The effect of dark energy on large-scale structure

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The effect of dark energy on large-scale structure

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Introduction

Dark energy is a phenomenon only observable on the largest scales (at the moment). We therefore expect to observe some change to the evolution of the large scale structure to its presence.

Furthermore, unlike Dark Matter, we will (probably) never observe Dark Energy in a table-top or particle accelerator experiment. Therefore, large-scale surveys may be our only source of information as to its nature.

Lecture 1: Physics of large-scale clustering and dark energy

Lecture 2: Observations of large scale structure

Lecture 3: Plans for the future

Perturbation equations: Newtonian Approximation

metric: $ds^2 = (1 + 2\phi(t,x))dt^2 - \delta_{ij}dx^i dx^j$

continuity equation: $\dot{\rho} \equiv \frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + (v^i\partial_i)\rho = -\rho(\nabla . v)$

Euler equation: $\dot{v}^i = -\partial^i\phi - \frac{1}{\rho}\partial^i p$

Decompose the gradient of the velocity field

$$\partial_j v_i = \omega_{ij} + \sigma_{ij} + H\delta_{ij}$$

Antisymmetric "vorticity"

Symmetric "shear"

Trace "Expansion rate"

Perturbation equations: 1st order Theory

Divergence of Euler equation:

$$\frac{\partial}{\partial t}(\nabla . v) + (v^j \partial_j)(\nabla . v) + (\partial_i v^j)(\partial_j v^i) + \nabla^2 \phi = -\partial_i \left(\frac{1}{\rho} \partial^i p\right)$$

becomes

$$3\dot{H} + 3H^2 - 2(\sigma^2 - \omega^2) + \nabla^2 \phi = -\partial_i \left(\frac{\partial^i p}{\rho} \right)$$

Consider only 1st order corrections to background variables

$$\delta\dot{H} = -2H_b \delta H - \frac{1}{3}\nabla^2 \delta\phi - \frac{1}{3}\frac{\nabla^2 \delta p}{\rho_b} = -2H_b \delta H - \frac{4\pi G}{3}\delta\rho - \frac{1}{3} c_s^2 \frac{\nabla^2 \delta\rho}{\rho_b}$$

Transform to perturbations in density (full relativistic result)

$$\underbrace{\ddot{\delta}}_{\text{"acceleration"}} + \underbrace{H_b \dot{\delta}(2 - 3(2w - c_s^2))}_{\text{"friction"}} - \frac{3}{2}\underbrace{H_b^2 \delta(1 - 6c_s^2 - 3w^2 + 8w)}_{\text{"inertia" or "gravity"}} = \underbrace{-\left(\frac{k}{a}\right)^2 c_s^2 \delta}_{\text{"pressure"}}$$

"acceleration" "friction" "inertia" or "gravity" "pressure"

Perturbation statistics: correlation function ξ(r)

overdensity field

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

definition of correlation function

$$\begin{aligned} \xi(\mathbf{x_1}, \mathbf{x_2}) &\equiv \langle \delta(\mathbf{x_1}) \delta(\mathbf{x_2}) \rangle \\ &= \xi(\mathbf{x_1} - \mathbf{x_2}) \\ &= \xi(|\mathbf{x_1} - \mathbf{x_2}|) \end{aligned}$$

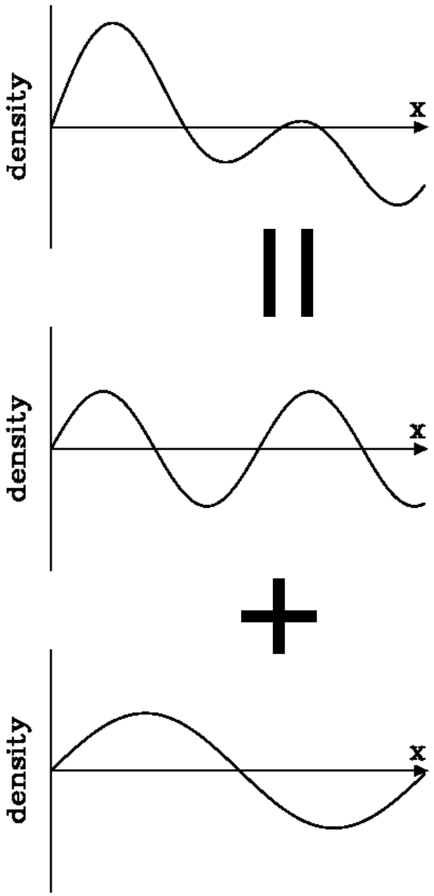
from statistical homogeneity

from statistical isotropy

can estimate correlation function using galaxy (DD) and random (RR) pair counts at separations ~r

$$1 + \xi(r) = \frac{\langle DD \rangle_r}{\langle RR \rangle_r}$$

Perturbation statistics: Fourier decomposition



Observed fluctuations

Small scale fluctuation – high frequency (k)

Large scale fluctuation – low frequency (k)

- The observed fluctuations can be decomposed into waves with different frequency (or wavelength)
- Standard Fourier decomposition is given by

$$\delta(\mathbf{k}) \equiv \int \delta(\mathbf{r}) e^{i\mathbf{k}.\mathbf{r}} d^3 r$$

$$\delta(\mathbf{r}) = \int \delta(\mathbf{k}) e^{-i\mathbf{k}.\mathbf{r}} \frac{d^3 k}{(2\pi)^3}$$

Perturbation statistics: power spectrum

definition of
power spectrum

$$\langle \delta(\mathbf{k_1})\delta(\mathbf{k_2})\rangle = (2\pi)^3 \delta_D(\mathbf{k_1} - \mathbf{k_2}) P(k_1)$$

power spectrum is the Fourier analogue of the correlation function

$$P(k) \equiv \int \xi(r) e^{i\mathbf{k}.\mathbf{r}} d^3 r$$

$$\xi(r) = \int P(k) e^{-i\mathbf{k}.\mathbf{r}} \frac{d^3 k}{(2\pi)^3}$$

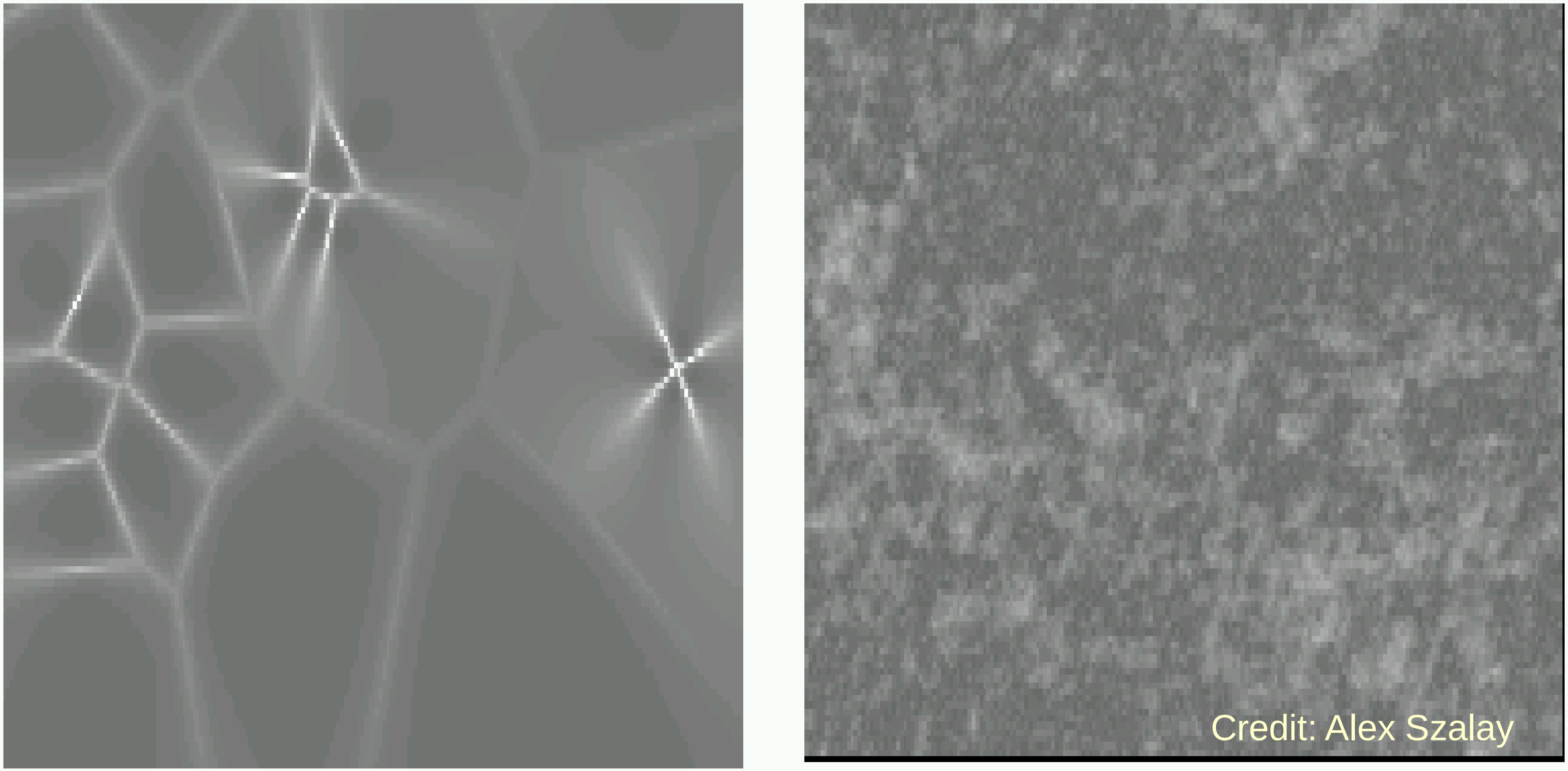
sometimes written in dimensionless form

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$$

Why are 2-pt statistics so important?

Because the central limit theorem implies that a density distribution is asymptotically Gaussian in the limit where the density results from the average of many independent processes; and a Gaussian is completely characterised by its mean (overdensity=0) and variance (given by either the correlation function or the power spectrum)

Are 2-pt statistics all there is to know?



Same 2pt, different 3pt

Power spectrum vs correlation function

The power spectrum and correlation function contain the same information; accurate measurement of each will gives the same constraints on cosmological models.

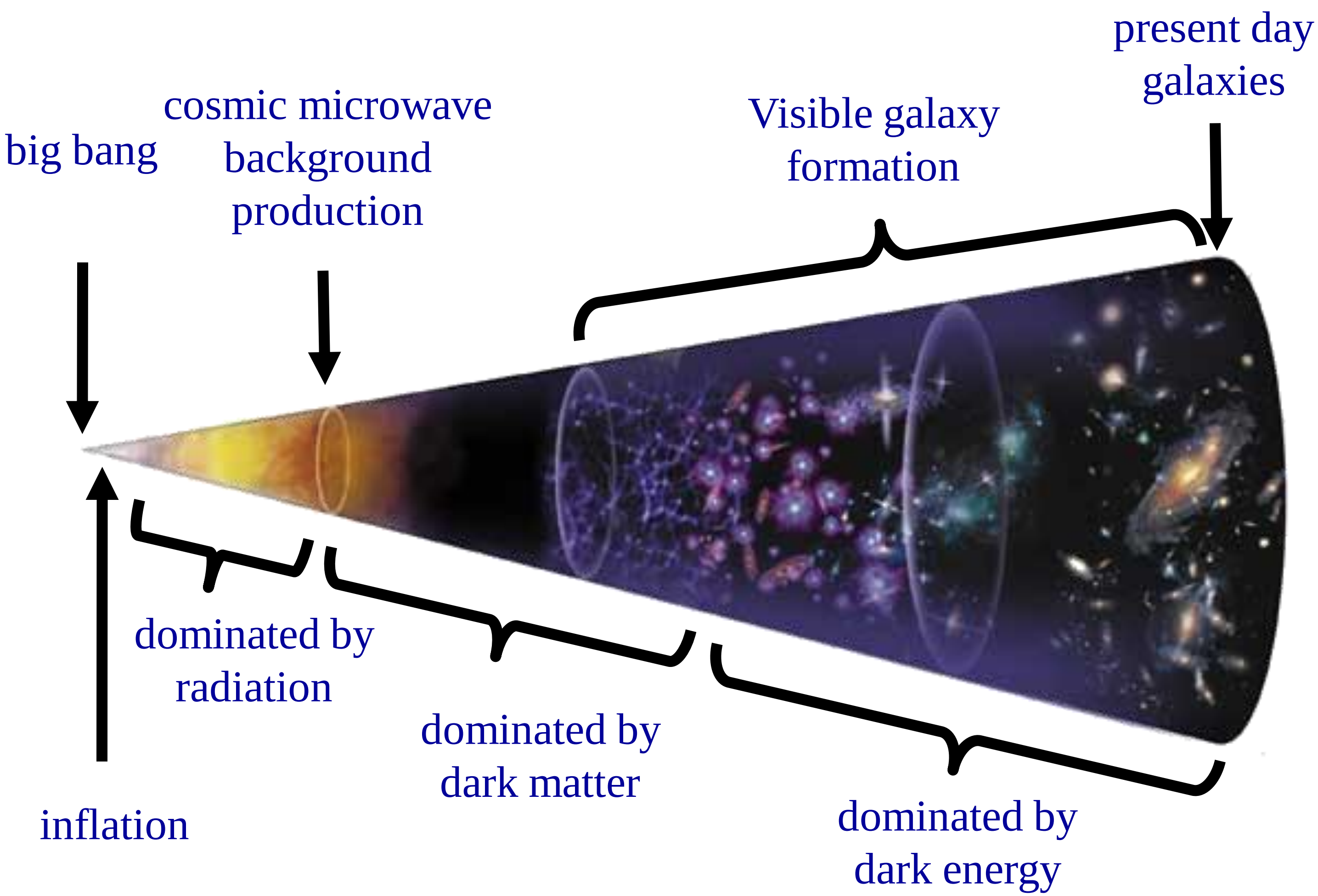
Both power spectrum and correlation function can be measured relatively easily (and with amazing complexity)

The power spectrum has the advantage that different modes are uncorrelated (as a consequence of statistical homogeneity).

Models tend to focus on the power spectrum, so it is common for observations to do the same ...

Physics of Structure-growth

Timeline of structure growth

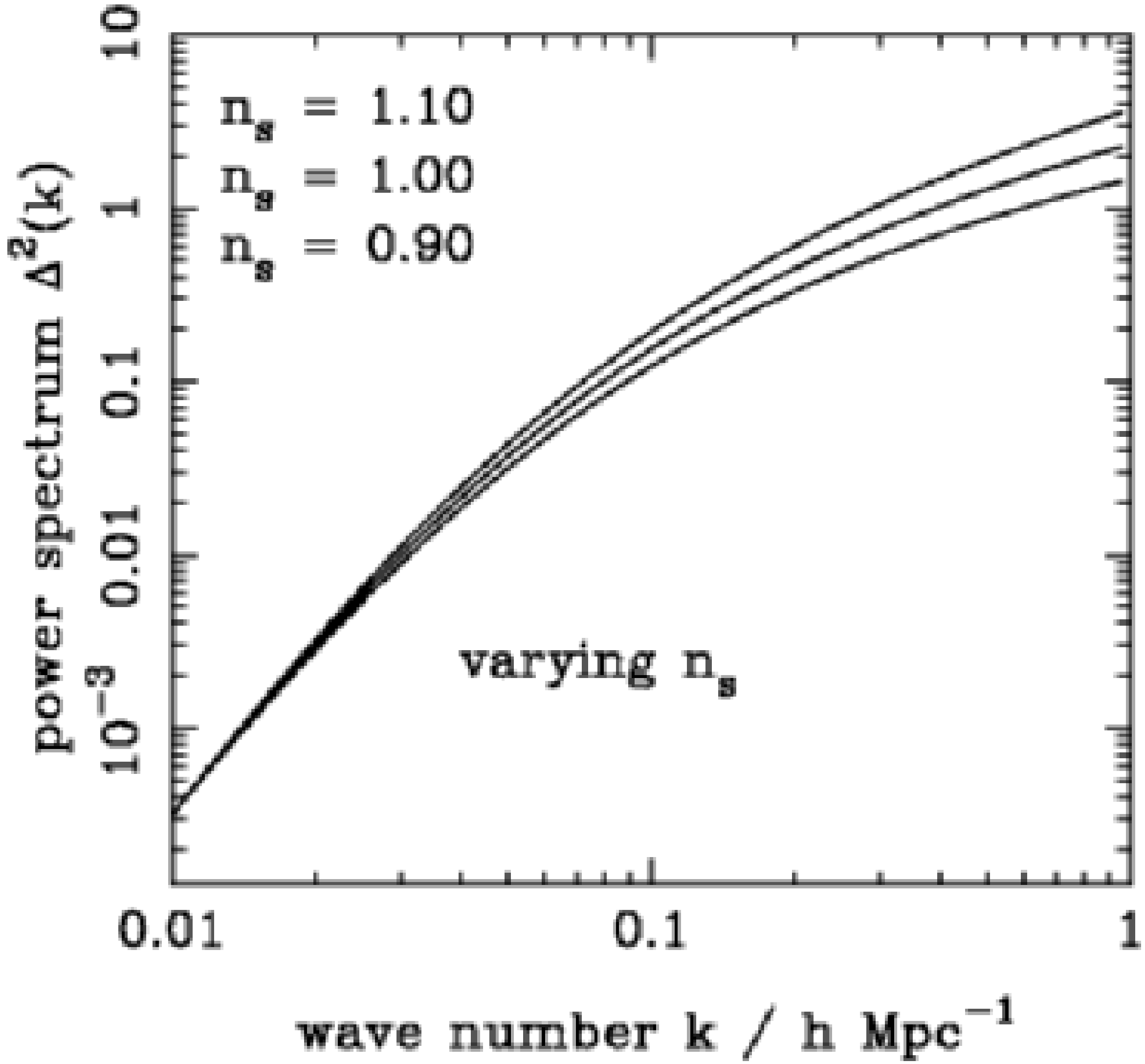


Why is there structure?

Inflation (a period of accelerated expansion of the early Universe driven by a scalar field) was invented as a toy model to solve some serious problems with "standard" cosmology:

- why is the energy density of the Universe close to critical density? – driven there by inflation
- why do causally disconnected regions appear to have the same properties? – they were connected in the past
- what are the seeds of present-day structure? - Quantum fluctuations in the matter density are increased to significant levels

Power spectrum dependence on inflation



We do not consider inflation here. We just assume an adiabatic scale-invariant power spectrum of fluctuation created by quantum processes deep in the Inflationary era,

$$P(k) = k^n$$

$$(n \approx 1)$$

Jeans length

After inflation, the evolution of density fluctuations in the matter depend on scale and the composition of that matter **(CDM, baryons, neutrinos, etc.)**

An important scale is the **Jeans Length** which is the scale of fluctuation where pressure support equals gravitational collapse,

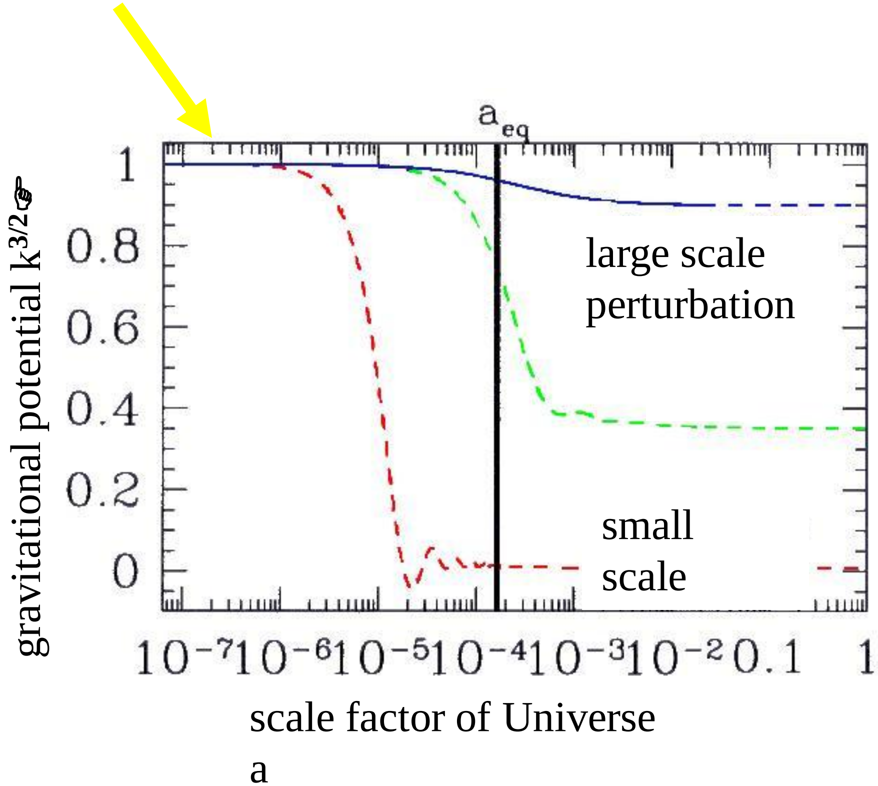
$$\lambda_J = \frac{c_s}{\sqrt{G\rho}}$$

where c_s **is the sound speed** of the matter, and ρ **is the density of matter**.

Perturbation evolution in the early Universe

primordial fluctuations

- arise due to inflation
- simplest inflationary model assumes that the potential is independent of scale
- Gaussian statistics



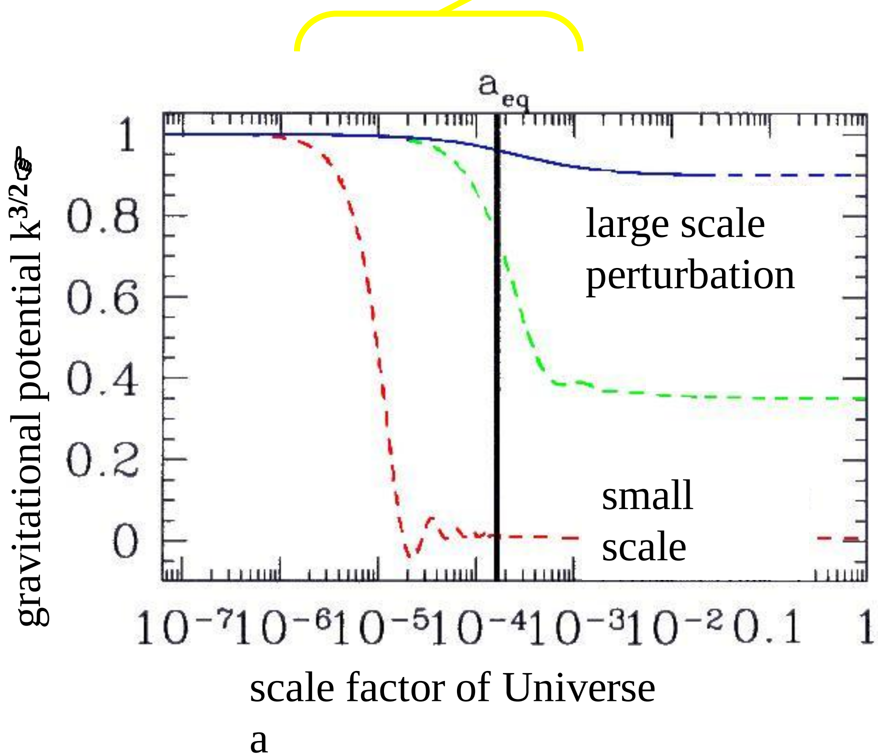
Perturbation evolution in the early Universe

"F=ma" for perturbation growth
(ignoring Hubble friction)

$$\ddot{\delta} = (\text{gravity} - \text{pressure})\,\delta$$

depends on Jeans scale

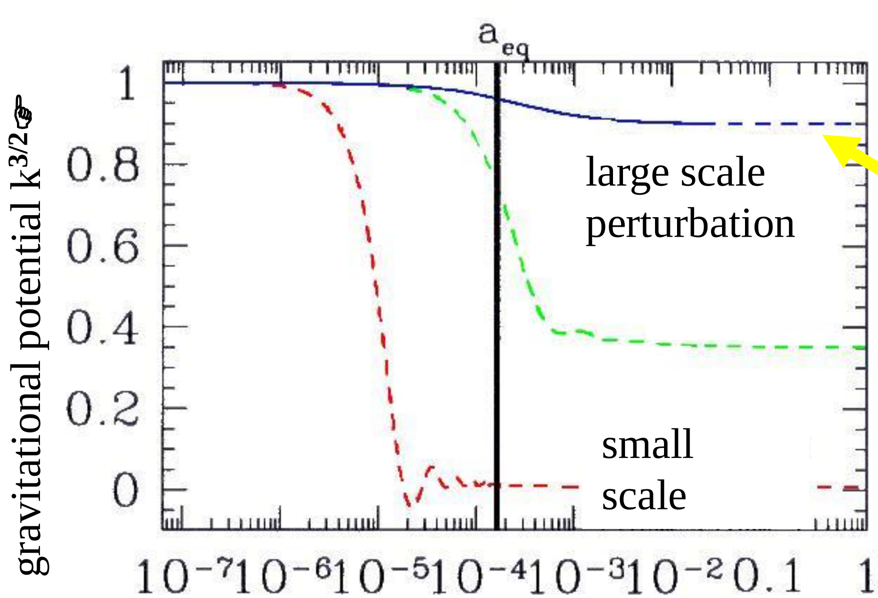
In radiation dominated epoch small perturbations are suppressed.
Cut-off dependent on matter density times the Hubble parameter $\Omega_m h$



+ effects from

- baryon oscillations (Ω_b)
- neutrinos (Ω_ν)

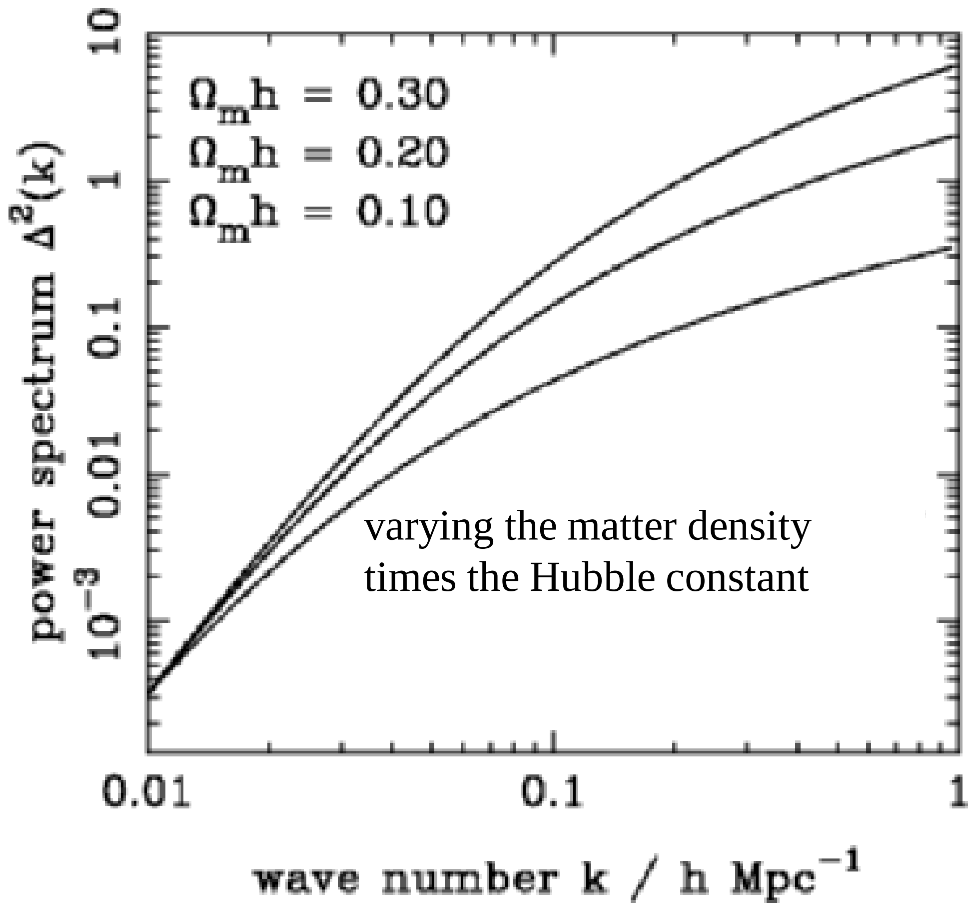
Perturbation evolution in the early Universe



scale factor of Universe
a

linear growth in
matter dominated
epoch – same on all
scales

the power spectrum turn-over

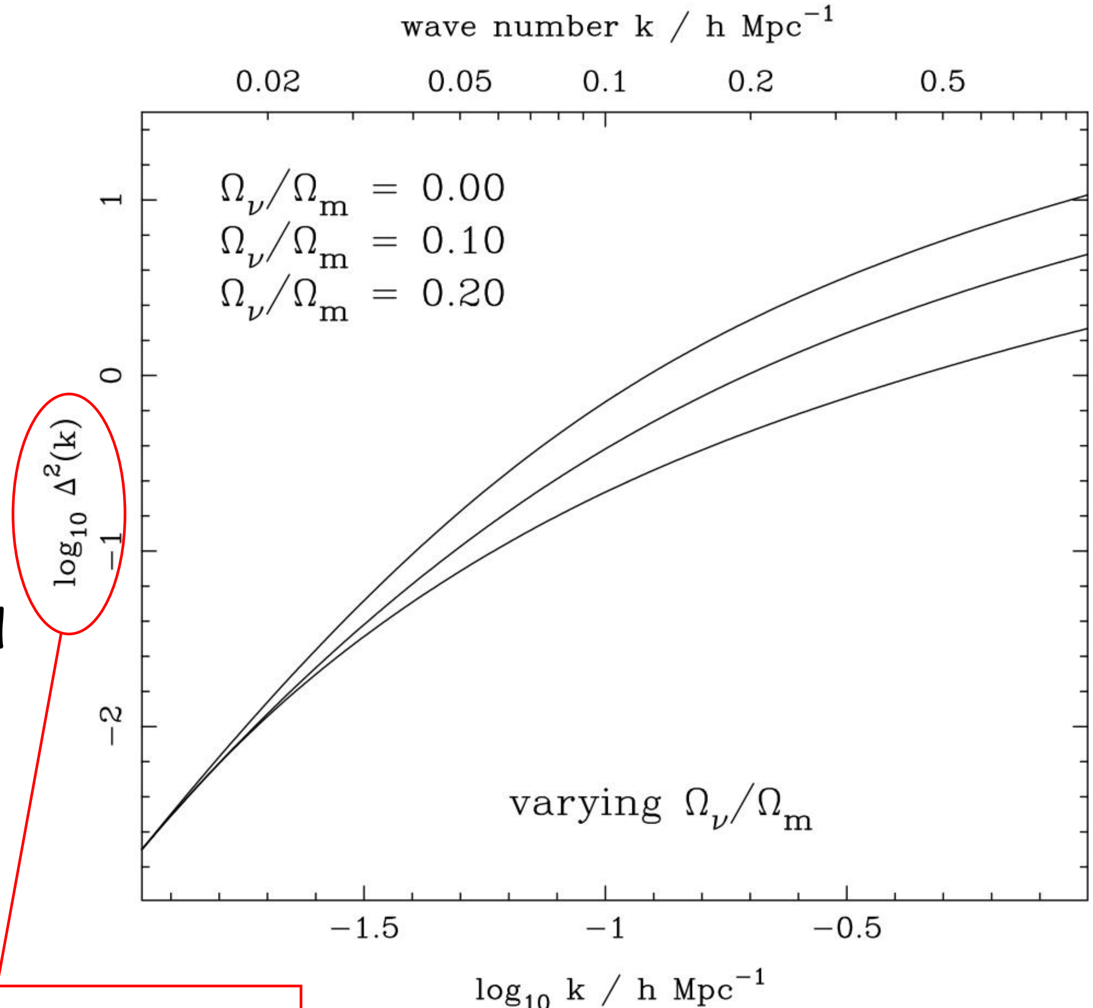


In radiation dominated Universe, pressure support means that small perturbations cannot collapse. Jeans scale changes with time, leading to smooth turn-over of matter power spectrum.

However, turn-over is degenerate with a scale dependent galaxy bias ⋯

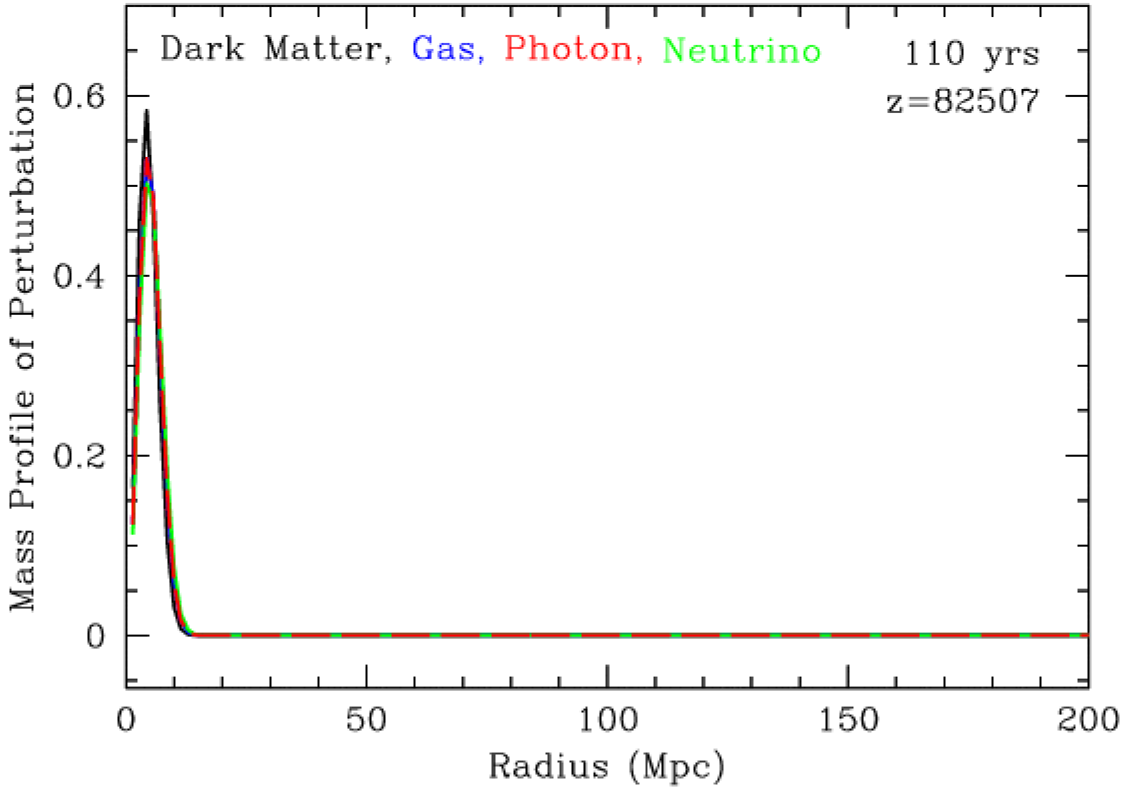
The effect of neutrinos

The existence of **massive neutrinos** can also introduce a suppression of T(k) on small scales relative to their Jeans length. Degenerate with the suppression caused by **baryons matter-radiation damping** (Silk damping) and **baryon damping** (free-streaming after recombination)



Δ□□k□ □ □ □□□□ □
π□□P(k)

Imprint of baryons: forming the acoustic peaks

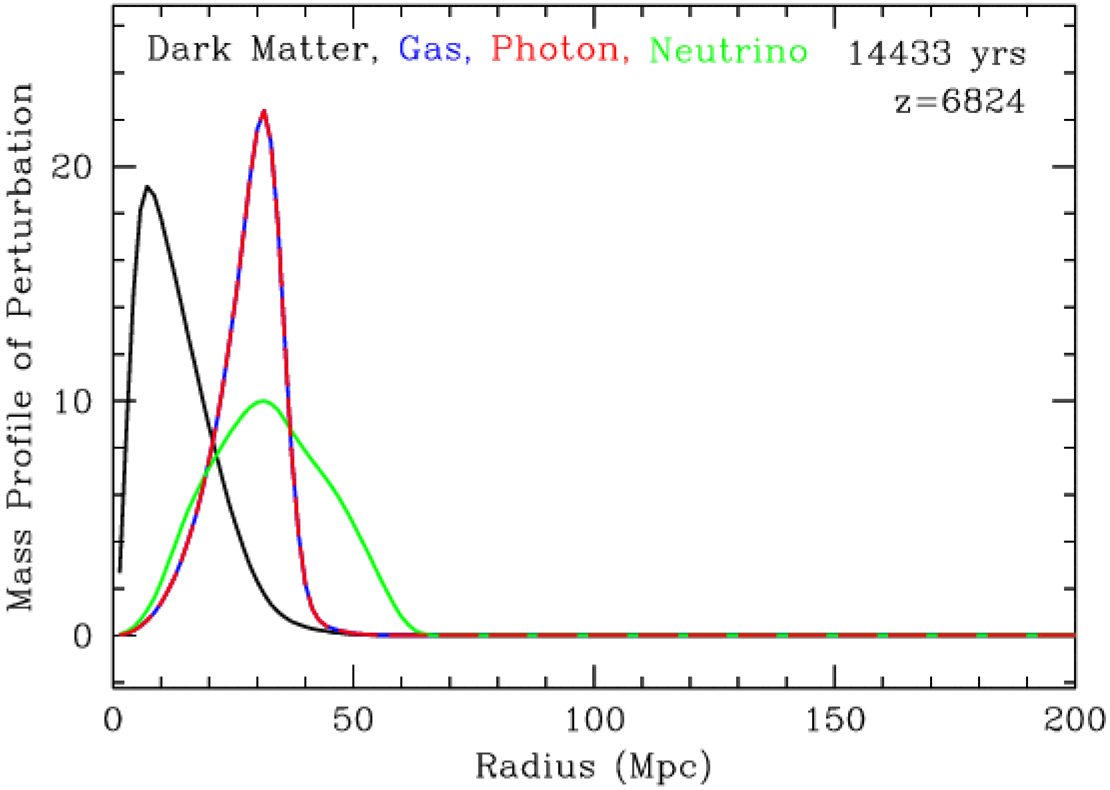


Ω_m=0.3, Ω_v=0.7, h=0.7, Ω_b/Ω_m=0.16

position-space description: Bashinsky & Bertschinger
astro-ph/0012153 & astro-ph/02022153

plots by Dan Eisenstein

Imprint of baryons: forming the acoustic peaks



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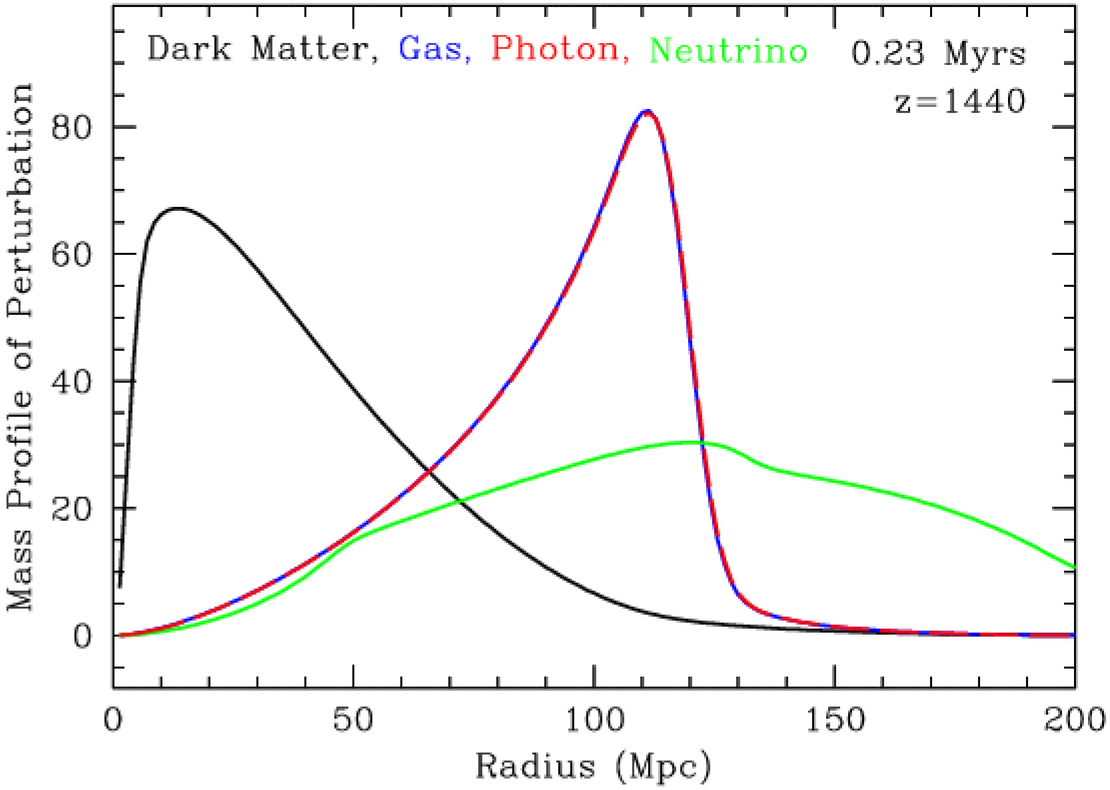

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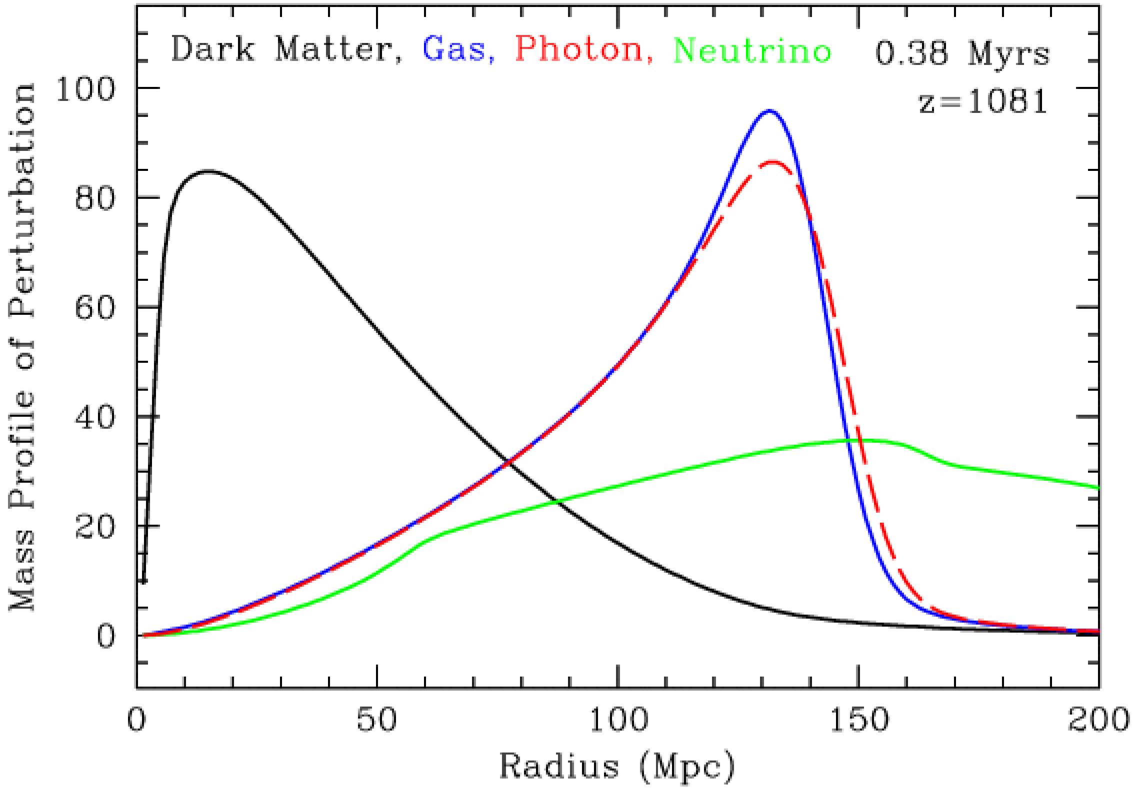

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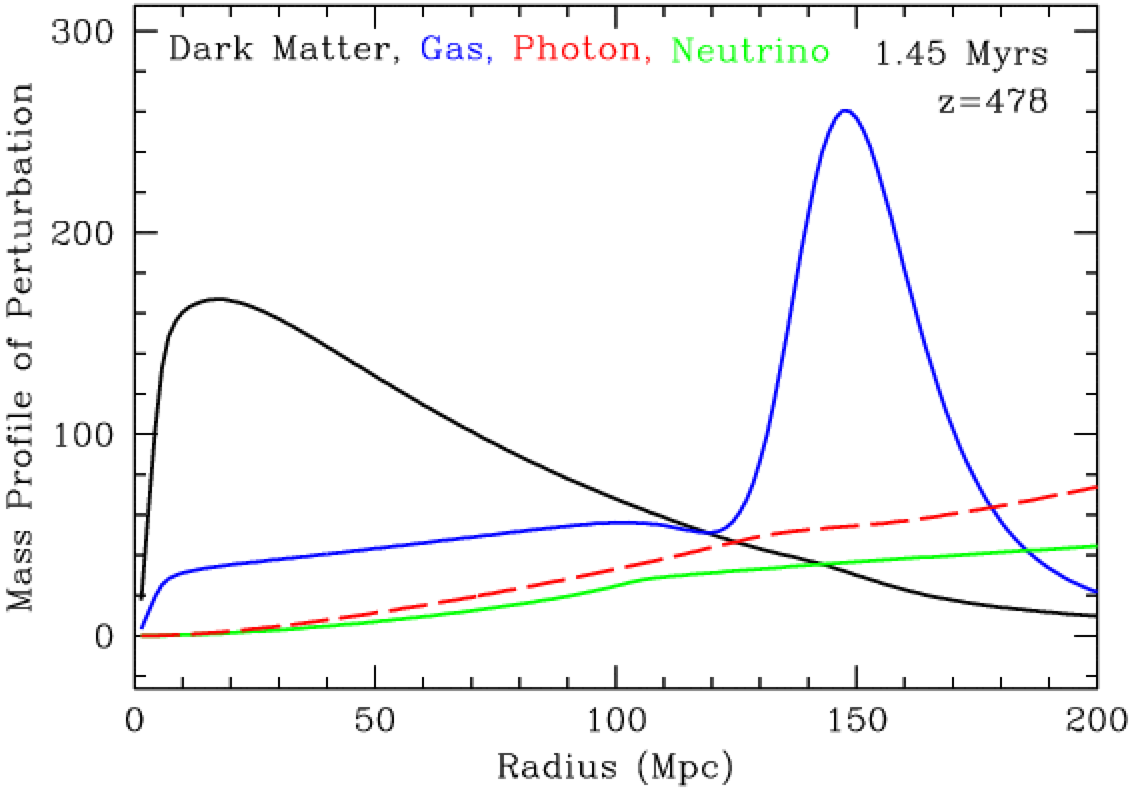

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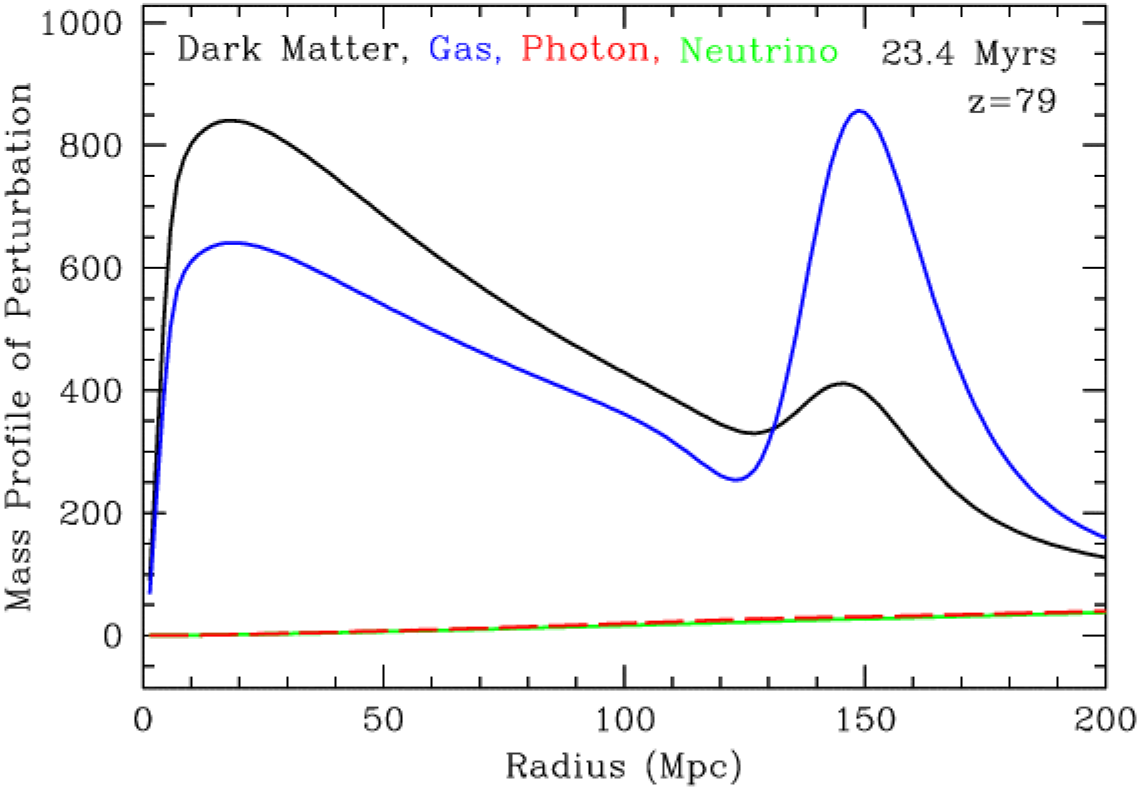

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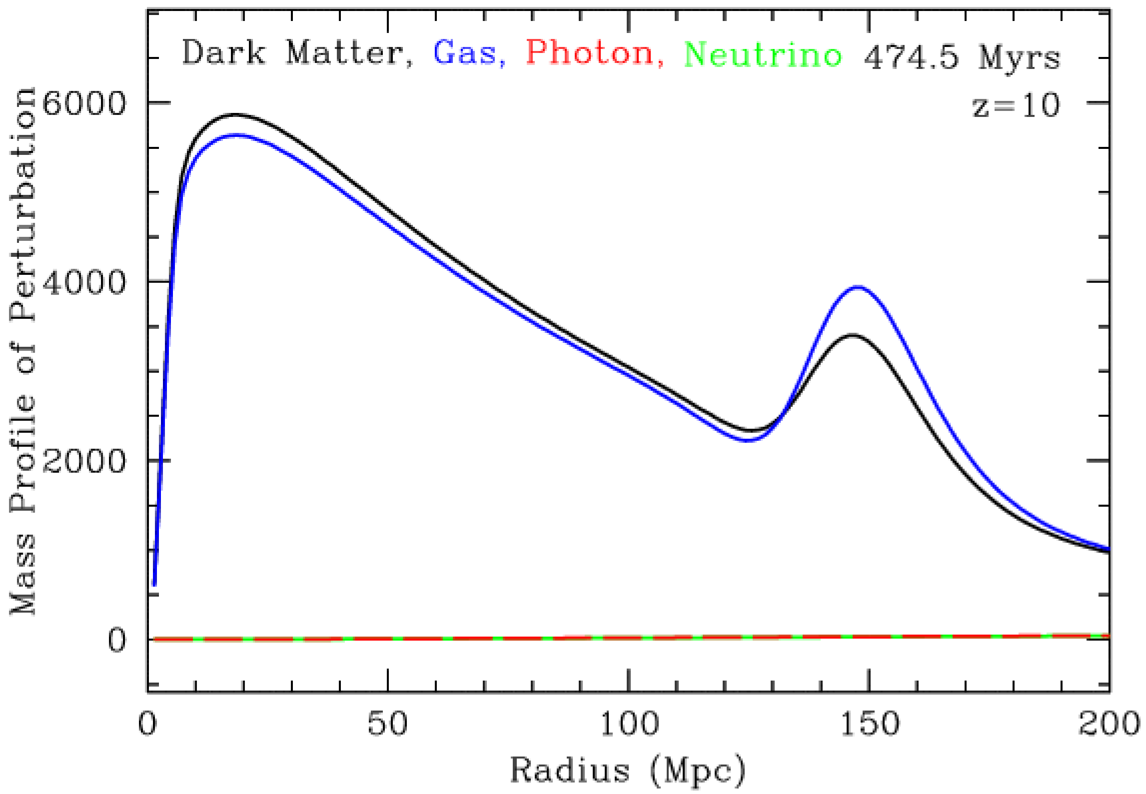

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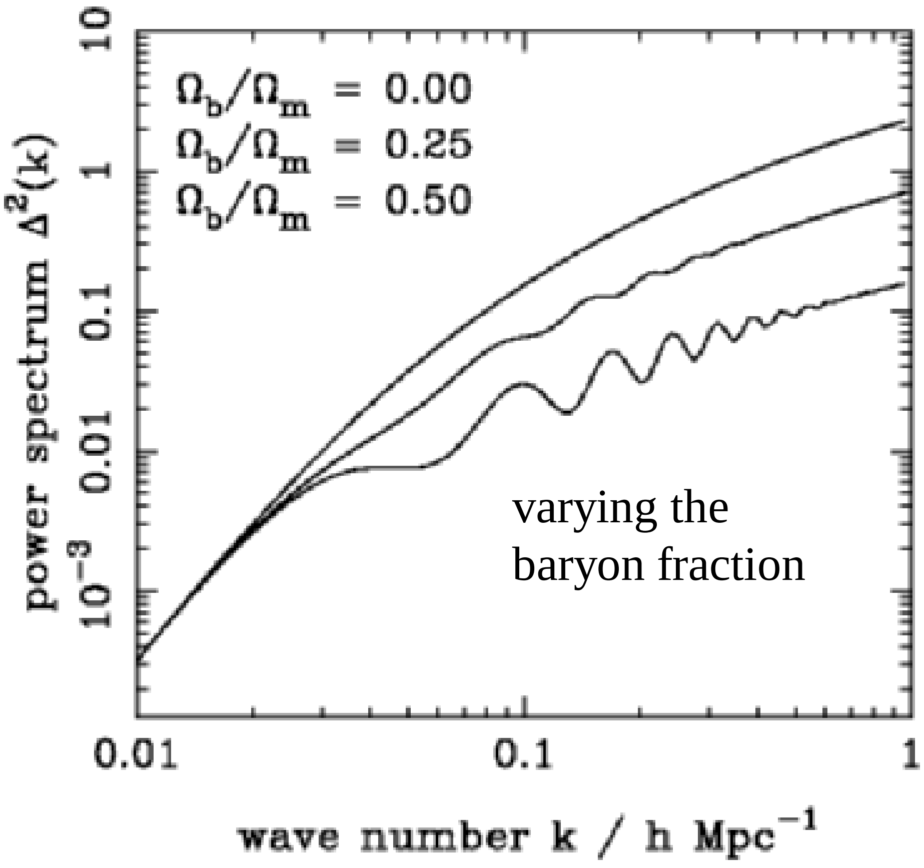
Baryon oscillations in the large-scale matter power spectrum



"Wavelength" of baryonic acoustic oscillations is determined by the comoving sound horizon at recombination

$$k_{\rm bao} = 2\pi/s$$

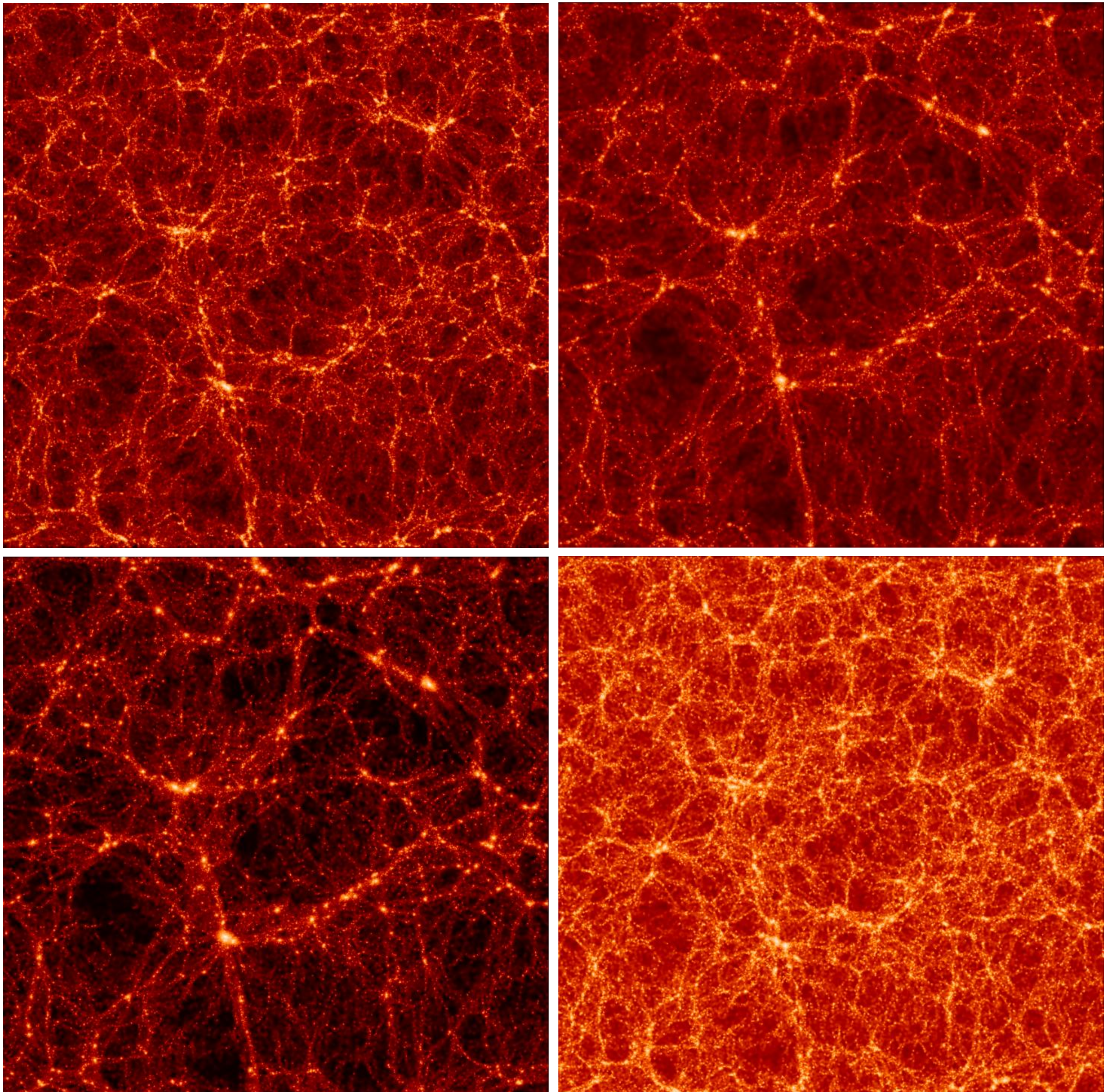
At early times can ignore dark energy, so comoving sound horizon is given by

$$s = \frac{1}{H_0\Omega_m^{1/2}}\int_0^{a_*} da \frac{c_s}{(a+a_{\rm eq})^{1/2}}$$

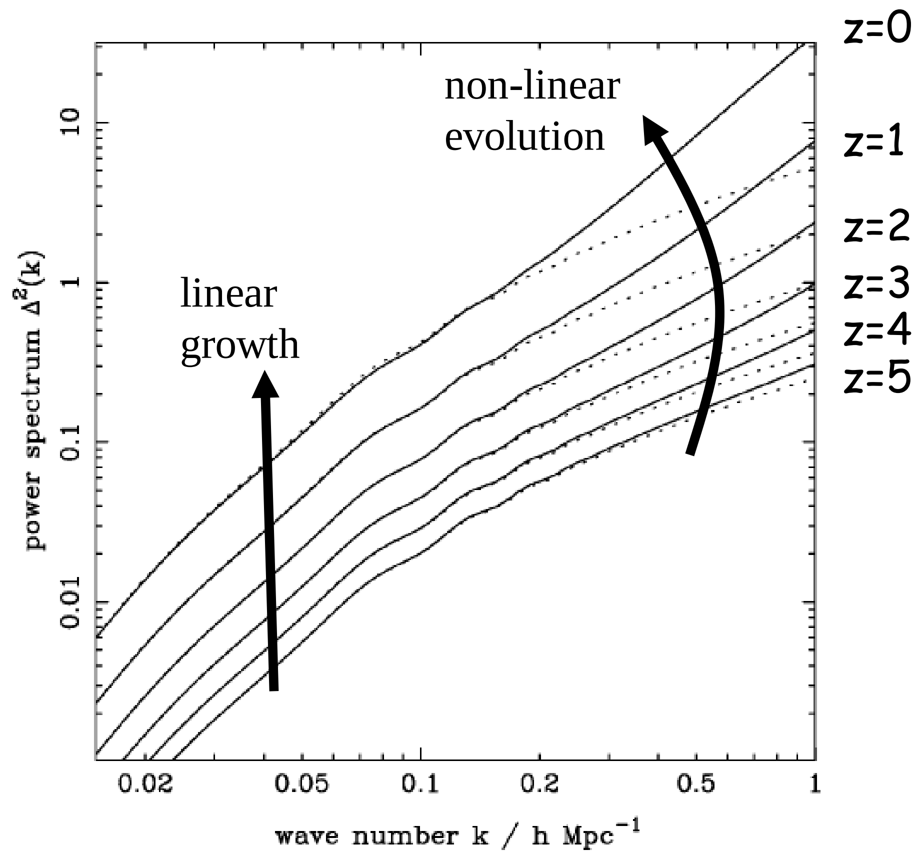
Sound speed c_s

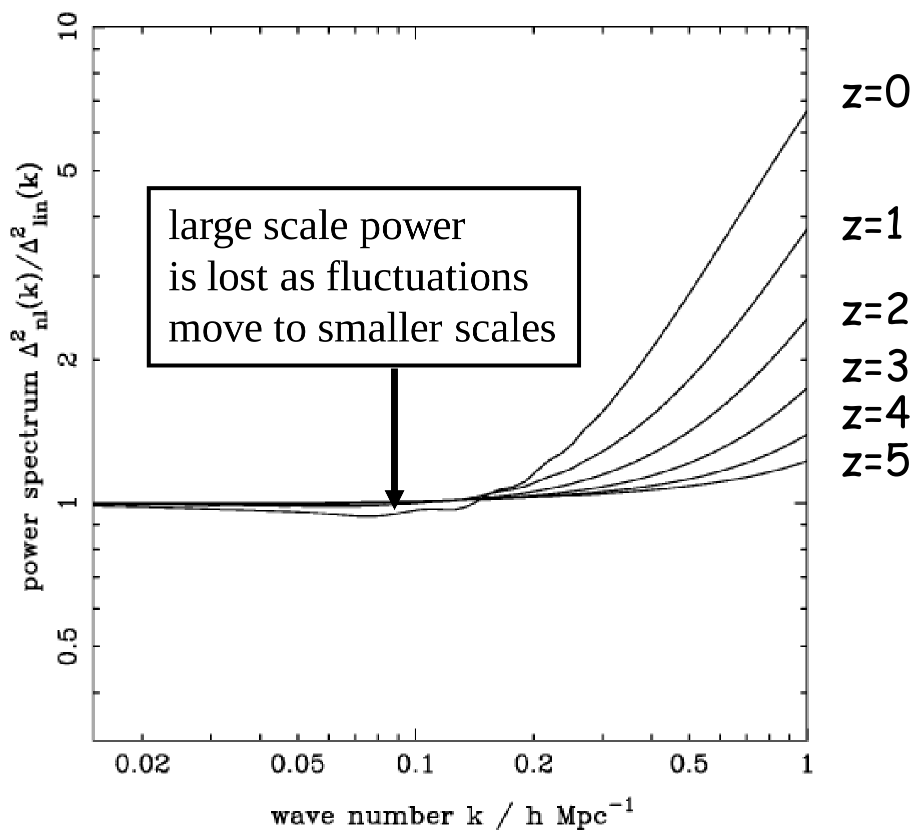
Gives the comoving sound horizon ~110h^{-1}Mpc, and BAO wavelength 0.06hMpc^{-1}

Simulations



Using differential clustering measurements





P(k) calculated from Smith et al. 2003, MNRAS, 341,1311 fitting formulae

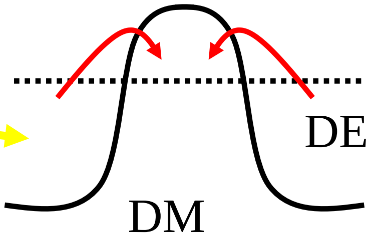
Structure formation depends on dark energy

Dark energy affects the growth rate of dark matter perturbations in a number of ways:

- A faster expansion rate in the past would have made it harder for objects to collapse (overall change in linear growth rate)
- On large scales, if the dark matter reacts to the perturbations in the dark energy, the linear growth rate become scale dependent, and the power spectrum of fluctuations is altered (Ma, Caldwell, Bode & Wang 1999)
- On small scales, if the dark energy properties can lead to changes in non-linear structure growth

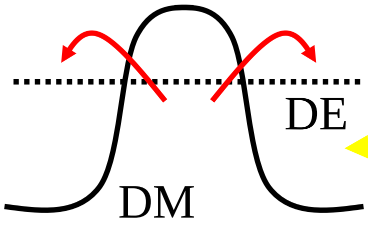
Dependence on sound speed

on large scales dark energy must follow Friedmann equation – this is what it was invented for!



low sound speed means that large scale DE perturbations are important

quintessence has ultra light scalar field so high sound speed



high sound speed means that DE perturbations are rapidly smoothed

The effect of the sound speed provides a potential test of gravity modifications vs stress-energy.

Cosmological evolution of energy densities

Friedmann equation & cosmology equation

$$E^2(a) = \frac{1}{a^2}\left(\frac{da}{dH_0 t}\right)^2 = \Omega_M a^{-3} + \Omega_K a^{-2} + \Omega_X a^{f(a)}$$

$$\frac{1}{a}\frac{d^2 a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

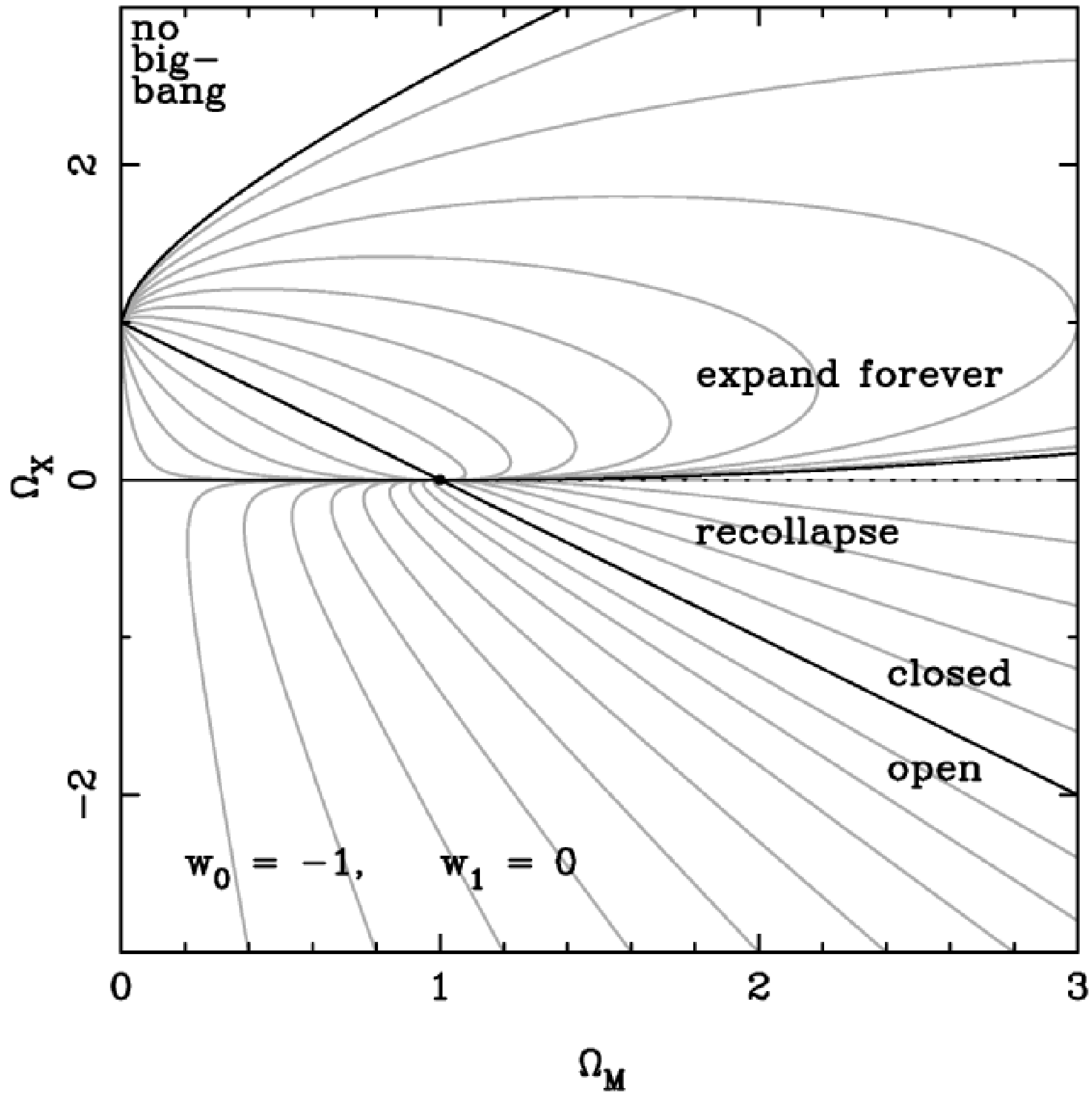
equation of state of dark energy p = w(a)❐

$$f(a) = \frac{-3}{\ln a}\int_0^{\ln a}\left[1+w(a')\right] d\ln a'$$

gives evolution of densities

$$\Omega_M(a) = \frac{\Omega_M a^{-3}}{E^2(a)}, \quad \Omega_X(a) = \frac{\Omega_X a^f(a)}{E^2(a)}.$$

Cosmological evolution of energy densities



Spherical perturbation: leading to linear growth

cosmology equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

$$\frac{1}{a_p}\frac{d^2a_p}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a_p^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

Consider homogeneous spherical perturbation – evolution is "same" as "mini-universe"

homogeneous dark energy means that this term depends on scale factor of background

"perfectly" clustering dark energy – replace a with a_p

Spherical perturbation: leading to linear growth

cosmology equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

$$\frac{1}{a_p}\frac{d^2a_p}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a_p^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

definition of δ

$$\delta = \frac{a^3}{a_p^3} - 1$$

to first order in perturbation radius

$$a_p = a(1-\delta/3)$$

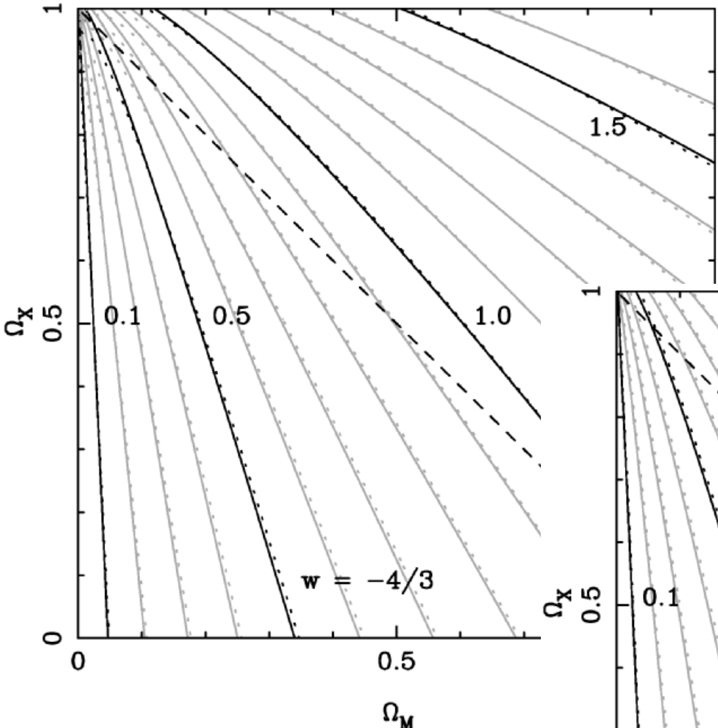
gives

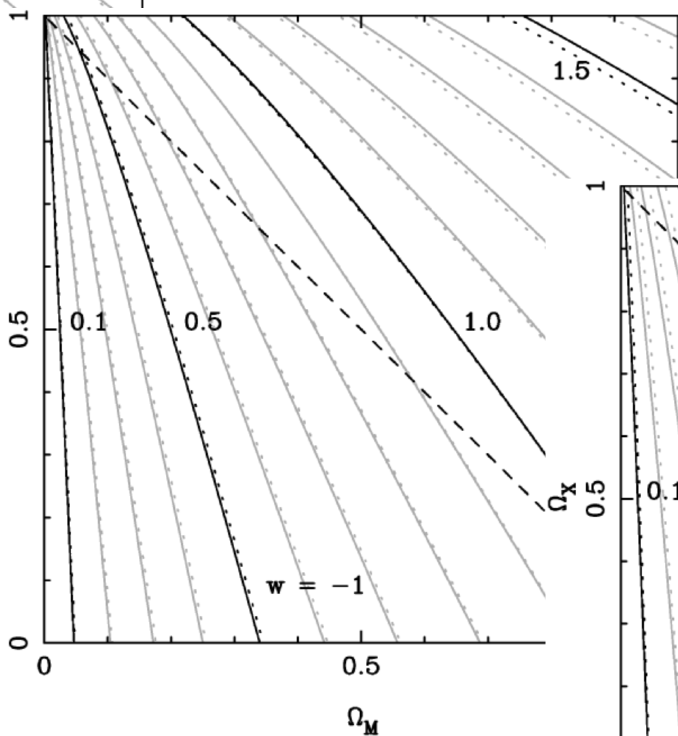
$$\frac{d^2\delta}{dt^2} + \frac{2}{a}\frac{da}{dt}\frac{d\delta}{dt} - \frac{3}{2}\Omega_M a^{-3}\delta = 0$$

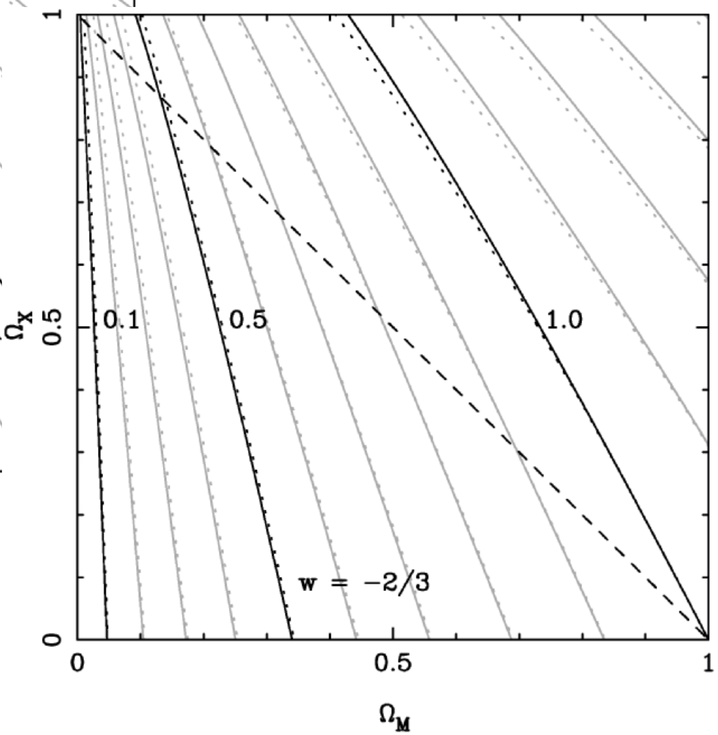
can also be derived using the Jeans equation

only has this form if the dark energy does not cluster – derivation of equation relies on cancellation in dark energy terms in perturbation and background

linear growth factor







linear growth factor to present day
relative to standard EdS value

Spherical perturbation: leading to top-hat collapse

cosmology equation

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

$$\frac{1}{a_p}\frac{d^2a_p}{dt^2} = -\frac{H_0^2}{2}\left[\Omega_M a_p^{-3} + [1+3w(a)]\Omega_X a^{f(a)}\right]$$

homogeneous dark energy means that this term depends on scale factor of background

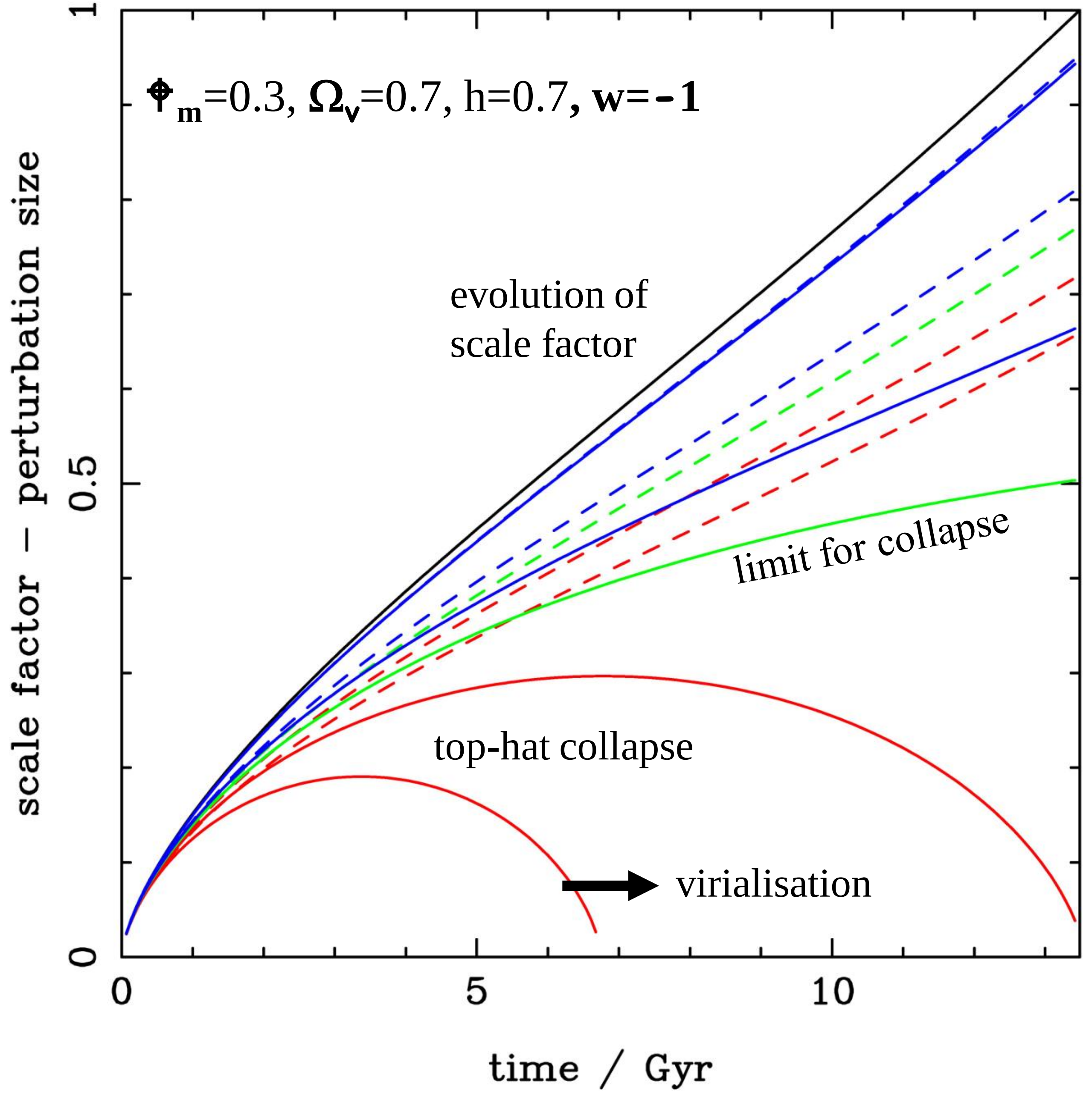
depends on the
equation of state of dark energy **p = w(a)** ❐

$$f(a) = \frac{-3}{\ln a}\int_0^{\ln a}\left[1+w(a')\right] d\ln a'$$

"perfectly" clustering dark energy – replace a with a_p

can solve differential equation and follow growth of perturbation directly from coupled cosmology equations

Evolution of scale factor and perturbations



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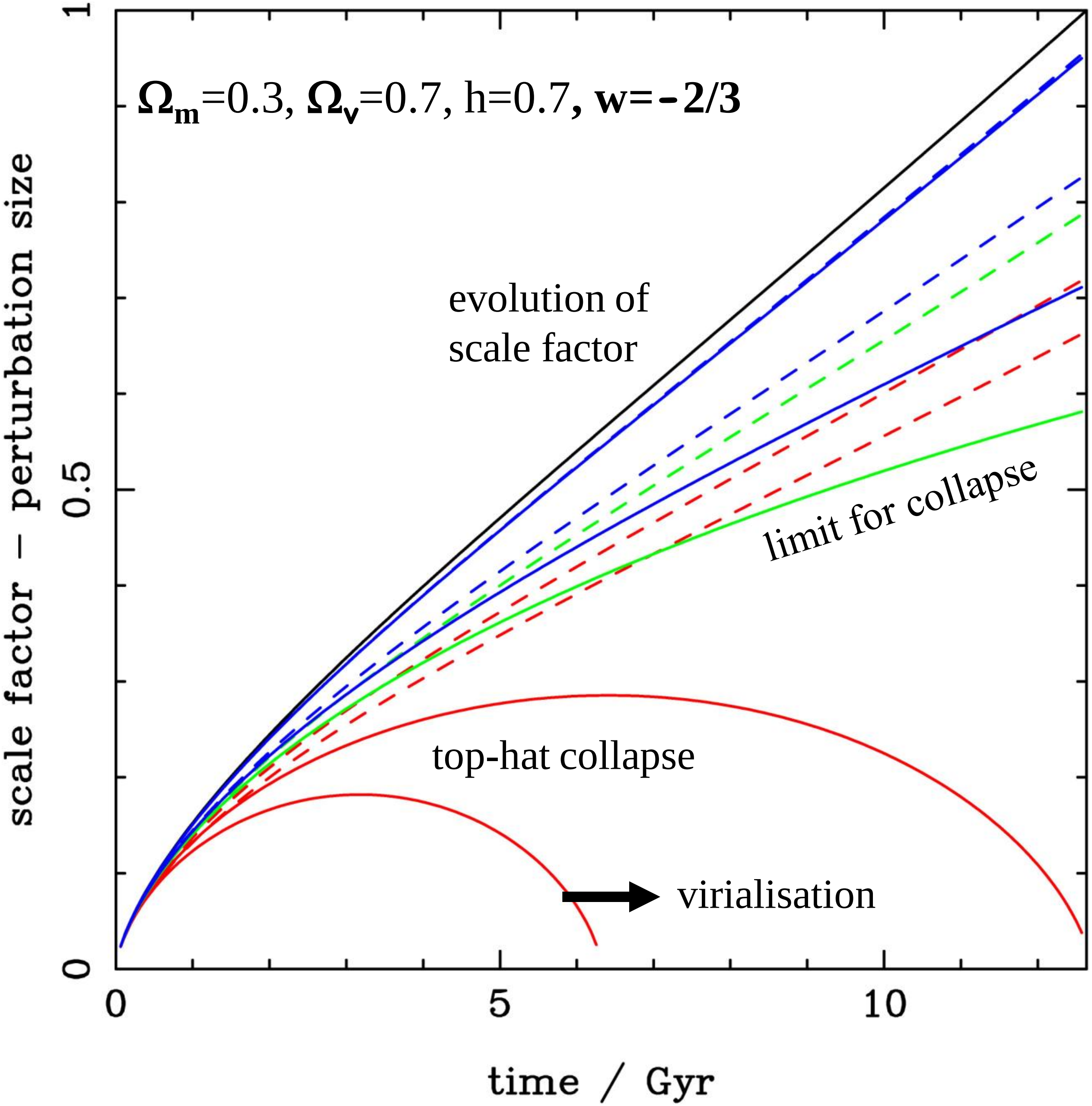
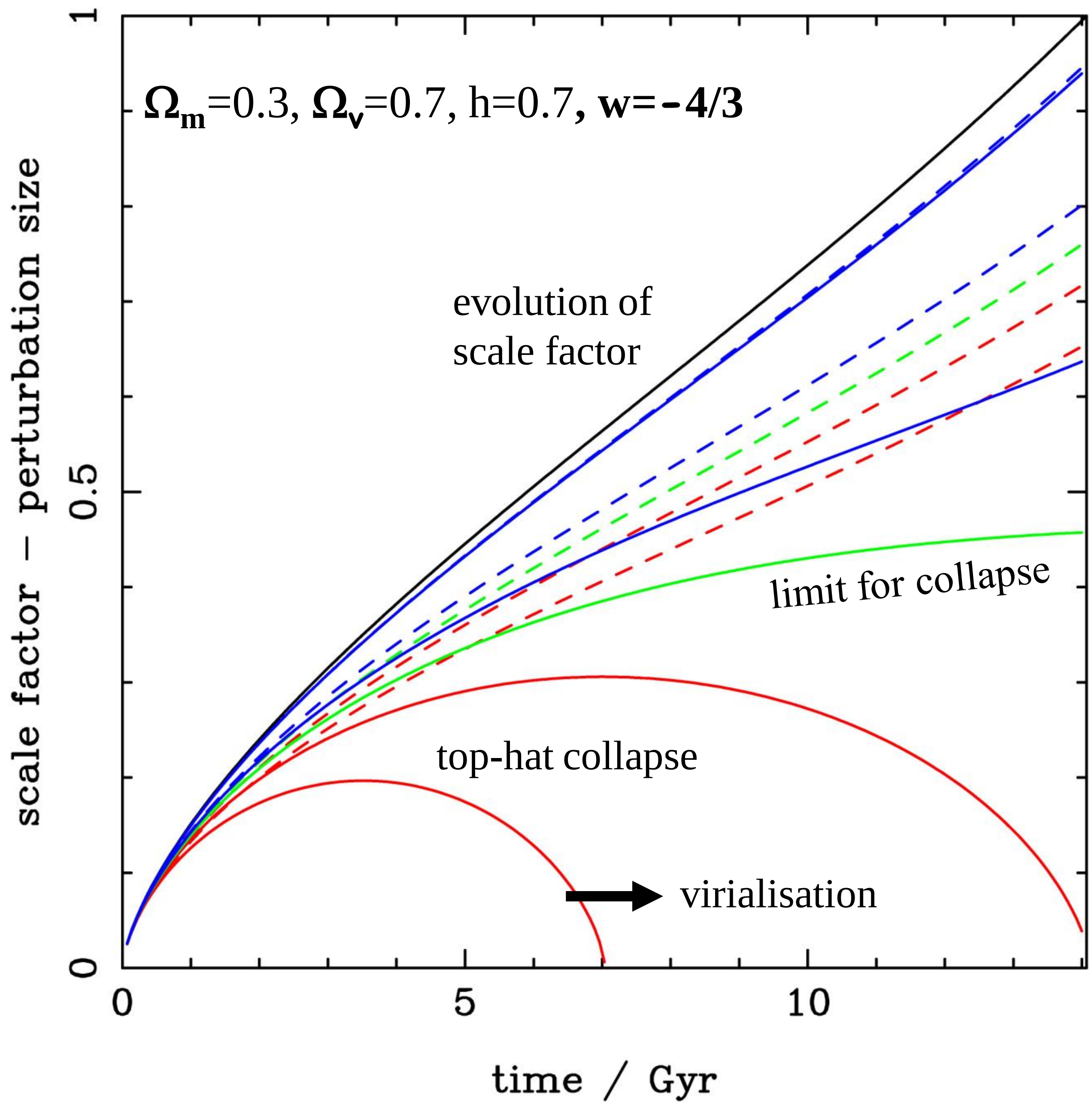


Evolution of scale factor and perturbations



Aside: energy evolution in a perturbation

in a standard cosmological constant cosmology, we can write down a Friedmann equation for a perturbation

$$\left(\frac{da_p}{dH_0 t}\right)^2 = \Omega_M a_p^{-1} + \epsilon_p + \Omega_\Lambda a_p^2$$

for dark energy "fluid" with a high sound speed, this is not true – energy can be lost or gained by a perturbation

the potential energy due to the matter U_G and due to the dark energy U_X

$$U_G = -\frac{3GM^2}{5R}$$

$$U_X = [1 + 3w(a)]\frac{4\pi GM}{10}\rho_X R^2$$

Aside: energy evolution in a perturbation