Cosmological Constant & other Dark Thoughts

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A Bit of History

1917 Einstein : $3H^2 + 3\frac{k}{a^2} = 8\pi G_N \rho$

pick $k = 1$, $\rho = \lambda = \text{const}$: $H = 0$ a solution!

$$a = \sqrt{\frac{3}{8\pi G_N \lambda}}$$

$$ds^2 = -dt^2 + \frac{3}{8\pi G_N \lambda} d\Omega_3$$

3-sphere

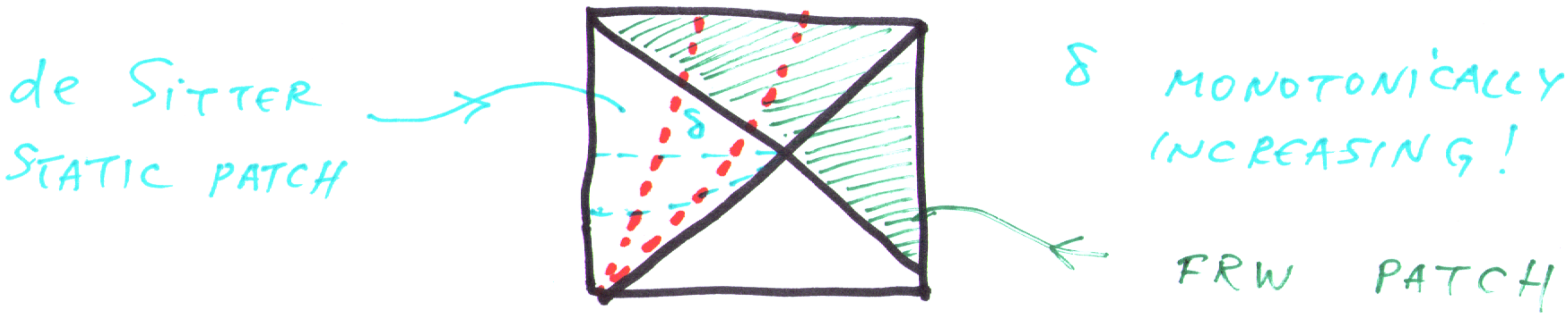
$$M = \lambda V = 2\pi a^3 \lambda \propto \frac{1}{\sqrt{G_N^3 \lambda}}$$

Mass = Geometry (Mach!)

1917, de Sitter : $H = \frac{1}{a_E} = \sqrt{\frac{8\pi G_N \lambda}{3}}$

$$ds^2 = -(1-H^2r^2)\,dt^2 + \frac{dr^2}{1-H^2r^2} + r^2 d\Omega_2$$

Staticity a mirage :



EXPONENTIALLY EXPANDING UNIVERSE IN DISGUISE :

$$ds^2 = -d\tau^2 + e^{2H\tau} d\vec{x}^2$$

1920's : HUBBLE, SLIPHER, FRIEDMANN

UNIVERSE IS EXPANDING!

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$

1923: Einstein: "If there is no quasi-static world, then away with the cosmological term!"

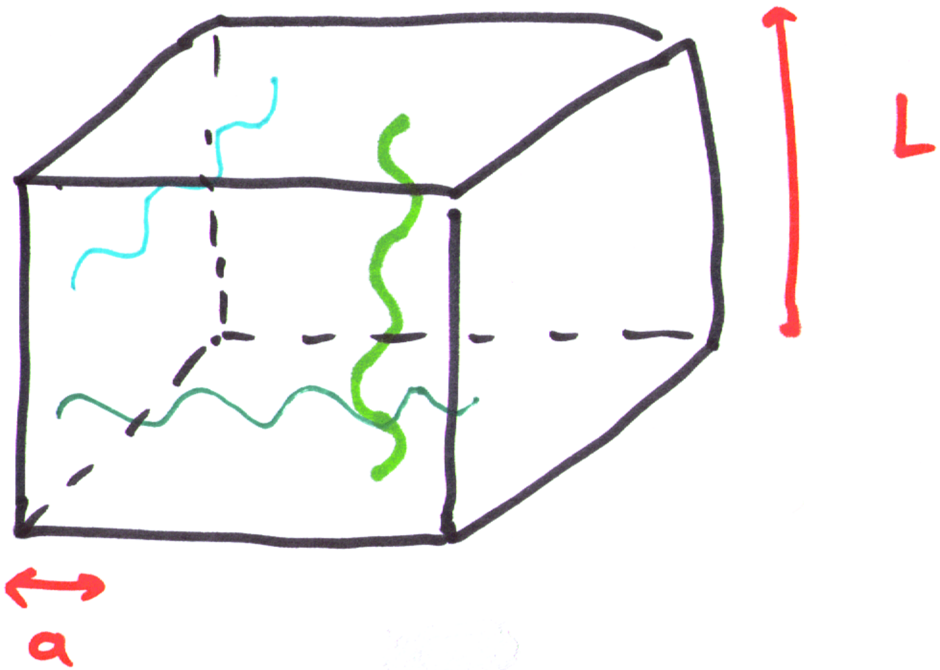
Idea: set $\lambda = 0$ and ignore it henceforth.

Blunder in a blunder!

Problem

We can't ignore λ! Vacuum energy!

oscillators in a box of size L & lattice spacing a ≡ QFT



KK levels: $a_n = n\,a \qquad 1 < n < \frac{L}{a}$

Occupation #: $\frac{1}{2} \leq \nu \leq \frac{L}{a_n}$

Energy levels: $E_n = \frac{\nu_1 + \nu_2 + \nu_2}{a_n}$

VACUUM ENERGY:

$$E_{vac} = \sum_n E_n(vac) \approx \sum_n \frac{1}{a_n}$$

$$\approx \frac{1}{a} \sum_{n_1 n_2 n_3}^{L/a} 1 \sim \frac{L^3}{a^4}$$

Vacuum Energy density: $\lambda = \frac{E_{vac}}{L^3}$

$$\lambda \simeq \frac{1}{a^4}$$

UV cutoff!

So even if we pick a classical theory with $\lambda = 0$ quantum effects induce "misalignment" shifting λ to $\frac{1}{a^4}$ where a, the lattice spacing, is the UV cutoff in configuration space!

SM valid down to $a \sim 1$ FERMI

$$\Rightarrow \quad \lambda_{SM} \sim (\mathrm{TeV})^4 \gg (10^{-3}\,\mathrm{eV})^4$$

WHY IS λ SO SMALL?

More rigorously

$$\lambda_{total} = \lambda_{classical} + \lambda_{vac}$$

at tree level in QFT we can pick $\lambda_{vac} = 0$ by normal ordering. Then we can also choose $\lambda_{classical}$. But spacetime symmetries are incompatible @ normal order at loop levels. We need a symmetry to keep λ_{vac} small. The only thing available: SUSY: $\lambda_{fermion} + \lambda_{boson} = 0$

But: SUSY → ~~SUSY~~ at $\gtrsim$ TeV

$$\Longrightarrow \quad \lambda_{\text{fermion}} + \lambda_{\text{boson}} \gtrsim (\text{TeV})^4$$

Then we need to tune $\lambda_{\text{classical}}$ so that

$$\lambda_{\text{classical}} = -\lambda_{\text{vac}} + (10^{-3}\,\text{eV})^4$$

~60 ORDERS OF MAGNITUDE, LOOP AFTER LOOP → VERY

UNNATURAL!

Note:

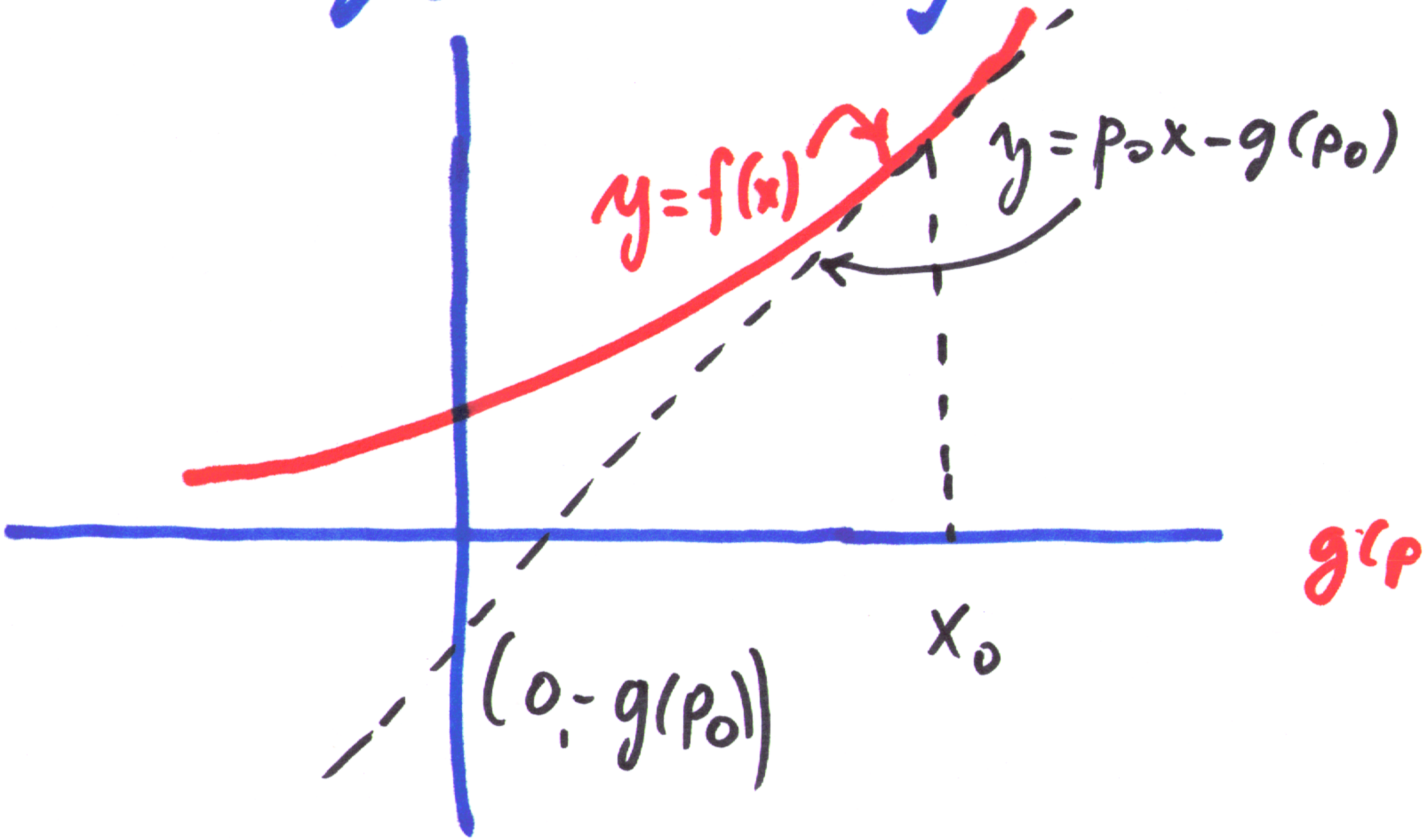
1) We CANNOT calculate $\lambda_{classical}$.
That is a BOUNDARY CONDITION

2) We CANNOT keep λ_{vac} small.
That is a RADIATIVE INSTABILITY

3) We CANNOT link $\lambda_{classical}$ and λ_{vac}, except by ANTHROPIC PRINCIPLE
This we do not know what it is...

Modified gravity v.s. Λ

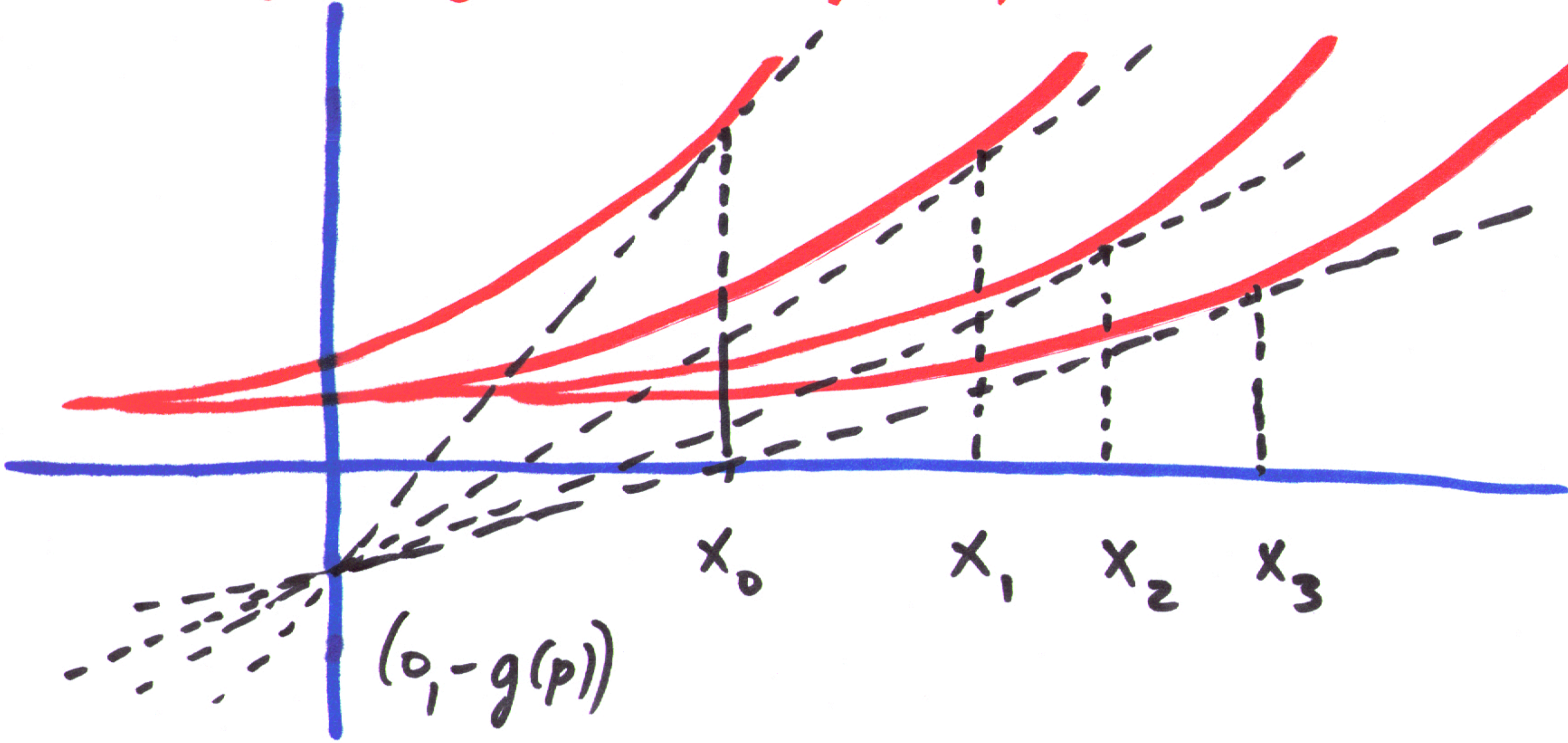
$\int \sqrt{g}\,\Lambda$: a Legendre transf.



$$p = \frac{\partial y}{\partial x}$$

$$g(p) = xp - y$$

Now: forget $f(x)$! Can reconstruct it by solving $g(y') = xy' - y$?



Solution not unique if we don't know x_k!

In GR: $x = \sqrt{\det g}$ a nonpropagating pure gauge DOF: can be ANYTHING!

We need a boundary condition!

GR: a Landscape! Einstein already "blundered" in and out of it (1919)

Unimodular gravity:

$$R^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu} R = 8\pi G_N \left(T^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu} T\right)$$

But: $\nabla_\mu T^{\mu}{}_{\nu} = 0 \rightarrow \partial_\mu (R + 8\pi G_N T) = 0 \rightarrow R + 8\pi G_N (T + 4\Lambda) = 0$

$$\therefore\ R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu} R = 8\pi G_N \left(T^{\mu}{}_{\nu} + \Lambda\, \delta^{\mu}{}_{\nu}\right)$$

$$\Lambda_{tot} = \langle T^{0}{}_{0} \rangle + \Lambda$$

Weinberg no-go

Assume: 4D gravity, finitely many fields, Poincaré symmetry

field eqs $$\frac{\delta \mathcal{L}}{\delta \Phi_m} = 0$$

gravity $$\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = 0$$

Field eqs are trivial; diffeomorphisms then set $\mathcal{L} = \sqrt{g_4}\,\lambda$ and gravity eq demands $\lambda = 0$

That is a fine tuning!

What if gravity equations are not <u>all independent</u>?

$$2 g_{\mu\nu} \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = \sum_m f_m(\Phi) \frac{\delta \mathcal{L}}{\delta \Phi_m}$$

If we had such a theory, then

$\dfrac{\delta \mathcal{L}}{\delta \Phi_m} = 0$ would IMPLY $\lambda = 0$!

BUT: this eq implies a symmetry:

$$\delta g_{\mu\nu} = 2 \epsilon g_{\mu\nu} \qquad \delta \Phi_m = -\epsilon f_m$$

If field space is smooth we can field redefine $\Phi_m \to \tilde{\Phi}_m = \tilde{\Phi}_m(\Phi_m)$ s.t.

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu}\,, \quad \delta\tilde{\Phi}_0 = -\epsilon\,, \quad \delta\tilde{\Phi}_m = 0 \quad m>0$$

Diff eq can be solved to yield

$$\mathcal{L} = \sqrt{g_4}\,\lambda\, e^{4\tilde{\Phi}_0}$$

So either we set $\lambda = 0 \to$ FINE TUNING or $\tilde{\Phi}_0 \to -\infty$. BUT: radiative stability requires $\mu_{eff} = \mu e^{\tilde{\Phi}_0}$, so all $\mu_{eff} \to 0 \to$ NO-SCALE !

THIS IS NOT OUR WORLD!

Possible caveats:

1) Field space not smooth $\to \exists$ special places! (fine tuning, anthropics)

2) ~~Poincare~~ $\to$ de Sitter (holographic ideas? to date no working model)

3) No 4D EFT $\to \infty$ many fields (misaligned SUSY $\to$ not clear it works)

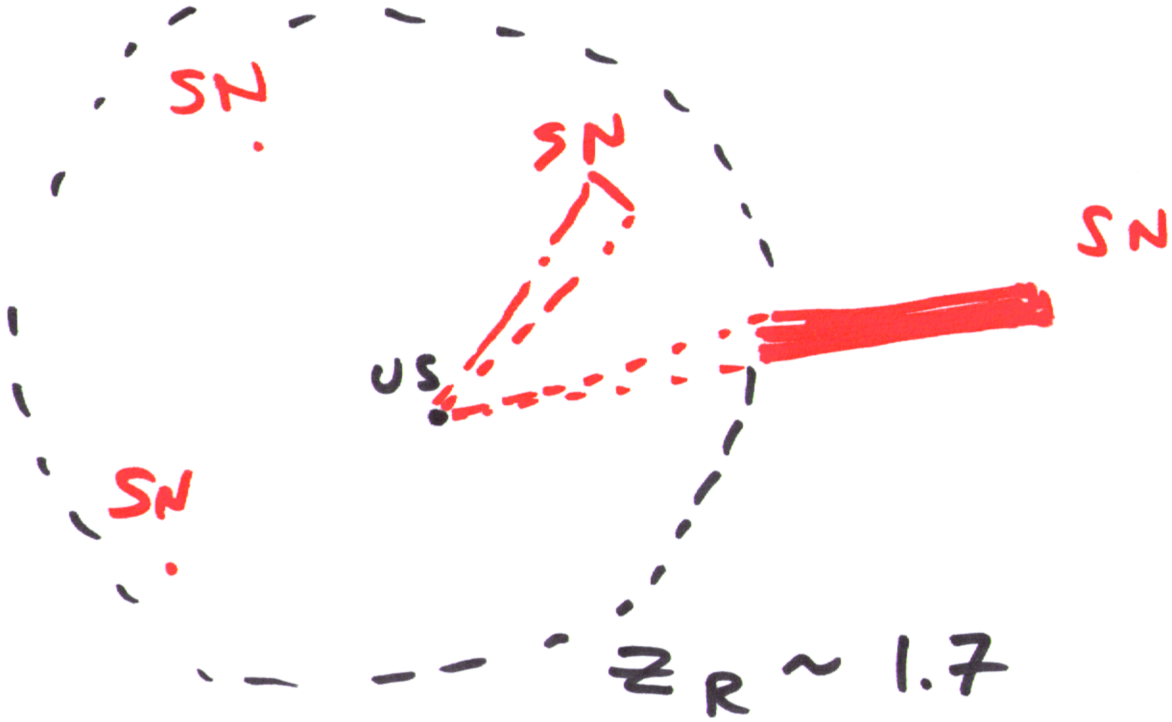
4) No 4D gravity (DGP, f(R): problems @ new degrees of freedom, strong coupling)

So: what are we seeing?

Given that we don't really have a compelling solution, it may be a good idea to stay open-minded... Maybe we get lucky...

Maybe not Geometric?

Our universe is OPAQUE? Dust: AGUIRRE

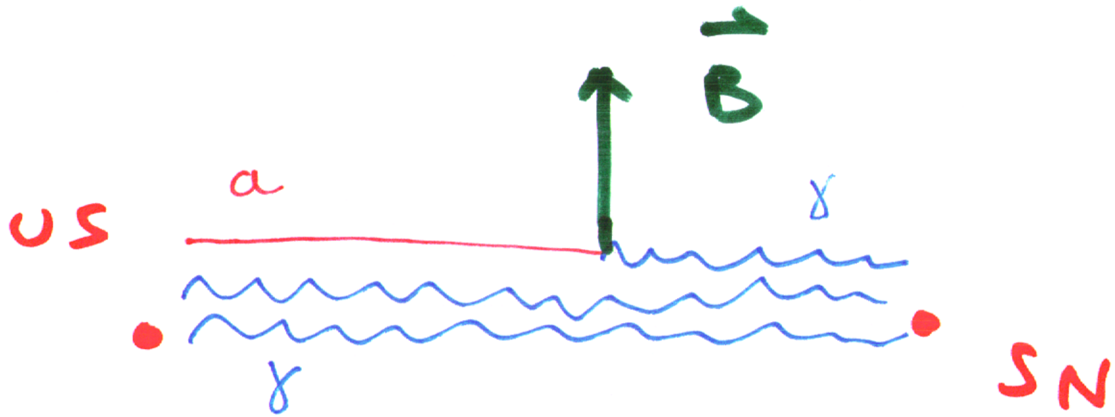


BUT:

1) WHY IS $z_R \sim 1.7$?
2) DUST IN EQUILIBRIUM - IT WOULD GLOW — BOUNDS EXCLUDE IT!

Axion - Photon Oscillations & Dimming

Csaki, Terning, NK



1) NO EQUILIBRIUM — NO GLOWING !
2) TRANSITION NATURALLY CONTROLLED BY MICROPHYSICS AND BY RANDOM $\vec{B}$ FIELDS

CANNOT ACCOUNT FOR ALL OF DIMMING BUT IT <u>CAN</u> CONTRIBUTE SIGNIFICANTLY TO w, EVEN MAKING IT LOOK $w < -1$!

Maybe not λ ?

Quintessence:

- Scalar fields ϕ @ $V(\phi)$ and $\langle V \rangle \sim (10^{-3}\,eV)^4$ now.
- FINE TUNED: $\langle V \rangle_{now}$ & $\left\langle \frac{\partial^2 V}{\partial \phi^2} \right\rangle_{now}$
- AN EXCEPTION: AXIONIC QUINTESSENCE @

$$V(\phi) = \mu^4 \left(1 + \cos\left(\frac{\phi}{M_P}\right)\right)$$

Frieman, Hill, Stebbins, Waga
Nomura, Watari, Yanagida

FOR AXIONIC QUINTESSENCE, PERTURBATIVE COUPLINGS ARE DERIVATIVE, SO POTENTIAL IS NONPERTURBATIVE AND HENCE STABLE UNDER RADIATIVE CORRECTIONS

BUT: NOT CLEAR THAT $f_\phi > M_P$ IS NATURAL, YET THIS BORDERLINE CASE $f_\phi \sim M_P$ MAY BE OK

axion (or more generally pNGB) models are useful benchmarks of $w \neq -1$

Chameleonic Q ?

Khoury, Weltman

Generically : BADLY fine tuned !

$$V(\phi) = \mu^4 f\left(\frac{\phi}{M}\right) \qquad \phi \ll M \ll M_4$$

$$\simeq \mu^4 + \frac{\mu^4}{M^2}\phi^2$$

$$m^2 = \frac{\mu^4}{M^2} >> H_0^2$$

AT HORIZON SCALE ϕ DECOUPLED
→ THIS IS THE SAME AS λ !

ONE EXCEPTION: Logarithmic V (NK)

(Next lecture)

WHAT IS INTERESTING IS POSSIBLE LABORATORY SIGNATURES AS DEVIATIONS FROM NEWTON'S LAW

Maybe not 4D GR?

$f(R)$: eg: $f(R) = M_4^2 \left(R - \frac{H^4}{R}\right)$

Capozziello et al, Carroll et al, Nojiri et al

BUT: ALL SUCH MODELS ARE SCALAR TENSOR THEORIES!

Whitt; Teysandier & Tourenc; Maeda; Chiba; Flanagan; Iglesias, Padilla, Park & NK.

$$\int d^4x \sqrt{g}\, f(R) = \int d^4x \sqrt{g} \left(\Phi R - V(\Phi)\right)$$

$$\Phi = \partial_R f(R)$$

$$V(\Phi) = \left(R\,\partial_R f(R) - f(R)\right)\Big|_{R=R(\Phi)}$$

Legendre transform!

Thus all these Models are really quintessence theories, possibly with direct matter couplings!

Confusions about Palatini reformulation are COMPLETELY resolved: arXiv:0708.1163

$$S_{\text{Palatini}} = \int d^4x \sqrt{g} \left(\Phi R_p - \frac{w}{\phi} (\nabla\Phi)^2 - V(\Phi) - \mathcal{L}_m \right)$$

$$+ \int d^4x \sqrt{g}\, \Phi\, \lambda^{\mu}{}_{\nu\rho} \nabla_\mu g^{\nu\rho}$$

Lagrange multipliers

IDENTICAL TO THE STANDARD METRIC FORMULATION!

Modified Source Gravity? NO!

Carroll et al Iglesias et al

$$\int d^4x \sqrt{g}\left(\frac{M_p^2}{2}R - \frac{Z(\omega)}{2}(\nabla\chi)^2 - V(\chi) - \mathcal{L}_{matter}\right)$$

$$\therefore \quad Z(\omega)\,\nabla^2\chi = \partial_\chi V + \frac{T}{2M_p}$$

$$Z \to 0 \quad \Rightarrow \quad \partial_\chi V + \frac{T}{2M_p} \to 0$$

a safe constraint, so we can have alternative cosmological evolution?

arXiv: 0708.1163

No: $g\chi^n \rightarrow \hat{g} = \frac{g}{Z^{n/2}}$

$$g\chi T^\mu{}_\mu \rightarrow g = \frac{1}{\sqrt{Z}\, M_P}$$

STRONG COUPLING PROBLEM!

It comes back in the matter sector even if we pretend we can ignore it in the χ sector.

example: $\mathcal{L} = \frac{1}{2}(\partial \mathcal{H})^2 + \frac{m^2}{2}\mathcal{H}^2$ & $\partial_x V + \frac{T}{2M_p} = 0$

$$\therefore \mathcal{L}_{eff} = \frac{1}{2} e^{-\frac{\chi_*}{M_p}} (\tilde{\nabla}\mathcal{H})^2 \left(O(1) + O(1)\, e^{-\chi_*/M_p} \frac{(\tilde{\nabla}\mathcal{H})^2}{\Lambda^4} + \dots \right)$$

where $\Lambda^4 = 3\Omega_\Lambda M_p^2 H_0^2$ is scale of dark energy

So: at $k_S \sim \Lambda$ derivative corrections DOMINATE over the standard vacuum terms:

$$(\tilde{\nabla}\mathcal{H})^2 < \Lambda^4$$

implies the SM in this theory breaks down at 10^{-3} eV $\rightarrow$ NOT what we see in Nature!

arxiv: 0708.1163

DGP: B(rane) I(nduced) G(ravity)

Dvali, Gabadadze, Porrati

$$S_{eff} = \int d^4x \sqrt{g_4}\left(\frac{M_4^2}{2}R - \mathcal{L}_{matter}\right) + \int d^5x \sqrt{g_5}\, M_5^3 R_5 + G.H.$$

Cod 1: 1) Does NOT solve C.C.

2) Has so-called self accelerating solutions

3) BUT THESE HAVE GHOSTS!

Charmousis, Gregory, NK, Padilla
Koyama; Tanaka et al
Gregory, NK, Myers, Padilla

Cod 2: refreshingly different   Kiley, NK.

1) No Ghosts!

2) Does not solve CC yet, but...

3) Problem is different: vacuum energy does not affect 4D curvature but the transition scale from 4D $\to$ 5D $\to$ 6D!

4) Vacuum energy needs to be BIG, not small...

5) Could we find theories @ IR fixed points which protect perturbativity ??...

(Last lecture)

Summary

1) We do not know what DE is
2) It is OK to be open-minded and explore various possibilities
3) Beware of MONSTERS: light modes, tachyons, ghosts, strong coupling,...
4) Make sure your theory does something NEW, to be different from λ, and that it is not EXCLUDED!
5) Be cautious with your philosophy...