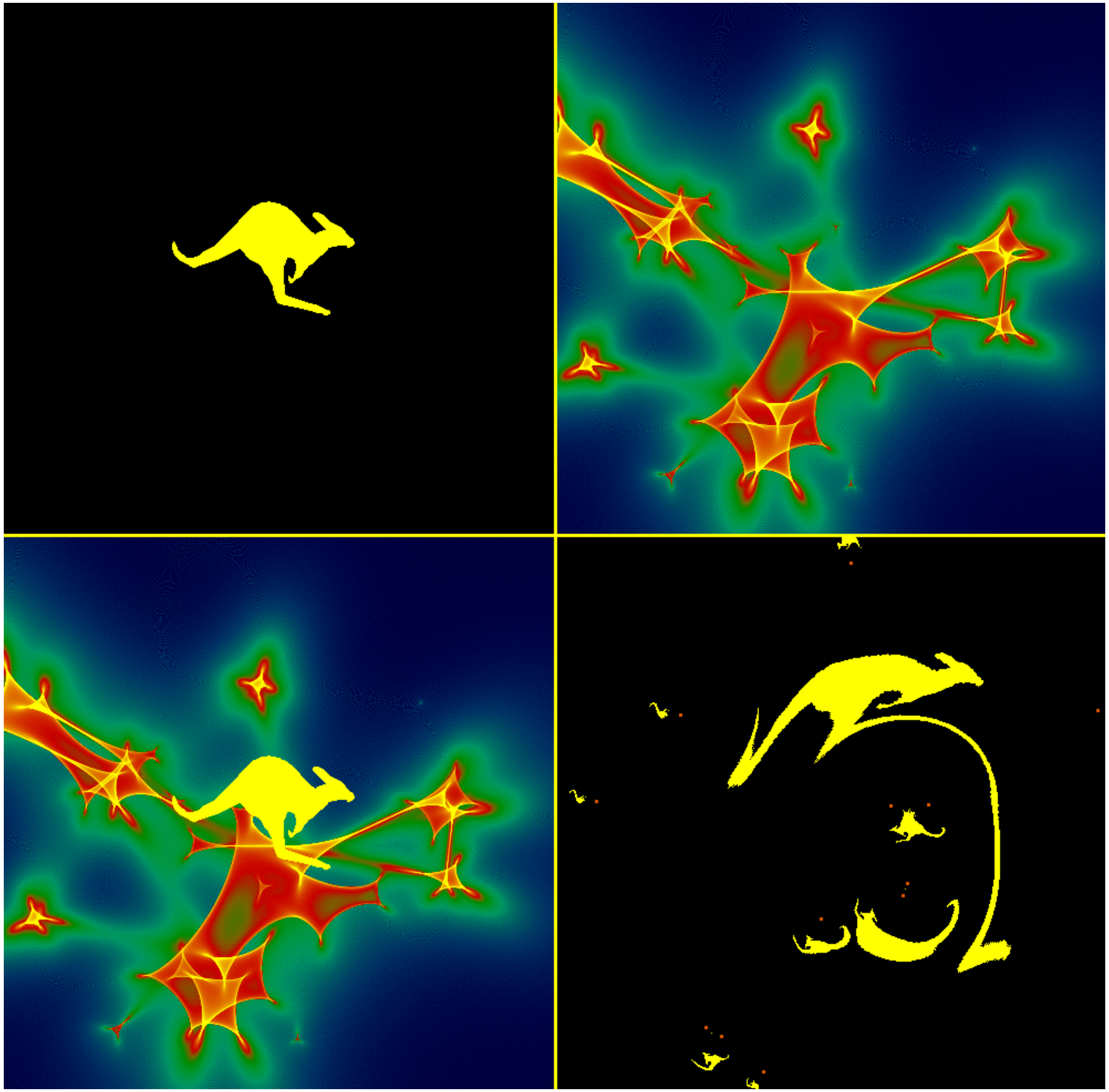
κ is the *convergence*

$$\kappa(\vec{\theta}) \equiv \Sigma(\vec{\theta})/\Sigma_{\rm crit}$$

- κ depends on source distance, as well as lens properties. *It is not directly observable*
- We will see how this equation allows us to invert image data (via ϕ) to obtain the surface mass density

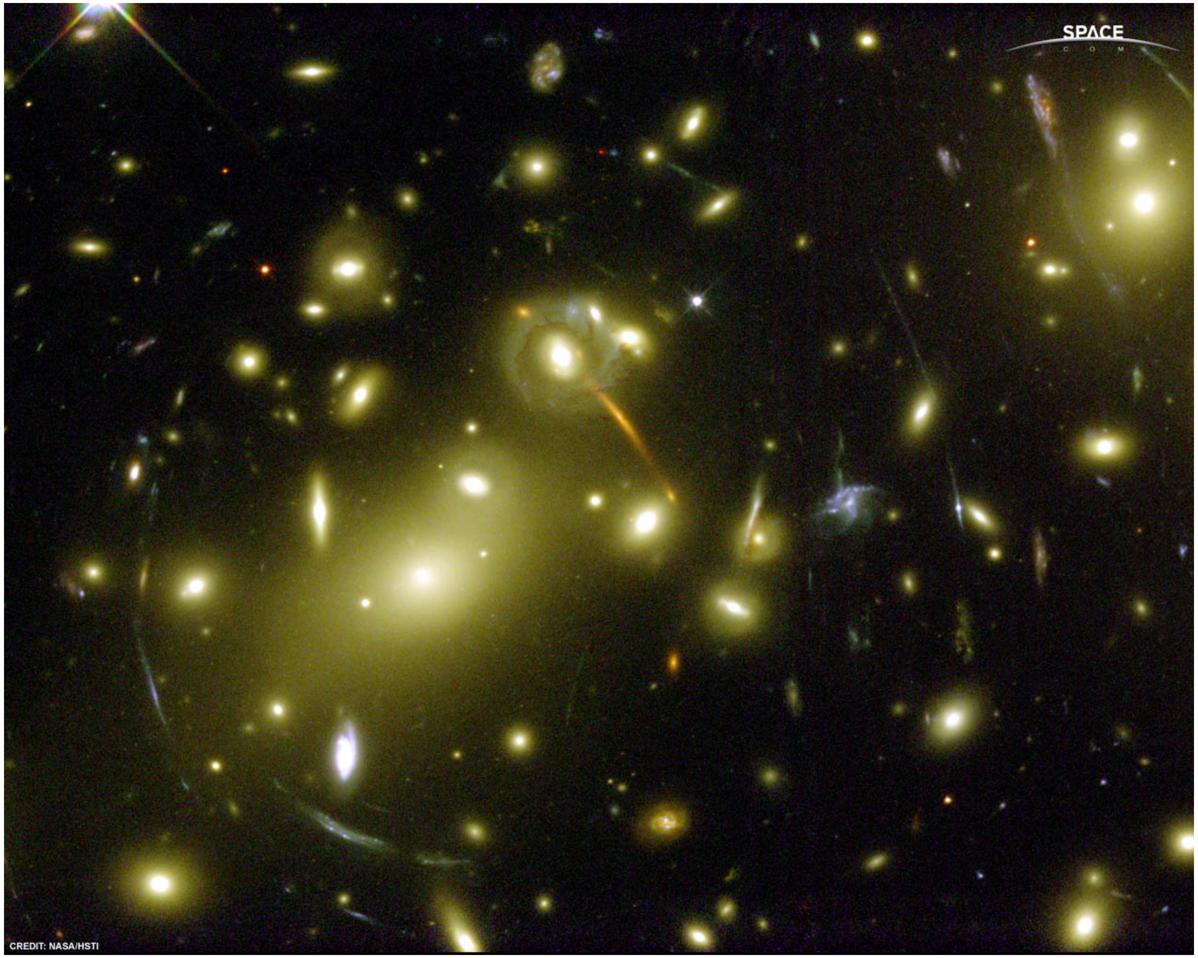
Caustics and critical lines

- $\kappa > 1$ is a sufficient (but not necessary) condition for ‘infinite’ magnification.
- Magnification is never infinite (to do so assumes a point source and geometrical optics), but can be very large…
- Highly magnified images occur if the source is close to a *caustic*, image on a *critical line*



J. Wambsganss

Arcs



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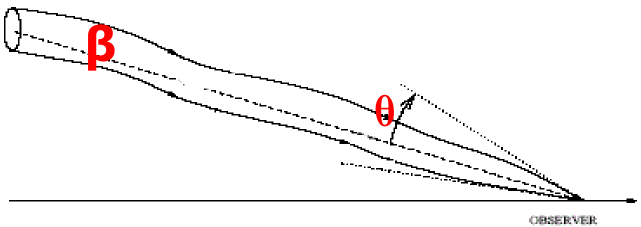
HST

Amplification, Magnification & Shear

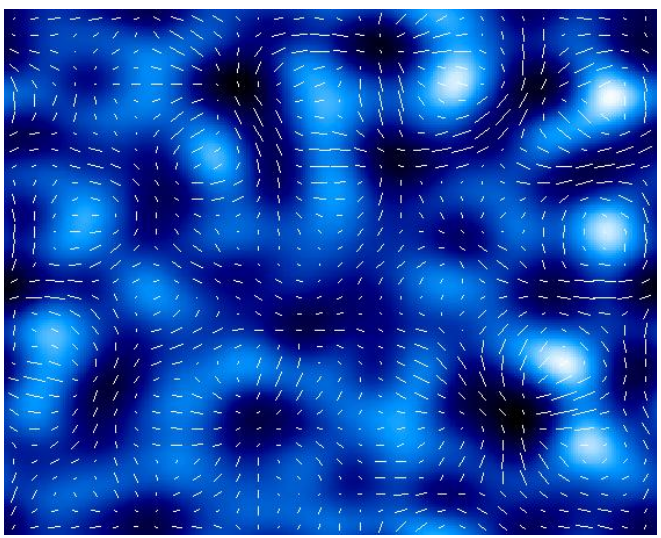
$$\vec{\beta} = \vec{\theta} - \vec{\alpha}$$

- Define the (inverse) amplification matrix:



$$A_{ij} \equiv \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \phi_{,ij} \qquad (\phi_{,ij} \equiv \partial^2 \phi / \partial\theta_i \partial\theta_j)$$

We can decompose this as follows:



$$A_{ij} = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

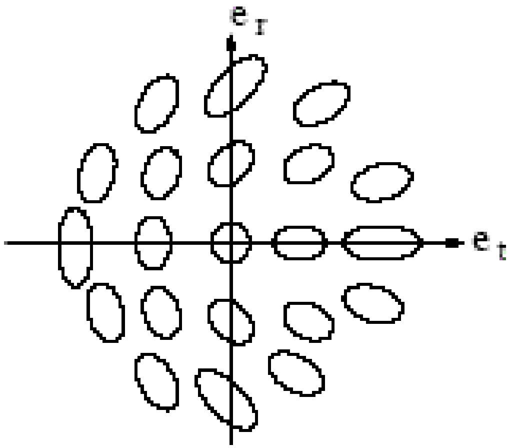
$$\begin{aligned} \kappa &= \frac{1}{2}(\phi_{,11} + \phi_{,22}) \\ \gamma_1 &= \frac{1}{2}(\phi_{,11} - \phi_{,22}) \equiv D_1 \phi \\ \gamma_2 &= \phi_{,12} \equiv D_2 \phi \end{aligned}$$

$$D_i D_i = (\nabla^2)^2 / 4$$

Complex Shear

The *complex shear* is

$$\gamma = \gamma_1 + i\gamma_2$$



Lensing preserves surface brightness, so the amplification of the source is

$$A = \frac{1}{\det A_{ij}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

Note that the overall magnification of the image is usually unobservable, so

$$A_{ij} = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

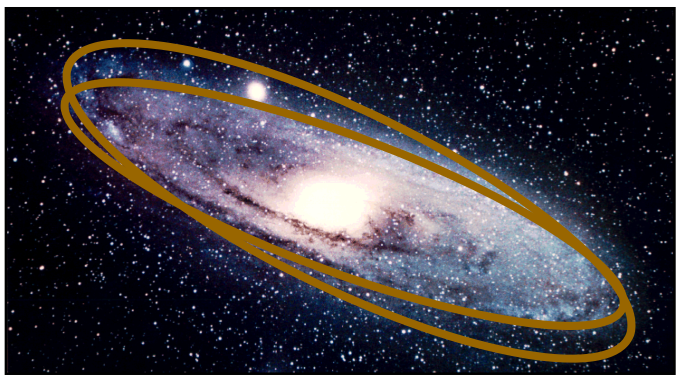
where g is the *reduced shear*, which is what can be more easily measured. $g \equiv \gamma/(1-\kappa) \simeq \gamma$ for weak lensing, where $|\kappa| \ll 1$.

- Note that A_{ij} is symmetric - not all locally linear distortions are allowed.

Estimating Shear (more details later)

Measure the ellipticity e of a galaxy. It is related to the intrinsic ellipticity e^s and reduced shear g by

$$e = \frac{e^s + g}{1 + g^* e^s}$$



Average over sources: $\langle e^s \rangle = 0 \Rightarrow \langle e \rangle = g$ i.e. e is unbiased

Note: error in g dominated by scatter in e^s.

For 'Cosmic Shear' $g \sim 0.01$. scatter in $e^s \sim 0.3$

2D (Dark) Matter reconstruction: thin lens

- Given a set of weak shear estimates, how do we map the matter?
  - We can estimate shear, but we would like κ
  - From κ we can obtain the surface density Σ, if we know the source redshifts

Work in Fourier space in 2D on sky. Expanding

$$\kappa_{\vec{\ell}} \equiv \int d^2\vec{\theta}\,\kappa(\vec{\theta})\,\exp(i\vec{\ell}\cdot\vec{\theta})$$

etc, then since $2\kappa = \nabla^2\phi$ and $D_1 = (\nabla_1^2 - \nabla_2^2)/2$ and $D_2 = \nabla_1\nabla_2$,

$$\begin{aligned}
\kappa_{\vec{\ell}} &= -\frac{1}{2}\ell^2\phi_{\vec{\ell}} \\
\gamma_{1\vec{\ell}} &= -\frac{1}{2}(\ell_1^2 - \ell_2^2)\phi_{\vec{\ell}} \\
\gamma_{2\vec{\ell}} &= -\ell_1\ell_2\phi_{\vec{\ell}}
\end{aligned}$$

where $\ell^2 = \ell_1^2 + \ell_2^2 = |\vec{\ell}|^2$.

Estimator for κ_ℓ

The following are estimators of $\kappa_{\vec{\ell}}$:

$$\left(\frac{\ell^2}{\ell_1^2 - \ell_2^2}\right) e_{1\vec{\ell}} \qquad \left(\frac{\ell^2}{2\ell_1 \ell_2}\right) e_{2\vec{\ell}}.$$

Variance in these estimators is proportional to

$$\sigma_e^2 \frac{\ell^4}{(\ell_1^2 - \ell_2^2)^2} \text{ and } \sigma_e^2 \frac{\ell^4}{(2\ell_1 \ell_2)^2}$$

respectively, where σ_e^2 is the variance in the source ellipticity distribution.

Optimal (inverse variance weighted) estimator for $\kappa_{\vec{\ell}}$ is

$$\hat{\kappa}_{\vec{\ell}} = \left(\frac{\ell_1^2 - \ell_2^2}{\ell^2}\right) e_{1\vec{\ell}} + \left(\frac{2\ell_1 \ell_2}{\ell^2}\right) e_{2\vec{\ell}}.$$

Estimating convergence

- Multiplication in ℓ space = convolution in real space. Quick way to solve these equations is (Kaiser & Squires 1993):

Remember $D_i D_i = (\nabla^2)^2/4$, so

$$\begin{aligned}
\gamma_i &= D_i \phi \\
&= 2 D_i \nabla^{-2} \kappa \\
\Rightarrow D_i \gamma_i &= 2 D_i D_i \nabla^{-2} \kappa \\
\Rightarrow \kappa &= 2 D_i \nabla^{-2} \gamma_i
\end{aligned}$$

But we know the solution to ∇^{-2} in 2D:

$$\nabla^{-2} \gamma_i(\vec{\theta}) = \frac{1}{2\pi} \int d^2 \vec{\theta}' \gamma_i(\vec{\theta}') \ln |\vec{\theta}' - \vec{\theta}|$$

Differentiate and sum:

$$\hat{\kappa}(\vec{\theta}) = \frac{2}{\pi} \int d^2\vec{\theta}' \frac{[\gamma_1(\vec{\theta}')\cos(2\alpha) + \gamma_2(\vec{\theta}')\sin(2\alpha)]}{|\vec{\theta}' - \vec{\theta}|^2}$$

where α is the angle between $\vec{\theta}$ and $\vec{\theta}'$.

Tempting: Replace integral by a sum over galaxies, and γ by its estimator, e:

$$\hat{\kappa}(\vec{\theta}) = \frac{2}{\bar{n}\pi} \sum_g \frac{[e_1(\vec{\theta}_g)\cos(2\alpha) + e_2(\vec{\theta}_g)\sin(2\alpha)]}{|\vec{\theta}_g - \vec{\theta}|^2}$$

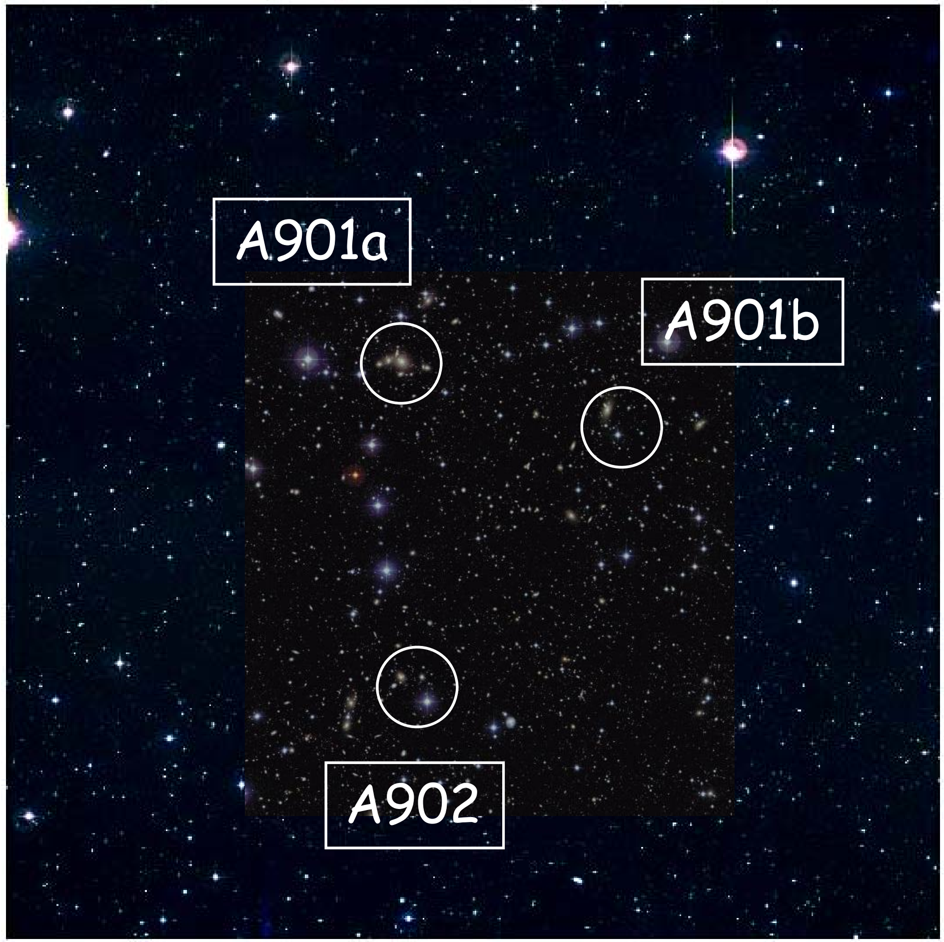
where $\bar{n}$ is the mean surface density of sources.

This is unbiased, but has *infinite noise* (from shot noise in γ). Solution is to *smooth*, at some point in the analysis.

Note - we can't get $\kappa_{\vec{\ell}=\vec{0}}$ - this is an example of the *mass-sheet degeneracy*.

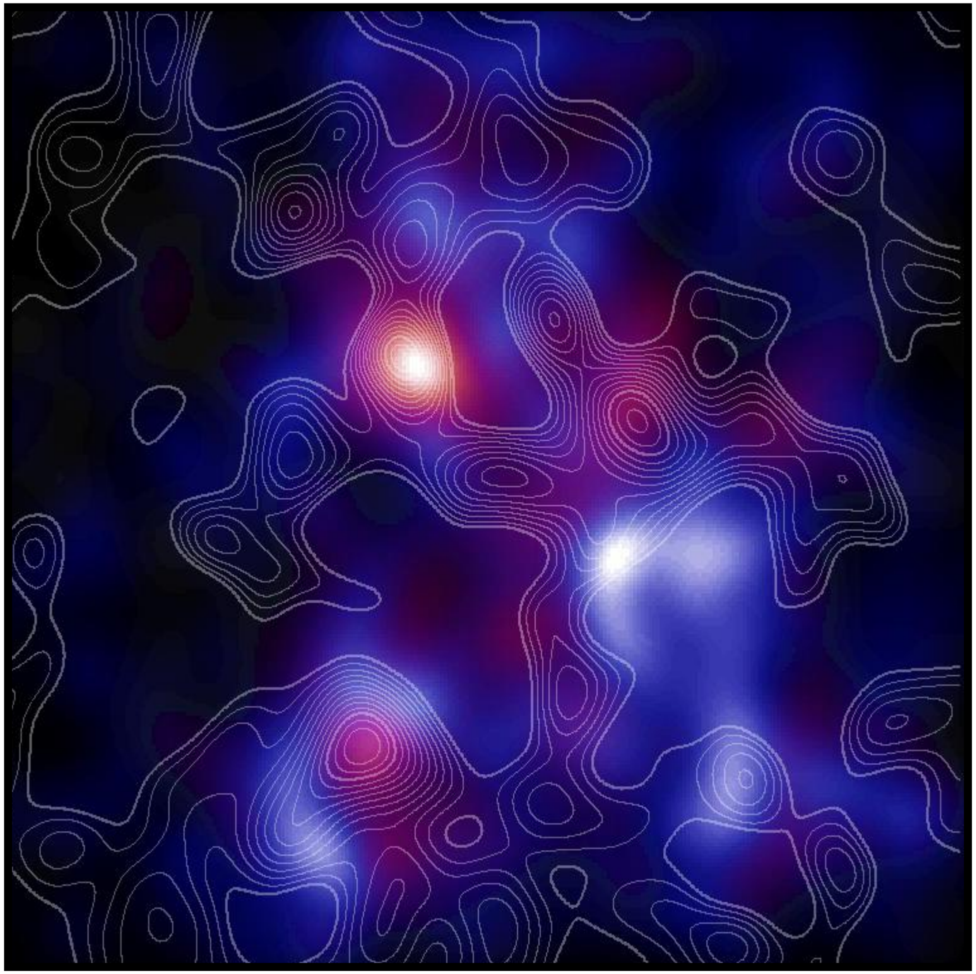
Supercluster Abell 901/2

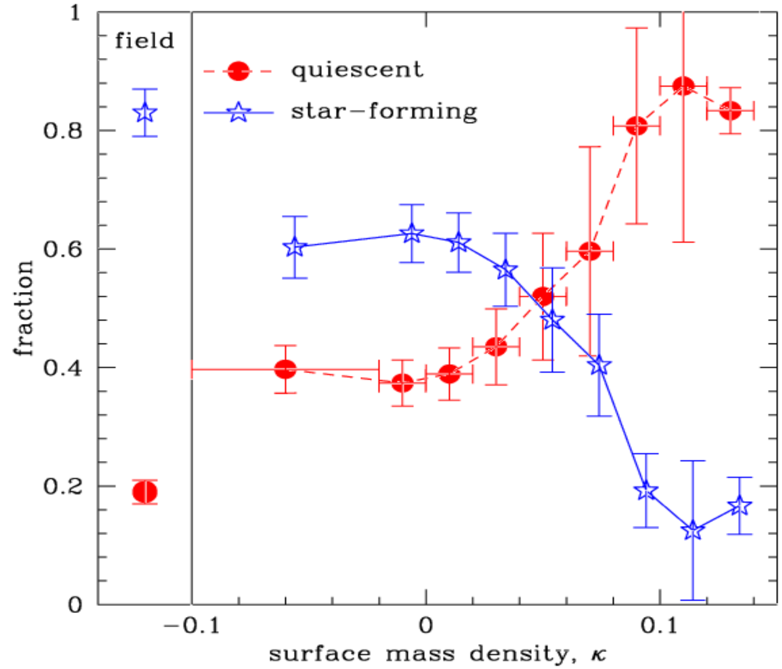
3Mpc/h



(Gray et al., 2002)

A901 cluster

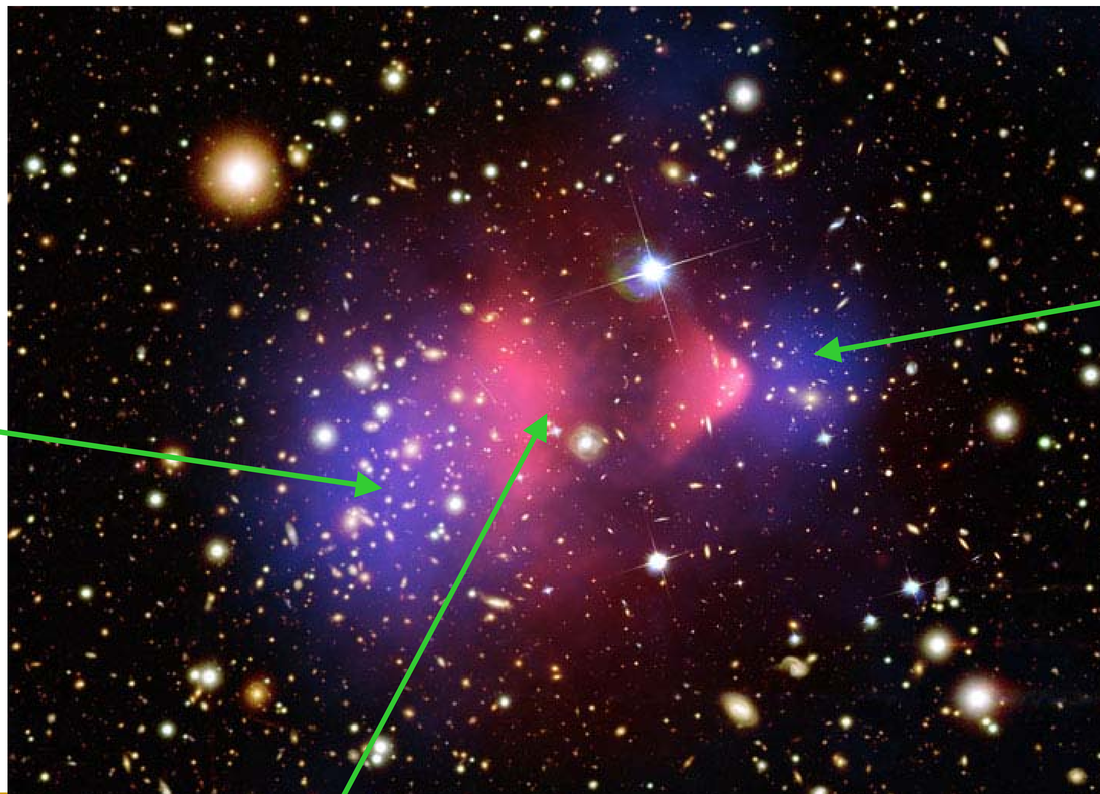




Gray et al 2004

‘Bullet cluster’

- Challenges MOND, TeVeS



Markevitch et al 2002

Clowe et al 2004

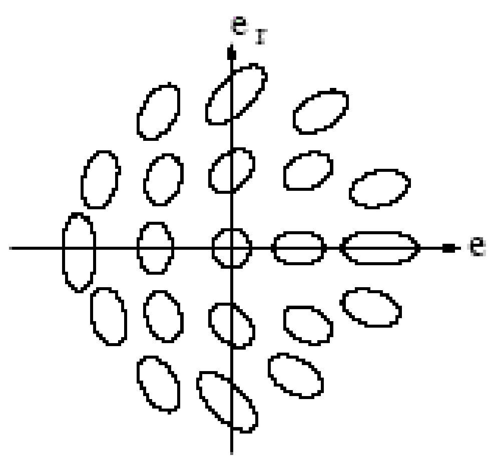
on shear and spin-weights

- Shear field is not a vector field, it is a *spin-weight 2* field

Rotating a coordinate system locally by ψ, a *spin-weight s* object transforms to

$$_sf \rightarrow e^{-is\psi} \quad _sf$$

- Scalar fields have *s=0*
- Shear is *s=2* because an axis rotation by $\pi/4$ changes γ by $\pi/2$



$s = \frac{1}{2}$

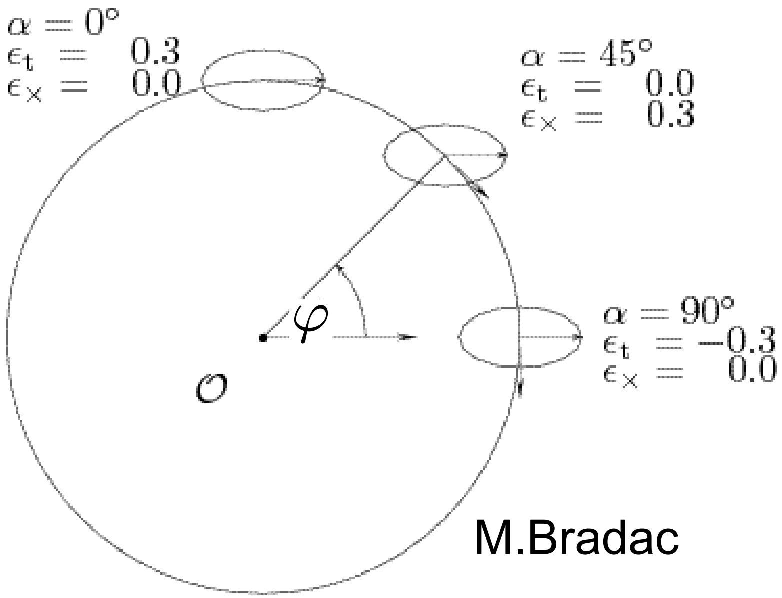
Tangential and cross shear

We now define the *tangential shear* by

$$\gamma_t \equiv -\mathcal{R}e(\gamma e^{-2i\varphi}),$$

and the *cross shear* by

$$\gamma_\times \equiv -\mathcal{I}m(\gamma e^{-2i\varphi})$$



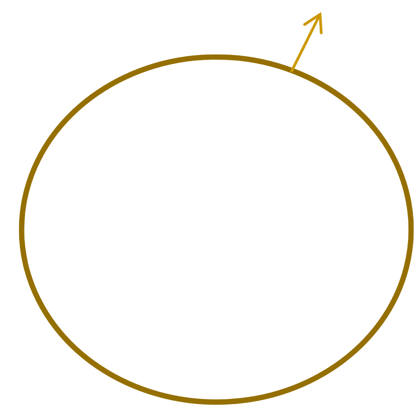
M.Bradac

Cluster masses from mean tangential shear

Important for Dark Energy – see later.

(This treatment is from Schneider, SAAS-FEE lectures)

The aim here is to estimate lensing masses from the shear field, with no assumptions about symmetry.
We start with Gauss' theorem in 2D:



$$\int d^2\vec{\vartheta}\,\nabla^2\phi = \oint ds\vec{\nabla}\phi\cdot\vec{n}$$

where $\vec{n}$ is an outward normal vector.

Since $\nabla^2\phi = 2\kappa$, then for a circular region,

$$2\int_0^\theta d^2\vec{\vartheta}\kappa(\vec{\vartheta}) = \theta\oint d\varphi\frac{\partial\phi}{\partial\theta}$$

LHS $\propto$ mass enclosed in the circle:

$$m(\theta) = \Sigma_{crit} \int d^2\vec{\vartheta}\,\kappa(\vec{\vartheta}) = \frac{\theta}{2}\Sigma_{crit} \oint d\varphi \frac{\partial \phi}{\partial \theta}$$

Hence

$$\frac{dm(\theta)}{d\theta} = \frac{m(\theta)}{\theta} + \frac{\theta}{2}\Sigma_{crit} \oint d\varphi\, \frac{\partial^2 \phi}{\partial \theta^2}$$

Now, on the x-axis,

$$\frac{\partial^2 \phi}{\partial \theta^2} = \phi_{,11} = \frac{1}{2}\left(\phi_{,11} + \phi_{,22}\right) + \frac{1}{2}\left(\phi_{,11} - \phi_{,22}\right) = \kappa(\vec{\theta}) + \gamma_1(\vec{\theta})$$

Since on this axis, $\gamma_1 = -\gamma_t$,

$$\frac{\partial^2 \phi}{\partial \theta^2} = \kappa(\theta) - \gamma_t(\theta)$$

and since this is now independent of φ explicitly, it holds generally, and the line integral is then expressible in terms of the average κ and γ_t around the circle:

$$\frac{dm(\theta)}{d\theta} = \frac{m(\theta)}{\theta} + \pi\theta\Sigma_{crit}\left[\langle\kappa(\theta)\rangle - \langle\gamma_t(\theta)\rangle\right]$$

Now, the mass enclosed can be written in terms of the average convergence on circles $\langle\kappa\rangle$, or in terms of the average in the circle, $\bar{\kappa}$:

$$m(\theta) = \Sigma_{crit}\left[\pi\theta^2\bar{\kappa}(\theta)\right] = \Sigma_{crit}\int_0^\theta d\vartheta\langle\kappa(\vartheta)\rangle 2\pi\vartheta$$

The second of these gives

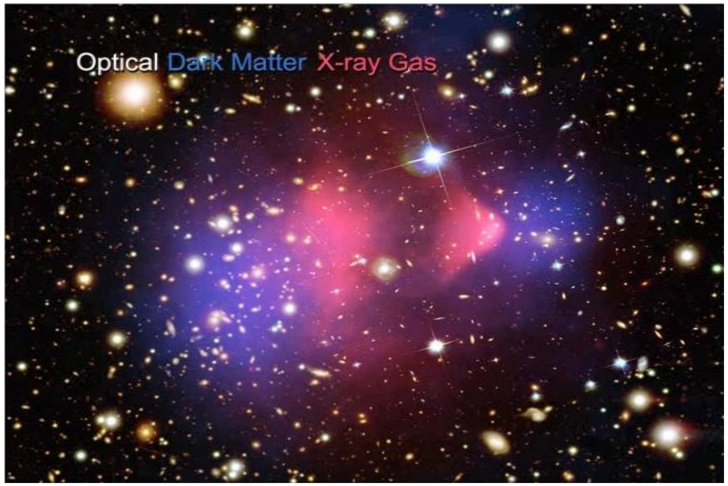
$$\frac{dm(\theta)}{d\theta} = 2\pi\theta\Sigma_{crit}\langle\kappa(\theta)\rangle$$

and substituting this and the first (for m) into the differential equation for m gives

$$\langle\gamma_t\rangle = \bar{\kappa} - \langle\kappa\rangle$$

For clusters, if we take a large radius where $\langle\kappa\rangle$ is small, we can estimate m from the average tangential shear, via $\bar{\kappa}$.

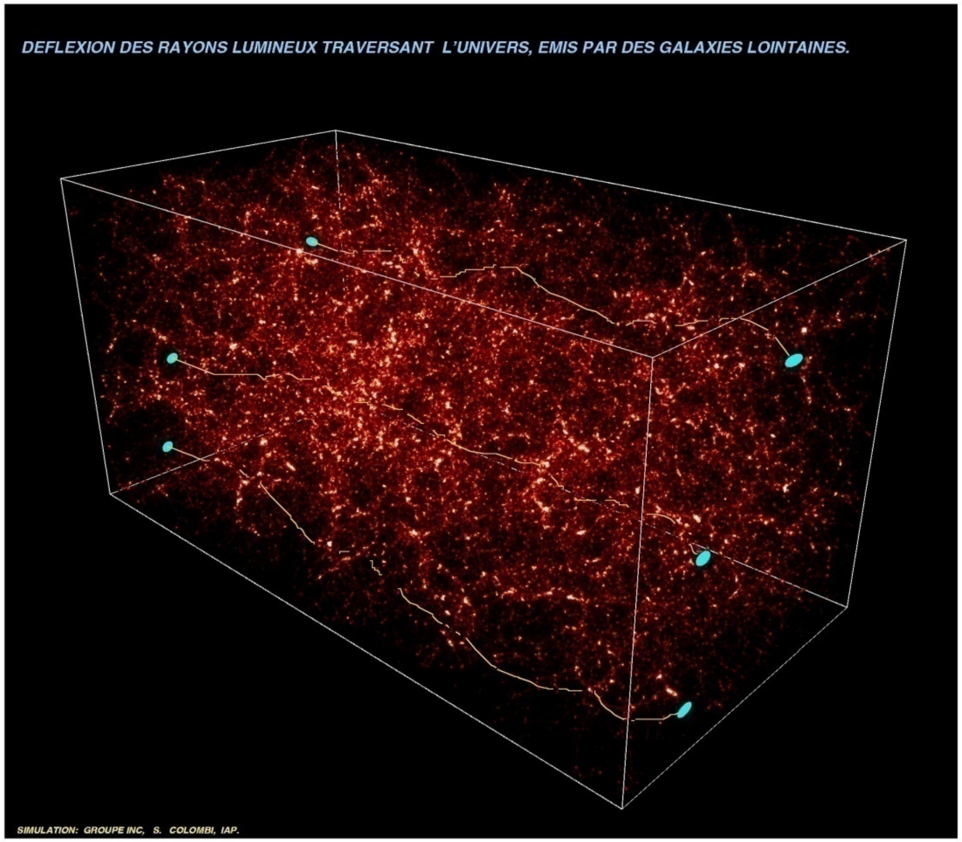
Conclusions of mass modelling of clusters



- Masses *mostly* agree well with those from other methods, where they are relaxed (lensing doesn't care)
- Clusters are rather centrally concentrated – consistent with NFW
- Clusters often show substructure in lensing maps
- Baryon fractions generally consistent with standard cosmological model
- Self-interacting dark matter strongly constrained – SIDM smooths the central profile, and reduces asymmetry, leading to too few arcs and severe distortions
- TeVeS: Note that the convergence in MOND-type gravity theories is not proportional to surface density, but they still have problems with the bullet cluster.

Cosmic Shear: lensing by clumpy Universe

- Problem: for cosmic shear, lens is all the way along the line-of-sight



Cosmological Lensing

For a weakly-perturbed Universe

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\Psi}{c^2}\right) a^2(t) \left[dr^2 + S_k^2(r) d\beta^2\right]$$

where Φ is the (Newtonian) gravitational potential.

We take $\Phi = \Psi$, relevant if no anisotropic stresses
Note: in general lensing depends on $\Phi + \Psi$ (exercise for student).

Convenient to work with conformal time $d\eta = cdt/a(t)$, and flat sky $d\theta_{x,y}$:

$$ds^2 = a^2(\eta) \left\{ \left(1 + \frac{2\Phi}{c^2}\right) d\eta^2 - \left(1 - \frac{2\Phi}{c^2}\right) \left[dr^2 + S_k^2(r)(d\theta_x^2 + d\theta_y^2)\right] \right\}.$$

Solve for $r(\eta)$ for unperturbed radial ray: $0 = d\eta^2 - dr^2 ...$

$$\boxed{\frac{dr}{d\eta} = -1.}$$

Geodesic equation:

$$\frac{d^2x^\lambda}{dp^2} + \Gamma^\lambda{}_{\mu\nu}\frac{dx^\mu}{dp}\frac{dx^\nu}{dp} = 0,$$

parametrised in terms of some p, and the affine connection is

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2}g^{\sigma\lambda}\left\{\frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\sigma\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right\}$$

Geodesic equation for η:

$$\frac{d^2\eta}{dp^2} = -2\frac{\dot{a}}{a}\dot{\eta},$$

By choosing the unit of p appropriately, we find

$$\frac{d\eta}{dp} = \frac{1}{a^2}.$$

Light propagation equation

Geodesic equation for θ_x (similarly for θ_y), in a flat universe

$$\frac{d^2\theta_x}{d\eta^2} - \frac{2}{r}\frac{d\theta_x}{d\eta} = -\frac{2}{c^2r^2}\frac{\partial\Phi}{\partial\theta_x}.$$

Simplifies to

$$\frac{d^2\vec{x}}{d\eta^2} = -\frac{2}{c^2}\vec{\nabla}\Phi$$

where $\vec{x} = r\vec{\theta}$ is the transverse displacement

EXERCISE: Generalise the result for the propagation of light to a non-flat Universe:

$$\frac{d^2\vec{x}}{d\eta^2} + k\vec{x} = -\frac{2}{c^2}\vec{\nabla}\Phi.$$

Cosmological lensing potential

- Remarkably, we can write the shear as the D_i gradients of a potential, just as with a thin lens:

Solve

$$\frac{d^2\vec{x}}{d\eta^2} = -\frac{2}{c^2}\vec{\nabla}\Phi$$

Integrate along *radial ray* (Born approximation), and use $dr = -d\eta$:

$$x_i = r\theta_i - \frac{2}{c^2}\int_0^r dr'\,\frac{\partial\Phi}{\partial x_i'}(r-r').$$

(I reversed the order of integration).

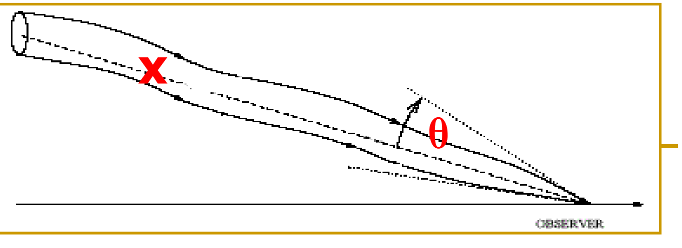


Image distortions: nearby rays

- Make Taylor expansion of gradient:

$$\Delta x_i = r\Delta\theta_i - \frac{2}{c^2}\Delta\theta_j \int_0^r dr' r'(r-r') \frac{\partial^2 \Phi}{\partial x'_i \partial x'_j}$$

or

$$\Delta x_i = r\Delta\theta_j(\delta_{ij} - \phi_{,ij})$$

and Δx_i becomes $r\beta_i$ at the source.

and

$$\phi(\vec{r}) \equiv \frac{2}{c^2}\int_0^r dr' \frac{S_k(r-r')}{S_k(r)\,S_k(r')} \Phi(\vec{r'})$$

is the *Cosmological Lensing Potential*. We have generalised this to the non-flat case for completeness.

Recap: cosmological lensing

- Cosmological lensing potential: $$\phi(\vec{r}) \equiv \frac{2}{c^2} \int_0^r dr' \frac{(r-r')}{rr'} \Phi(\vec{r'}) \quad \text{(flat)}$$

$$\begin{aligned}
\kappa(\vec{r}) &= \frac{1}{2}(\phi_{,11} + \phi_{,22}) \\
\gamma_1(\vec{r}) &= \frac{1}{2}(\phi_{,11} - \phi_{,22}) \equiv D_1\phi \\
\gamma_2(\vec{r}) &= \phi_{,12} \equiv D_2\phi
\end{aligned}$$

Using Poisson's equation

$$\nabla^2_{3D}\Phi = \frac{3H_0^2\Omega_m}{2a(t)}\delta$$

where $\delta = \rho/\bar{\rho} - 1$ is the fractional matter overdensity, the convergence is

$$\kappa(\vec{r}) = \frac{3H_0^2\Omega_m}{2c^2} \int_0^r dr' \frac{r'(r-r')}{r} \frac{\delta(\vec{r'})}{a(r')}$$

Averaging over source redshift distribution

If the source probability distribution is $p(r)$, then the convergence averaged over a line-of-sight is

$$\tilde{\kappa} = \int_0^\infty dr' \kappa(\vec{r'}) p(r')$$

Reversing the order of integration,

$$\tilde{\kappa} = \frac{3H_0^2\Omega_m}{2c^2} \int_0^\infty dr\, r\, \frac{g(r)}{a(r)}\, \delta(\vec{r})$$

where

$$g(r) \equiv \int_r^\infty dr'\, p(r') \left(\frac{r' - r}{r'} \right).$$

Cosmology theory does not predict δ(**r**), but only its statistical properties, e.g. Power spectrum P(k), or the correlation function.

Connection to cosmological parameters

- Consider 2-point quantities (e.g. power spectrum, correlation function)
- Relate to the 3D matter power spectrum P(k):

$$\langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^D(\mathbf{k} - \mathbf{k}') P(k).$$

where Fourier transform of $\delta(\mathbf{x})$ is $\delta_{\mathbf{k}}$.

$$\begin{aligned} \kappa_\ell &= \int d^2\boldsymbol{\Theta}\, \kappa(\boldsymbol{\Theta}) e^{-i\boldsymbol{\ell}.\boldsymbol{\Theta}} \\ &= A \int_0^\infty dr\, r \frac{g(r)}{a(r)} \int d^2\boldsymbol{\Theta}\, \delta(r\boldsymbol{\Theta}, r) e^{-i\boldsymbol{\ell}.\boldsymbol{\Theta}} \end{aligned}$$

$$A \equiv 3H_0^2 \Omega_m / 2c^2$$

$$\langle \kappa_\ell \kappa^*_{\ell'} \rangle = A^2 \int_0^\infty dr\, G(r) \int_0^\infty dr'\, G(r') \int d^2\Theta d^2\Theta' \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{d^3\mathbf{k}'}{(2\pi)^3}$$
$$\langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle \exp(ik_\| r - ik_\|' r') \exp(i\mathbf{k}_\perp.\Theta - i\mathbf{k}'_\perp.\Theta') \exp(-i\boldsymbol{\ell}.\Theta + i\boldsymbol{\ell}'.\Theta')$$

$\langle \delta_k \delta^*_{k'} \rangle$ term gives a delta function

$$G(r) \equiv r g(r)/a(r)$$

This introduces $P(\sqrt{k_\|^2 + \mathbf{k}_\perp^2}) \simeq P(|\mathbf{k}_\perp|)$.

$k_\|$ integral gives another delta function.

Θ, Θ' integrations give more delta functions

$$\langle \kappa_\ell \kappa^*_{\ell'} \rangle = (2\pi)^2 \delta^D(\boldsymbol{\ell} - \boldsymbol{\ell}') P_\kappa(|\ell|).$$

$$P_\kappa(\ell) = \left(\frac{3H_0^2\Omega_m}{2c^2}\right)^2 \int_0^\infty dr \left[\frac{g(r)}{a(r)}\right]^2 P(\ell/r; r).$$

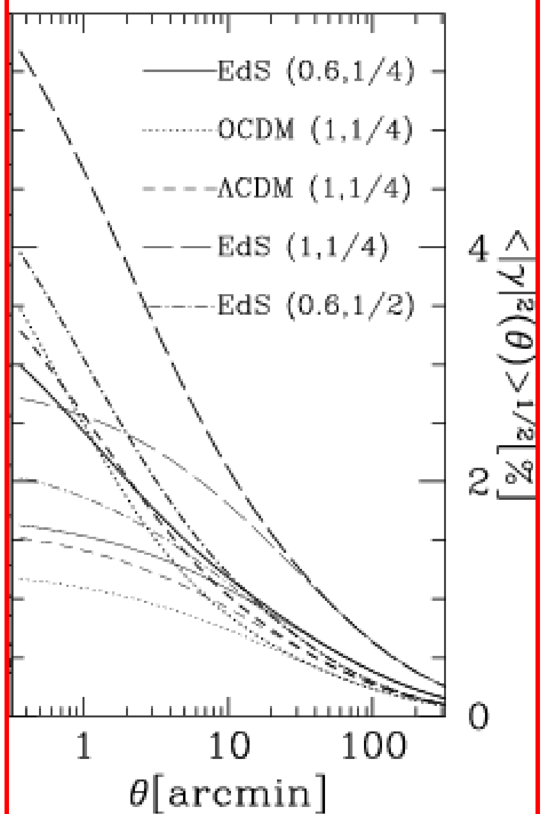
Convergence Power Spectrum

Example 2-point statistic: shear correlation function

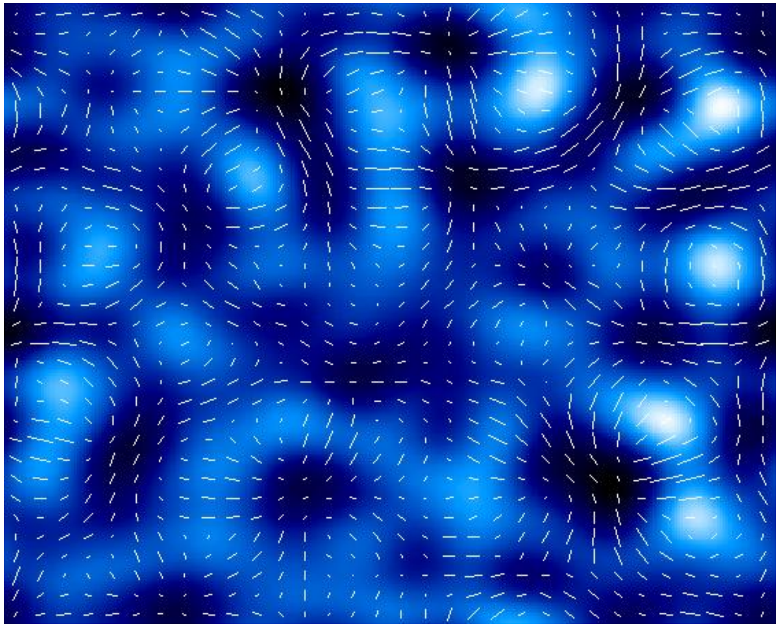
- Shear correlations are observable

Exercise: Show that $P_\gamma(\ell) = P_\kappa(\ell)$.

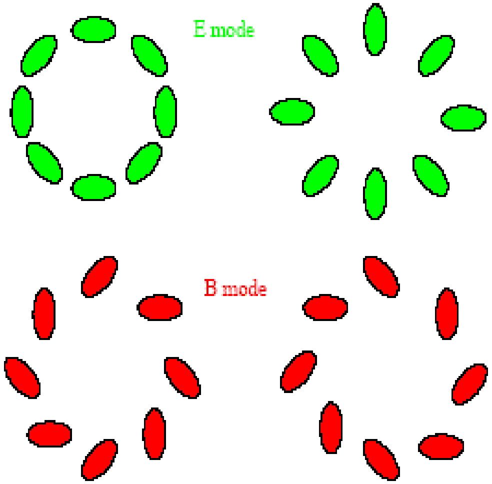
$$\begin{aligned}
\langle \gamma\gamma^* \rangle_\theta &= \int \frac{d^2\ell}{(2\pi)^2}\, P_\gamma(\ell)\, e^{i\boldsymbol{\ell}.\boldsymbol{\Theta}} \\
&= \int \frac{\ell d\ell}{(2\pi)^2}\, P_\kappa(\ell) e^{i\ell\theta\cos\varphi} d\varphi \\
&= \int \frac{d\ell}{2\pi} \ell P_\kappa(\ell) J_0(\ell\theta)
\end{aligned}$$



E- and B-modes



Jain & Seljak

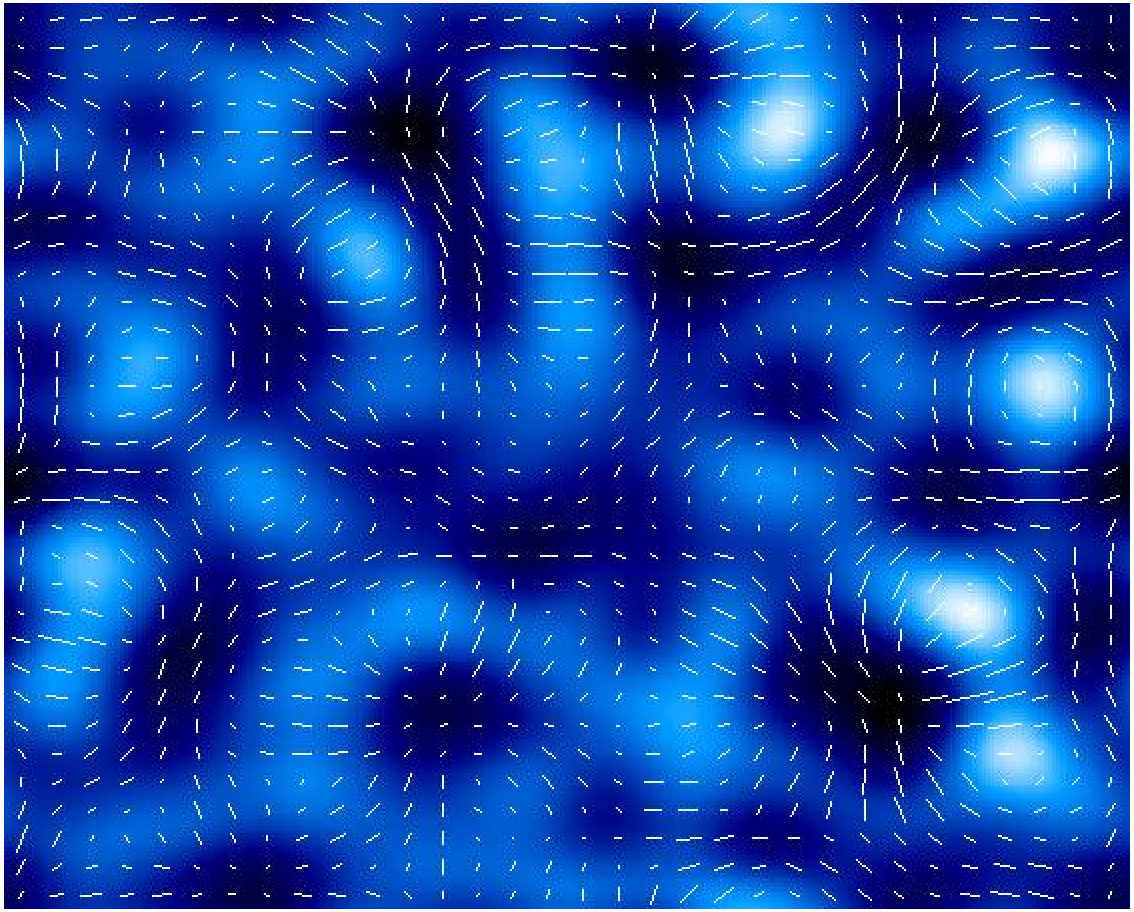


Lensing essentially produces only E modes

Presence of B-modes indicates something is wrong

B modes from galaxy clustering, 2nd-order effects (both small), imperfect PSF modelling, optics systematics, intrinsic alignments of galaxies

Cosmic shear maps



Statistical properties depend on

a) how clumpy the Universe is (via P)

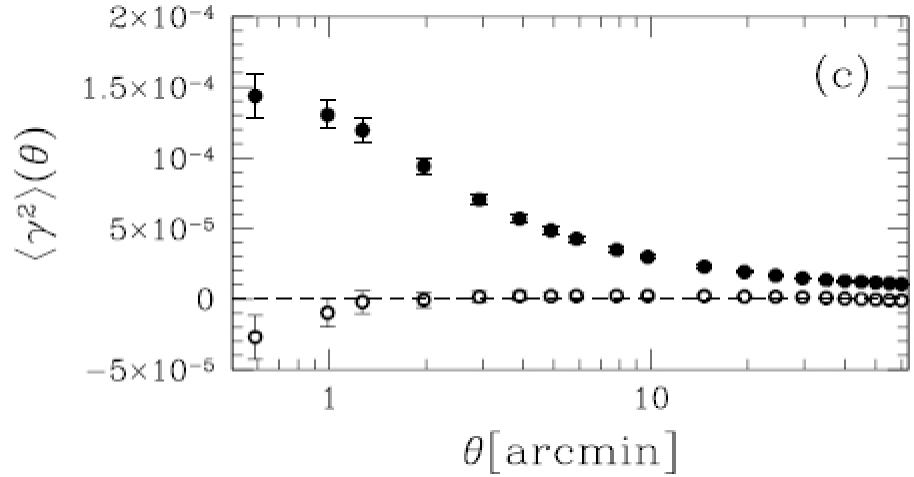
b) the source distances (via g(r))

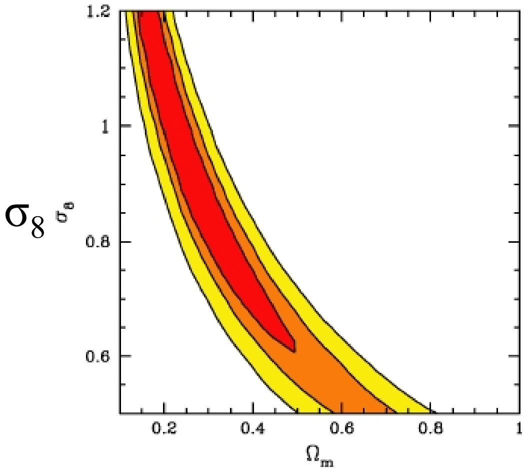
c) the r(z), $S_k(r)$ relations

(we measure p(z), not p(r))

⇒ can probe cosmology

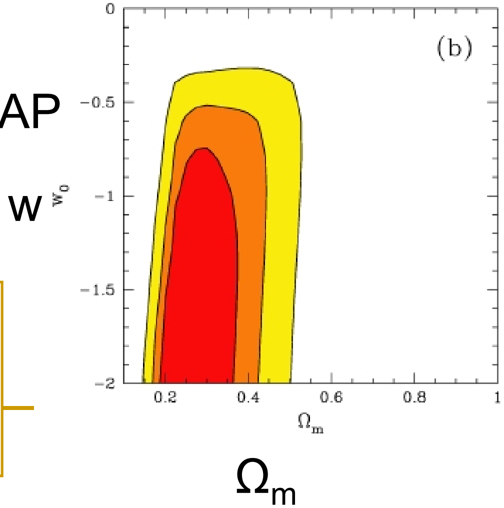
Recent results: CFHTLS





22 sq deg; median z=0.8. σ_8 high compared with WMAP

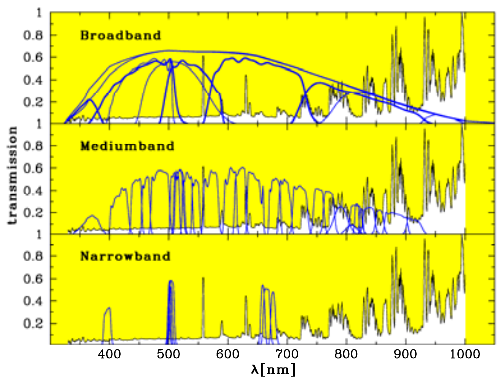
Hoekstra et al 2005; see also Semboloni et al 2005



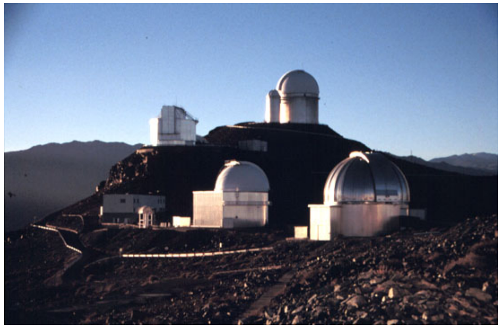
But: original estimate of *p(z)* was from Hubble Deep Field galaxies – and HDF is very small and subject to large sample variance

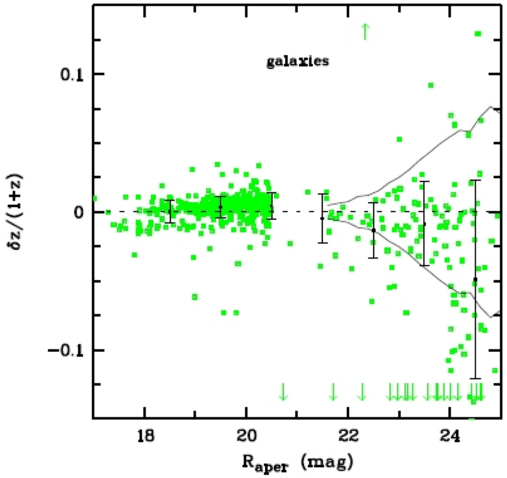
Challenges I: photometric redshifts

- Spectroscopy is too slow.
- Can estimate redshifts to within 0.02-0.1, depending on number and wavelength of bands, and type of galaxy.



COMBO-17





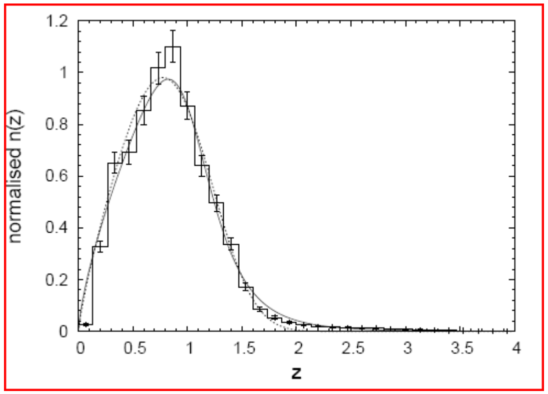
Wolf et al 2004

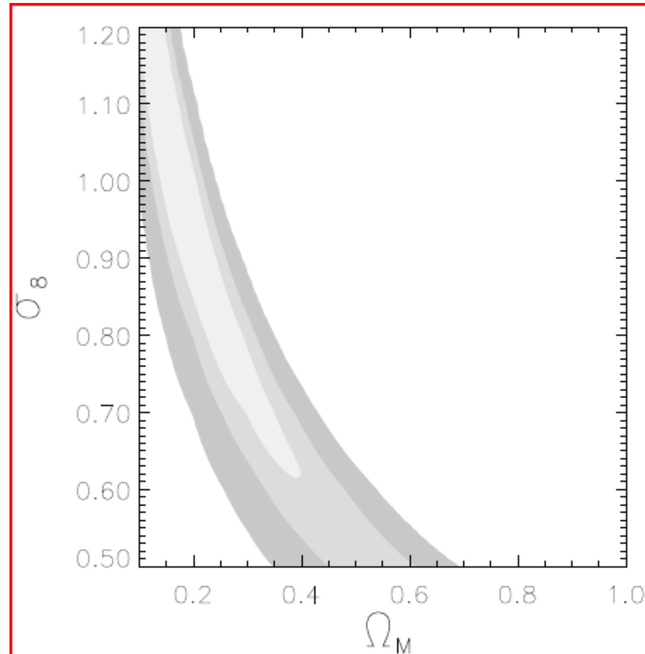
For accurate Dark Energy studies, the photo-z distribution has to be calibrated *very* accurately – with systematic error in the median z of a few times 10^{-3} or better. This needs ~10^5 spectroscopic redshifts (WFMOS?)

Reanalysis of recent data

- 100 square degree survey (Benjamin et al 2007). Better p(z), from CFHTLS photozs

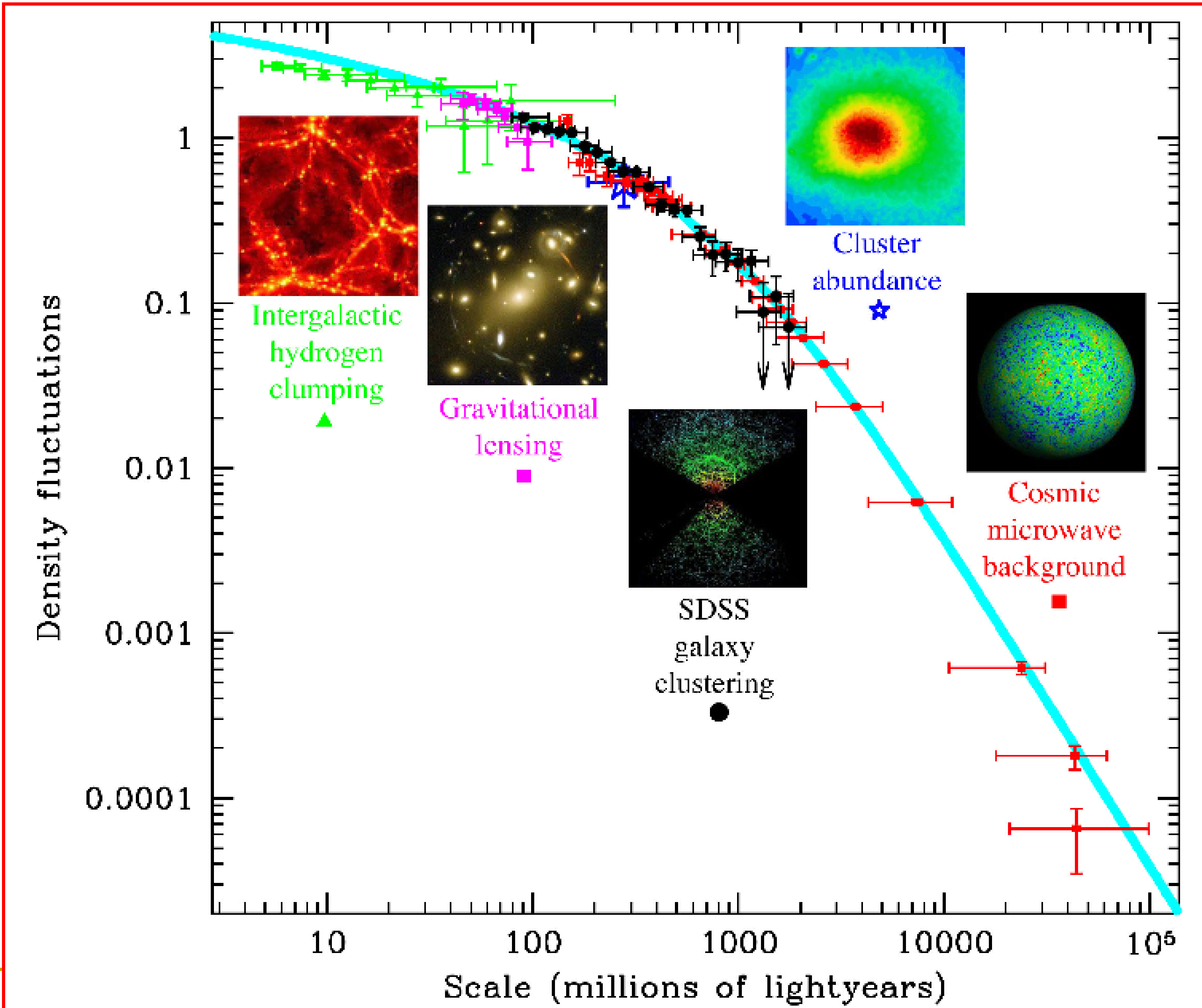
$$\sigma_8 = 0.84 \pm 0.07 \left(\frac{\Omega_m}{0.24}\right)^{0.59}$$





Removes serious tension between lensing results and WMAP.

Reason: poor redshift distributions used previously



Tegmark 2004

3D lensing

- With photo-zs, much more is possible:
  - 3D gravitational potential and matter density reconstruction
  - Better cosmological parameter estimation
  - Better control of systematics

3D matter density reconstruction

- Taylor 2001

Can invert lensing potential:

$$\phi(\vec{r}) = \frac{2}{c^2}\int_0^r dr' \left(\frac{1}{r'} - \frac{1}{r}\right)\Phi(\vec{r'})$$

to

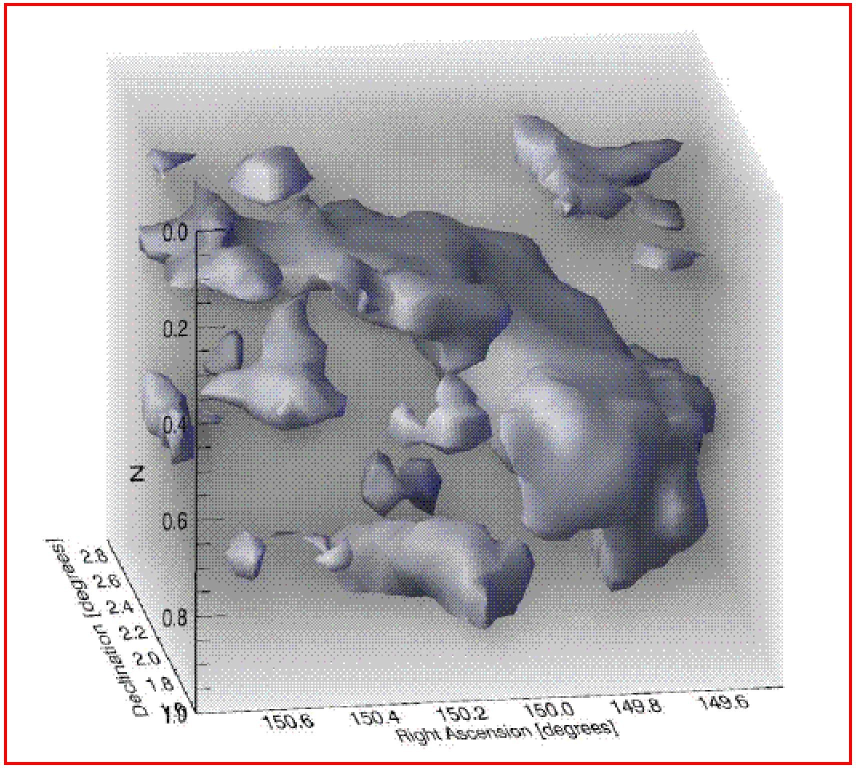
$$\Phi(\vec{r}) = \frac{c^2}{2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\phi(\vec{r})\right]$$

and hence to the mass overdensity:

$$\delta(\vec{r}) = \frac{a(t)c^2}{3H_0^2\Omega_m}\nabla^2_{3D}\left\{\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\phi(\vec{r})\right]\right\}$$

3D reconstruction: COSMOS field

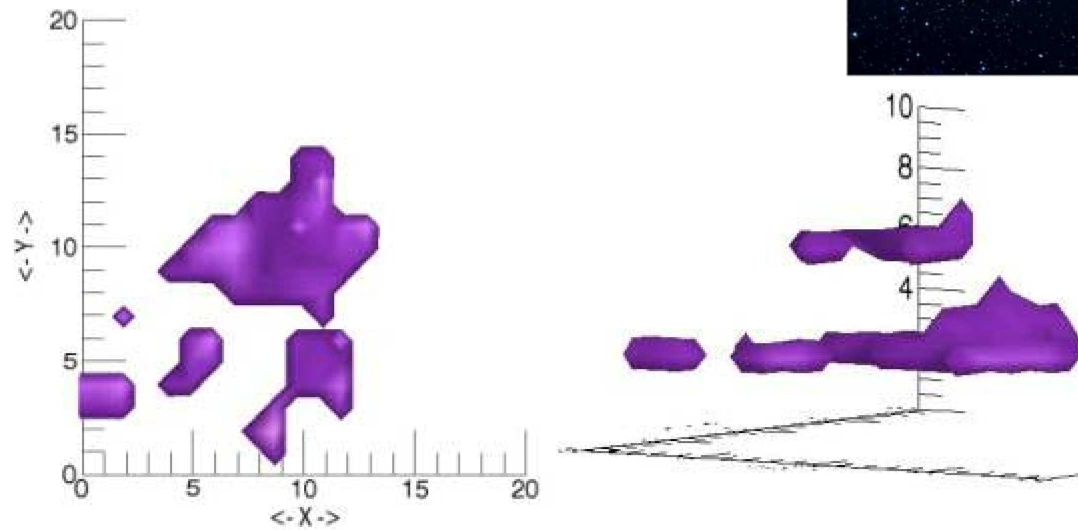
- Massey et al 2007



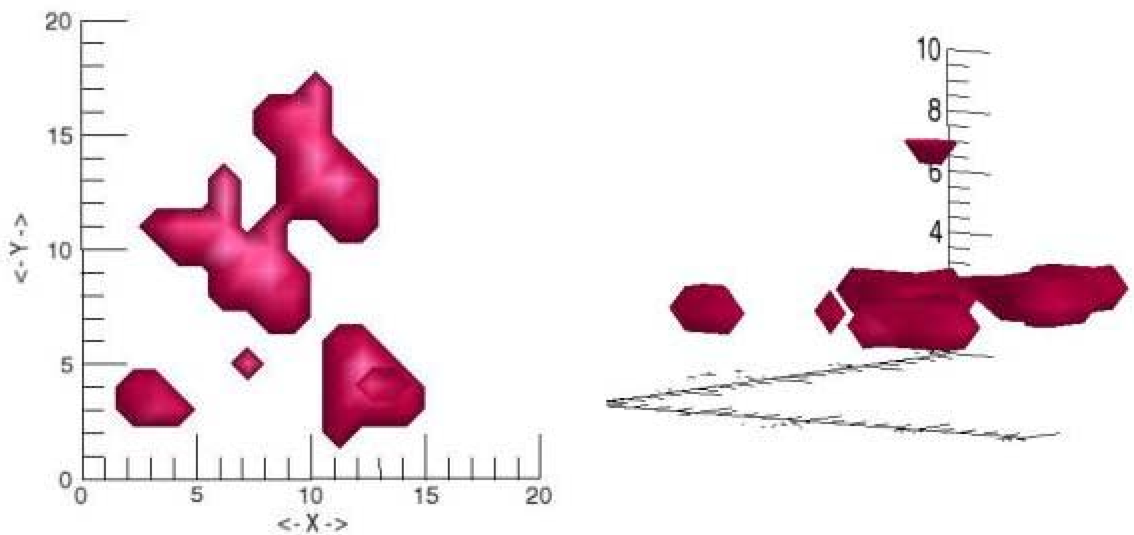
Beware! Mass-sheet degeneracy in 3D; poor resolution in z (200 Mpc)

3D Mass Reconstruction – A901

- Potential Field:



- Galaxy density:



Taylor et al, 2004

Cosmology and Dark Energy

- Measurable Effects of Dark Energy:
  - Distance-redshift relation

$$r = \int_0^z dz' \, \frac{c}{H(z')}$$

where the Hubble parameter is given by

$$H^2(a) = H_0^2 \left[\Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_q \exp\left(3 \int_1^a \frac{da'}{a'} \left[1 + w(a') \right] \right) \right]$$

  - Growth rate of perturbations

Assuming DE is smooth,

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m \delta = 0$$

Dark energy from clusters: the shear-ratio test

Recall that for a circular aperture,

$$\langle\gamma_t\rangle = \bar{\kappa} - \langle\kappa\rangle,$$

and that

$$\kappa = \frac{\Sigma}{\Sigma_{crit}} = \frac{4\pi G D_L D_{LS} \Sigma}{c^2 D_S}$$

so

$$\langle\gamma_t\rangle = \left[\bar{\Sigma} - \langle\Sigma\rangle\right] \frac{4\pi G a(z_L) S_k(r_L) a(z_S) S_k(r_S - r_L)}{c^2 a(z_S) S_k(r_S)}.$$

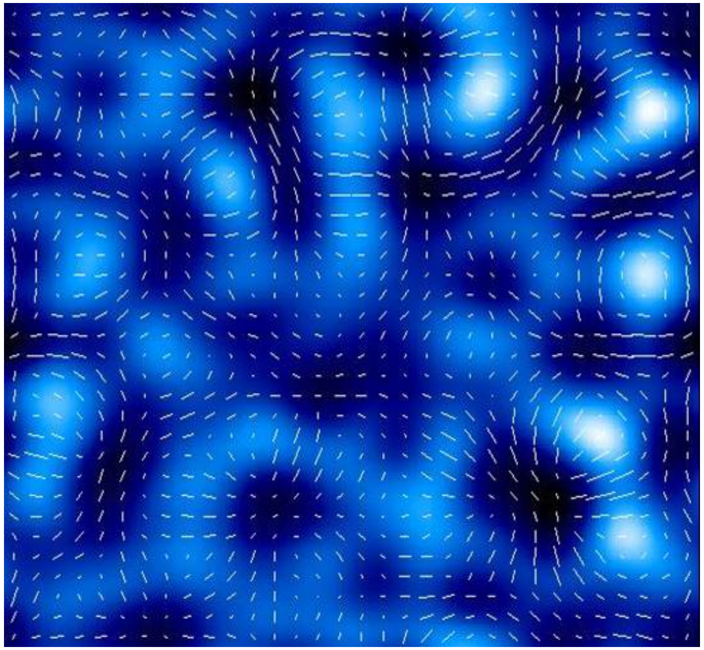
All the cluster physics is in the term in square brackets.
If we take *ratios* of average tangential shears, the cluster distribution drops out, leaving

$$\frac{\langle\gamma_t\rangle_1}{\langle\gamma_t\rangle_2} = \frac{S_k(r_2) S_k(r_1 - r_L)}{S_k(r_1) S_k(r_2 - r_L)}$$

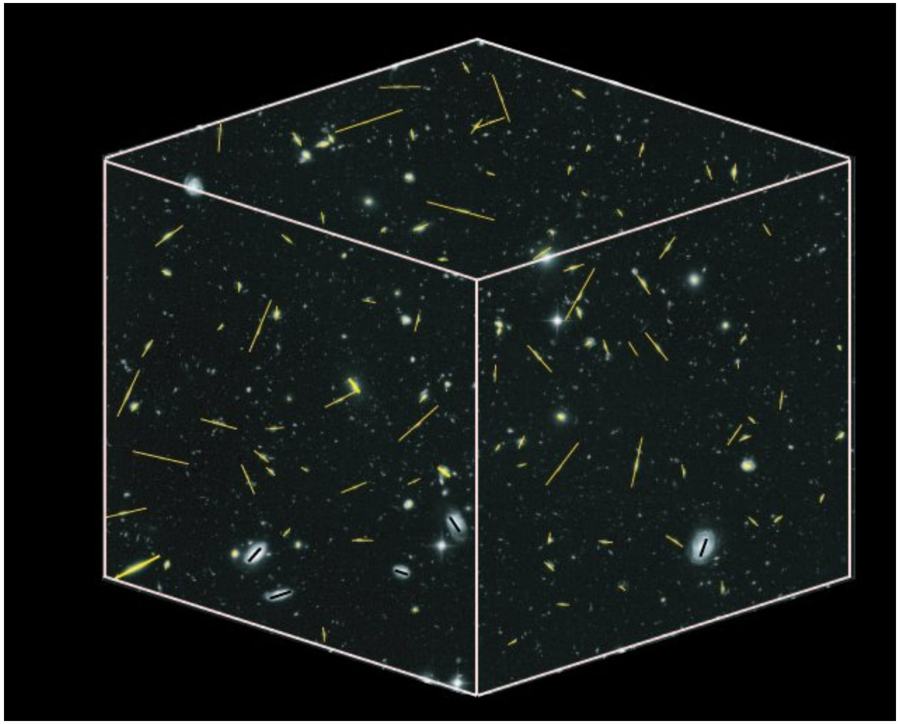
for two source shells 1 and 2.

3D Statistical Analysis

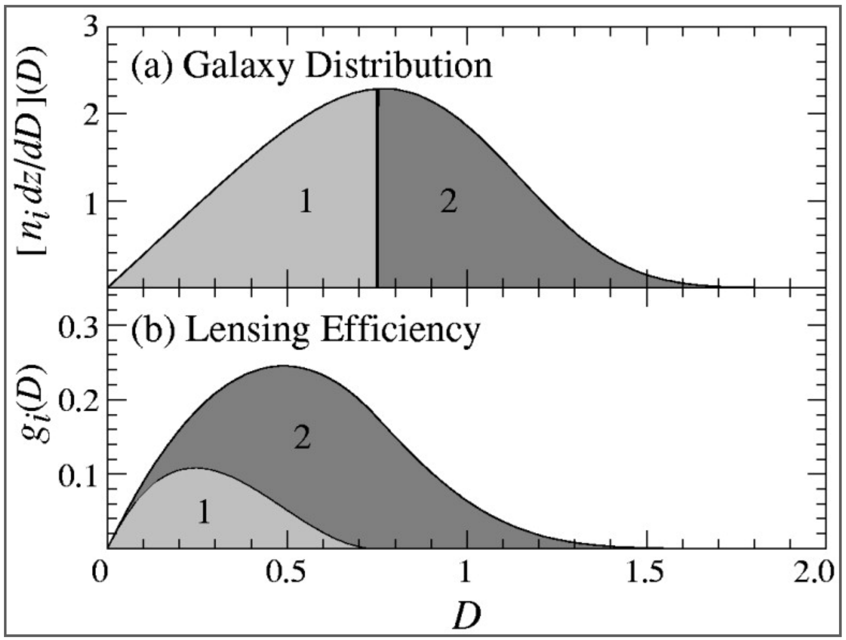
- With 3D source positions, why project at all?
- Treat survey as a discretely-sampled, very noisy, 3D field

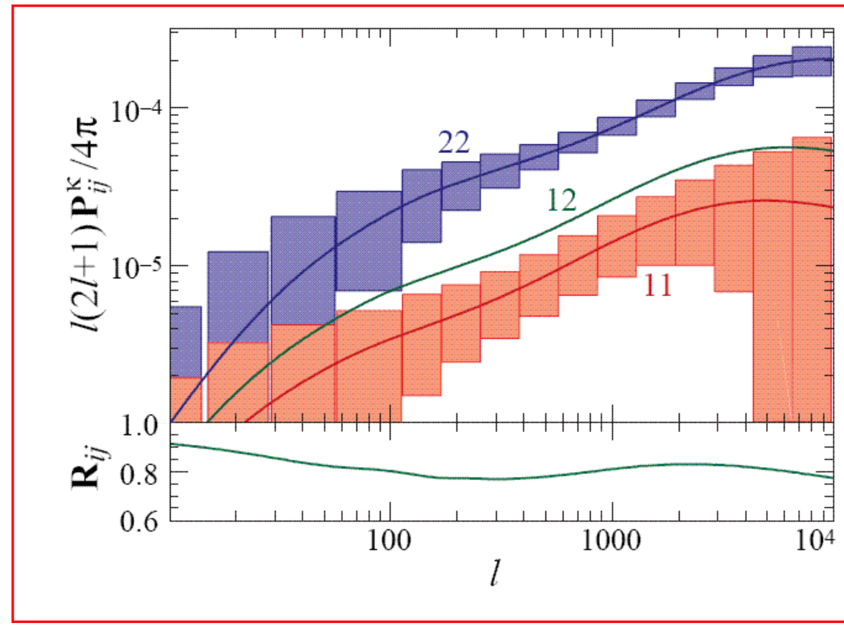


+z →



Steps to 3D: lensing in slices (Tomography)





Hu 1999

Dividing the source distribution improves parameter estimation

Need to go beyond linear theory

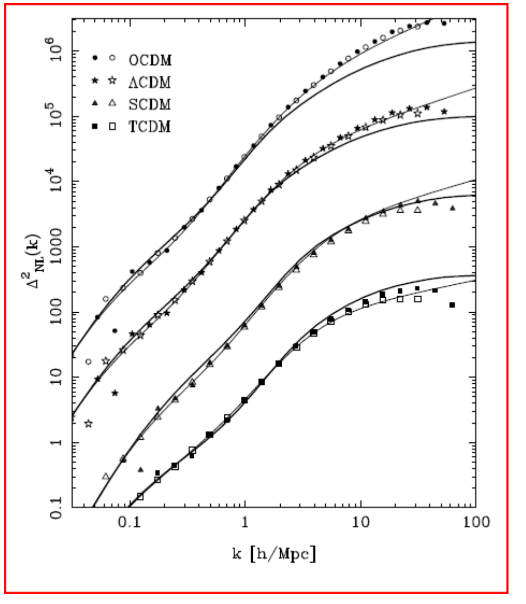
- Need to go beyond linear theory to get good signal-to-noise in weak lensing

Linear

Nonlinear

Nonlinear Power Spectrum

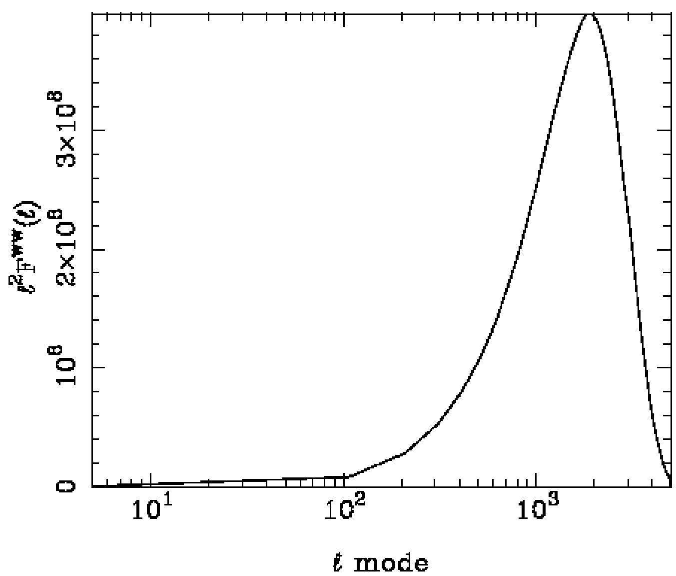
- Nonlinear P(k) is quite accurately known, from N-body simulations
- Baryons? Affect high k
- k > 10, or 2? Debate



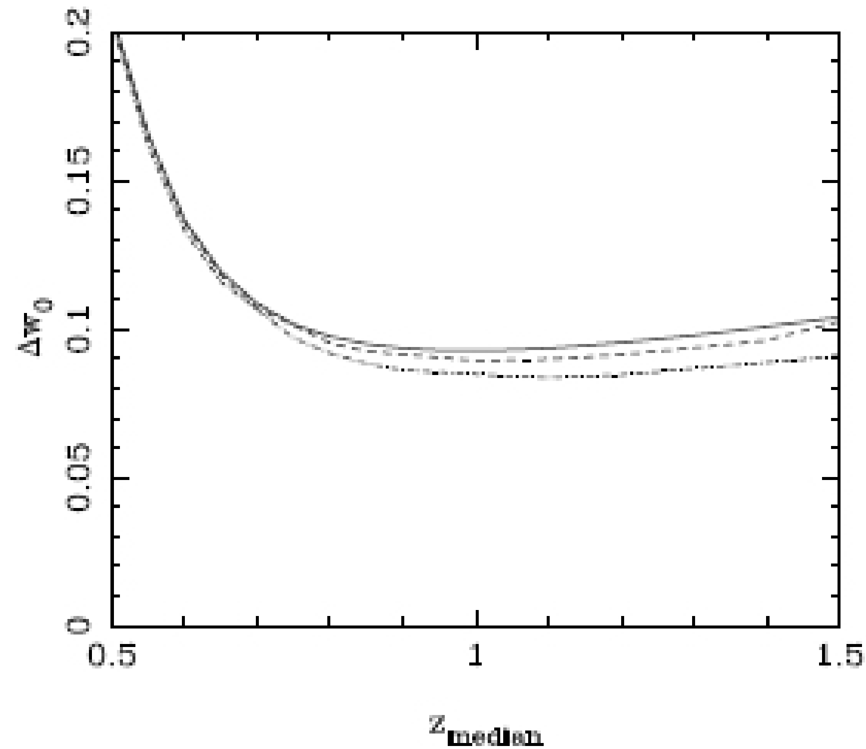
Smith et al 2003

Where does signal come from?

- Most signal for $w=p/\rho c^2$ from $\ell \sim 1000$



- Best to target z~1 for measuring w



COMBO 17 – Dark Energy results: 3D shear only

- First 3D shear power spectrum analysis
- Two fields (0.5 sq deg)
- Smaller error bars than 2D (from galaxies with photozs)

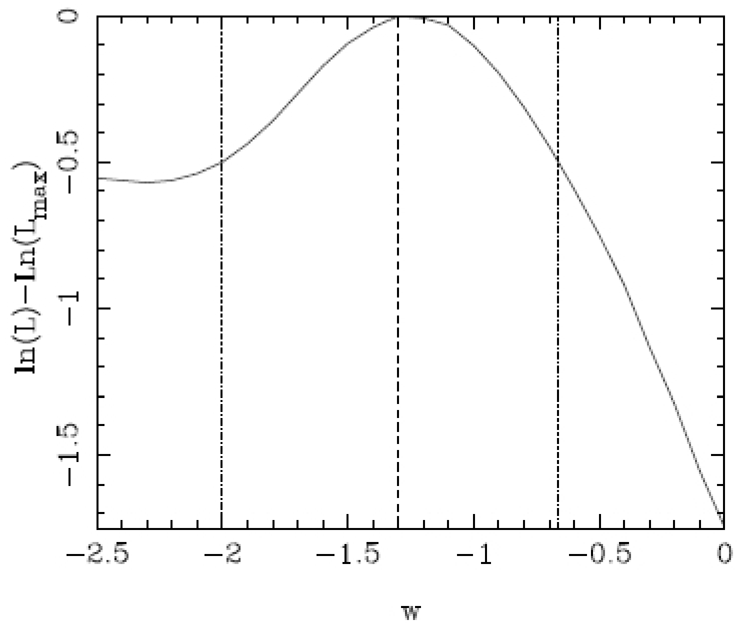


Figure 2. The one-parameter maximum likelihood constraint on w from the CDFS and S11 fields using the 3D cosmic shear analysis. The dashed line shows the most likely value and the dot-dashed show the one-parameter 1-σ constraints.

Kitching, AFH et al 2007

COMBO-17 Dark Energy results: geometric test, and both together

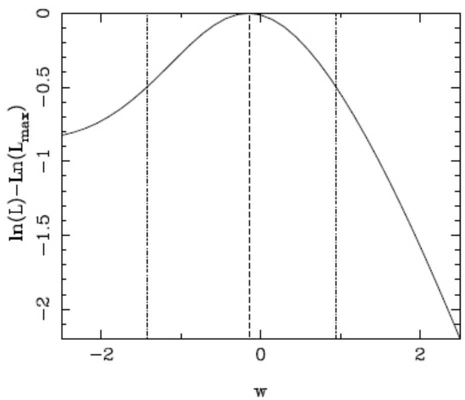
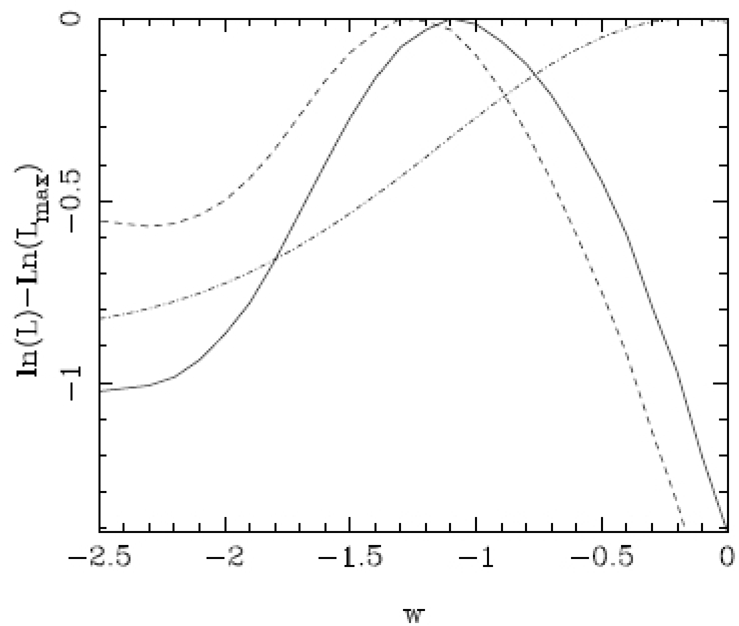


Figure 3. The dark energy geometric shear-ratio analysis applied to the supercluster Abell A901/2. The dashed line marks the maximum likelihood value, the dot-dashed lines show the one-parameter 1-σ limits. Note that the x-axis scale has been extended relative to Figures 2 and 3 to encompass the confidence limits of this analysis.



- Note: Conditional error only
- From 0.75 square degrees only

w = -1.1 +/- 0.6

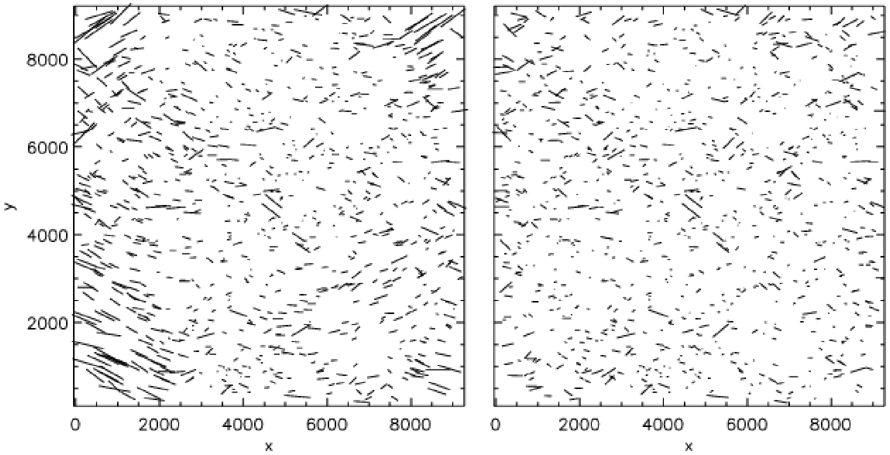
Not a competitive error, but proof of concept for future large 3D surveys

Kitching, AFH et al 2007

Challenges and prospects for weak lensing

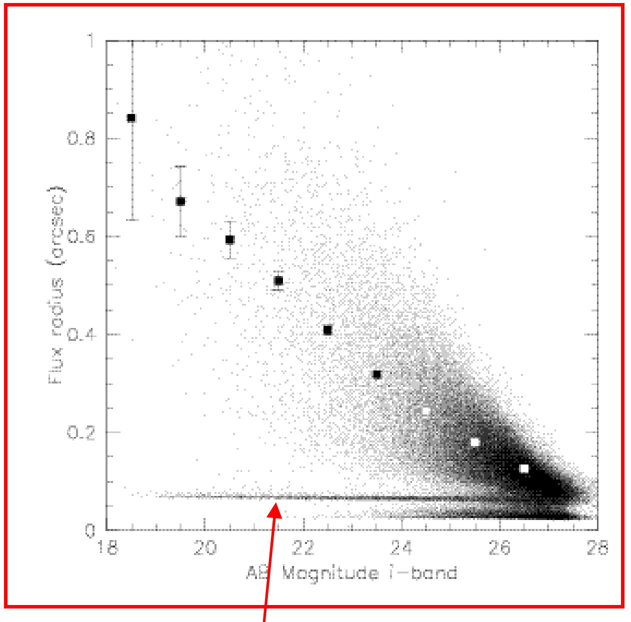
Challenges II: Image quality

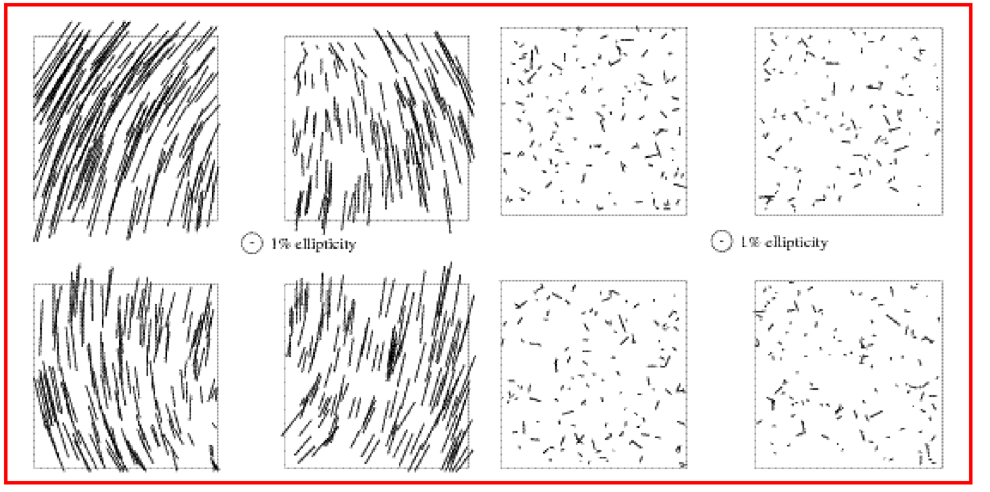
- Shear is ~1%
- Telescope optics & atmosphere may distort images to ~10%
- Use stars to correct for the Point Spread Function (PSF) distortions



Correcting for telescope distortions

- Can be even worse…





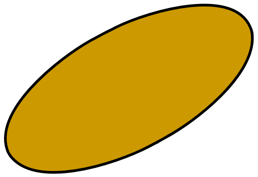
Weak lensing has been done successfully with this.

Select stars to correct the PSF

Relating ellipticity to shear: KSB method

- Measure moments of the light distribution:

$$Q_{ij} \equiv \int d^2\Theta \theta_i \theta_j I(\Theta) W(\Theta)$$



$$Q^s_{ij} = A_{il} Q_{lm} A_{jm}$$

KSB:

$$e'_1 \equiv \left(\frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} \right); \qquad e'_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}}$$

$$e' = \frac{e'^s + 2\gamma + g^2 e'^{s*}}{1 + |g|^2 + 2Re(ge'^{s*})}$$

Seitz & Schneider:

$$e = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

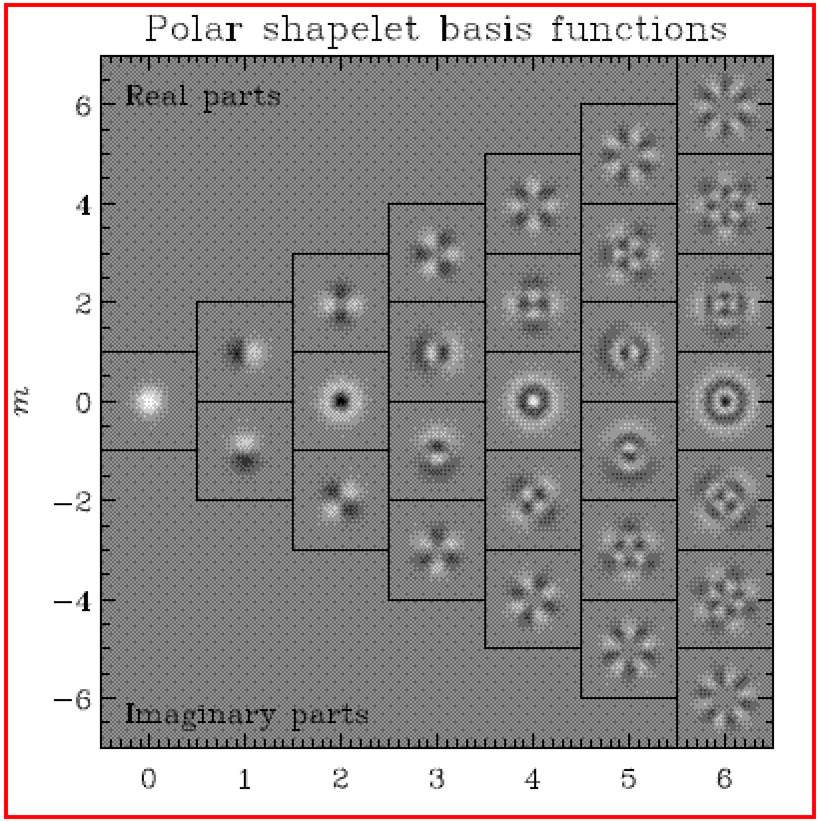
$$e_S = \frac{e - g}{1 - g^* e}$$

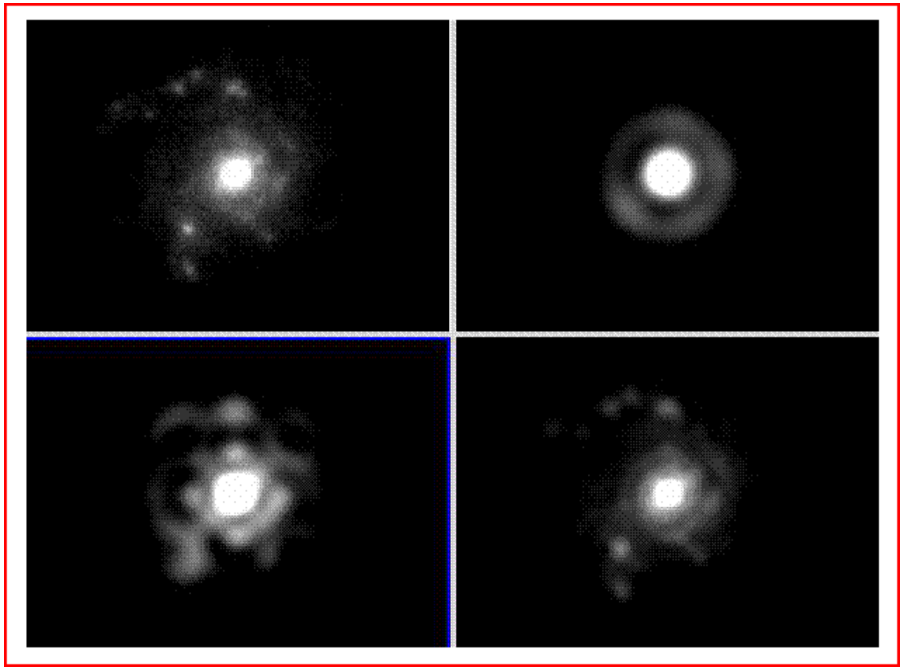
(Cleaner transfor mation)

Shapelets

Alternative shape measurement

- Refregier, Massey, Bacon



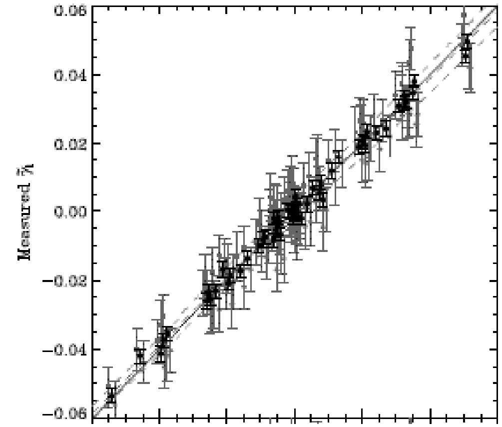


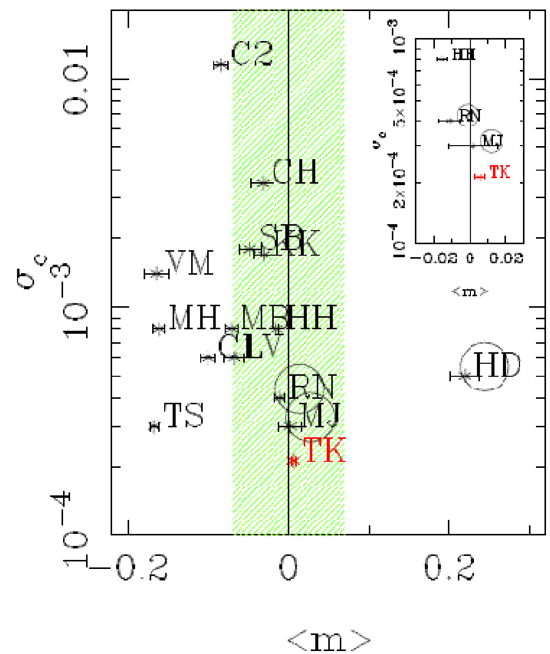
Shear moves power from one shapelet to another, diagonally by 2

Systematics control:

$$\gamma_1 - \gamma_1^{\mathrm{true}} = q(\gamma_1^{\mathrm{true}})^2 + m\gamma_1^{\mathrm{true}} + c_1$$

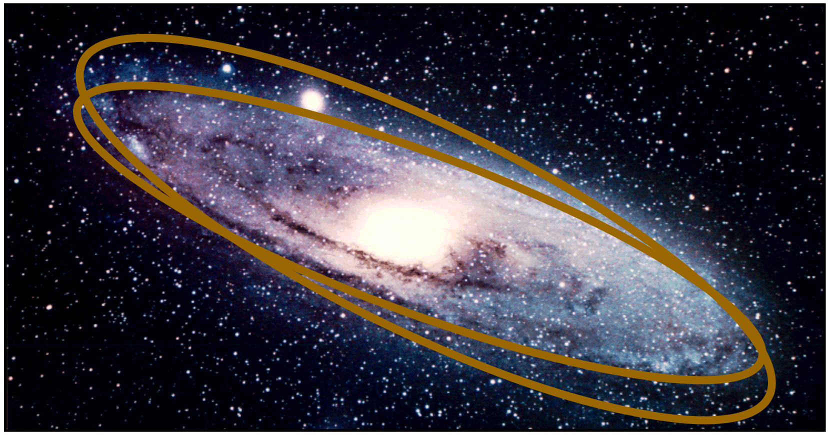
- Need to measure this to a systematic uncertainty of ~0.3% (of 1%)
- Currently ~1% appears achievable (STEP programme: Heymans et al; Massey et al)
- New model-fitting methods (lensfit) are hitting target
- Better PSFs help





Challenges III: Physical systematics

- Ellipticity of galaxy e = e(source) + γ
- $e_s \sim 0.3$; $\gamma \sim 0.01$
- Estimate γ by averaging over many galaxies
- $e \sim e_s + \gamma$
- Hence $\langle ee^* \rangle = \langle \gamma\gamma^* \rangle$

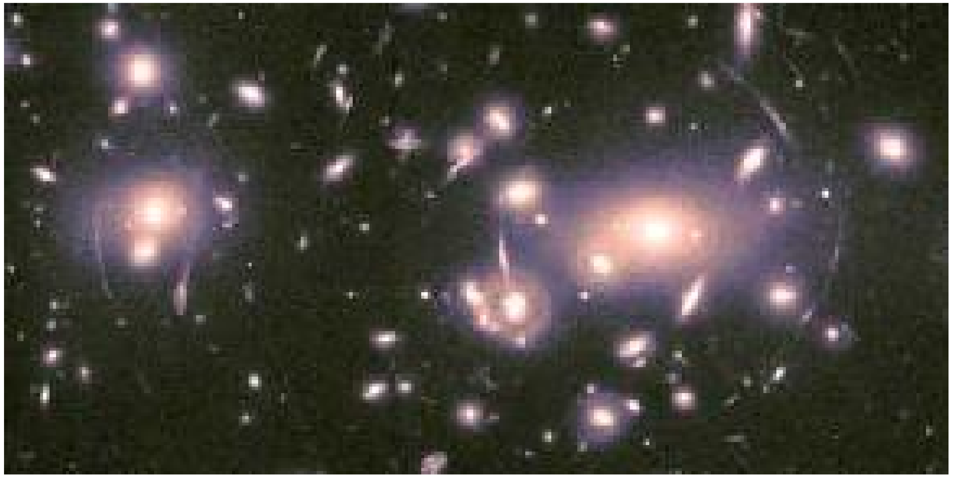


Provided galaxies are not intrinsically aligned $<e_s e_s^*>=0$

Astrophysical complications

- Intrinsic alignments

• Lensing analysis assumes orientations of source galaxies are uncorrelated



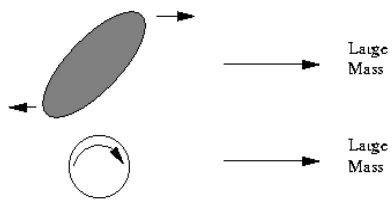
Weak lensing $e \cong e_s + \gamma$

$\langle ee^* \rangle = \langle e_s e^*_s \rangle + \langle \gamma\gamma^* \rangle$

Intrinsic alignments

$$\langle ee^* \rangle = \langle e_s e^*_s \rangle + \langle \gamma\gamma^* \rangle$$

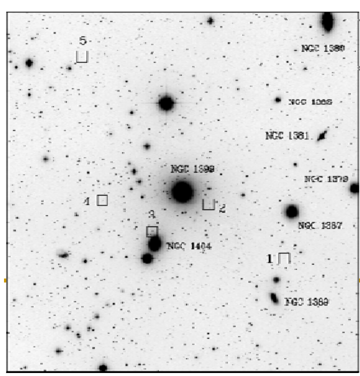
$\langle e_s e^*_s \rangle$ Theory: Tidal torques



Heavens, Refregier & Heymans 2000, Croft & Metzler 2000, Crittenden et al 2001 etc

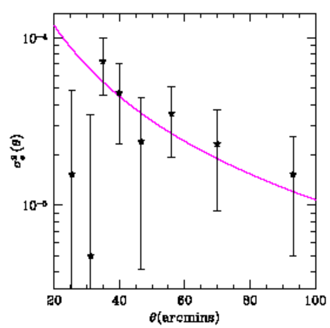
SOLUTION:
Downweight/discard pairs with similar photometric redshifts
(Heymans & Heavens 2002; King & Schneider 2002a,b)

REMOVES EFFECT ~COMPLETELY



Brown et al 2000

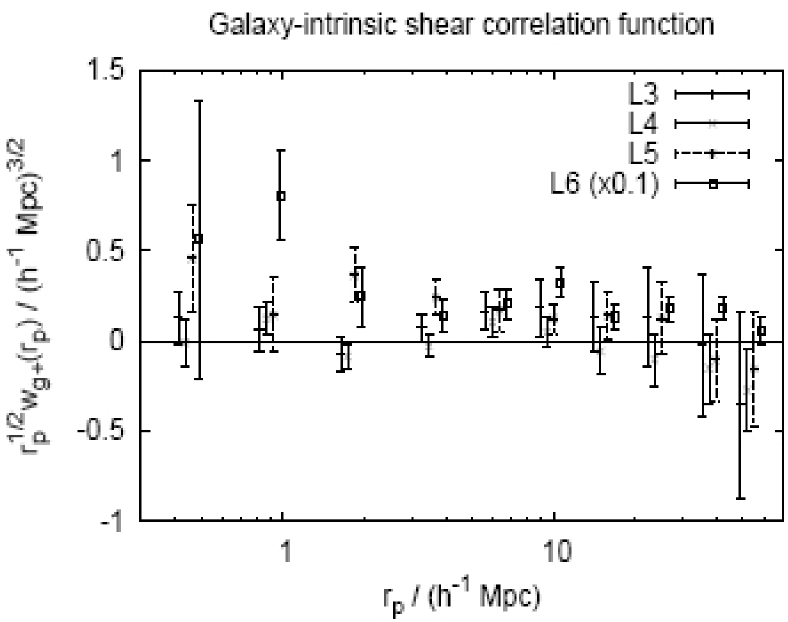
Heymans et al 2003



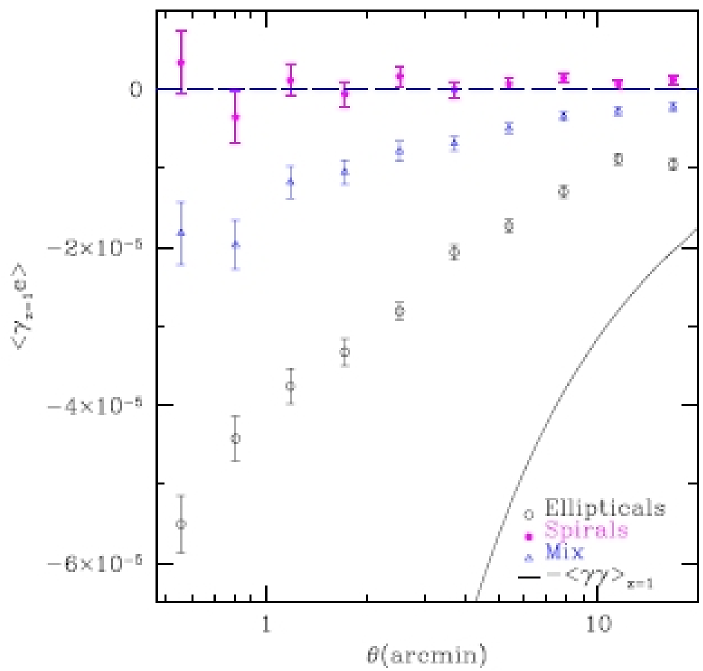
Shear-intrinsic alignments

$$\langle ee^* \rangle = \langle e_s e^*_s \rangle + \langle \gamma\gamma^* \rangle + 2\langle e\gamma^* \rangle$$

- Tidal field contributes to weak shear (of background)
- Tidal field could also orient galaxies (locally) (Hirata and Seljak 2004)



SDSS: Mandelbaum et al 2005



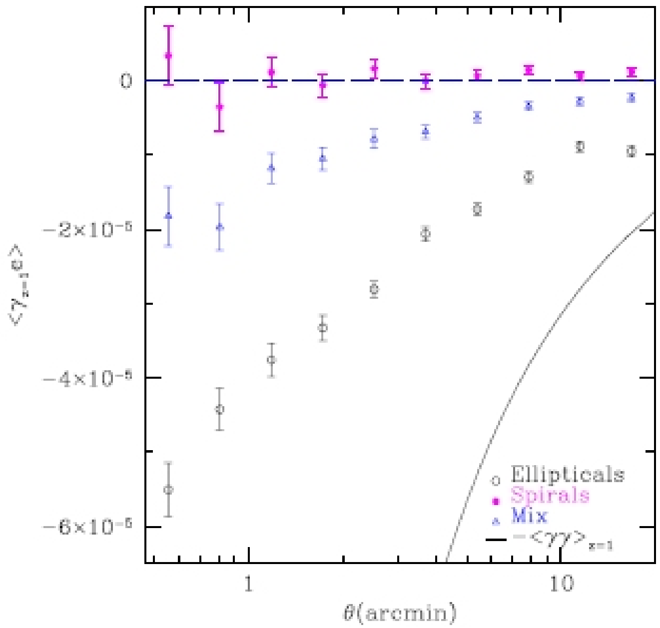
Theory: Heymans, AFH et al 2006

Expect 5-10% contamination

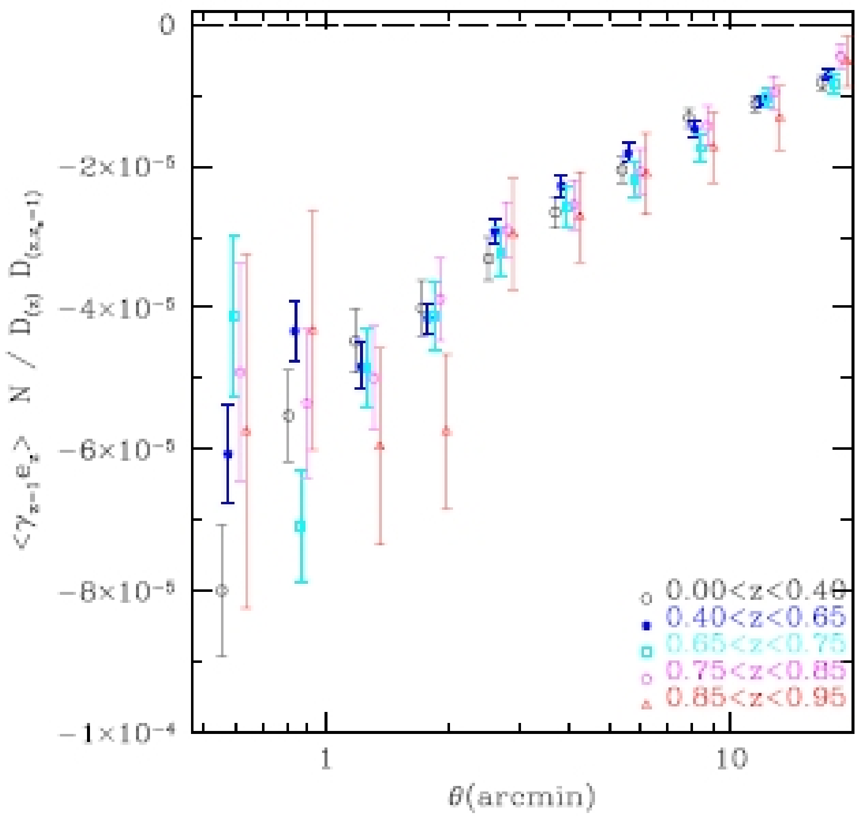
Removing shear-intrinsic ellipticity contamination

- Solution not as easy as intrinsic alignments
  - massive galaxies largely responsible
  - B-mode signature
  - Signal has different redshift dependence from weak lensing (Hirata & Seljak 2004, King et al 2006, Heymans et al 2006)

Removal of intrinsic-shear



Contamination signal expected to be proportional to
$D_L \, D_{LS}/D_S$

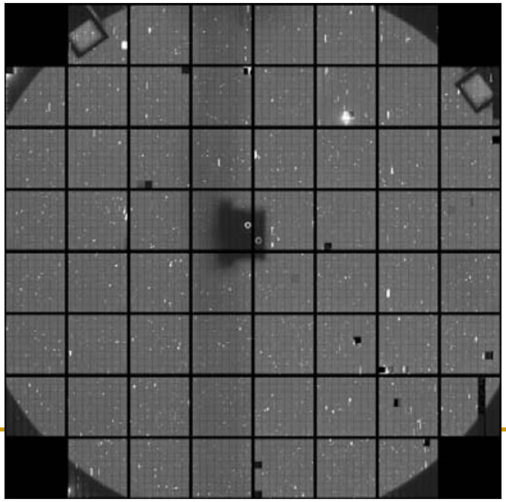


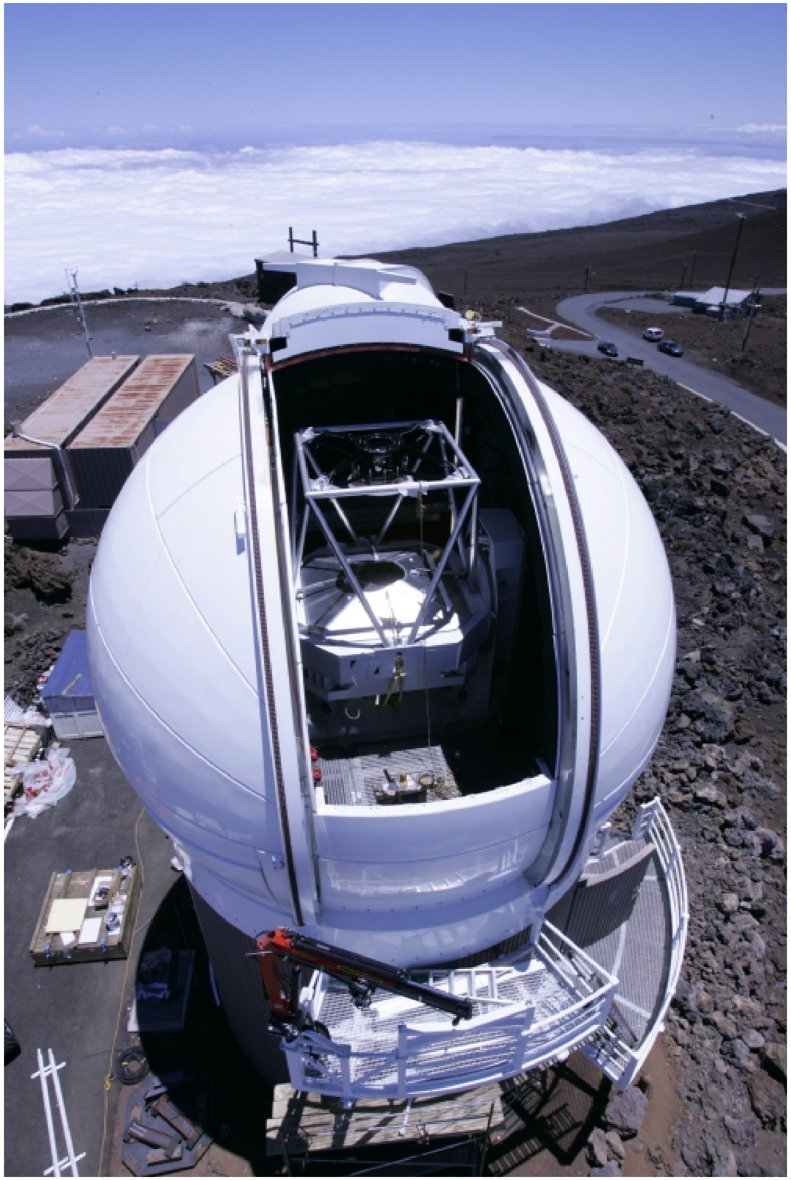
Just requires alignment of galaxies w.r.t. tidal field to be independent of redshift

Model it, or 'null' it (at expense of worse noise; Joachimi & Schneider 2008)

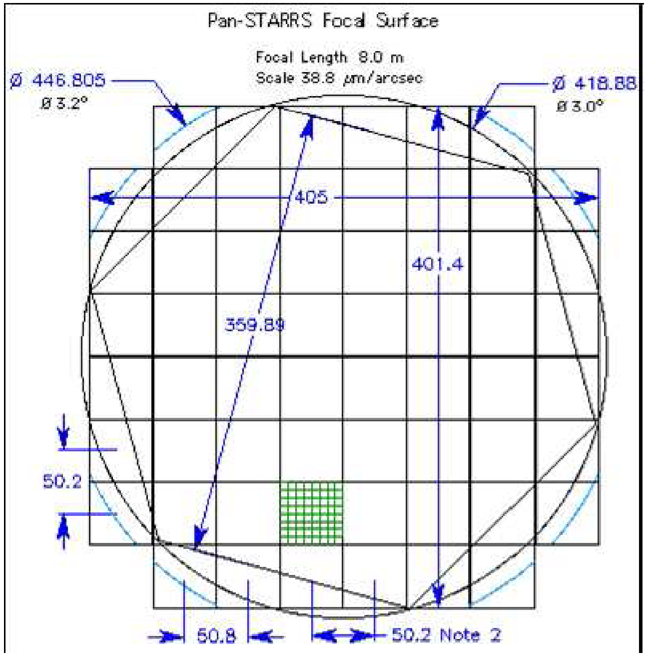
Pan-STARRS 1

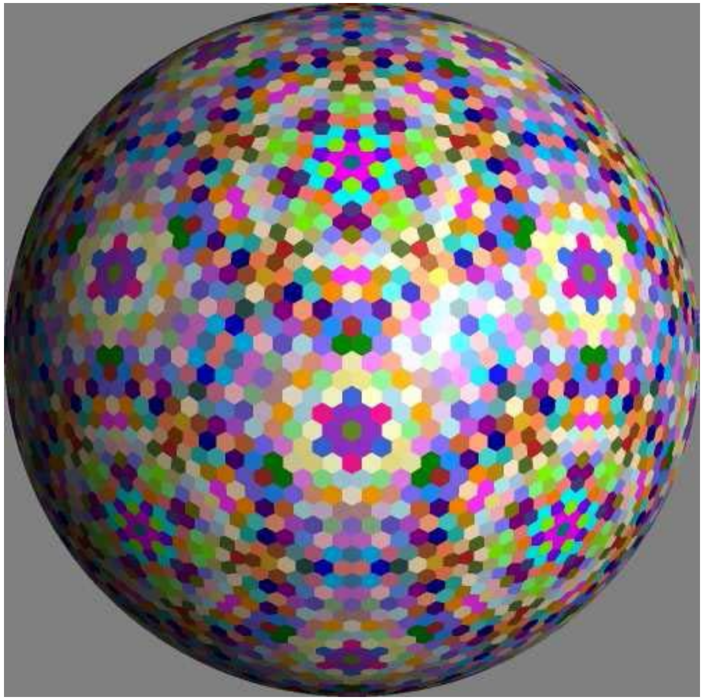
- 1.8 m telescope on Maui, Hawaii
- 7 square degree 1.4 Gpixel camera
- grizy filters
- 3π steradian survey to median z ~ 0.6; deeper 70 sq deg MDS
- Very good image quality: ~2% PSF distortions
- Due to start operating in early 2009
- US/UK/Germany





Pan-STARRS 1 camera and tiling





Other large-area surveys: CFHTLS (finished), DES, HSC, LSST/PS4, EUCLID, SNAP

Forecasting errors: the Fisher Matrix

- The Fisher Matrix is:

D = data

μ = mean of data

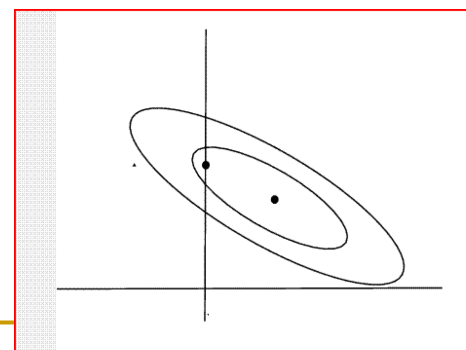
C = covariance matrix of data

$$\mathbf{F}_{ij} = \langle \mathcal{L}_{,ij} \rangle = \frac{1}{2}\mathrm{Tr}[\mathbf{C}^{-1}\mathbf{C}_{,i}\mathbf{C}^{-1}\mathbf{C}_{,j} + \mathbf{C}^{-1}\mathbf{M}_{ij}],$$

where we have defined the matrix $\mathbf{M}_{ij} \equiv \langle \mathbf{D}_{,ij} \rangle = \mu_{,i}\,\mu_{,j}^{t} + \mu_{,j}\,\mu_{,i}^{t}$ (Tegmark, Taylor, Heavens 1997).

Note – NO DATA!

Can analyse experimental design



Conditional and Marginal Errors

- The *conditional error* on θ_i is (at least)

  This assumes all other parameters are known

$$\frac{1}{\sqrt{\mathbf{F}_{ii}}}$$

- The *marginal error* on θ_i is (at least)

  This assumes all other parameters are also estimated from the data

$$\sqrt{(\mathbf{F}^{-1})_{ii}}$$

The conditional error is almost never relevant and should not be quoted. The marginal error is no smaller than the conditional error. Obey this rule with real data too!

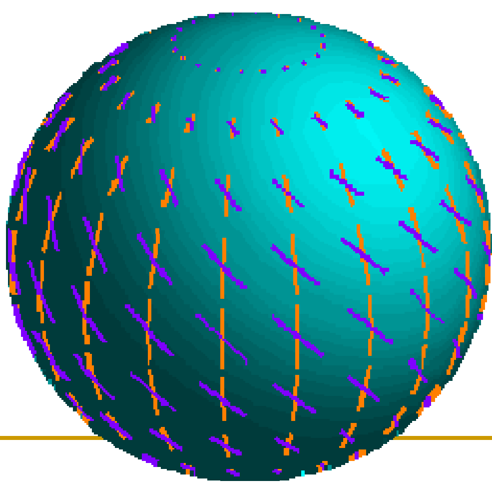
Full 3D cosmic shear on the full sky

- Shear is referred to a coordinate system. How do we correlate shears on a curved sky?

On small scales,

$$\gamma = \frac{1}{2}(\nabla_1 - i\nabla_2)(\nabla_1 - i\nabla_2)\phi$$

gives γ with respect to a locally cartesian coordinate system.



Natural to refer shear components to spherical coordinate system.

Generalise to (effectively) covariant derivative on a sphere at fixed r:

$$\gamma(\mathbf{r}) = \frac{1}{2}ðð\phi(\mathbf{r})$$

‘edth’ operator; Newman & Penrose 1966

3D shear analysis: Natural expansions

- Natural to expand ϕ in spherical harmonics and spherical Bessel functions (flat space)

$$\phi(\mathbf{r}) = \int_0^\infty dk \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \phi_{\ell m}(k) j_\ell(kr) Y_{\ell m}(\theta, \varphi)$$

- Now

$$\eth\, {}_sY_{\ell m} = [(\ell - s)(\ell + s + 1)]^{\frac{1}{2}}\, {}_{s+1}Y_{\ell m}$$

- So, expand shear in spin-weight 2 spherical harmonics

$$\gamma(\mathbf{r}) = \int_0^\infty dk \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} {}_2\gamma_{\ell m}(k) j_\ell(kr)\, {}_2Y_{\ell m}(\theta, \varphi)$$

Shear to density field

- Can then relate coefficients of γ directly to coefficients of dark matter fluctuations:

$j_\ell(kr)Y_\ell^m(\theta, \varphi)$ is an eigenfunction of Laplace's equation, so coefficients of δ and Φ are related very simply by $-k^2$.

(Modifications if not flat)

Shear 3D power spectrum:

Expand the shear field in radial waves $j_\ell(kr)$ and transverse waves $(\ell)\, e^{i l.\theta}$

$$\gamma(k,\vec{\ell}) \propto H_0^2 \Omega_m \int_0^\infty dz\, dz_p\, p(z_p \mid z)\, \bar{n}(z)\, j_\ell(kr)$$

$$\int_0^{r(z)} dr' \left(\frac{1}{r} - \frac{1}{r'}\right)(1+z') \int dk'\, j_\ell(k'r')\, \delta(k',\vec{\ell};t')$$

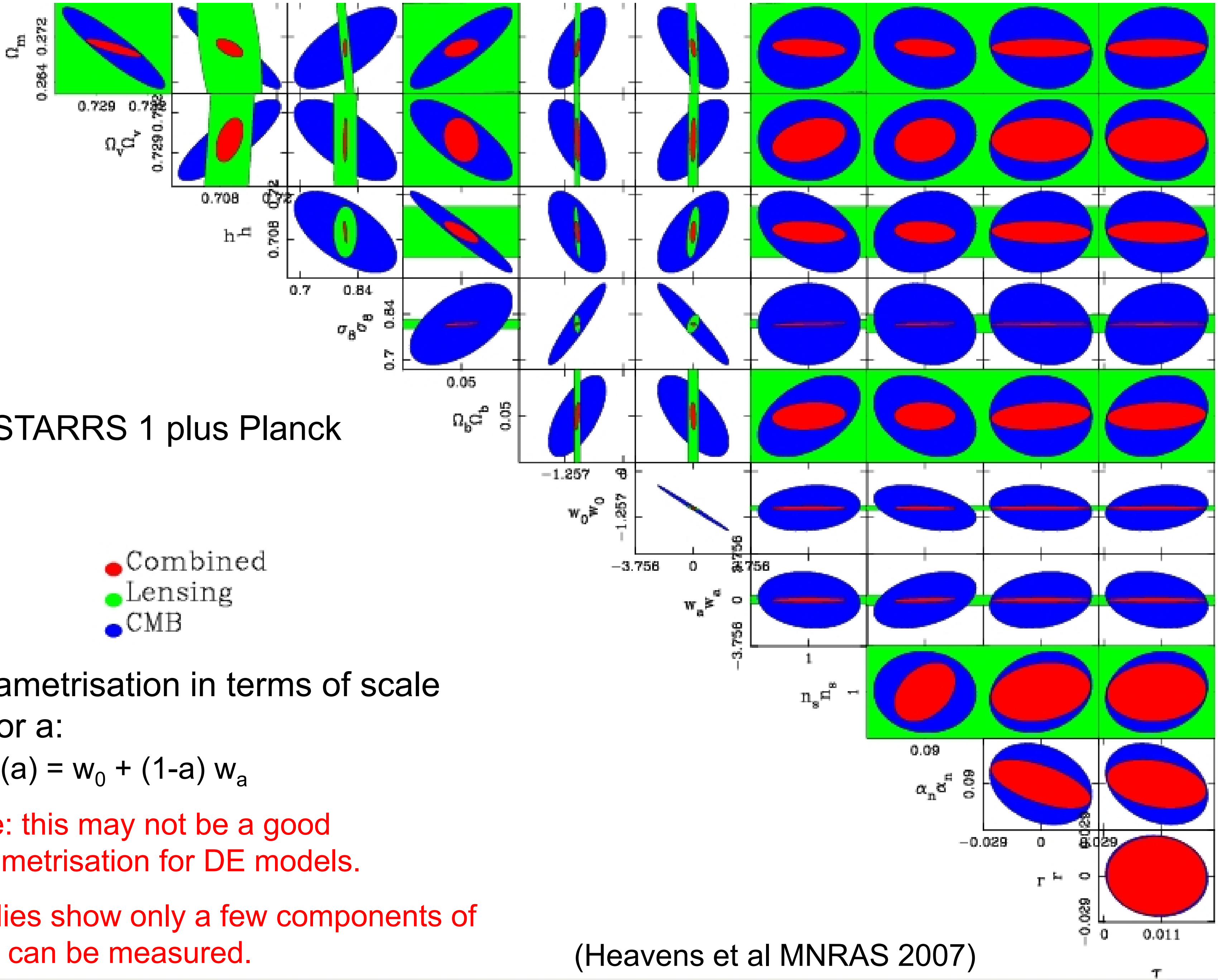
Integral nature of lensing

z and r

Include photo-z errors

Transform of density field

Small-angle surveys (Heavens et al MNRAS 2006)



Pan-STARRS 1 plus Planck

Combined
Lensing
CMB

Parametrisation in terms of scale factor a:

$w(a) = w_0 + (1-a)\, w_a$

Note: this may not be a good parametrisation for DE models.

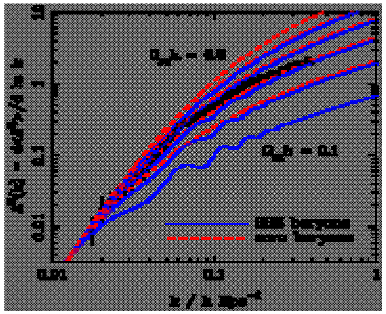
Studies show only a few components of w(a) can be measured.

(Heavens et al MNRAS 2007)

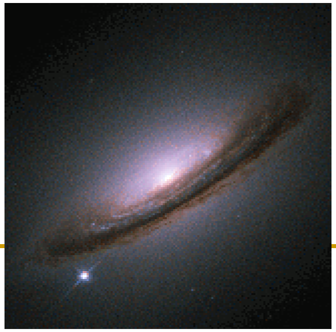
Combination with other experiments

- CMB: Planck

- BAO: WFMOS 2000 sq deg to z=1

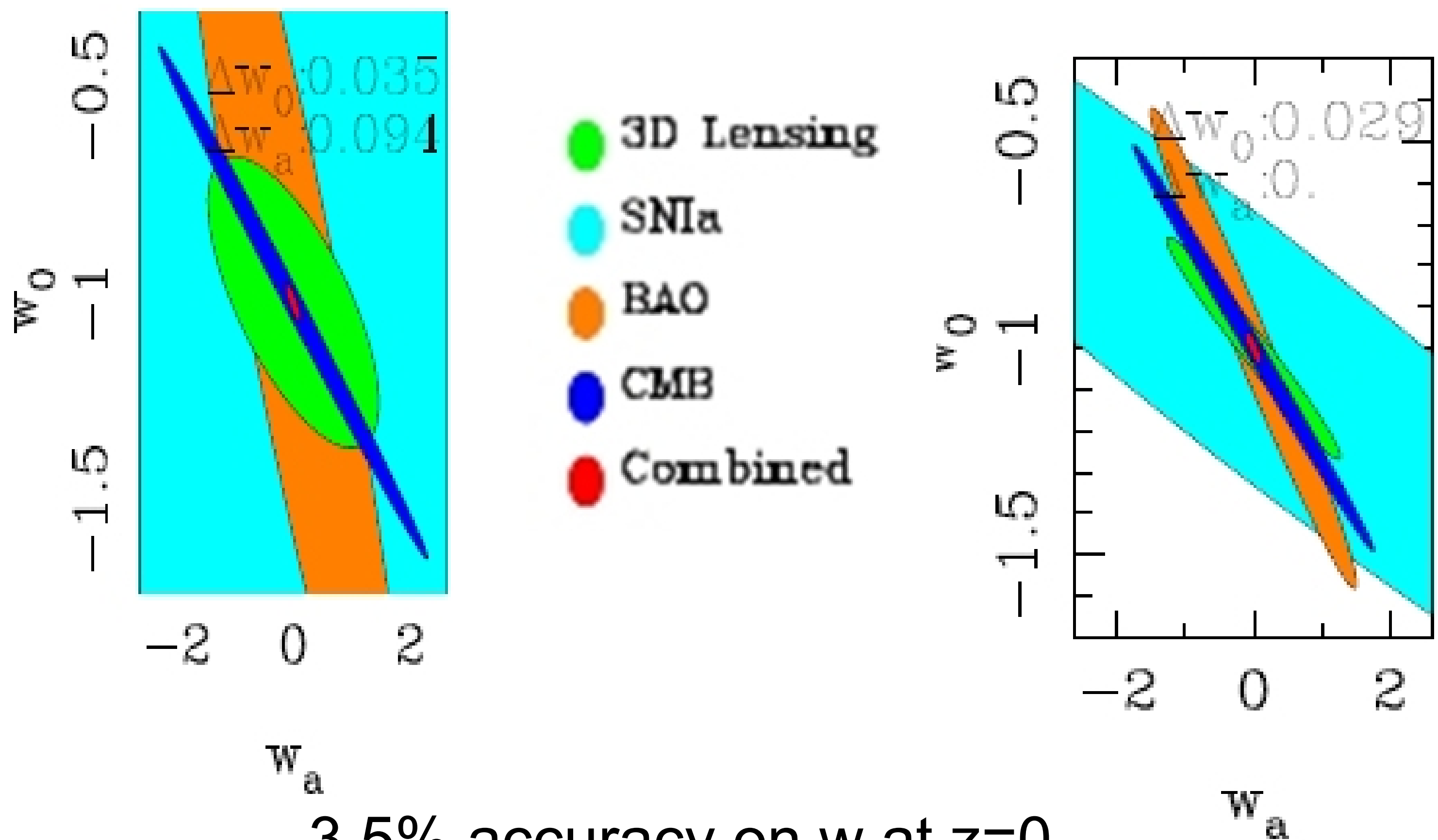


- SNe: 2000 to z=1.5



Combining 3D lensing, CMB, BAO, SNe

(10000 sq deg lensing survey: one third of PS1)



Flat

3.5% accuracy on w at z=0

1% on w(z) at z~0.4

Future: Euclid, formerly DUNE

- Other gravity theories will yield an expansion rate *H(a)*
- Any H(a) can be mimicked by choosing Dark Energy with a certain w(a)
- How do we tell whether GR+DE is preferred over modified gravity?
- …via weak lensing and the growth rate
- Needs a very extensive space-based survey: Euclid would be excellent
- 4 bands from space (+ ground)
- 20,000 square degrees
- 35 sources per square arcminute; median redshift z~1
- Potential: Error on w = 0.01, Error on w_a=0.06

Modified Gravity or Dark Energy?

- Modified Gravity theory will give a certain *H(a)*.
- We can always find an 'equation of state' (strictly just p/ρ) to mimic this in GR:

Friedmann:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}$$

and

$$\frac{d}{da}\left(\rho_q a^3\right) = -p_q a^2 = -w(a)\rho_q a^2$$

- Solve for any given *H(a)*. Exercise:

$$w(a) = -\frac{1}{3}\frac{d}{d\ln a}\ln\left[\frac{1}{\Omega_m(a)} - 1\right]$$

- which depends on *H(a)* via the critical density

$$\rho_{crit}(a) = \frac{3H^2(a)}{8\pi G}$$

Supernovae cannot unambiguously distinguish GR from modified gravity [via r=c∫dz/H(z)]

Reproducing the expansion history with effective w(a)

- Flat DGP expansion history is very close to GR + Dark Energy with

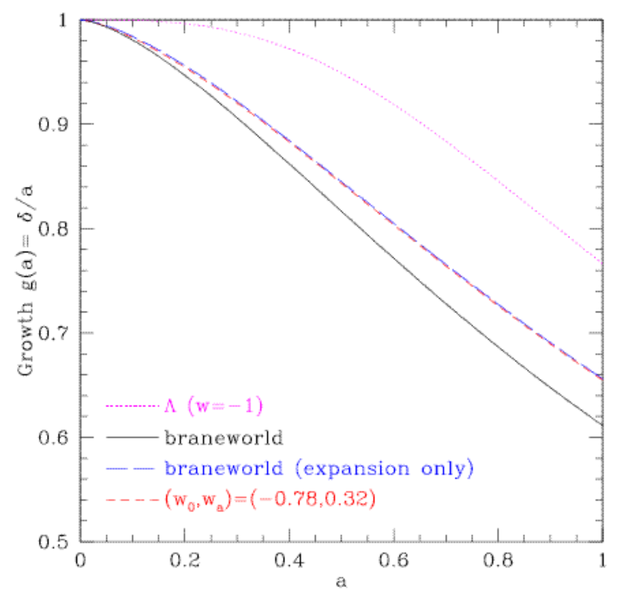
$$w_0 = -0.78 \qquad w_a = 0.32$$

where

$$w(a) = w_0 + w_a(1-a)$$

$$H^2 - H/r_c = (8\pi/3)\rho,$$

$$r_c = H_0^{-1}/(1-\Omega_m)$$



Minimal Modified Gravity

- However, gravity theory affects the growth rate, so weak lensing can distinguish GR from modified gravity in principle.
- A convenient parametrisation for the growth rate is (Linder 2005)

$$\frac{\delta}{a} = \exp\left\{ \int_0^a \frac{da'}{a'} \left[\Omega_m(a')^\gamma - 1 \right] \right\}$$

- $\gamma \cong 0.55$ (GR)
- $\gamma \cong 0.68$ (flat DGP)
- Main question: is there any evidence that γ deviates from GR value?

Bayesian Evidence

- Bayesian method to answer such questions
- Let models be M,M' and data be D. Let model parameters be θ (or θ')
- What we want is p(M|D)
- Bayes' theorem: p(M|D)=p(D|M)p(M)/p(D)
- If we take a *non-commital prior* on models, p(M) = constant,
- p(M|D) α p(D|M) α ∫ dθ p(D|θ,M) p(θ|M)

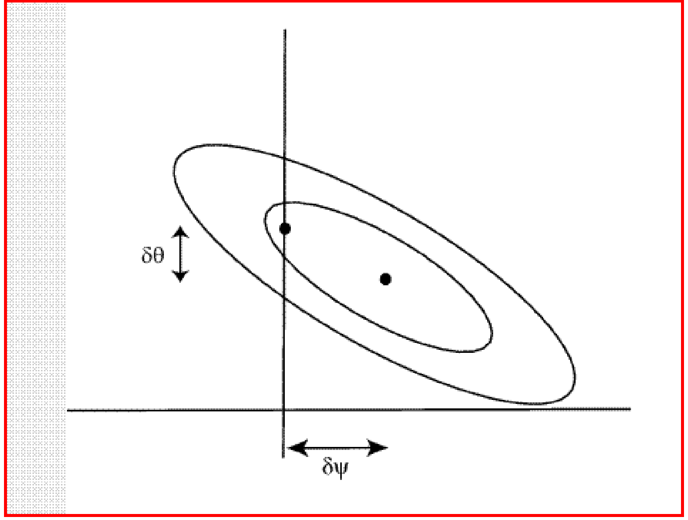
- p(M|D) α ∫ dθ L(D|θ,M) = 'EVIDENCE' (here with flat priors)
- B = ratio of evidences

EVIDENCE = Likelihood integrated over parameter space

Laplace approximation

- Like Fisher, but for evidences
- Assume likelihood is a multivariate gaussian
- Integrate analytically
- Include biases in parameter estimates

$$\delta\theta'_\alpha = -(F'^{-1})_{\alpha\beta} G_{\beta\zeta} \delta\psi_\zeta$$



G is part of F

$$B = (2\pi)^{-p/2} \frac{\sqrt{\det F}}{\sqrt{\det F'}} \exp\left(-\frac{1}{2}\delta\theta_\alpha F_{\alpha\beta}\delta\theta_\beta\right) \prod_{q=1}^{p} \Delta\theta_{n'+q}$$

Prior ranges for parameters

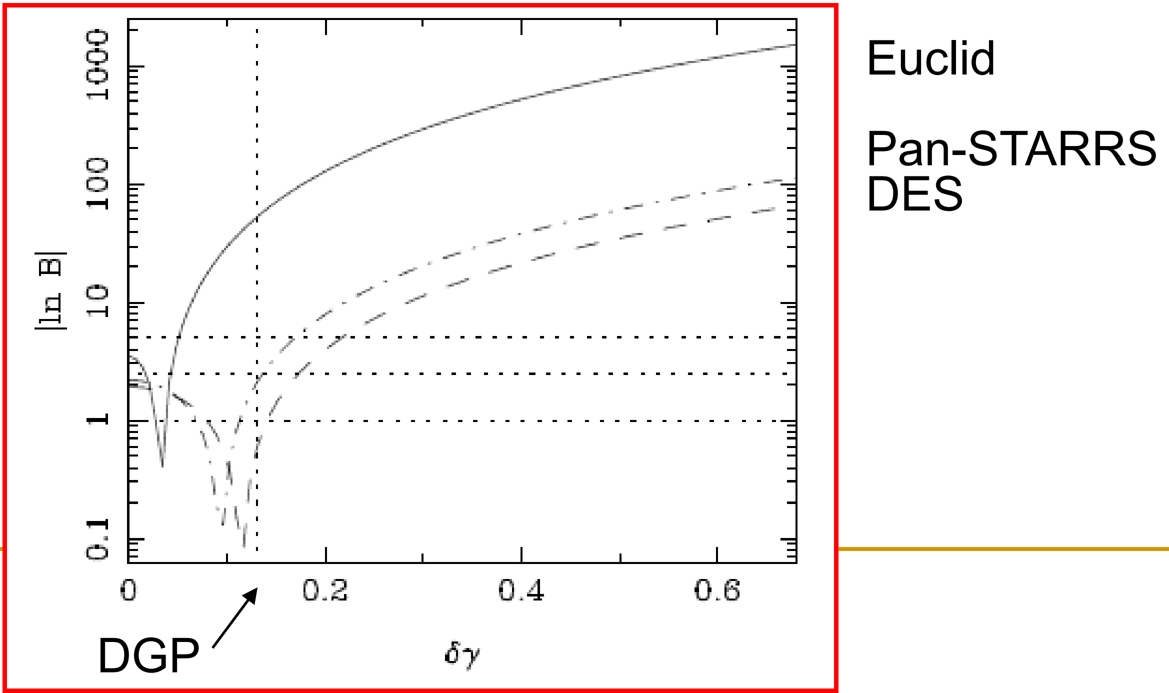
Heavens, Kitching, Verde, astroph/0703191

Prospects

Compare GR with Dark Energy with a modified gravity model *with the same expansion history.*

We take flat DGP braneworld model as example.

- Pan-STARRS 1 + Planck+BAO+SNe: |lnB|=3.8 (DGP/GR)
- Euclid + Planck + BAO + SNe: |lnB|=63 (DGP/GR)
- Euclid should be able to find evidence for gravity theory beyond GR, if it is there.



Conclusions

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- Post-WMAP/2dF/SDSS, Dark Energy and Dark Matter are key scientific goals of cosmology
- Lensing is powerful because it detects mass directly, and the connection with fundamental theory is very direct (simple physics)
- Mass reconstruction puts limits on Dark Matter interactions, and baryon fraction
- Lensing in 3D is very powerful for Dark Energy: accuracies of ~2% on *w* potentially possible with Pan-STARRS 1
- Main challenges - systematics:
  - Accurate shape measurement
  - Unbiased photo-zs
  - Removal of intrinsic-shear contamination
- Can test GR vs modified gravity models from extra dimensions, by probing geometry *and* growth rate. Euclid should be able to do this.