Instead of familiar landscapes...

Charting a Landscape of Modified Gravity

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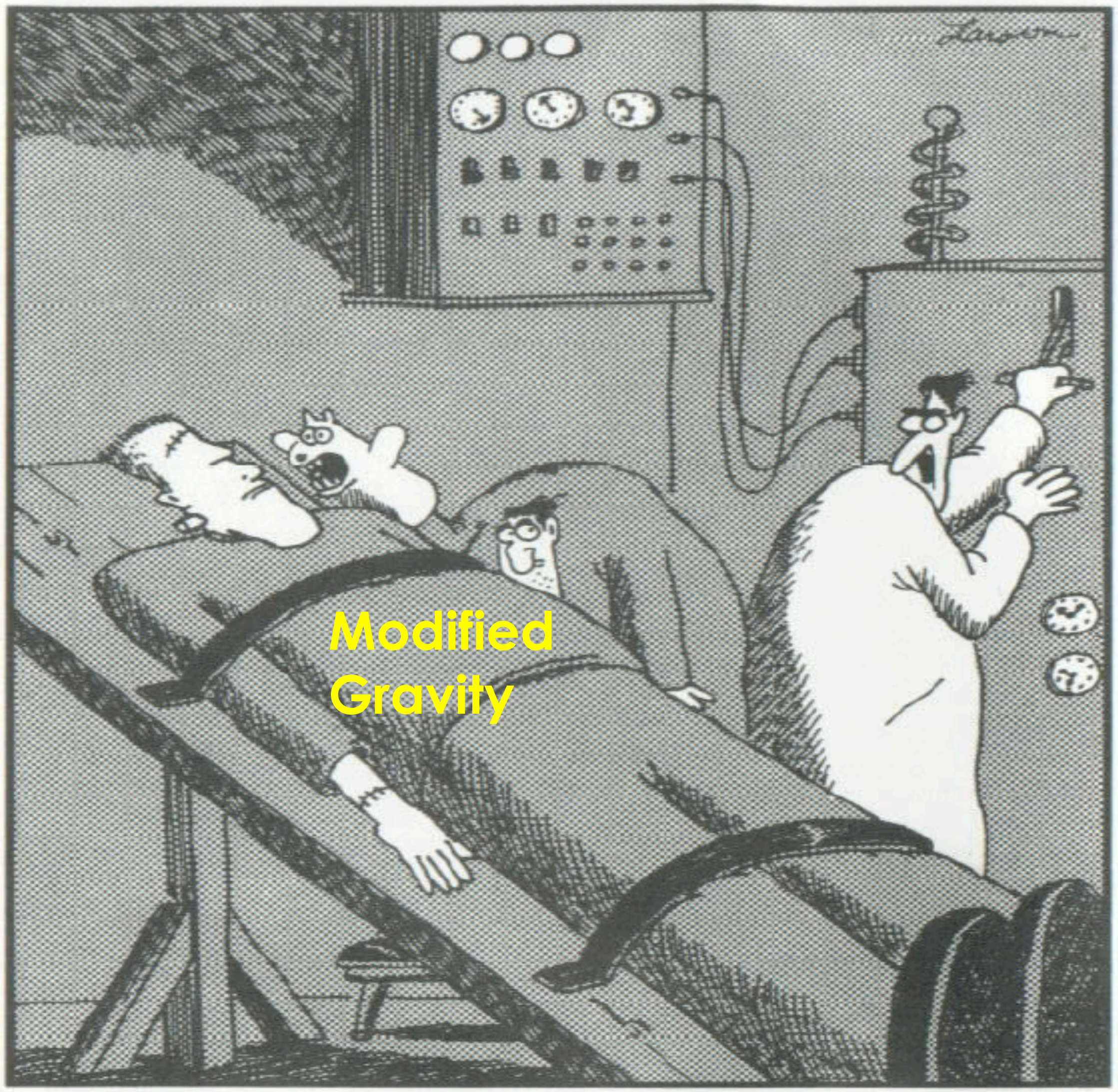
Based on: work with D. Kiley, JHEP 05 (2007) 045.

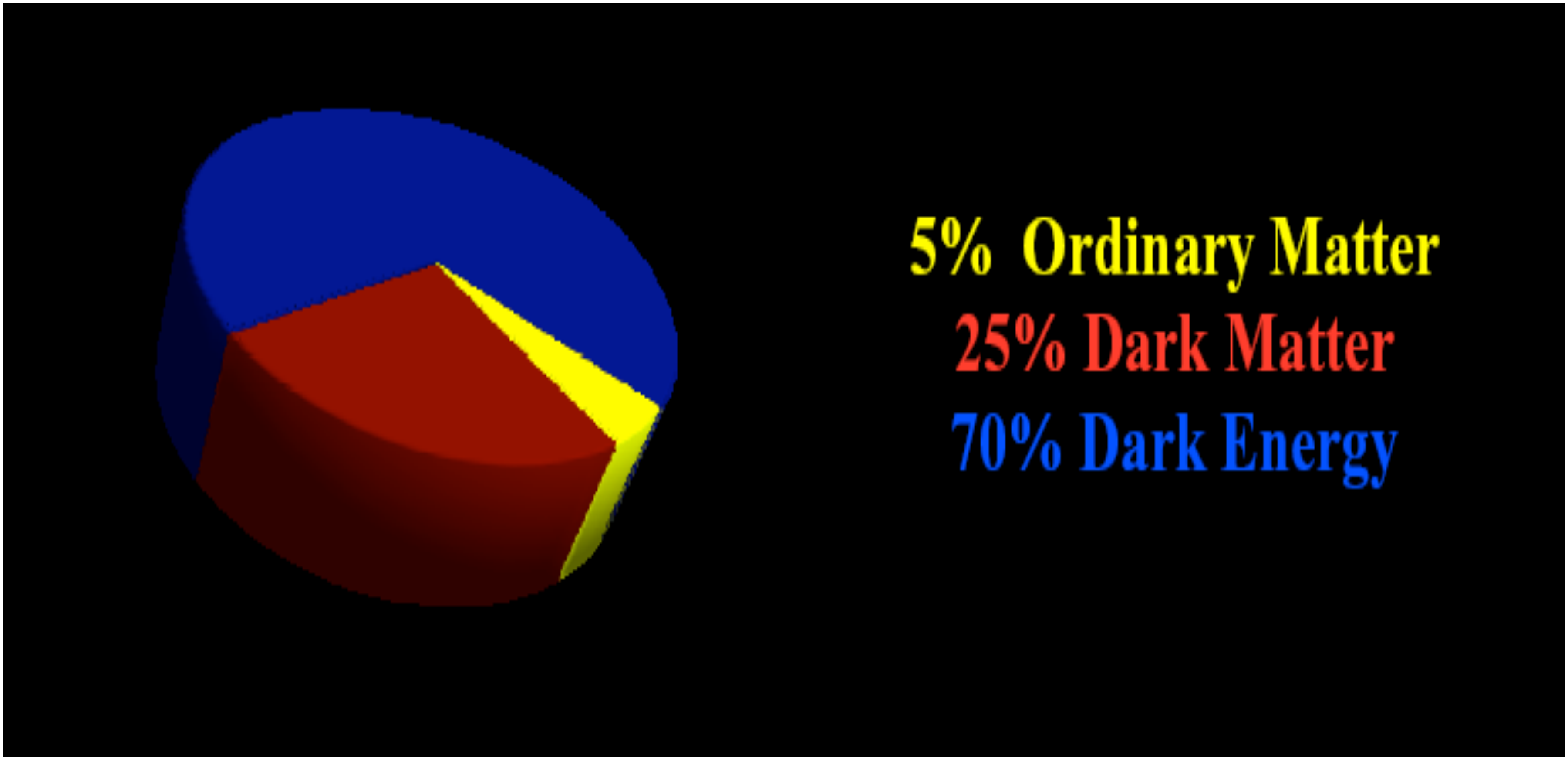
Overview

- Who cares about modified gravity?
- Lightning review of Codimension-1 BIG
- Codimension-2
  - Shock therapy: electrostatics on cones
  - Resolved brane and gravitational see-saw
  - A different kind of **Landscape** …
  - …with a different vacuum energy problem!
- In lieu of a summary…



"Igor! Get that Wolfman doll out of his face! …
Boy, sometimes you really are bizarre."

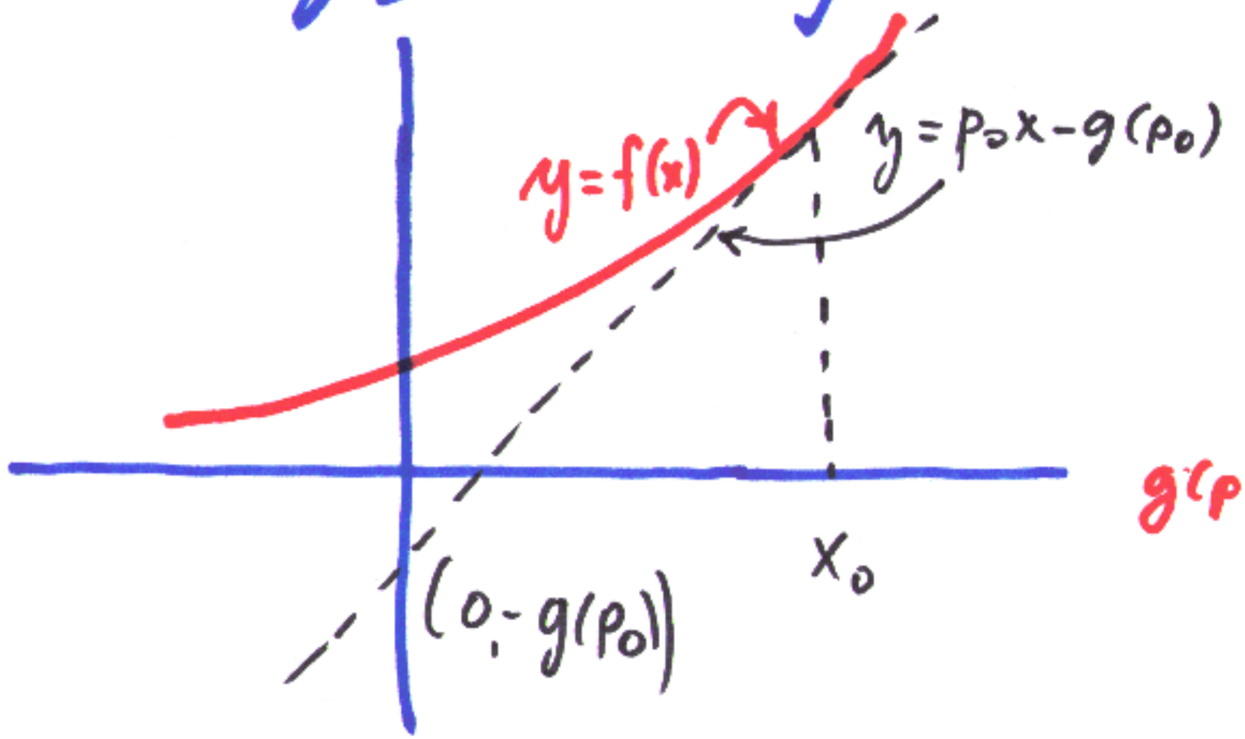
Splitting the cosmic pie



What is 95% of the contents of our Universe?!

Modified gravity v.s. Λ

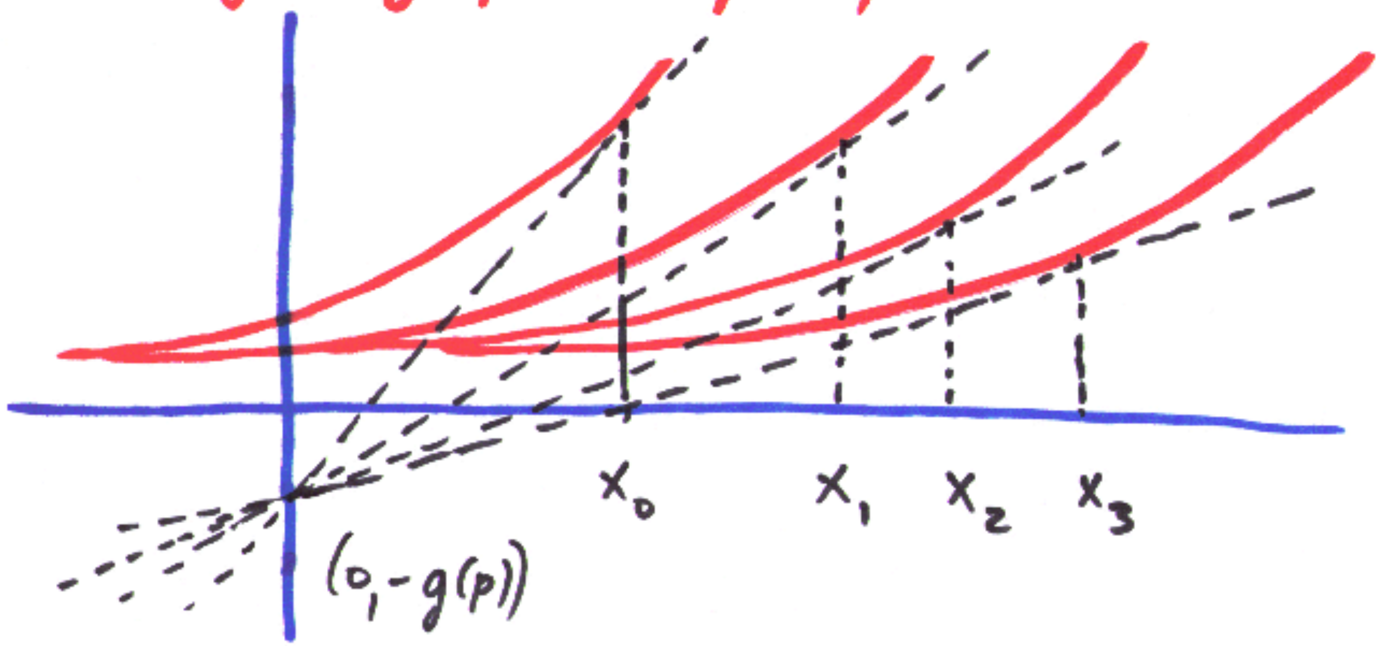
$\int \sqrt{g}\,\Lambda$: a Legendre transf.



$$p = \frac{\partial y}{\partial x}$$

$$g(p) = xp - y$$

Now: forget $f(x)$! Can reconstruct it by solving $g(y') = xy' - y$?



Solution not unique if we don't know x_k!

In GR: $X = \sqrt{\det g}$ a nonpropagating pure gauge DOF: can be **ANYTHING!**

We need a boundary condition!

GR: a Landscape! Einstein already "blundered" in and out of it (1919)

Unimodular gravity:

$$R^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu} R = 8\pi G_N \left(T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu} T\right)$$

But: $\nabla_\mu T^{\mu}{}_{\nu} = 0 \rightarrow \partial_\mu (R + 8\pi G_N T) = 0 \rightarrow R + 8\pi G_N (T + 4\Lambda) = 0$

$$\therefore \; R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu} R = 8\pi G_N \left(T^{\mu}{}_{\nu} + \Lambda\, \delta^{\mu}{}_{\nu}\right)$$

$$\Lambda_{tot} = \langle T^{0}{}_{0} \rangle + \Lambda$$

- Why modify gravity: change it so it won't admit unimodular formulation !
- Very tricky! (except for the "trivial" alterations involving light scalars in various disguises…)
- Interesting setups: where helicity-2 $\gamma_{\mu\nu}$ is composite (examples: DGP, KR)
- Here: codimension-2 BIG: ∞ bulk, no 4D $\gamma_{\mu\nu}$ - only bulk resonance
- Key: get $\frac{1}{r}$ for scales 0.1 mm $< r <$ 1000 MPc…

Cod-1 DGP: a brief review

- Action:

$$S=\int d^5x\sqrt{g_5}\frac{M_5^3}{2}R_5+\int d^4x\sqrt{g_4}(\frac{M_4^2}{2}R_4-M_5^3K^A{}_A-\lambda-\mathcal{L}_{\rm SM})$$

- Assume ∞ bulk: 4D gravity has to be mimicked by the exchange of bulk DOFs!
- How do we then hide the 5th dimension???
- Gravitational perturbations: assume flat background & perturb; while perhaps dubious this is simple, builds up intuition…

Masses and filters

- **Propagator:**

$$G(p)|_{z=0} = \frac{1}{2M_5^3 p + M_4^2 p^2}\left(\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

- **Gravitational filter:**
  - Terms ~ M_5 in the denominator of the propagator dominate at LOW p, suppressing the momentum transfer as **1/p** at distances $r > M_4^2/2M_5^3$, making theory look 5D.
  - Brane-localized terms ~ M_4 dominate at HIGH p and render theory 4D, suppressing the momentum transfer as **1/p²** at distances shorter than $r_c < M_4^2/2M_5^3$.

VDVZ

- Terms $\sim M_5$ like a mass term; resonance composed of bulk modes, with 5 DOFs → massive from the 4D point of view. So the resonance has extra longitudinal gravitons; discontinuity when $M_5 \to 0$ similar to $m_g \to 0$ (van Dam, Veltman; Zakharov; 1970):
- Fourier expansion for the field of a source on the brane:

$$\tilde{h}_{\mu\nu}(p, z=0) = \frac{\tilde{T}_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\tilde{T}^{\lambda}{}_{\lambda}}{2M_5^3\, p + M_4^2\, p^2}$$

- Take the limit $M_5 \to 0$ and compare with 4D GR:

$$\tilde{h}_{\mu\nu}(p) = \frac{\tilde{T}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\tilde{T}^{\lambda}{}_{\lambda}}{M_4^2\, p^2}$$

Strongly coupled scalar gravitons

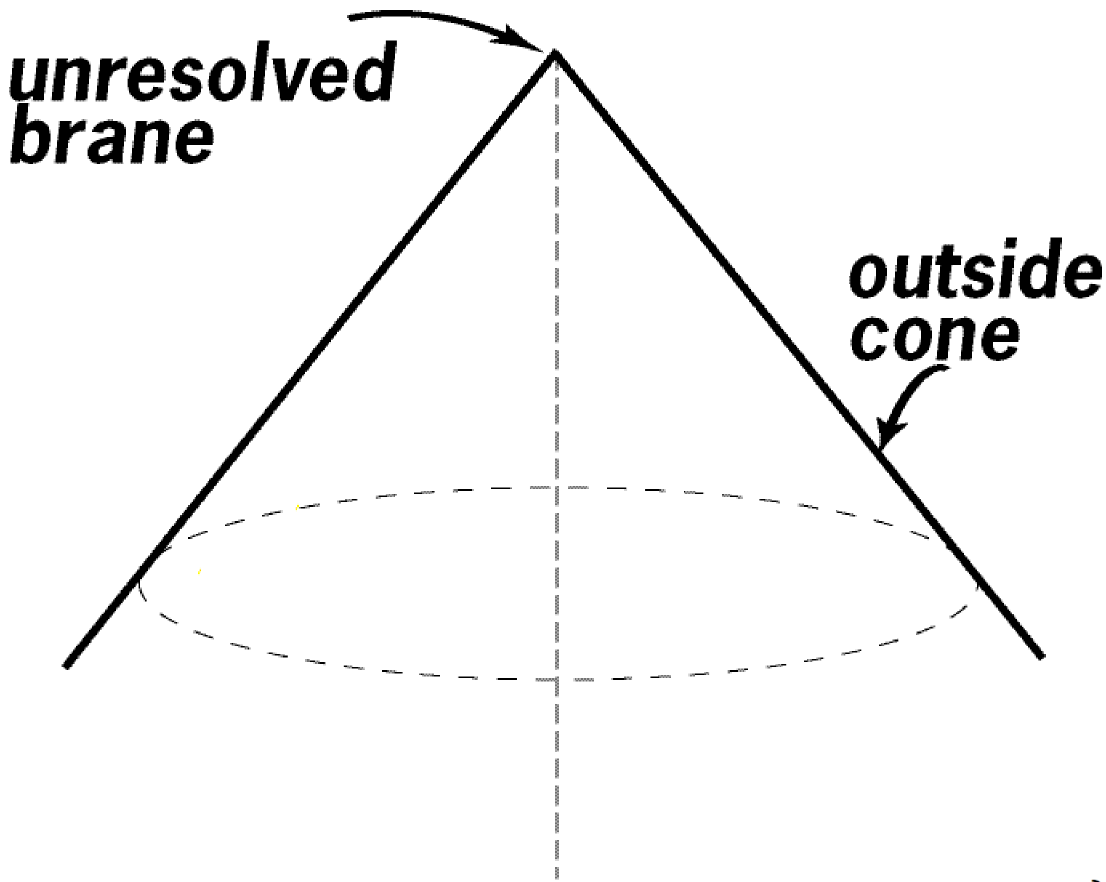
- However: naïve linear perturbation theory in massive gravity on a flat space breaks down → nonlinearities yield continuous limit (Vainshtein, 1972).
- There exist examples of the absence of vDVZ discontinuity in curved backgrounds (Kogan *et al*; Karch *et al*; 2000).
- The reason: the scalar graviton becomes strongly coupled at a scale much bigger than the gravitational radius. (Arkani-Hamed, Georgi, Schwartz, 2002):
- EFT analysis of DGP (Porrati, Rattazi & Luty, 2003): a naïve expansion around flat space suggests a breakdown of EFT at $r_* \sim$ ***1000 km;*** But inclusion of curvature pushes it down to **~ 1 cm** (Rattazi & Nicolis, 2003)…

- Onward to Codimension-2 case
- It is known that Codimension-2 objects allow for the possibility of *offloading* vacuum energy by generating deficit angle (masses in 3D spacetimes, cosmic strings in 4D, 3-branes in 6D…)

COULD THIS HELP WITH THE CC???

Unresolved vacuum

- A conical singularity in two infinite extra dimensions:

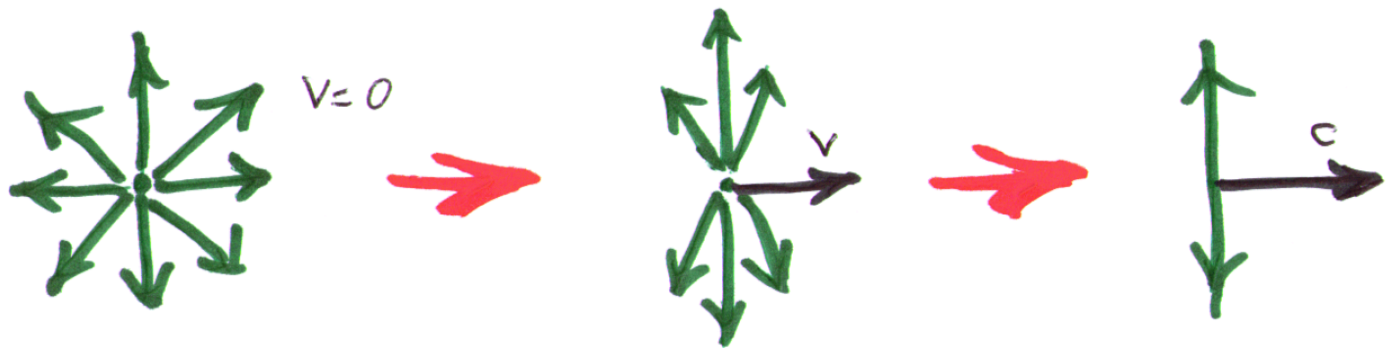


Sundrum, '98

Deficit angle: $\Delta\phi = 2\pi b \qquad b = \frac{\lambda}{2\pi M_6^4}$

What about gravity along the brane?

- A simple diagnostic: perturb with a relativistic particle and see the response
- The problem always linearizes: in any covariant theory it reduces to a single linear differential equation!
- Reason: Lorentz contraction!



- Vacuum before & after, a NULL junction condition in between, resynchronizing clocks!

Pirani; Penrose; Dray & 't Hooft

In more detail…

- Consider the geometry of a mass point, which is a solution of some gravitational field equations, which obey
  - Analyticity in m
  - Principle of relativity
- Then pick an observer who moves VERY FAST relative to the mass source. In his frame the source is boosted relative to the observer. Take the limit of infinite boost.

Only the first term in the expansion of the metric in m survives, since $p = m \cosh b = const$. All other terms are $\sim m^n \cosh b$, and so for $n > 1$ they vanish in the extreme relativistic limit!

Aichelburg-Sexl shockwave

- In flat 4D environment, the exact gravitational field of a photon found by boosting linearized Schwarzschild metric (Aichelburg, Sexl, 1971).

$$ds_4^2 = dudv - \delta(u) f du^2 + dy^2 + dz^2$$

- Here $u,v = (x \pm t)/\sqrt{2}$ are null coordinates of the photon.
- For a particle with a momentum p , f is, up to a constant

$$f_{4D}(\Omega) = \frac{p}{\pi M_4^2} \ln(\frac{\mathcal{R}}{\ell_0})$$

where $R = (y^2 + z^2)^{1/2}$ is the transverse distance and l_0 an arbitrary integration parameter.

Dray-'t Hooft trick

- Shock the geometry with a discontinuity in the null direction of motion v using orthogonal coordinate u, controlled by the photon momentum. Field equations linearize, yield a single field eq. for the wave profile the Israel junction condition on a null surface. The technique extends to DGP, and other brany setups! (NK, 2005)
- Idea: pick a spacetime and a set of null geodesics.
- Trick: substitute

$$v \to v + \Theta(u) f\,,$$

$$dv \to dv + \Theta(u) df$$

discontinuity

change to

$$\hat{v} = v + \Theta(u) f$$

$$dv \to d\hat{v} - \delta(u) f du$$

$$v, dv \to \hat{v}, d\hat{v} - \delta(u) f du$$

A side remark

- There are confusions about cod-2 in the literature, claiming that "only tension can live of cod-2 branes"…
- WRONG! The point is simply that gravitational nonlinearities spill into the bulk - imagine a string threading a bead…
- Exact solutions can in fact be found (Kiley, NK):
  - Photon fields (shock waves)
  - Black holes threaded by cod-2 branes
  - In generic cases, nonlinearities resolve the δ-function and probe the internal structure of the defect
- But defect is defined NOT by δ-source, but by deficit angle - and this remains unaffected!

- Dray and 't Hooft trick on cod-2 brane:

$$ds_6^2 = 4dudv + d\vec{x}_\perp^2 + d\rho^2 + (1-b)^2\rho^2 d\phi^2 - 4\delta(u) f(\vec{x}_\perp, \rho) du^2$$

$$\tau^\mu{}_\nu = \frac{2p}{\sqrt{g_4}} g_{4\,uv} \delta(u)\delta(\vec{x}_\perp)\delta^\mu{}_v \delta^u{}_\nu$$

- Substitute into field eqs & turn the crank

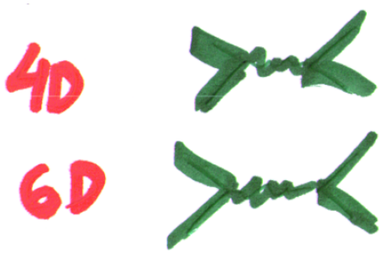
$$\nabla_4^2 f + \frac{M_4^2}{M_6^4}\nabla_\perp^2 f \delta^{(2)}(\vec{y}) = \frac{2p}{M_6^4}\delta^{(2)}(\vec{x}_\perp)\delta^{(2)}(\vec{y})$$

- A little more effort yields exact solution:

$$f = -\frac{p}{2\pi^2(1-b)M_6^4}\int_0^\infty dk \frac{kK_0(k\rho)J_0(k|\vec{x}_\perp|)}{1 + \frac{M_4^2}{\pi(1-b)M_6^4}k^2 \text{``}K_0(0)\text{''}}$$

- Notice that K(0) is singular! But bear with it...

- How should we read this?



$$\rightarrow \int dk\, k \frac{1}{k^2} = \int \frac{dk}{k}$$

$$\rightarrow \int dk\, k^3 \frac{1}{k^2} = \int dk\, k$$

- Our solution is

$$\int dk \frac{k K_0(k\rho)}{1 + \frac{M_4^2 \text{“}K_0(0)\text{”}}{\pi(1-b) M_6^4} k^2}$$

- Implies that f looks 4D when $k^2 > k_c^2$, and 6D otherwise

$$k_c^2 = \frac{\pi(1-b) M_6^4}{M_4^2 \text{“}K_0(0)\text{”}}$$

- But K(0) divergent! The shock looks 4D at all scales on the brane but has core and bulk singularities ➔ regulate!

Similar observation: Dvali, Gabadadze, Hou, Sefusatti

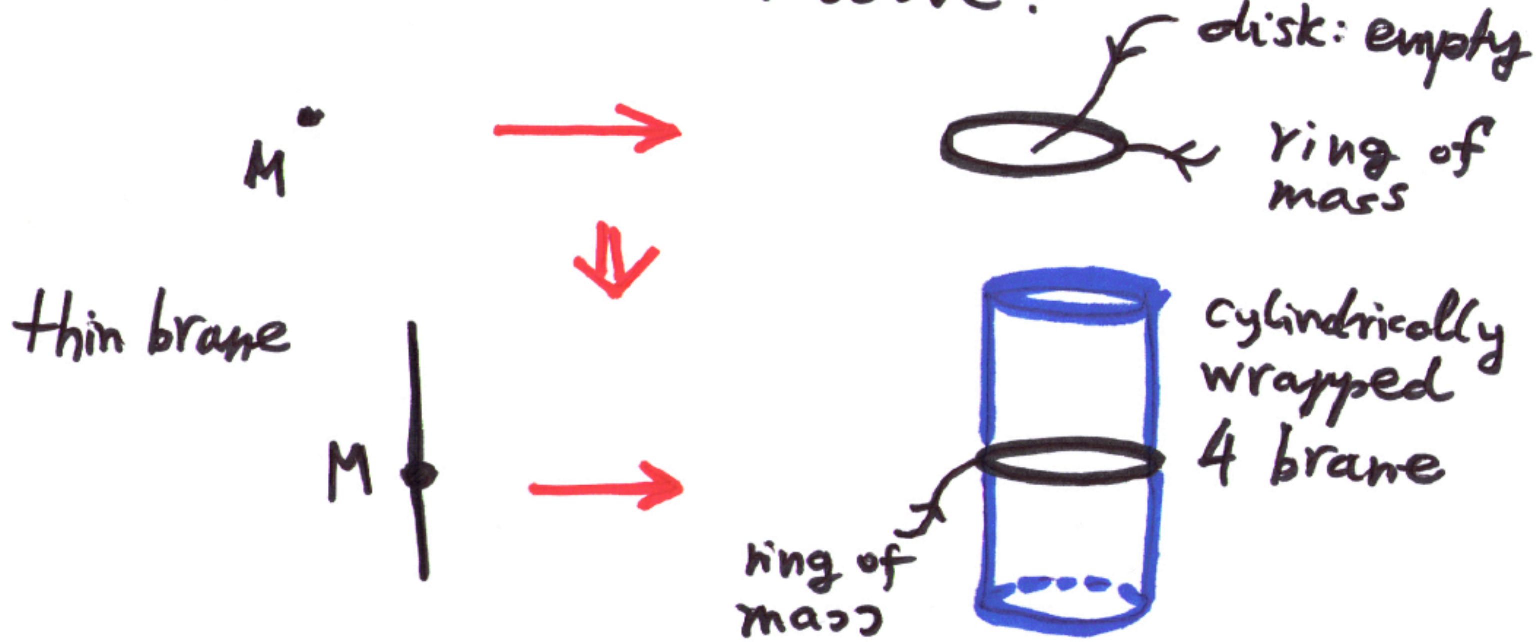
- To decide how well gravity is confined we MUST explore the regulated version of the codimension-2 brane

Mathematically:

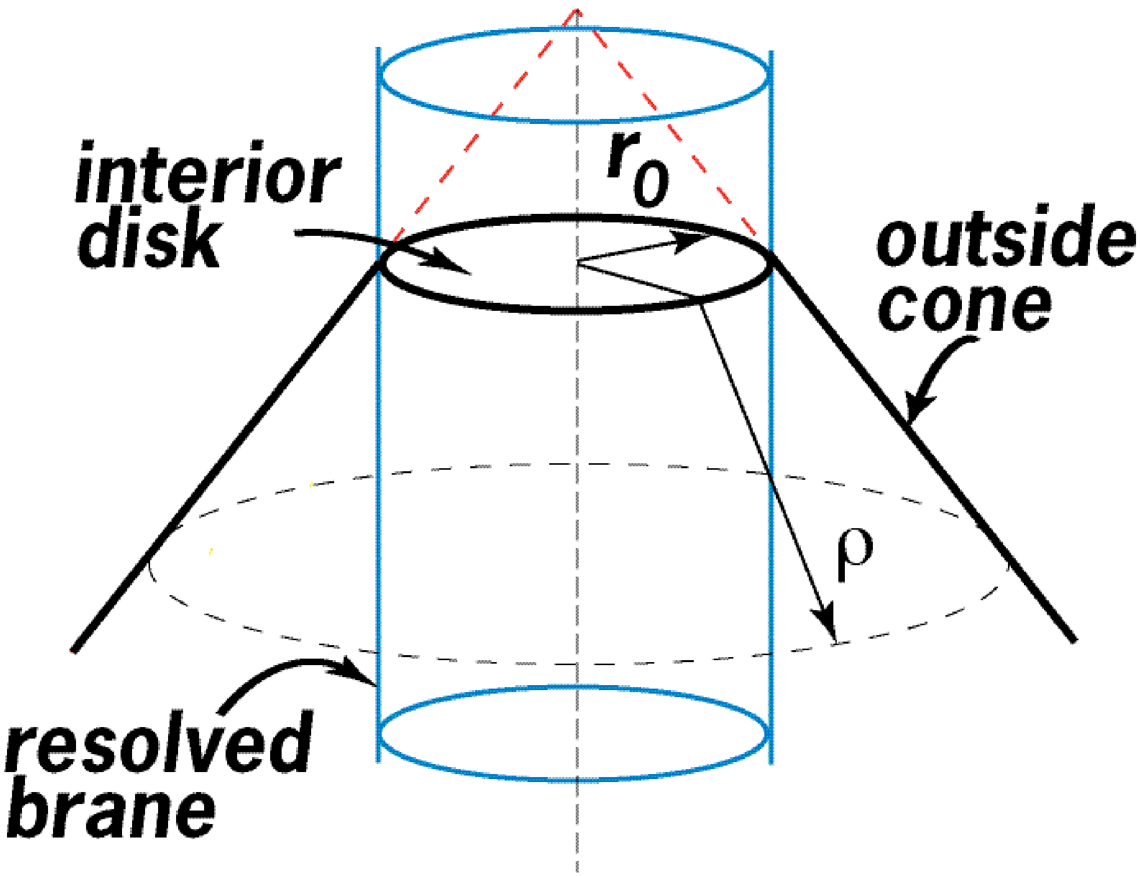
M

$$\nabla^2_{cone}\phi = -M\delta^{(2)}(\vec{y}) \rightarrow \phi \sim \ln|\vec{y}|$$

SINGULAR NEAR AND FAR – ILL POSED EXTERIOR PROBLEM! Clear how to resolve:



Brane mesa



A (Semiclassical) Landscape! To wrap a 4-brane, must cancel tension in the compact direction: add an axion and pick its flux

$$\Sigma = q\phi$$

$$q^2 = 2\lambda_5 r_0^2$$

$$\lambda = 4\pi r_0 \lambda_5$$

(Peloso, Sorbo, Tasinato)

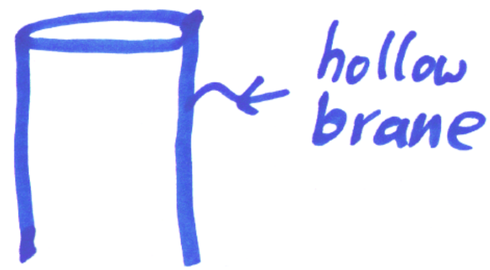
Vacua counted by integration constants!

Background solution:

$$ds^2_{6\,vac} = \eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + \left[\Big(1 - b\Theta(\rho - r_0)\Big)\rho + br_0\Theta(\rho - r_0)\right]^2 d\phi^2$$

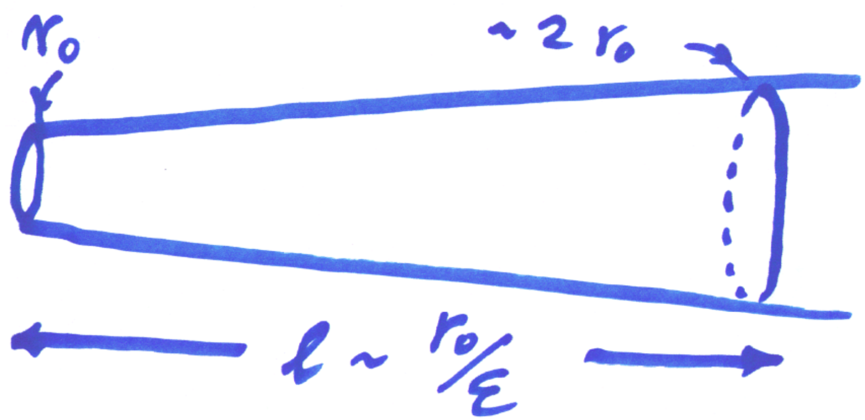
- Exterior: $$ds_2^2 = d\rho^2 + (1-b)^2\left(\rho + \frac{b}{1-b} r_0\right)^2 d\phi^2$$
- Critical case, b=1: $\lambda_{cr} = 2\pi M_6^4 \leftrightarrow \lambda_{5\,cr} = \frac{M_6^4}{2r_0}$

Bulk shrinks to a cylinder!



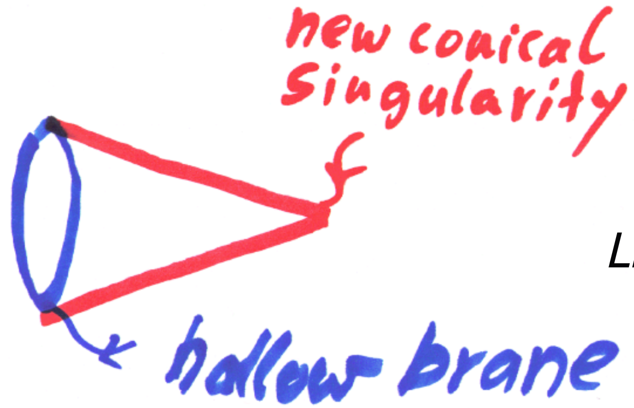
- Near-critical case, $\epsilon = 1 - b \ll 1$

Thin sliver: bulk "compactifies" out to l !



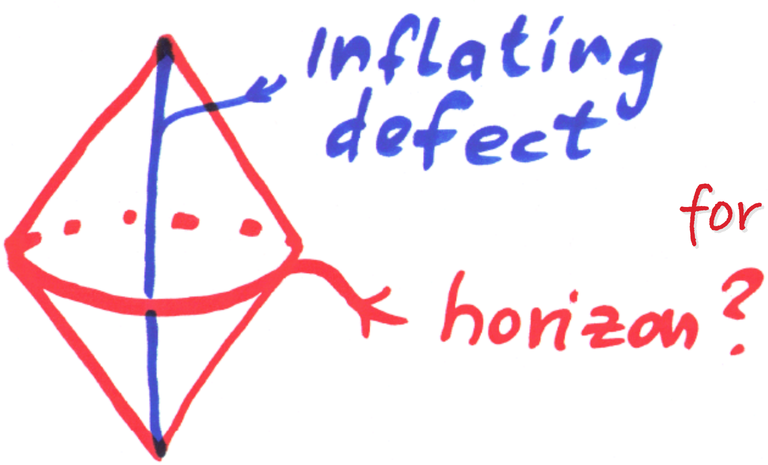
- Supercritical case, b>1: $ds_2^2 \to d\rho^2 + (b-1)^2\left(\rho - \frac{b}{b-1}r_0\right)^2 d\phi^2$

Singular teardrop, similar to Gell-Mann & Zwiebach's



Linet

- Unreliable because of the singularity - similar to supercritial cosmic strings in the '80s.
- Resolution: TOPOLOGICAL INFLATION! *Linde; Vilenkin*



for cod-2, what are exact solutions?!

Numerical study: Cho & Vilenkin; Gregory

- We can at least analyze subcritical cases, $b \leq 1$:

$$S_{matter} = -\int d^5x\sqrt{g_5}\mathcal{L}_{matter}$$

$$\Phi = \sum \Phi_n e^{in\phi}$$

$$M^2 = m^2 + \frac{n^2}{r_0^2}$$

- At distances $\ell \gg r_0$ this truncates to only the n=0 states which are AXIALLY SYMMETRIC masses on the cylinder
- Redo the shock wave analysis!

- Regulated exact solution, on the brane, is

$$f(\vec{x}_\perp, r_0) = -\frac{p}{2\pi^2 r_0}\int_0^\infty dk \frac{kI_0(kr_0)K_0(k\frac{r_0}{1-b})J_0(k|\vec{x}_\perp|)}{kM_6^4[I_0K_1 + I_1K_0] + M_5^3k^2I_0K_0}$$

- Arguments of I_n and K_n in the denominator are kr_0 and $kr_0/(1-b)$, respectively
- For $l \gg r_0$ the argument of I's is always small, so we can approximate $I_0=1$ and $I_1=0$. Care needed with K_n!

$$f(\vec{x}_\perp, r_0) = -\frac{p}{2\pi^2 r_0}\int_0^\infty dk \frac{kK_0(k\frac{r_0}{1-b})J_0(k|\vec{x}_\perp|)}{kM_6^4K_1(k\frac{r_0}{1-b}) + M_5^3k^2K_0(k\frac{r_0}{1-b})}$$

- Therefore:

4D! $\quad k > \frac{M_6^4}{M_5^3}\frac{K_1(k\frac{r_0}{1-b})}{K_0(k\frac{r_0}{1-b})} = k_c$

Sub-critical branes

- We have

$$b \sim \mathcal{O}(1)\,, \qquad \frac{kr_0}{1-b} \ll 1\,, \qquad K_1 \sim \frac{1-b}{kr_0}\,, \qquad K_0 \sim \ln(\frac{2(1-b)}{kr_0})$$

$$k_c^2 = \frac{(1-b)M_6^4}{M_5^3 r_0 \ln(\frac{2(1-b)}{kr_0})}$$

- Since $b = \frac{\lambda}{2\pi M_6^4}$, must tune λ to get the right k_c
- Must also arrange $M_5 \gg M_6$: standard DGP tuning
- However: graviscalars strongly coupled below crossover scale! No perturbative 4D regime! This is because 4D vacuum is not 5D vacuum: axion induces screening.
- But...

Near-critical branes much better behaved!

- We now have $\frac{kr_0}{1-b} \gg 1 \quad \rightarrow \quad \frac{K_1}{K_0} \rightarrow 1$

$$K_c = \frac{1}{r_c} = \frac{M_6^4}{M_5^3}$$

- To push the crossover out to present horizon size we need

$$M_5 \sim M_4 \sim 10^{19}\,\mathrm{GeV}\,, \qquad M_6 \sim \mathrm{TeV}$$

- Gravitational see-saw: when there's bulk sliver, the theory looks 5D - reducing and using cod-1 formulas we get exactly see-saw!

$$r_c = \frac{M_4^2}{2\pi M_6^4 r_0}$$

Static sources

- Shocks reveal the theory contains 4D GR, what else?
- Relativistic sources do not tickle graviscalars because as v goes to c conformal symmetry is restored and *T*=0
- Need to consider static rings of mass - arduous!!!
- Gauge-fix and turn the crank!

$$ds_6^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + d\rho^2 + (1 + \tilde{\Phi})\alpha(\rho)d\phi^2$$

$$h_{\mu\nu} = \gamma_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + \partial_\mu \partial_\nu \Psi - \frac{1}{4}\eta_{\mu\nu}\partial_4^2\Psi + \frac{1}{4}\eta_{\mu\nu}h_4$$

$$\xi = 0\,, \qquad \partial^\mu \gamma_{\mu\nu} = \gamma^\mu{}_\mu = \partial_\mu A^\mu = 0\,, \qquad \alpha = ((1 - b\Theta)\rho + br_o\Theta)^2$$

- To linear order ignore vectors, and project out helicities.

- For TT-tensors, we have

$$\nabla_6^2\gamma^\mu{}_\nu + \frac{M_5^3}{M_6^4}\partial_4^2\gamma^\mu{}_\nu\delta(\rho-r_0) = -\frac{2}{M_5^3}\left(\tau^\mu{}_\nu - \frac{1}{3}\left(\delta^\mu{}_\nu - \frac{\partial^\mu\partial_\nu}{\partial_4^2}\right)\tau^\alpha{}_\alpha\right)\delta(\rho-r_0)$$

- Difference from shocks: 1/3 on RHS - expected as this is a resonance built from massive modes!
- Solutions on the brane:

$$\gamma^\mu{}_\nu = -2\mathcal{G}_{TT}\left(\tau^\mu{}_\nu - \frac{1}{3}(\delta^\mu{}_\nu - \frac{k^\mu k_\nu}{k^2}\tau^\alpha{}_\alpha)\right) \quad \mathcal{G}_{TT} = \frac{1}{M_6^4\frac{K_1}{K_0}k + M_5^3k^2} = \frac{1}{M_5^3}\frac{1}{k_ck + k^2}$$

- Fourier-transform for a massive source:

$$\gamma^0{}_0 = \frac{8}{3}G_N\frac{\mathcal{M}}{|\vec{x}|}\,, \quad \gamma^j{}_k = -\frac{2}{3}G_N\frac{\mathcal{M}}{|\vec{x}|}(\delta^j{}_k + \frac{x^jx_k}{\vec{x}^2}) \quad \rightarrow \quad V_N = -\frac{4}{3}G_N\frac{\mathcal{M}}{|\vec{x}|}$$

- A BD theory with w=0 - an extra mode's present!… Maybe strong coupling effects could decouple helicity-0 mode on non-vacuum backgrounds?…

Vainshtein, 1972

Graviscalars

- The scalar sector contains one independent mode, the "radion" $X = \frac{1}{4}(h_4 - \partial_4^2 \Psi)$ which yields

$$X = -\frac{1}{6}\mathcal{G}_X \left(\tau^\alpha{}_\alpha + \frac{3M_5^3 r_0 k^2}{bM_6^4} \frac{1 - b - \frac{bM_6^4 r_0}{M_5^3}}{1 - b + k^2 r_0^2} \tau^\phi{}_\phi \right)$$

- On subcritical branes there's an Euclidean momentum pole at the crossover scale! No perturbative 4D regime!
- On near-critical branes,

$$\mathcal{G}_X = \frac{1}{M_6^4 k + M_5^3 \frac{r_0^2 k^2}{1-b+r_0^2 k^2}(k^2 + \frac{3}{4r_c r_0})}$$

- Perturbativity better: when $k^2 r_0^2 \gg 1 - b$ X is massive, and when $k^2 r_0^2 \ll 1 - b$, X is subgravitationally coupled!
- NOTE: no GHOSTS or TACHYONS! (`normal branch')

Cosmological constant?

- $1-b \ll 1$ is under control in quadratic theory…
- Conical magic:
  - Desensitizes 4D to 5D transition from tension: see-saw
  - Improves screening: scalar strong coupling scale higher!
  - Decouples the radion inside the throat
- What happens with (nonlinear) strong coupling?
- What about the vacuum energy problem?

Vacuum energy problem is Different!

- Need near-critical tensions to get crossover hierarchy

$$r_c = \frac{M_5^3}{M_6^4} < r_{throat} = \frac{r_0}{1-b} \simeq \frac{q/\sqrt{\lambda_5}}{1 - q\sqrt{\lambda_5}/M_6^4}$$

- Detuning λ_5 can invert the inequality; the theory ends up with strongly coupled graviscalars in the 4D regime
- A thought: can we protect this hierarchy by utilizing some strong coupling dynamics? E.g. can we get an IR fixed point, where RG drives us back to the tuned value?
- If so: an example of how to evade Weinberg's no-go in a scalar-tensor gravity - not the real world, but ...

In lieu of a summary...

Maybe the problem is that our theories give us **General Relativity** too easily ... forcing us into some kind of a landscape.

But maybe there are more exotic landscapes, criss-crossed by deep flat valleys where gravity adjusts to the environment and eats up the vacuum energy. We still need to be lucky to end up in one such place, but if we do radiative corrections don't evict us out...?