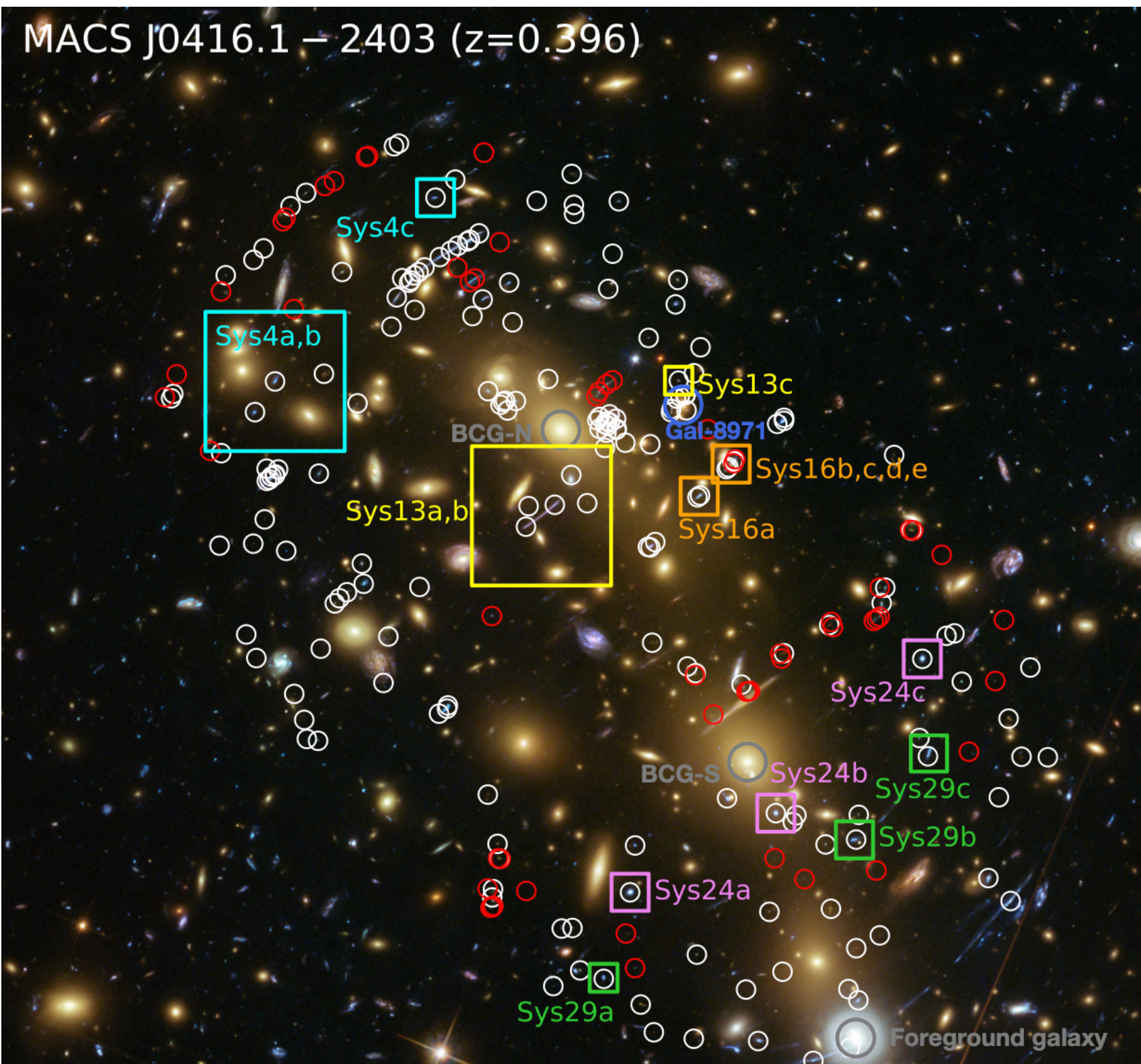


The next step in galaxy cluster strong lensing: modelling the surface-brightness distribution of multiply-imaged sources

Ana Acebron

with Claudio Grillo, Sherry Suyu, Giuseppe Angora, Pietro Bergamini, Gabriel Caminha, Sebastian Ertl, Amata Mercurio, Mario Nonino, Piero Rosati, Han Wang, Andrea Bolamperti, Massimo Meneghetti, Paolo Tozzi, Stefan Schuldt, Eros Vanzella

A new generation of *pointlike* SL models



Bergamini+2023

Large samples of multiple images

- HST, JWST, VLT
- Large redshift range
- Resolved knots within extended sources

Careful selection of cluster member galaxies

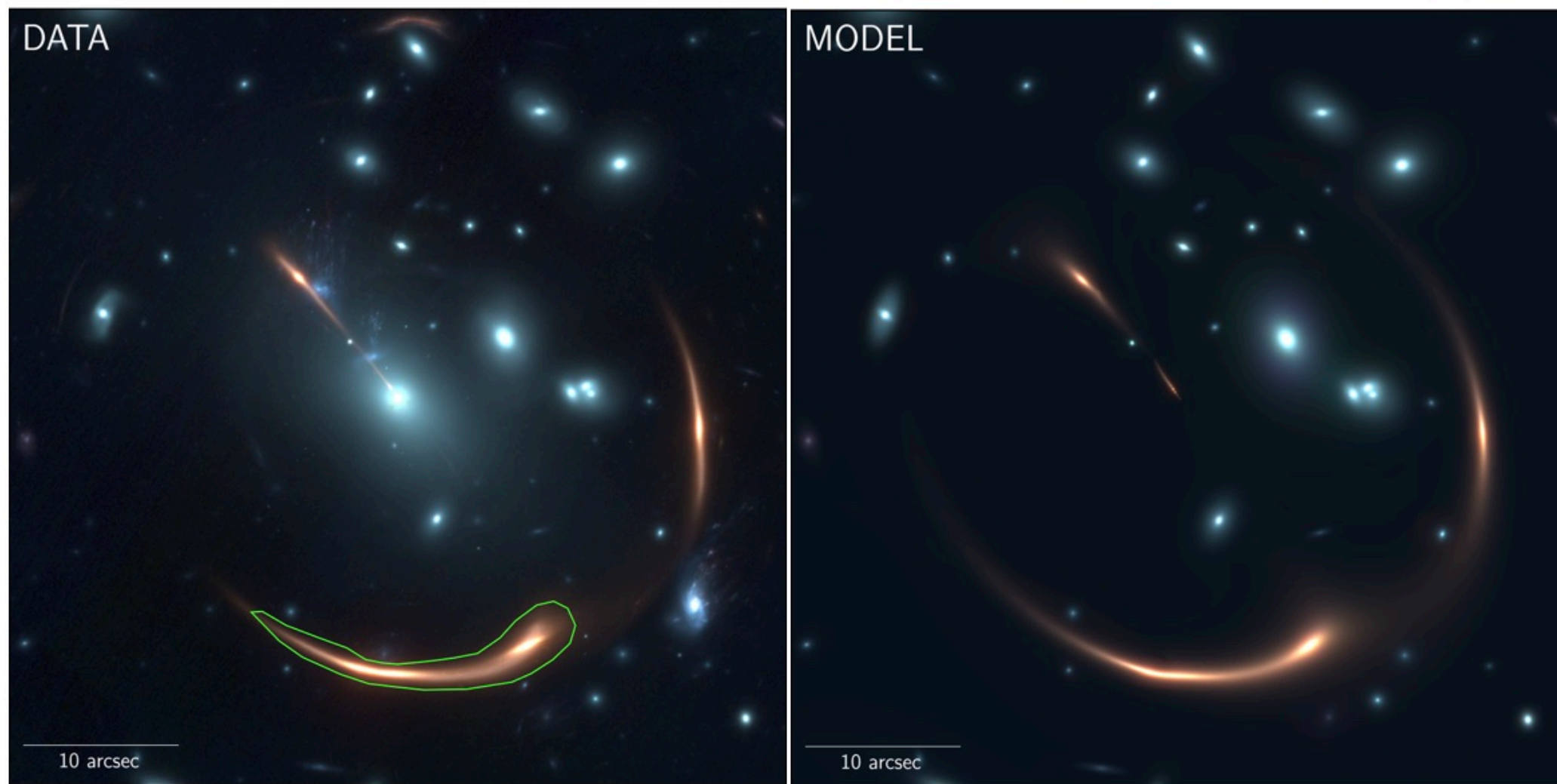
- Large z_{spec} samples
(e.g. *Grillo+2016; Richard+2021; Furtak+2023; Cha+2024; Rihtaršič+2025*)
- Sophisticated CNNs
(*Angora+2020*)

Accurate total mass models

- state-of-the-art pipelines
- ICM directly modelled
(*Bonamigo+2017, 2018*)
- calibrated sub-halo component with measured $\sigma_{\star}$
(e.g. *Bergamini+2019, Beauchesne+2021*)

A new generation of *pointlike* SL models

Going beyond the rms to quantify the goodness of cluster SL models



Acebron, Bergamini, Rosati+2025

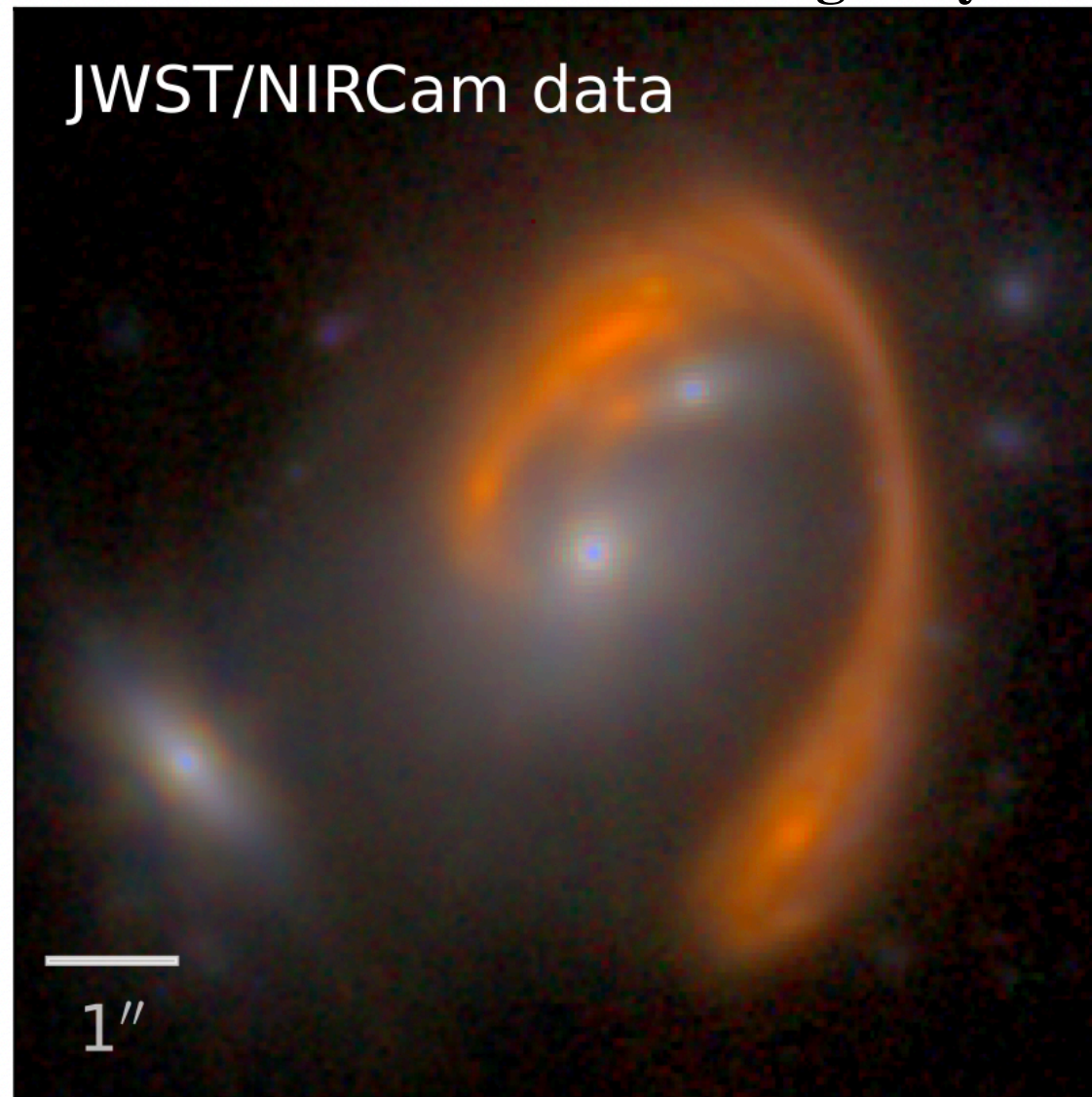
see also *Grillo+2016; Bergamini+2023*



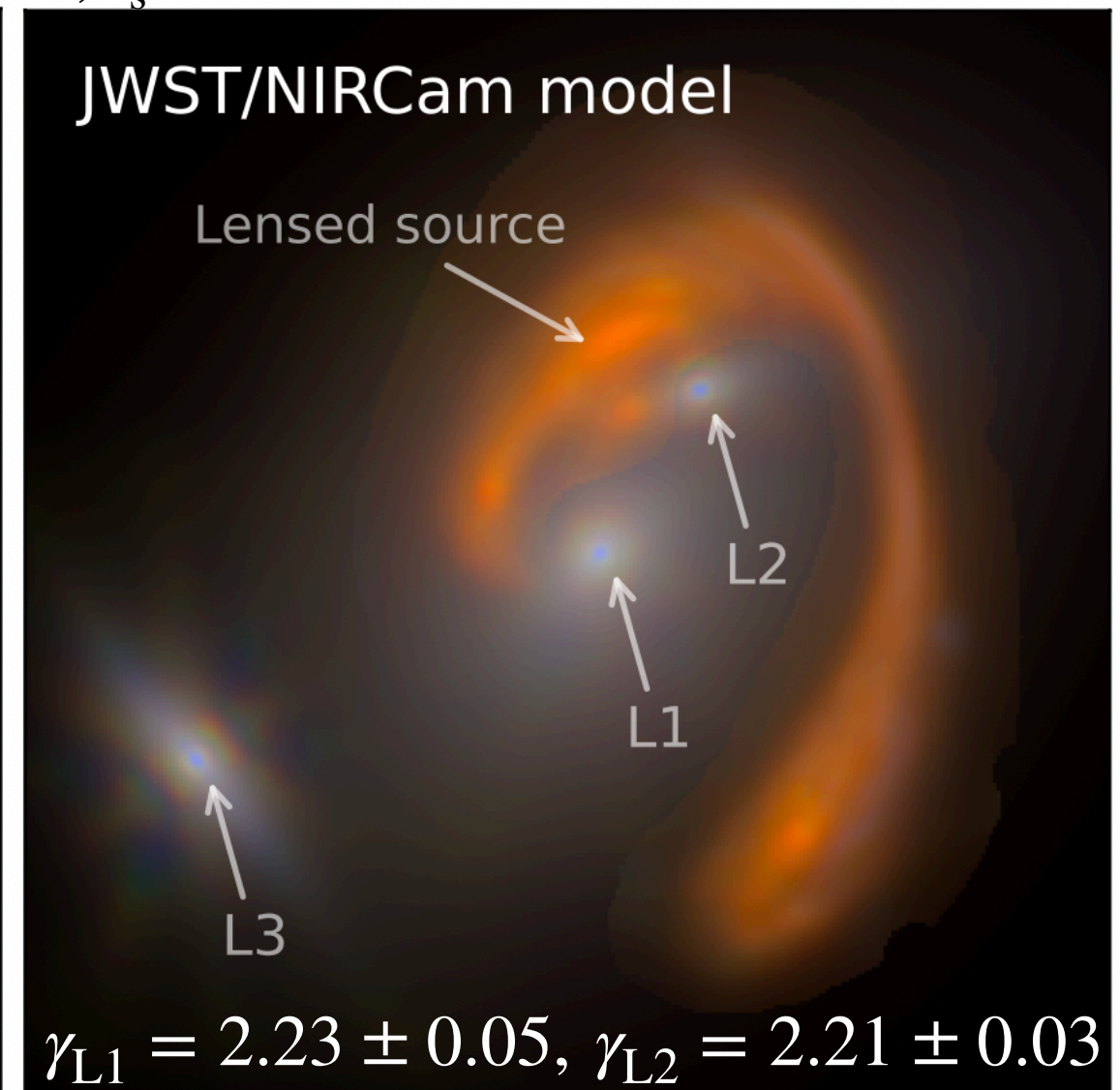
Our next step: directly include the observed surface-brightness distribution of strongly-lensed sources into our cluster lens models!

Extended SL modelling in lens galaxy clusters

‘El Anzuelo’ in El Gordo lens galaxy cluster, $z_s = 2.29$

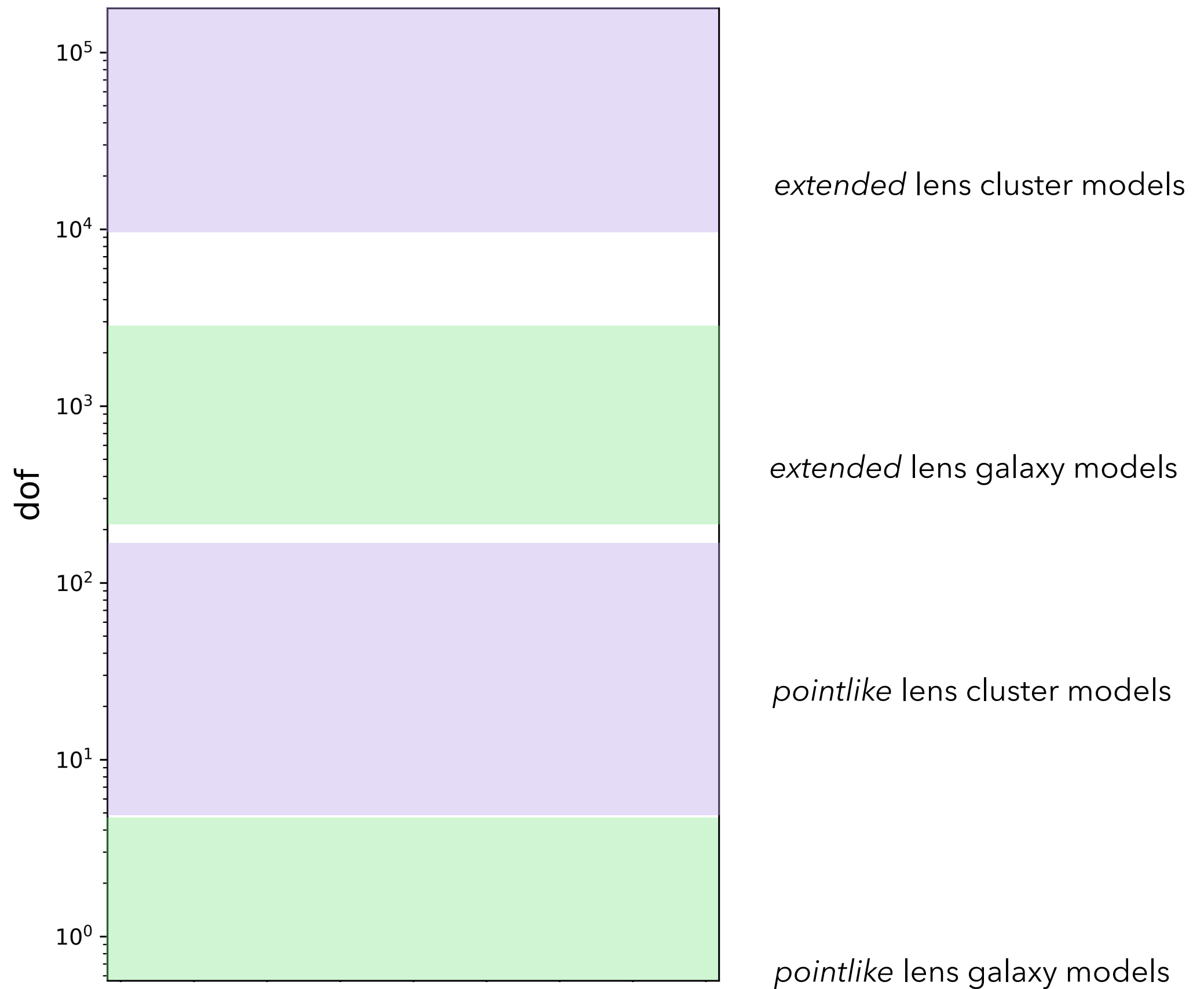


Galan+2024



However, the galaxy cluster total mass distribution is **fixed** or **approximated**

Many, many, many observables!



SDSS J1029+2623: an ideal test-bed

HST color-composite image

★ **Multi-band HST
imaging**

GO-12195 - P.I. Oguri

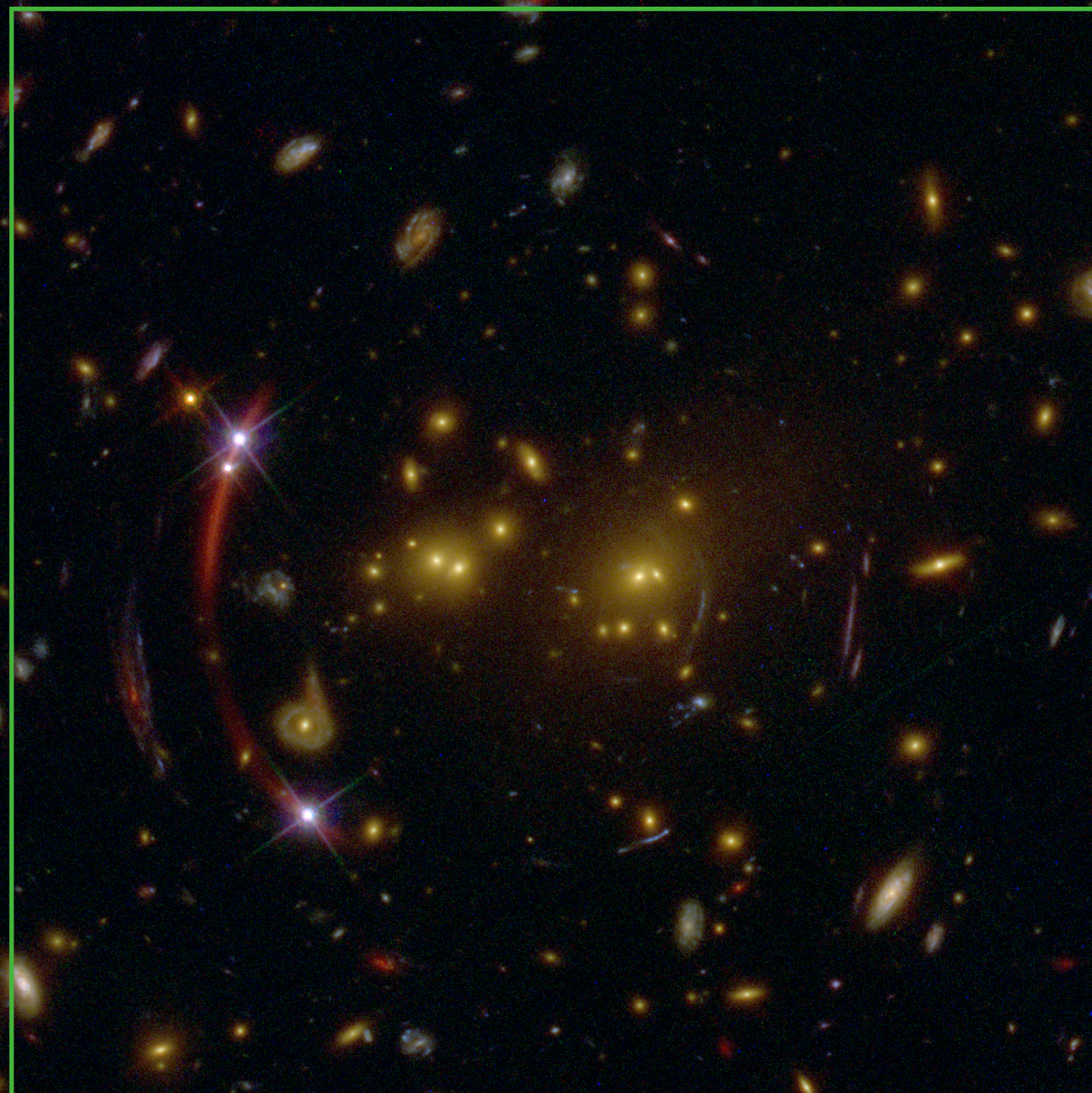
ACS/ F475W & F814W

WFC3/ F160W

★ **4.4h VLT/MUSE
exposure**

P.I. Grillo

$\sim 1 \text{ arcmin}^2$

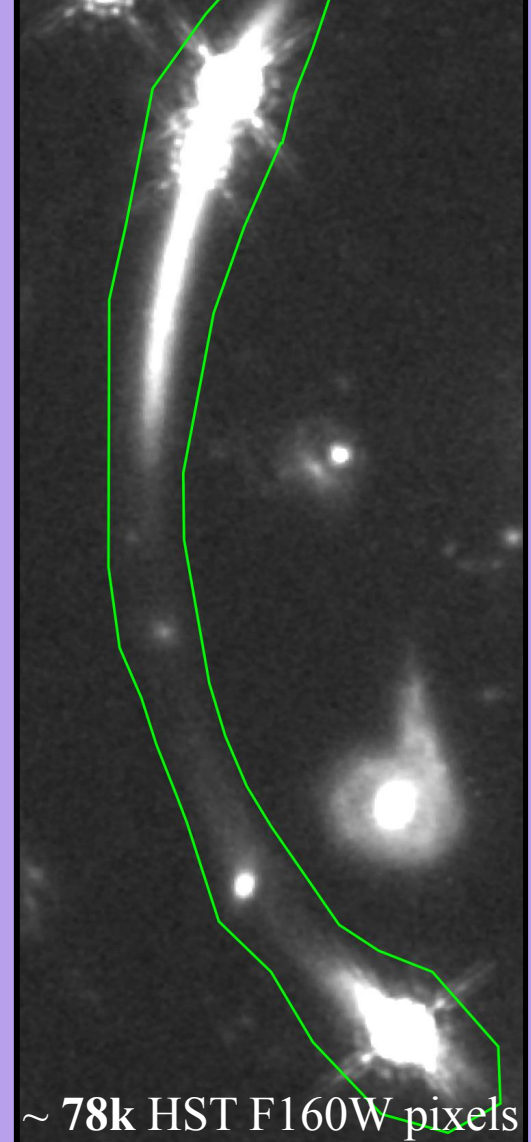


Acebron+2022a

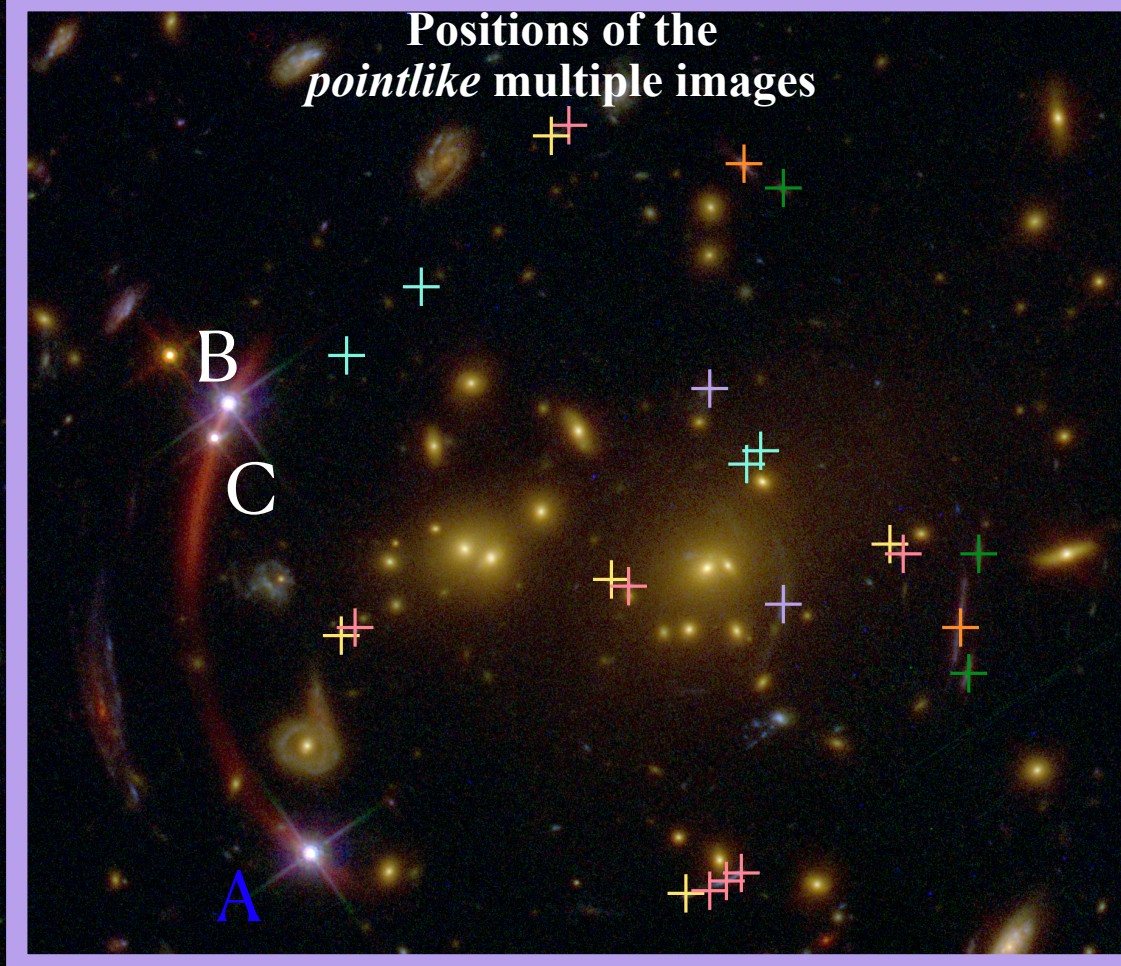


Secure identification of cluster members & multiple images

Surface brightness of the
host galaxy



Positions of the
pointlike multiple images



Acebron+2022a

Lenstool
Jullo+2007

GLEE
Suyu & Halkola, 2010; Suyu+2012

- multi-plane formalism
- extended SB modeling

Total mass distribution in a multi-plane formalism

GLEE *Suyu & Halkola, 2010; Suyu+2012*

Dark matter haloes

dPIE *Elíasdóttir+2007*

$$\rho(r) \propto r^{-2}$$

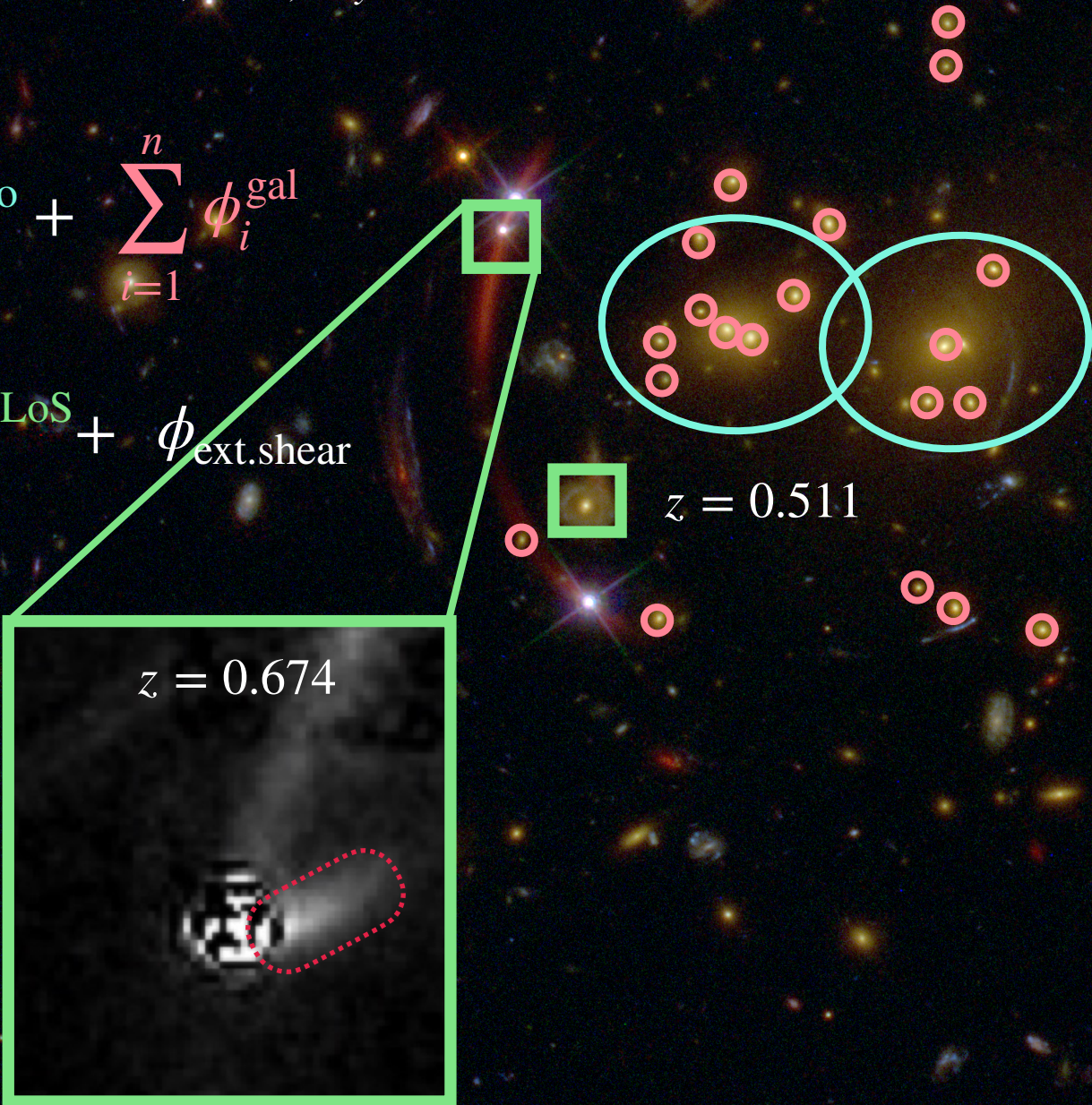
Cluster members

$$\theta_E = \theta_E^{\text{ref}} \left(\frac{L}{L_{\text{ref}}} \right)^\alpha$$

with $M_i^{\text{tot}}/L_i \propto L^{0.2}$

$$r_{\text{cut}} = r_{\text{cut}}^{\text{ref}} \left(\frac{L}{L_{\text{ref}}} \right)^\beta$$

$$\begin{aligned} \phi_{\text{tot}} = & \sum_{i=1}^N \phi_i^{\text{halo}} + \sum_{i=1}^n \phi_i^{\text{gal}} \\ & + \sum_{i=1}^{i=2} \phi_i^{\text{gal,LoS}} + \phi_{\text{ext.shear}} \end{aligned}$$

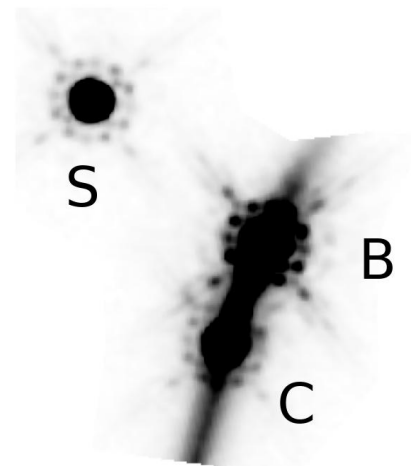


SDSS J1029+2623: methodology

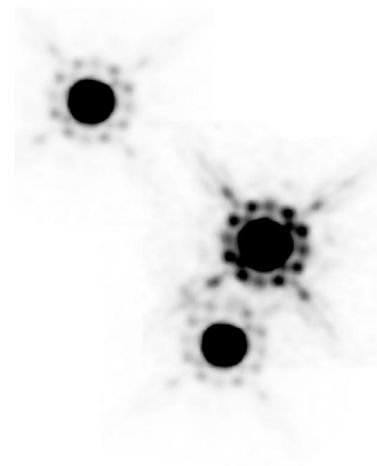
$$\chi^2_{\text{tot}} = \chi^2_{\text{img}} + \chi^2_{\text{ext}}$$

1. Light of QSOs images and other perturbers

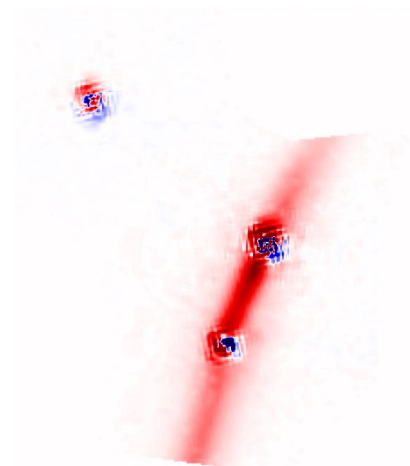
F160W



Model

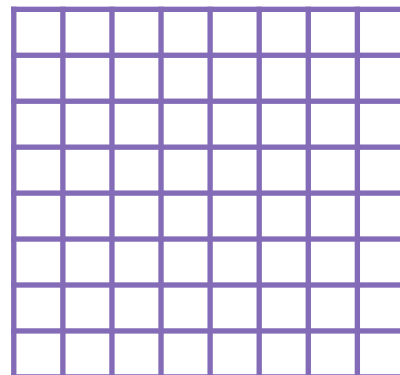


Normalised residuals



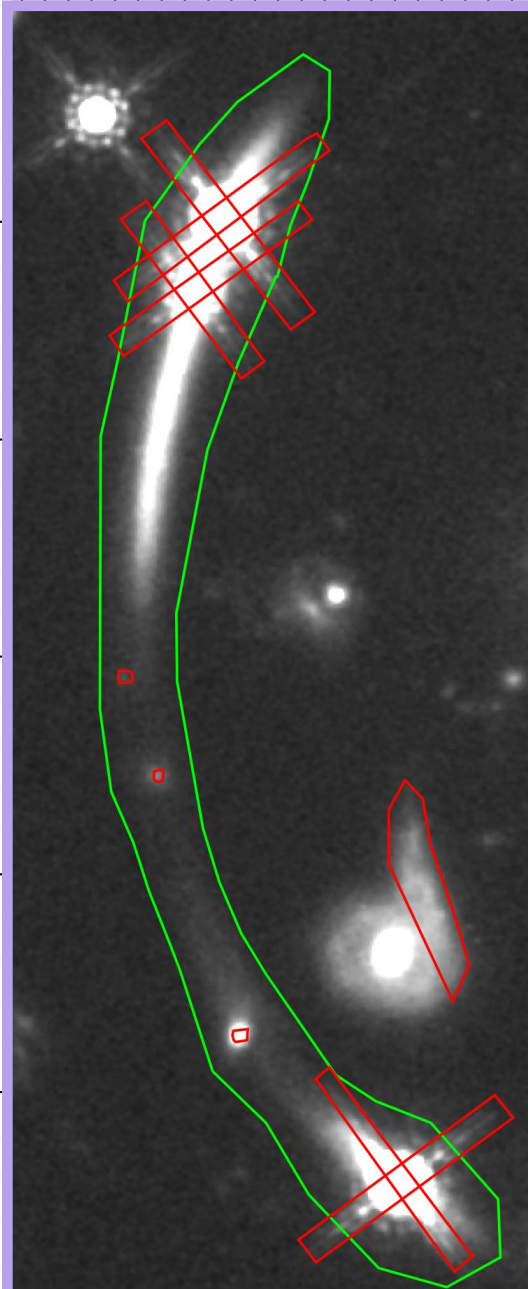
2. Extended surface brightness distribution of extended QSO host

$$I(\vec{\theta}) = I(\vec{\beta})$$

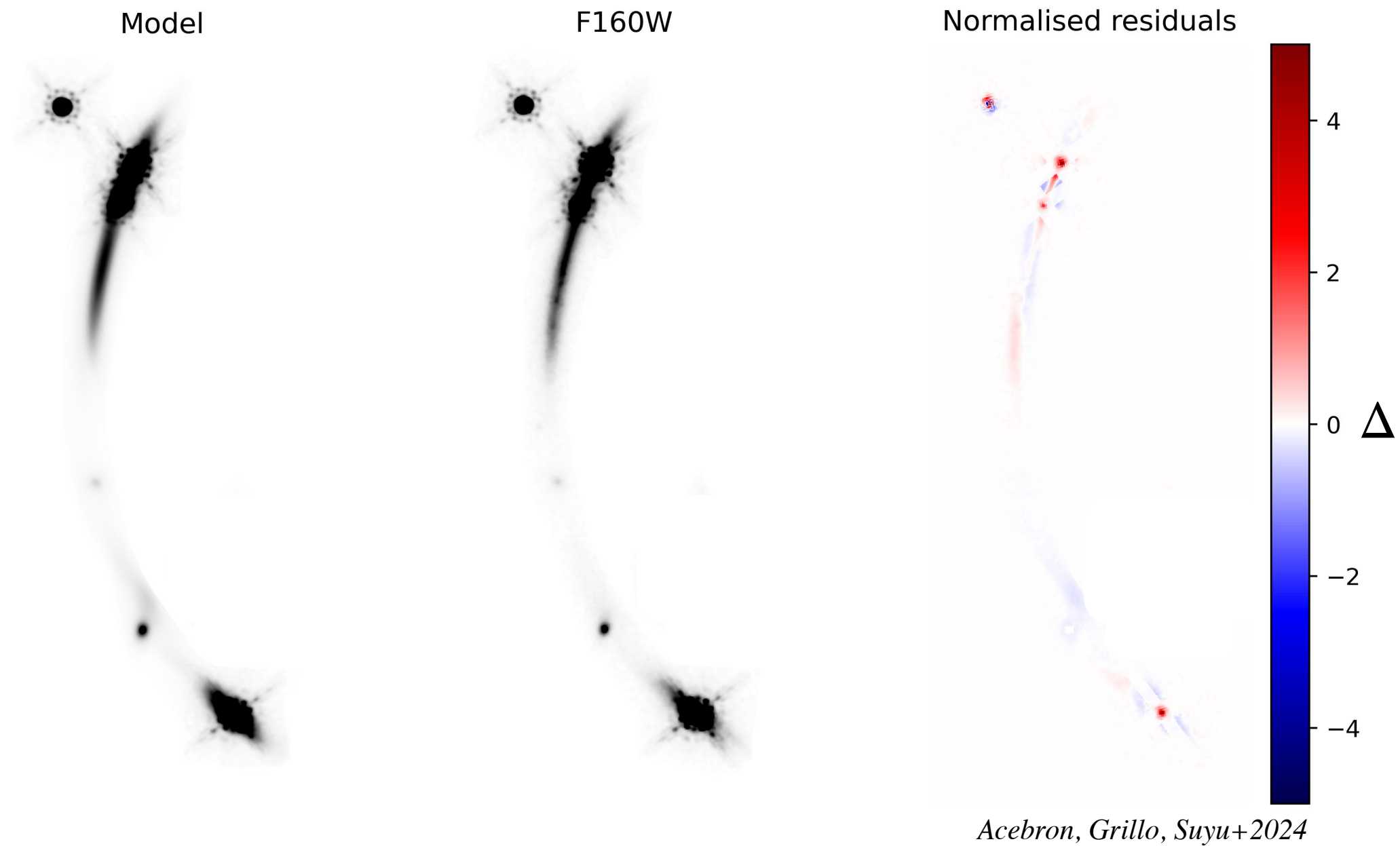


40 × 40 pixels

$$\chi^2_{\text{ext}} = (\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{pred}})^T \mathbf{C}_D^{-1} (\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{pred}})$$



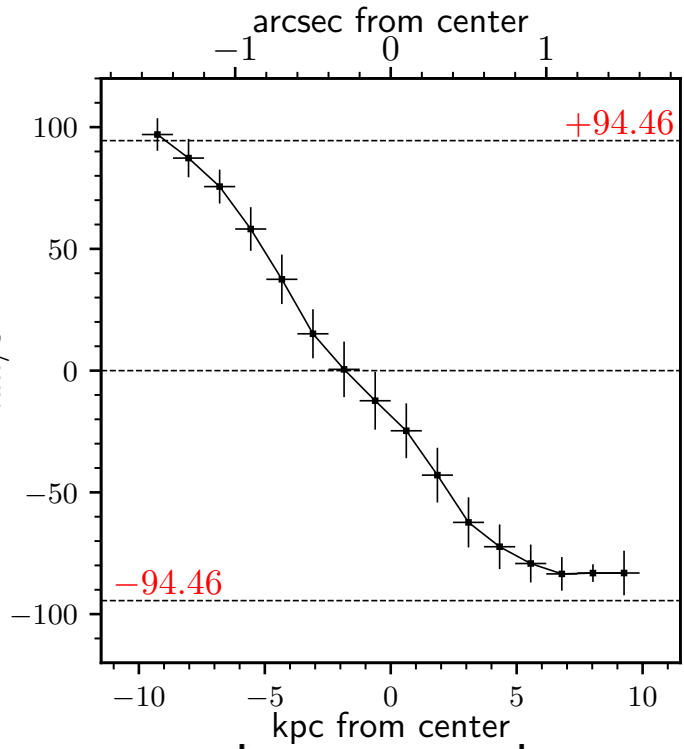
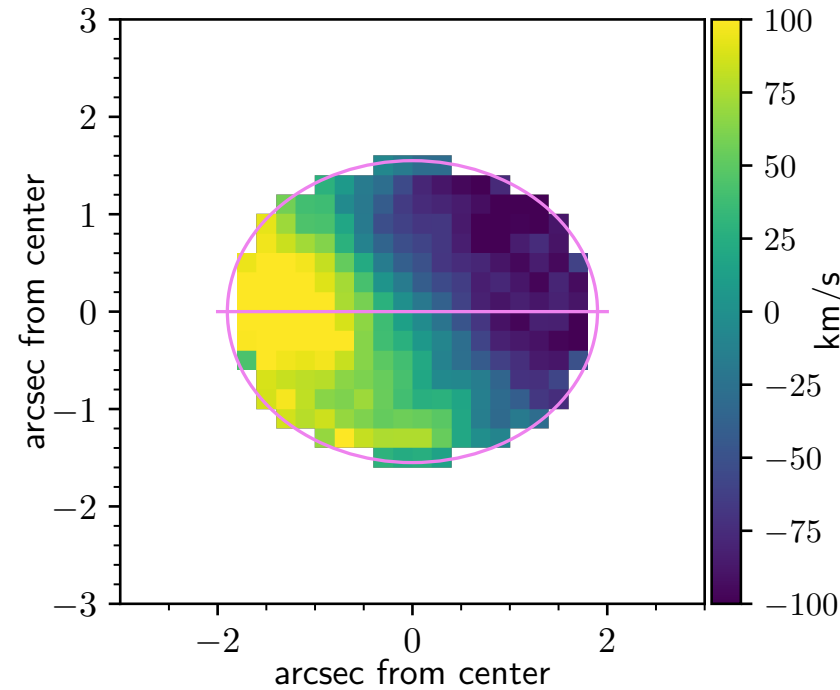
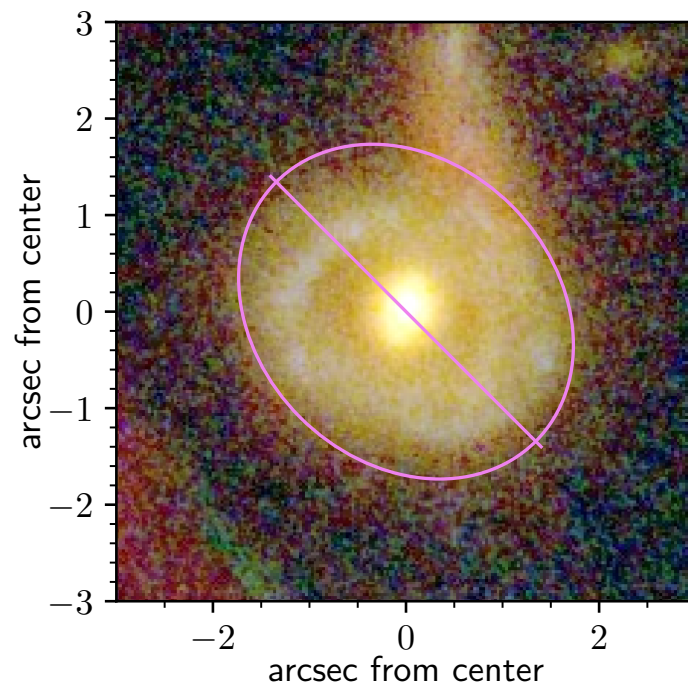
SDSS J1029+2623: extended surface brightness modelling



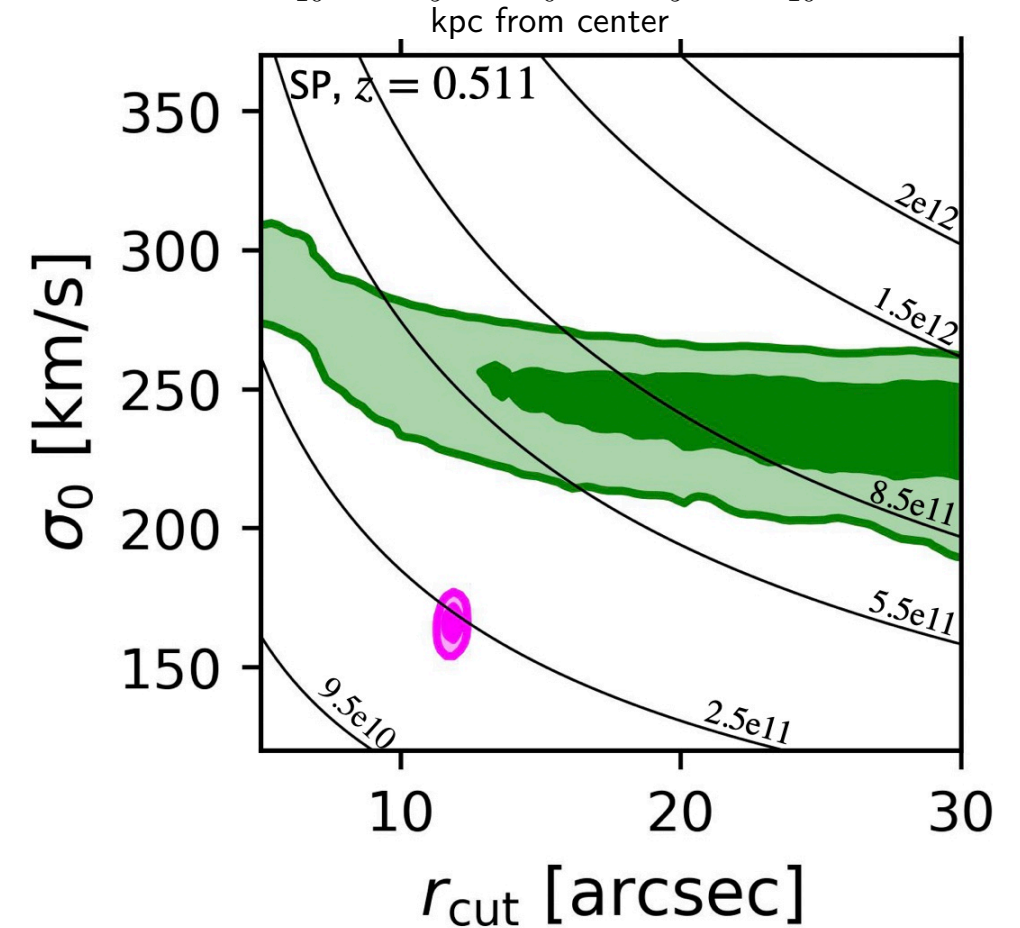
The host galaxy of the QSO is well reproduced, over **78k** HST pixels

SDSS J1029+2623

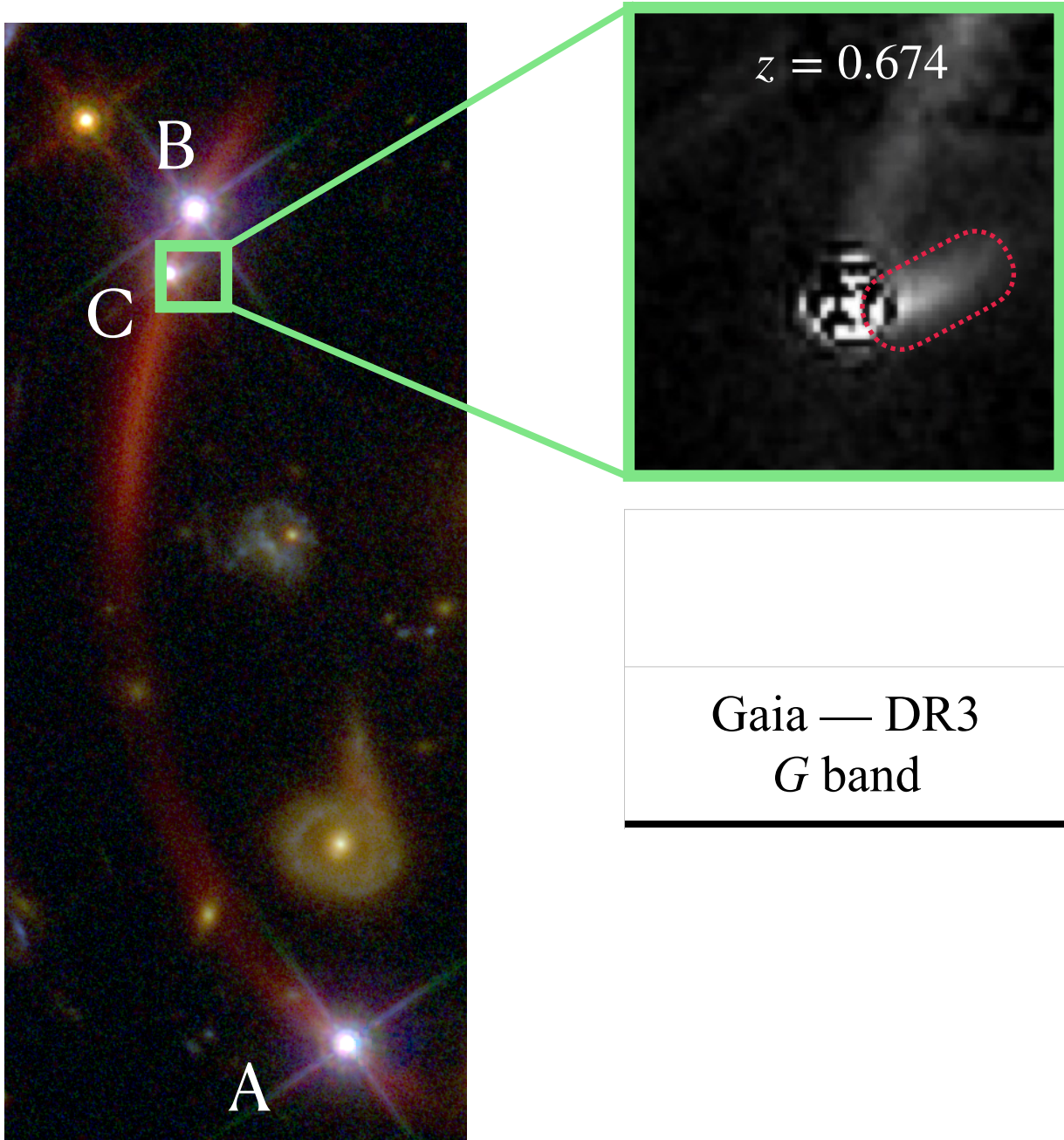
Characterisation of subhaloes along the LoS



<i>Pointlike</i> Lenstool	$(2.4^{+0.4}_{-0.4}) \times 10^{11} M_{\odot}$
Extended GLEE	$(1.1^{+0.1}_{-0.1}) \times 10^{11} M_{\odot}$



Characterisation of subhaloes along the LoS



We expect: $\mu_B \sim \mu_C < \mu_A$

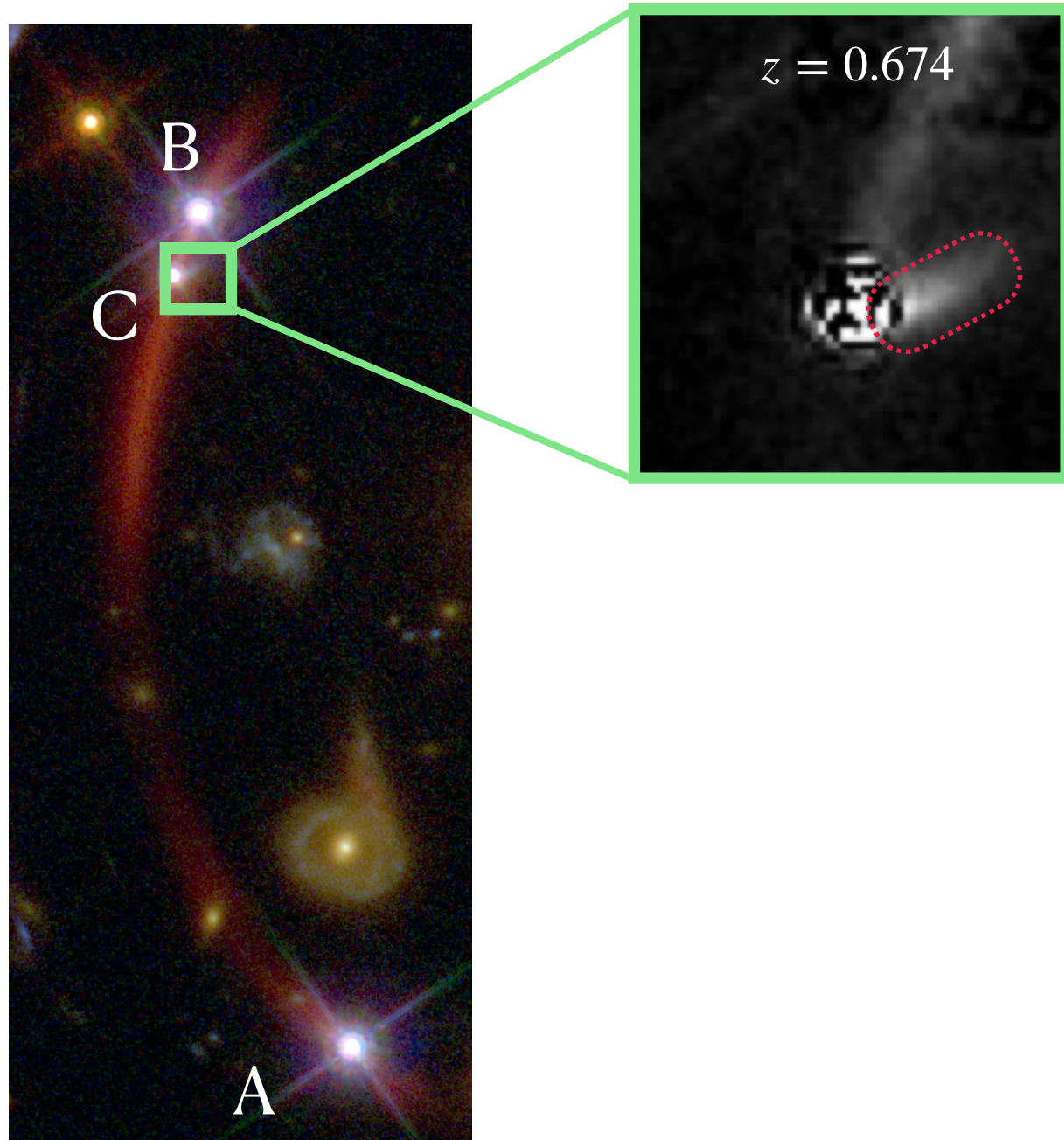
We observe: $\mu_A \sim \mu_B < \mu_C$

see also *Oguri+2013*

	$\Delta\text{mag}(\text{AB})$	$\Delta\text{mag}(\text{AC})$	$\Delta\text{mag}(\text{BC})$
Gaia — DR3 <i>G</i> band	0.36 ± 0.02	-1.60 ± 0.05	-1.96 ± 0.05

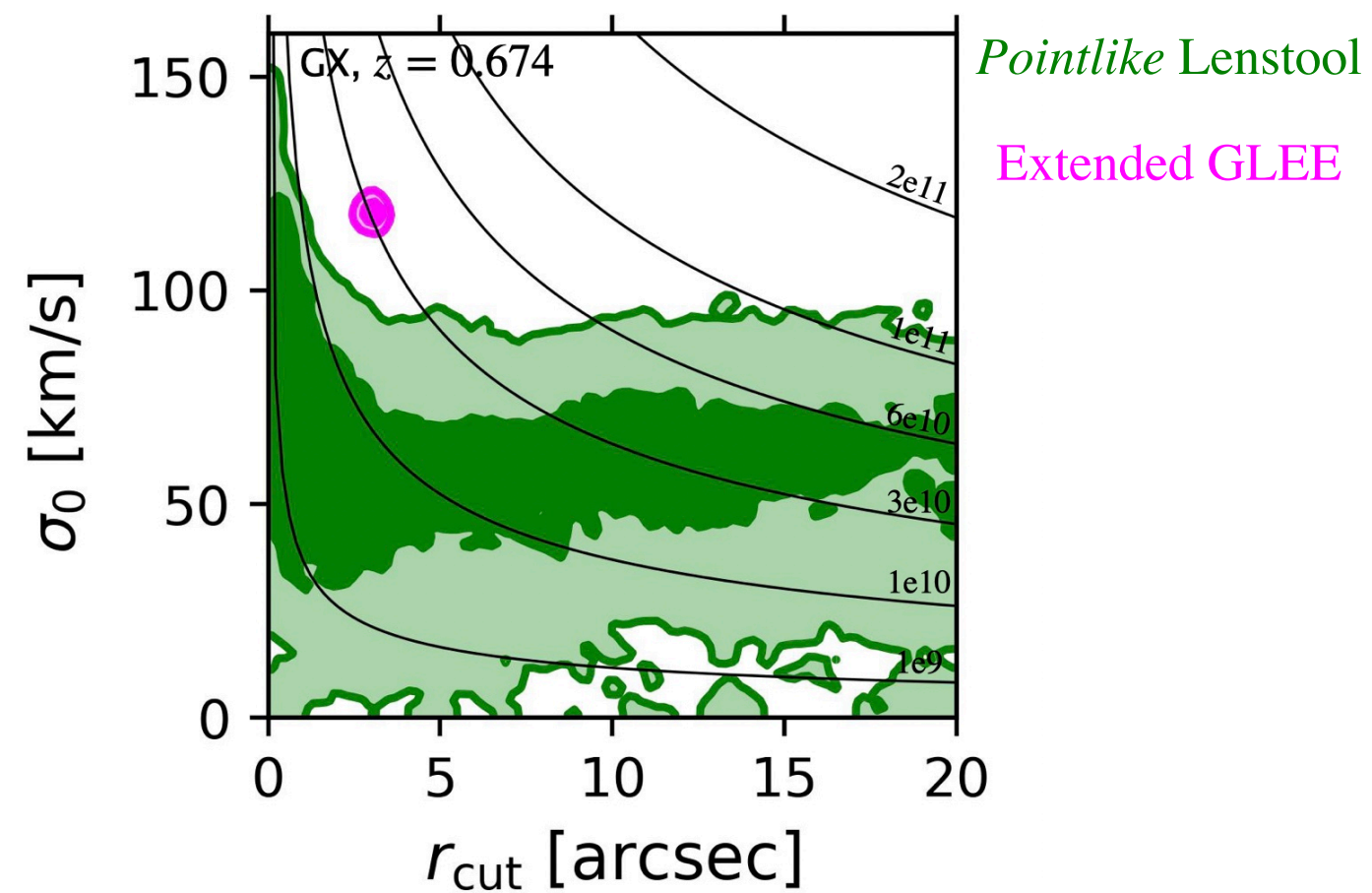
SDSS J1029+2623

Characterisation of subhaloes along the LoS



We expect: $\mu_B \sim \mu_C < \mu_A$

We observe: $\mu_A \sim \mu_B < \mu_C$

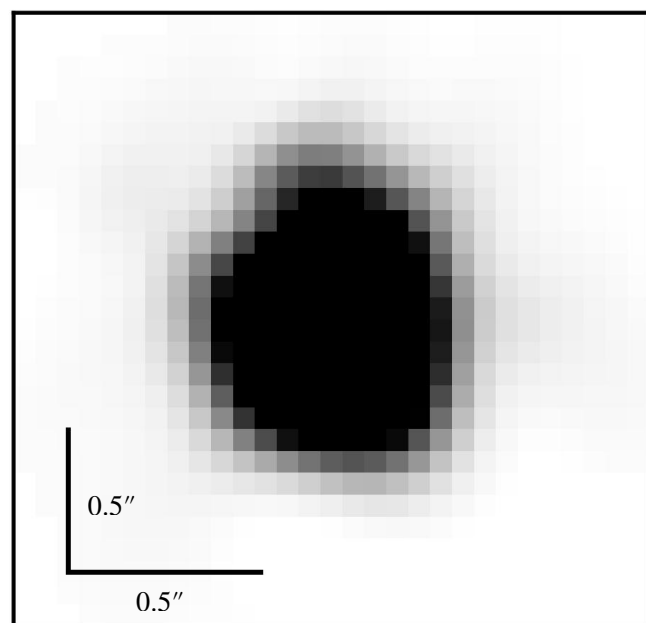


More accurate & precise reconstruction of the total mass of structures along the line of sight

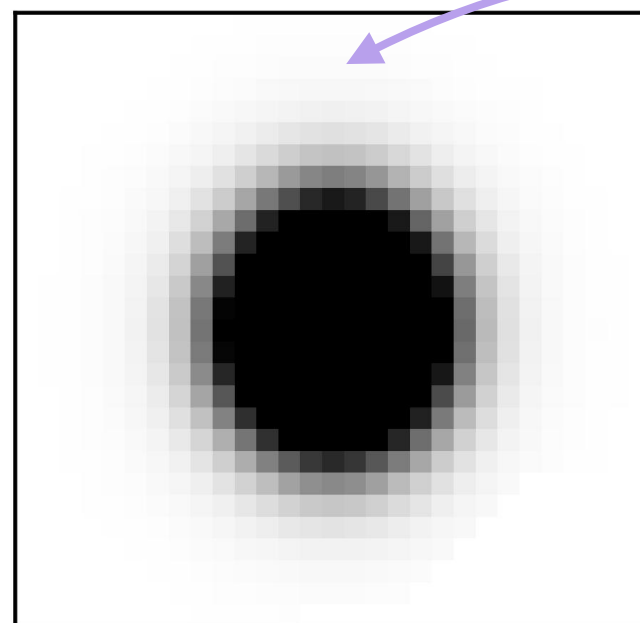
SDSS J1029+2623

Properties of the QSO host galaxy at $z = 2.1992$

Reconstructed



Model

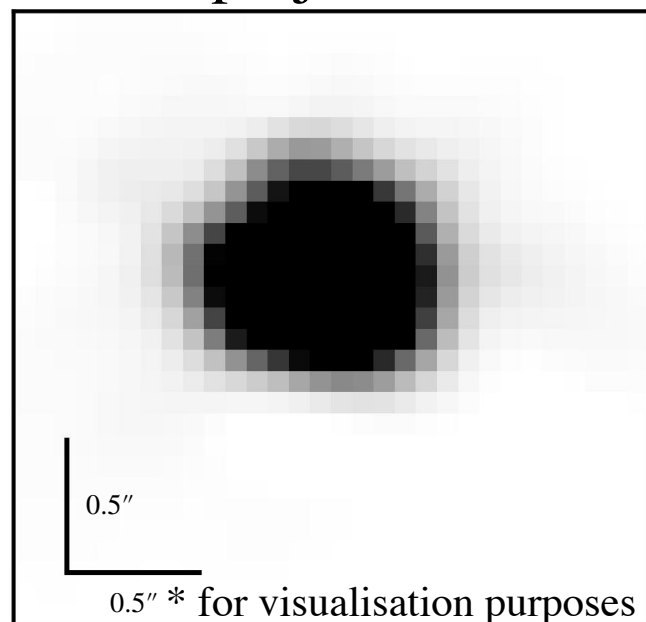


Single Sérsic profile

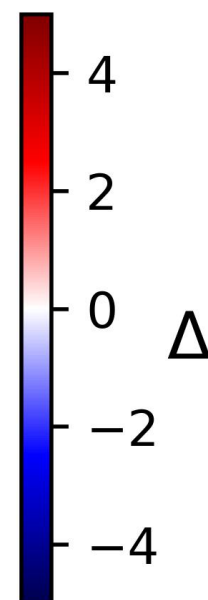
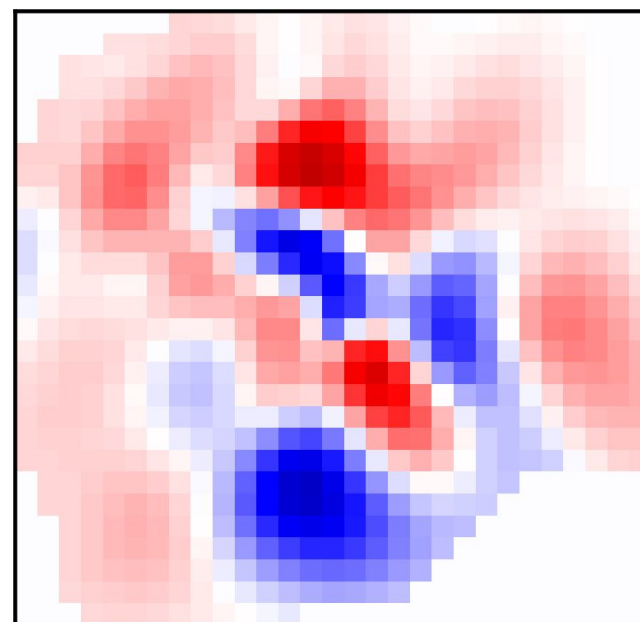
$$F160W_{\text{host}} = 23.3 \pm 0.1$$

$$R_{\text{eff}} = (2.39 \pm 0.03) \text{ kpc}$$

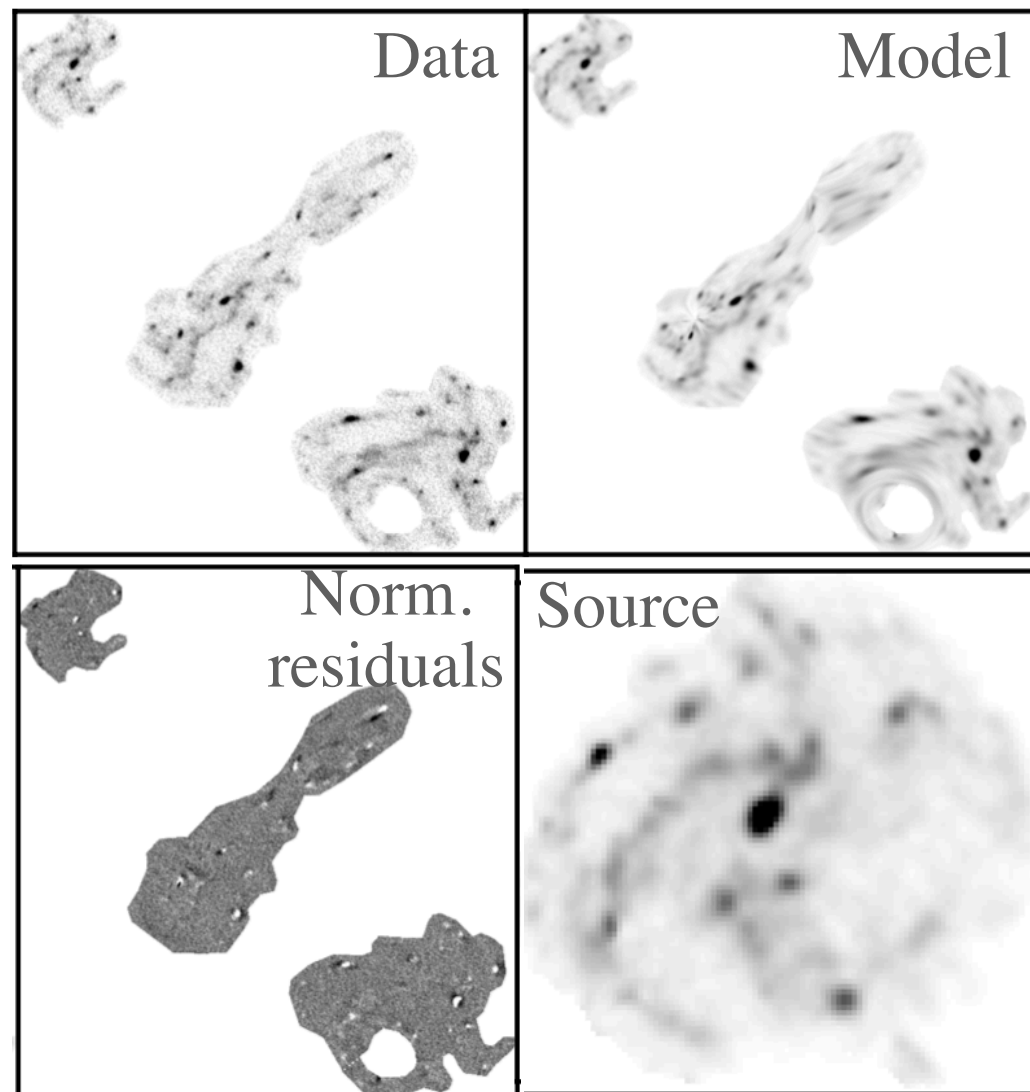
Reprojected *



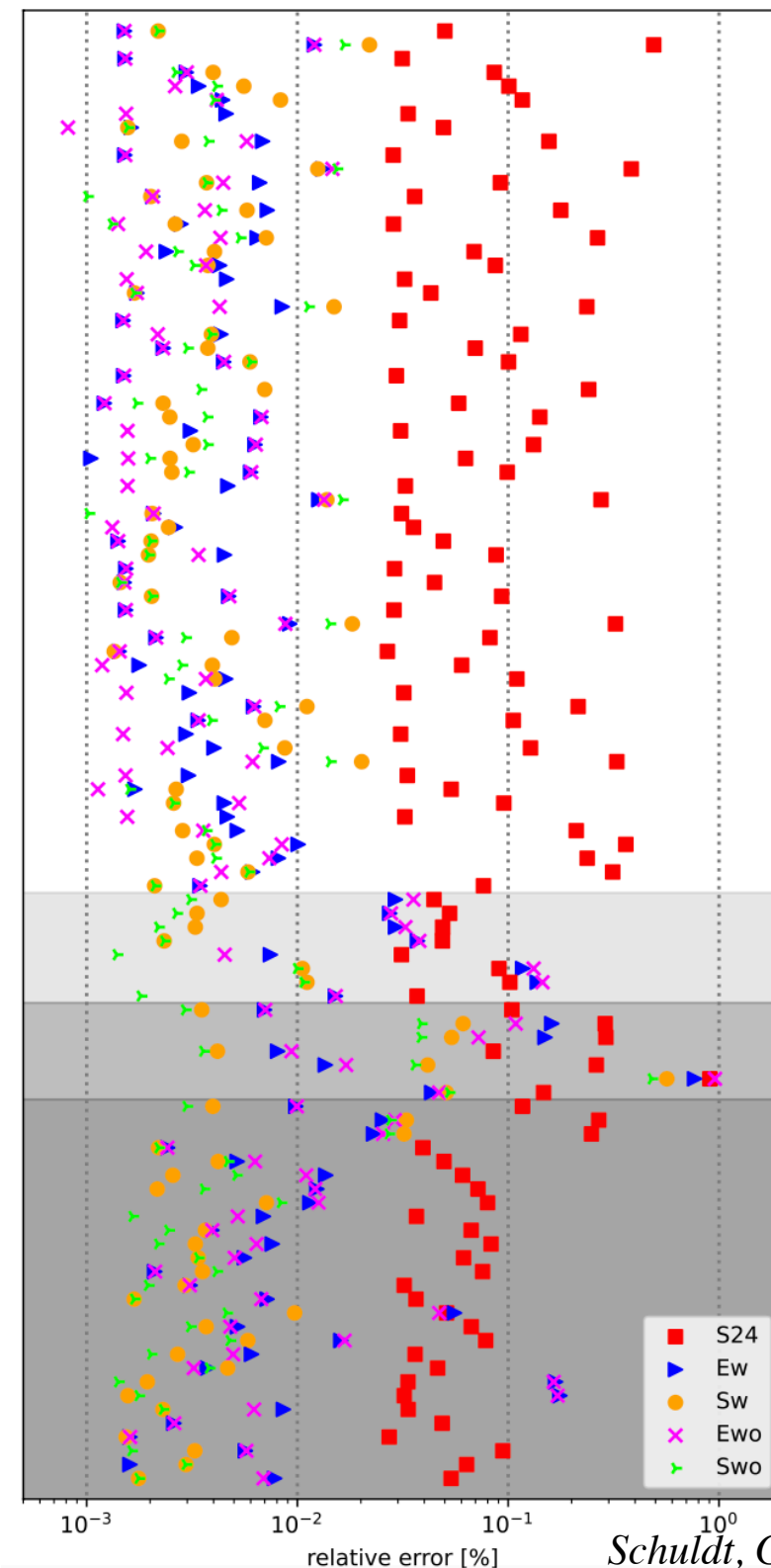
Normalised residuals



What about the impact of different weighting schemes of different types of observables?



In all cases, the precision of **magnification estimates improves between one and two orders of magnitude** compared to the pointlike approach



Pointlike
lens model
(Schuldt+24)

χ^2_{ext}
dominates

$\chi^2_{\text{ext}} \sim \chi^2_{\text{img}}$

Exciting opportunities & challenges

- ★ SDSS J1029+2623 as a **proof-of-concept** SL extended model: promising prospects for time-delay cosmography, substructure, and QSO host galaxy studies.
- ★ A **new generation of cluster strong lensing models**, including all available lensing observables, in the era of JWST, Euclid, Rubin/LSST & Roman.

Current bottleneck: computationally expensive, long running time

Next step: regime where systematic errors dominate and need to be quantified