
GW from Neutron Stars
誤記や説明不足あると思いますので、承知願います
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2013.9.27

Possible GW Sources

First Rank (Topics in previous school)

✓NS/BH Binary Merger

✓Supernovae

Second Rank (This talk) 3G sources ?

✓NS Oscillation

✓Rotating NS

-> Rich information of the interior

✓…

Example

Helioseismology

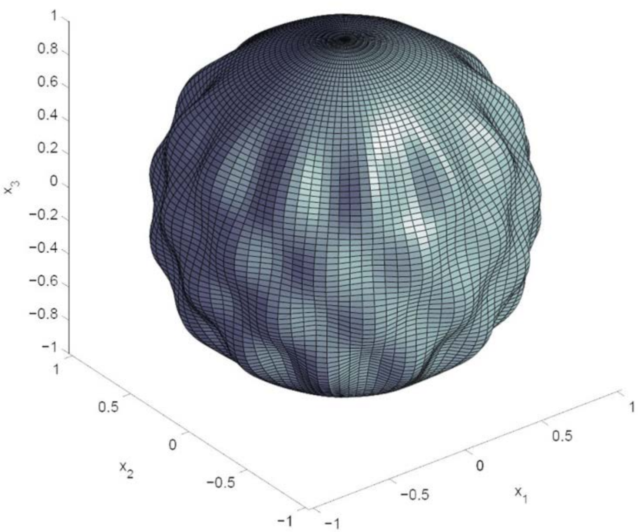
Acoustic wave(p-mode)

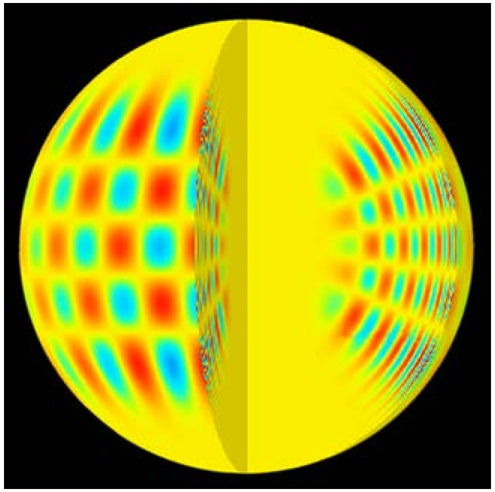
-> solar (rotation, structure)

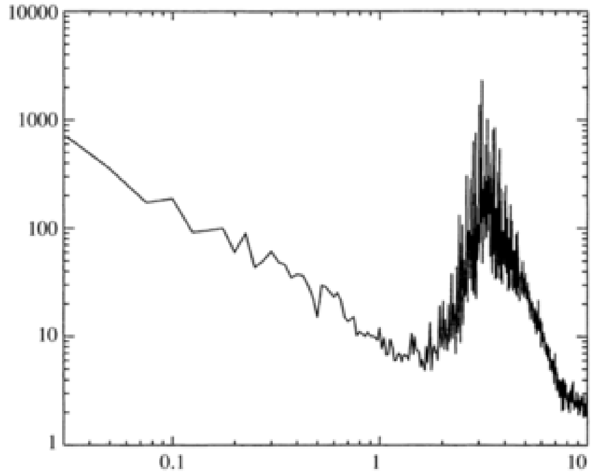
-> neutrino physics

$$\nu \approx R/a \approx 3mHz$$

$$T \approx 5\min$$







Contents

- NS Oscillation theory
- GW radiating NS in LMXB?
  (Continuous GW from Pulsars)
- Magnetar Giant Flare

Basics of Stellar Oscillation

- GR+Fluid Motion

$$\nabla^{\nu}\delta T_{\mu\nu}=0,\ \delta G_{\mu\nu}=8\pi\delta T_{\mu\nu}$$

->> Oscillation (from $\nabla^{\nu}\delta T_{\mu\nu}=0$), GW (from $\delta G_{\mu\nu}=8\pi\delta T_{\mu\nu}$)

- A model, e.g., spring

$$m\,d^2\xi/dt^2=-k\xi$$

$$2\pi\nu=\omega=(k/m)^{1/2}$$

High frequency for strong force

- Mode coupling by interaction

Resonance, Zero frequency

-> Unstable

$$\xi\Rightarrow\xi_1,\xi_2$$

$$\omega_1,\omega_2\Rightarrow\omega_1\pm\omega_2\approx 0$$

$$t\exp(-i\omega t),\exp(-i\omega t+\Gamma t)$$

- Damping(interaction)
- Excitation

Stellar oscillations in a simple NS model

Small perturbation from equilibrium

- f-mode   Fundamental(?) mode driven by gravity
- p-mode Acoustic or pressure (p) modes driven by internal pressure
- g-mode Gravity (g) modes driven by buoyancy
- s(w)-mode Space-time, GW in GR

+ + + + more modes in realistic models

Typical frequency of l=2

Mode Frequency /Damping time

- f-mode 1.5-3 KHz/ 0.1-0.5sec

$$\nu = \frac{1}{2\pi}\left(\frac{2l(l-1)}{2l+1}\frac{GM}{R^3}\right)^{1/2} = 1.7kHz\left(M/_{Ms}\right)^{1/2}\left(R/_{10km}\right)^{-3/2}$$

$$\Omega_K = \left(\frac{GM}{R^3}\right)^{1/2} \approx 1.1\times(2\pi\nu)$$

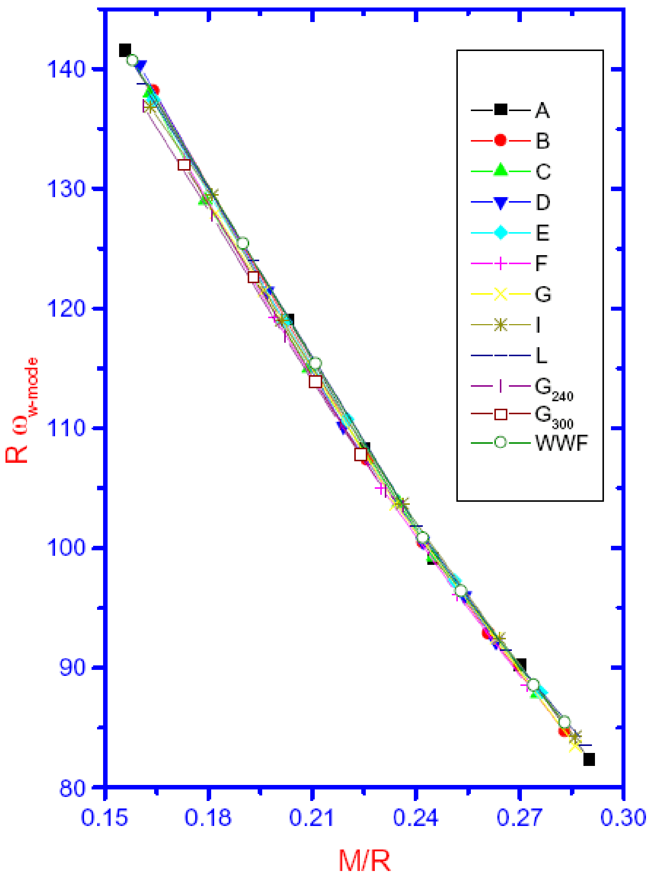
Breakup limit
1.7kHz, P=0.6ms

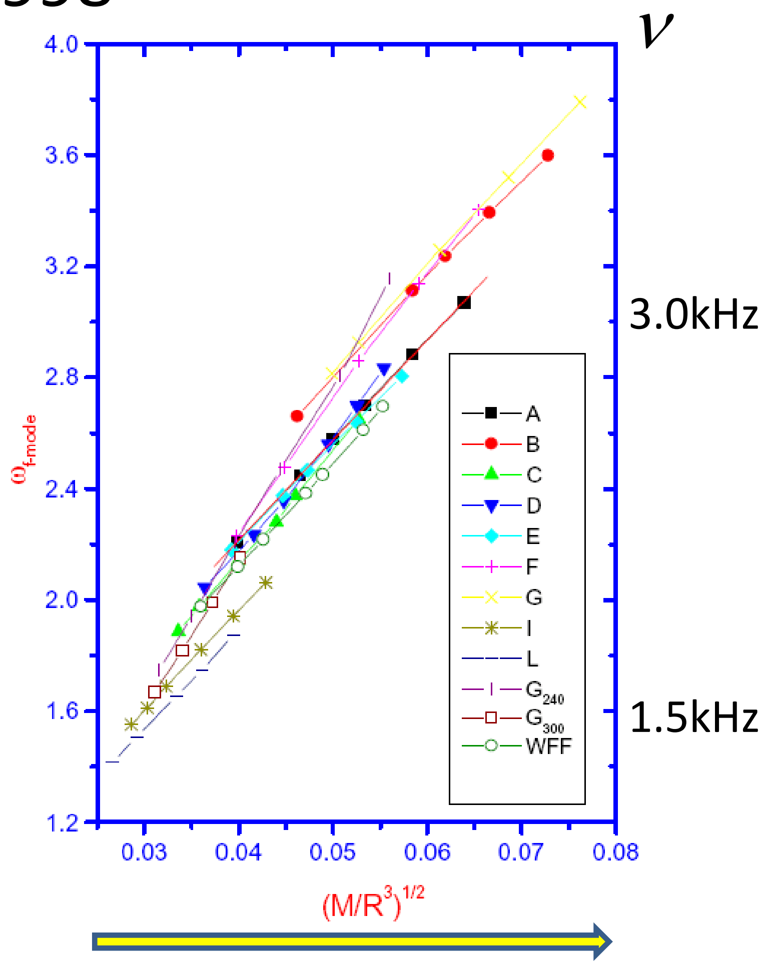
- p-mode 4-7KHz / sec
- g-mode 10Hz / year
- Spacetime(w)-mode 10KHz/0.01sec

$$\nu \sim \frac{c}{R} \sim 10kHz, \qquad \tau \sim \frac{R}{c} < 0.01\sec$$

Typical Oscillation Frequency (l=2)

Andersson & Kokkotas 1998





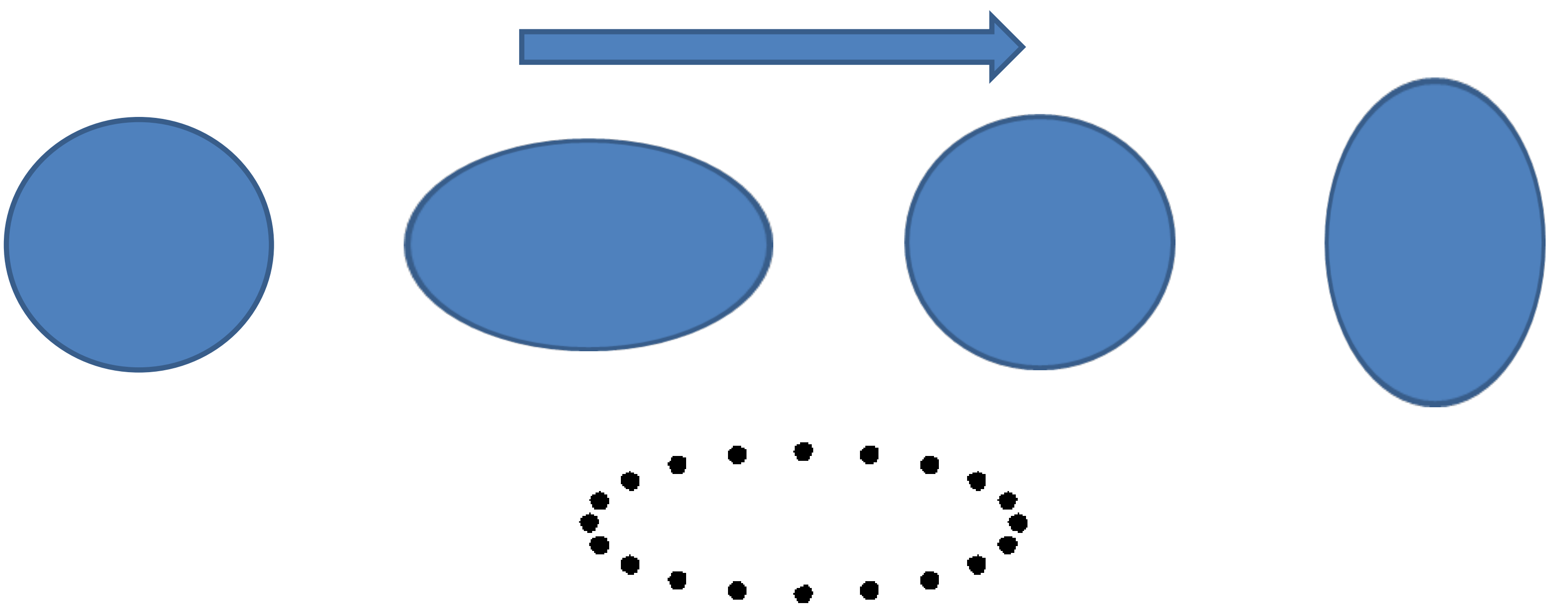
Relativistic, Massive, Compact

Quadrupole (l=2) f-mode Oscillation

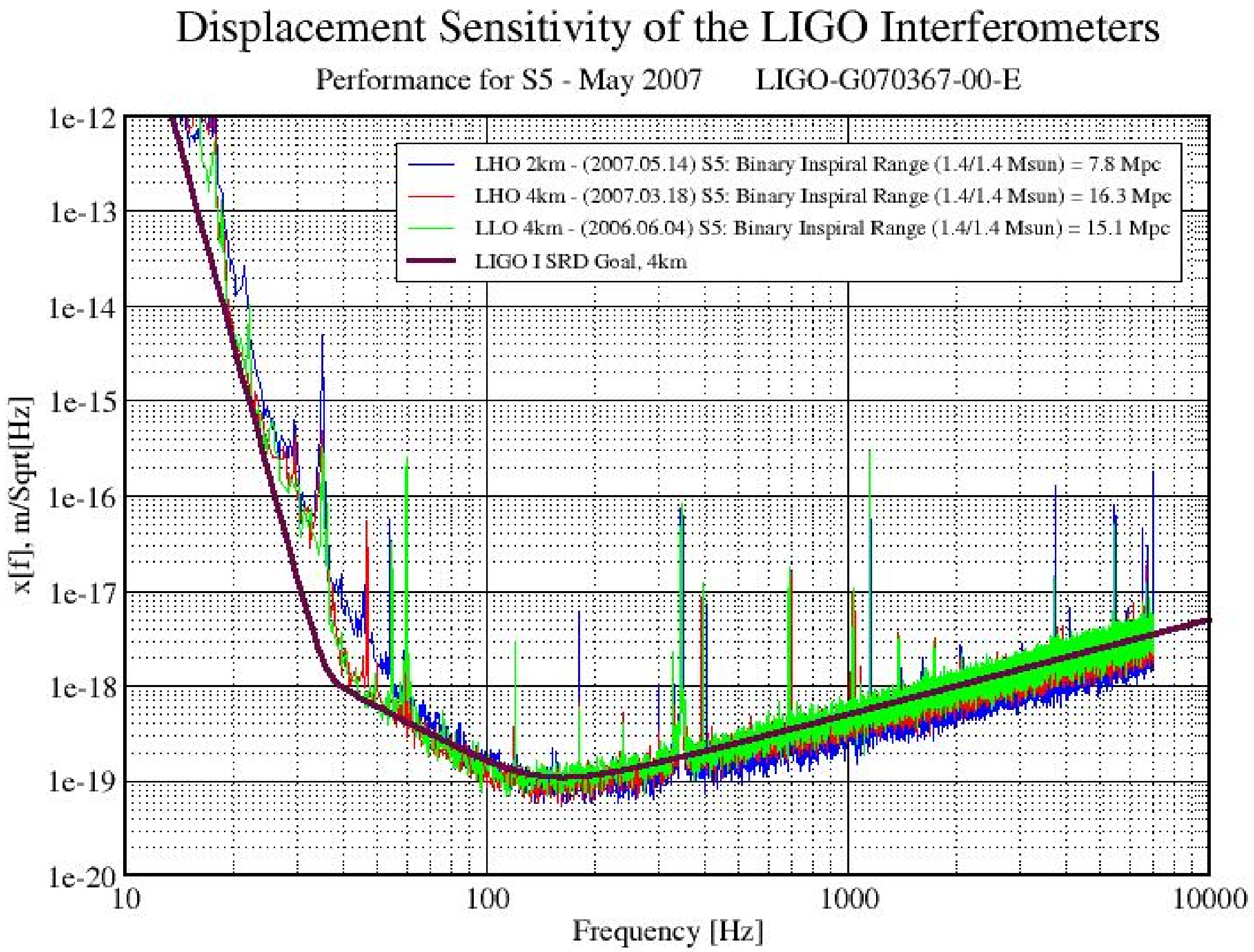
GW radiation

-> damping

$$\tau \sim 0.2\sec\left(M/_{Ms}\right)^{-3}\left(R/_{10km}\right)^{4}$$



Frequency Range for GW obs.

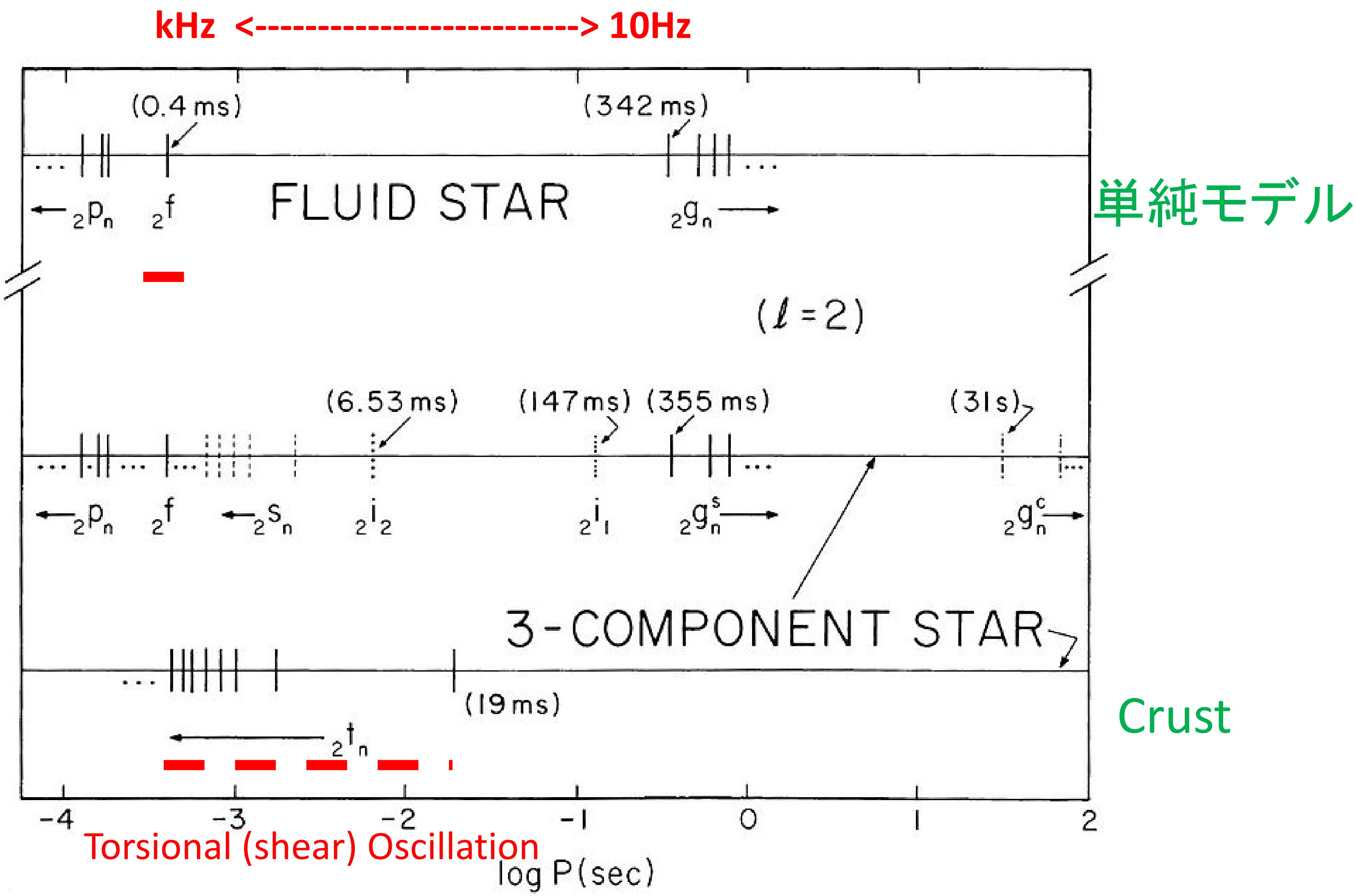


20 Hz <---------------------------------------> 6 kHz

～ 人間の聴覚範囲

Oscillation frequencies for Realistic NS

McDermott et al. (1988)



F-mode oscillation excited?

- SN-collapse

$R_{PNS} = 50km \sim R_b$ at (centrifugal) core bounce

$t \sim t_{dyn}(R_b) \sim 2 - 10ms$

Mixed variation

Fast Decay by fluid motion

- Hypothetical Phase transition?

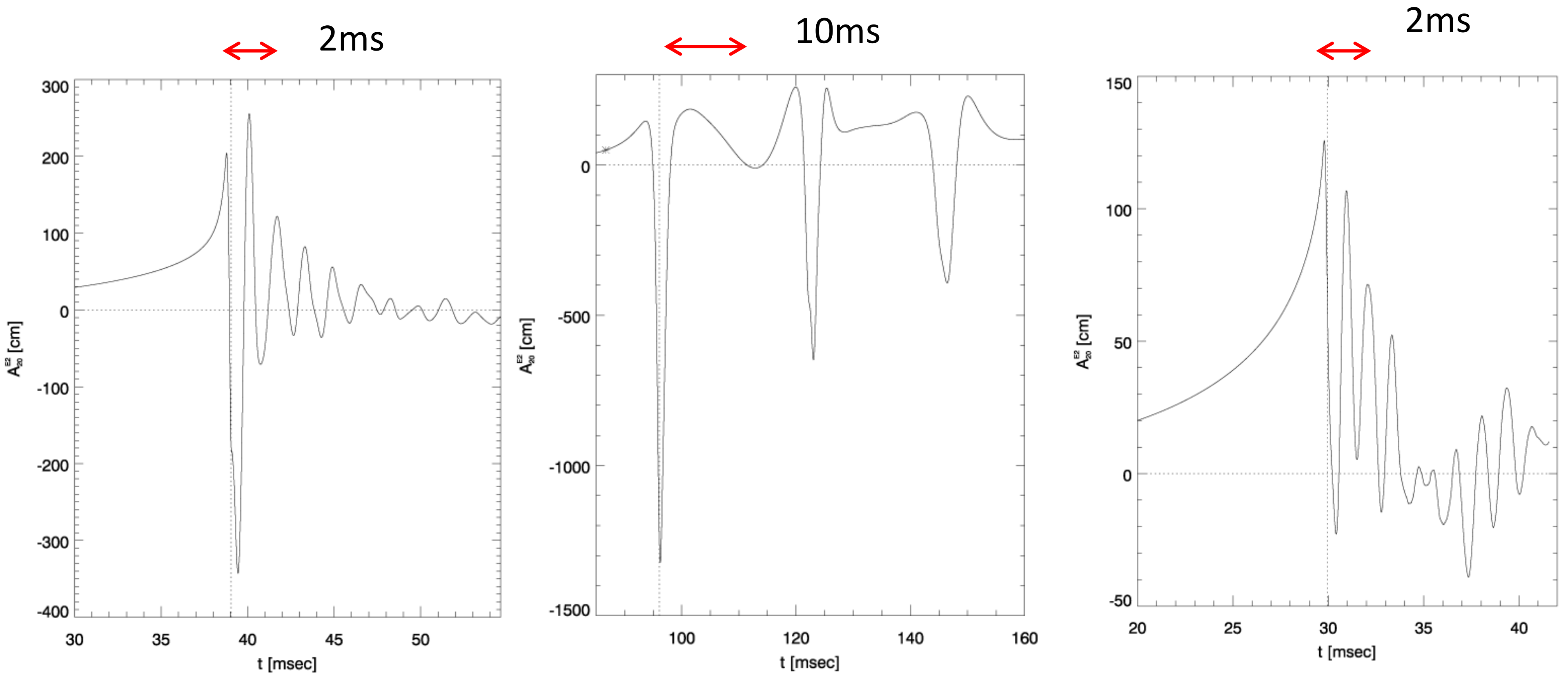
$\Delta E \sim 10^{46} ergs$

$\Rightarrow h \sim 10^{-21}(d / 10kpc)^{-2}$

????

Waveform from core collapse SN

Ref. simulations of a collapsing polytrope by Zwerger and Müller's [1997]



+Rotation

$$\omega \quad \Rightarrow \quad \omega - m\Omega$$

Non-axisymmetric modes
GW Radiation

Gravitational wave Radiation Reaction instability

- f-mode : Stable except for extreme rotation

$$\omega \sim \Omega_K \qquad \omega - m\Omega \sim 0 \quad \Leftrightarrow \Omega \sim \Omega_K$$

- r-mode : Coriolis force

unstable even for small rotation

$$\omega \approx 2m\Omega / l(l+1)$$

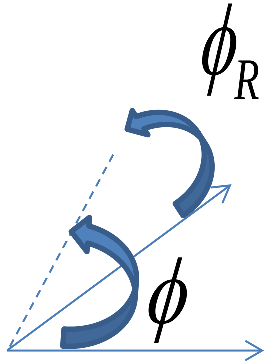
Physics for GRR instability

Frequency in inertial & fluid rest frames

$$\exp(-i(\omega t - m\phi)) = \exp(-i((\omega - m\Omega)t - m\phi_R))$$

$$\phi_R = \phi - \Omega t$$

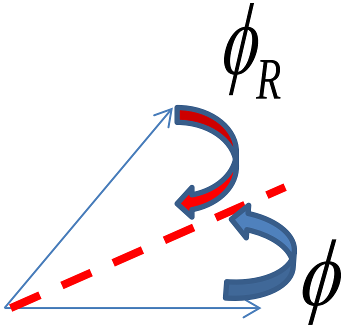
$$\omega, \qquad (\omega - m\Omega)$$



<u>Back Reaction Instability</u>

Instability Condition

Mismatched direction $\omega(\omega - m\Omega) < 0$



Positive(Negative) angular momentum for wave,
Negative(Positive) for fluid
Negative one as the reaction, i.e. instability

Growth rate determined by radiation rate
Instability damped by viscosity

Astrophysical Periodic Sources
LMXB

Accretion -> spin up / GW radiation -> down

Lx (/Fx)   Lgw (/ hgw)

Equilibrium? Maximum rotation frequency

Wagoner(84)

$$d^2 F_X \sim \frac{GM(dm/dt)}{r} \sim (\Omega_K / f)(fhd)^2$$

Amplitude

$$\Rightarrow h = 2\times 10^{-27} (f/300Hz)^{-1/2} (F_X/10^{-8})^{1/2}$$

Spin of LMXB(716Hz max) << break up limit (1.8kHz)

- Deformed NS/(mountain,   r-mode) radiating GW

Reference

Pulsars

Continuous GW source with known f

Deformed NS -> GW (Quadrupole radiation)

$$h \sim \varepsilon M R^2 f^2 d^{-1}$$

$$\Rightarrow h = 8\times 10^{-26}(\varepsilon/10^{-6})(f/kHz)^2(d/10kpc)^{-1}$$

Spin-down rate > GW Loss $\sim (hfd)^2$

Upper limit

Ex Crab $h < 8\times 10^{-25}, (\varepsilon < 6\times 10^{-4})$

Noise reduction $\propto (T\_obs)^{-1/2}$ -> obs

LMXB

Accretion -> spin up / GW radiation -> down

With small mountain (Bildsten 98)

$$\varepsilon \sim 10^{-7}(f/300Hz)^{-5/2}(F_X/10^{-8})^{1/2}(d/10kpc)$$

$$\Delta r = \varepsilon R \sim 10cm$$

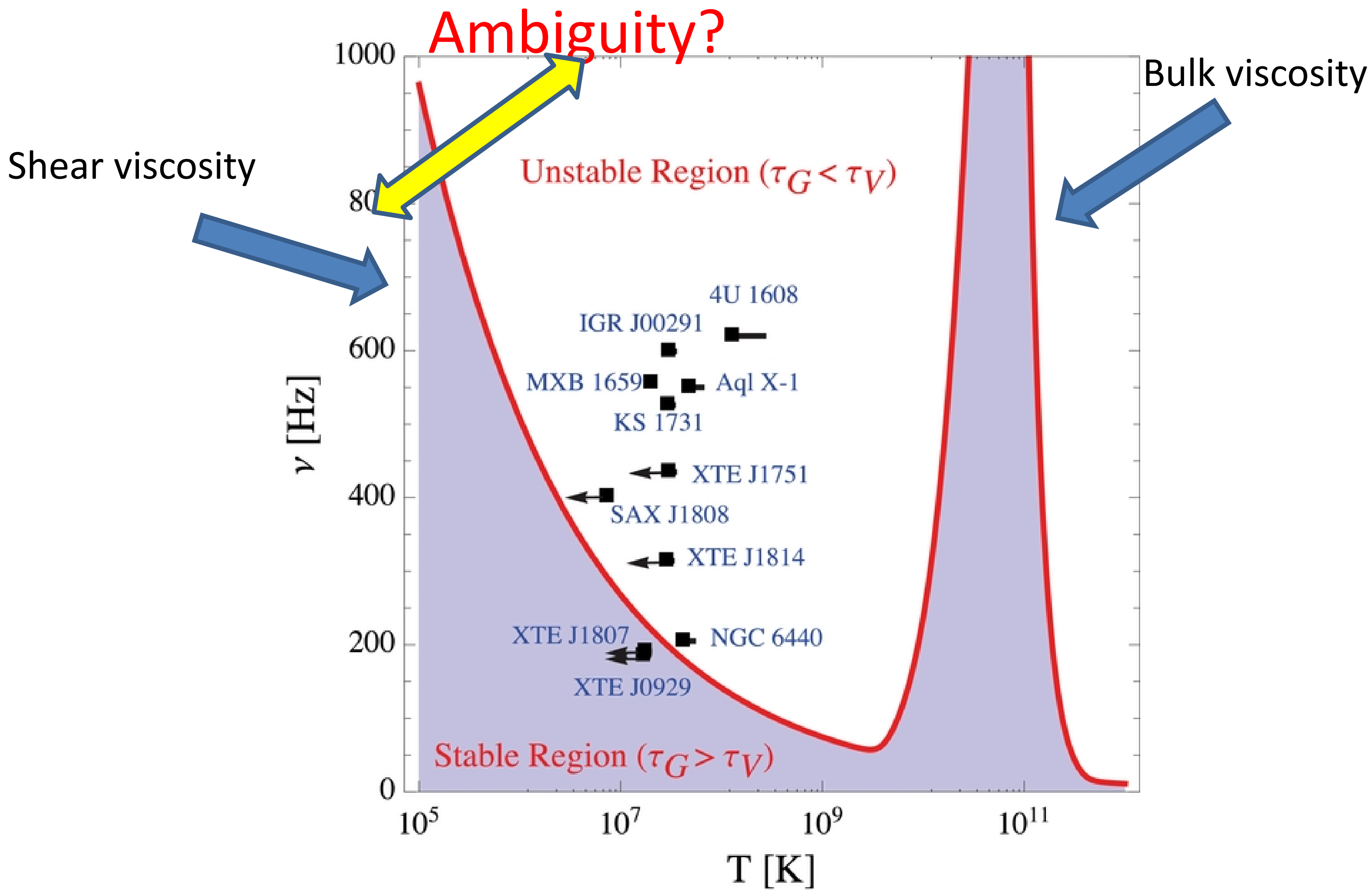
Cf. Magnetic deformation $\varepsilon \sim 10^{-8}(B/10^{13}G)^2$

LMXB <10^(8-9) G

GW obs.

Pulsar(isolated) <-> LMXB (binary) + parameters

R-mode instability in LMXB



e.g. Mahmoodifar+(13)

Problem for R-mode

(Theoretical many problems)

- Spin balance -> dimension less amplitude

$$\alpha /(\sim \varepsilon) \sim 10^{-5}(f/300Hz)^{-7/2}(F_X/10^{-8})^{1/2}(d/10kpc)$$

- Mahmoodifar+(13) fit obs. data

$$dJ/dt = -GWloss(\alpha) + (Nacc = 0)$$

$$\Rightarrow \alpha < 10^{-8} - 10^{-6}$$

Table 4

Upper Bounds on *r*-mode Amplitudes and NS Spin-down Rates

Source	$\alpha_{\rm th.eq}$ ($1.4\,M_\odot$)	$\alpha_{\rm th.eq}$ ($2.0\,M_\odot$)	$\alpha_{\rm th.eq}$ ($2.21\,M_\odot$)	$\dot{\nu}$ (Hz s^{-1}) ($1.4\,M_\odot$)	$\dot{\nu}$ (Hz s^{-1}) ($2.0\,M_\odot$)	$\dot{\nu}$ (Hz s^{-1}) ($2.21\,M_\odot$)	$\dot{\nu}_{\rm sd}$ (Hz s^{-1}) Observation
4U 1608−522	7.15×10^{-8}	6.60×10^{-8}	2.61×10^{-5}	-1.44×10^{-15}	-1.78×10^{-15}	-2.08×10^{-10}	
IGR J00291+5934	1.41×10^{-8}	1.32×10^{-8}	3.99×10^{-7}	-4.42×10^{-17}	-5.59×10^{-17}	-3.82×10^{-14}	-3×10^{-15}
MXB 1659−29	1.16×10^{-8}	1.07×10^{-8}	1.49×10^{-7}	-1.78×10^{-17}	-2.18×10^{-17}	-3.16×10^{-15}	
Aql X-1	3.49×10^{-8}	3.27×10^{-8}	2.26×10^{-6}	-1.49×10^{-16}	-1.89×10^{-16}	-6.74×10^{-13}	
KS 1731−260	2.35×10^{-8}	2.20×10^{-8}	6.44×10^{-7}	-4.81×10^{-17}	-6.09×10^{-17}	-3.90×10^{-14}	
XTE J1751−305	5.09×10^{-8}	4.76×10^{-8}	1.44×10^{-6}	-6.13×10^{-17}	-7.74×10^{-17}	-5.29×10^{-14}	-5.5×10^{-15}
SAX J1808−3658	1.28×10^{-8}	1.19×10^{-8}	3.30×10^{-8}	-2.19×10^{-18}	-2.74×10^{-18}	-1.57×10^{-17}	-5.5×10^{-16}
XTE J1814−338	1.76×10^{-7}	1.67×10^{-7}	4.49×10^{-6}	-7.49×10^{-17}	-9.73×10^{-17}	-5.26×10^{-14}	
NGC 6440	1.54×10^{-6}	1.45×10^{-6}	8.03×10^{-5}	-2.90×10^{-16}	-3.71×10^{-16}	-8.50×10^{-13}	

Notes. Upper bounds on the *r*-mode amplitude from the "thermal equilibrium" condition that are consistent with quiescent luminosity data are given for different

Cont.

- Small Spin down rate

  -> small amplitude

  -> EM loss with B=10^(8-9)G

  MSP

  (see Patruno + 12)

$$df/dt = -10^{-15}\,Hz/\sec$$
$$\Rightarrow dP/dt = (df/dt)\,f^{-2}$$
$$\sim 10^{-20}$$

- Smaller GW amplitude

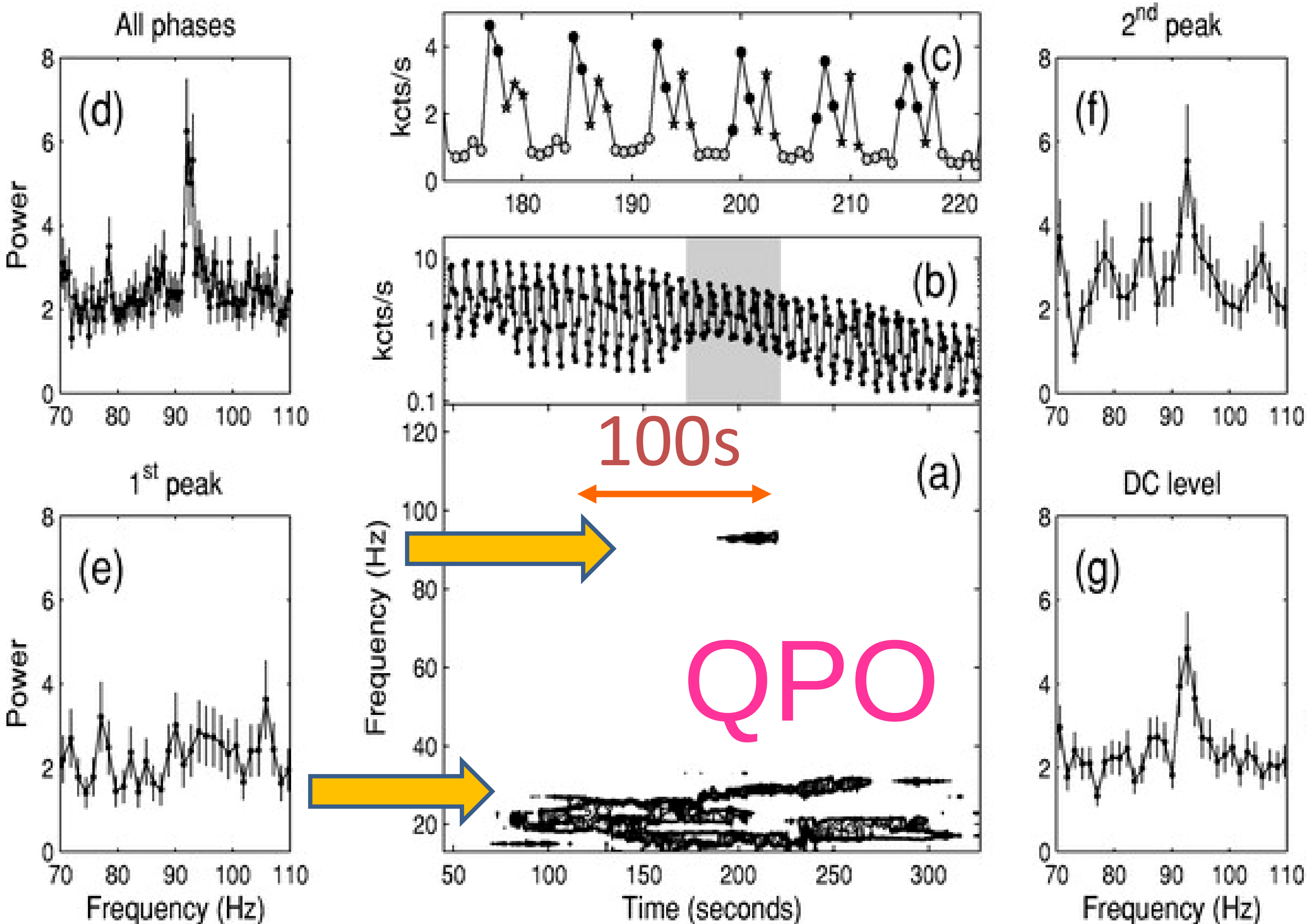
$$\alpha < 10^{-8} - 10^{-6}$$
$$\Rightarrow h \sim 2\times10^{-29} - 5\times10^{-28}$$

??????????????????????????????

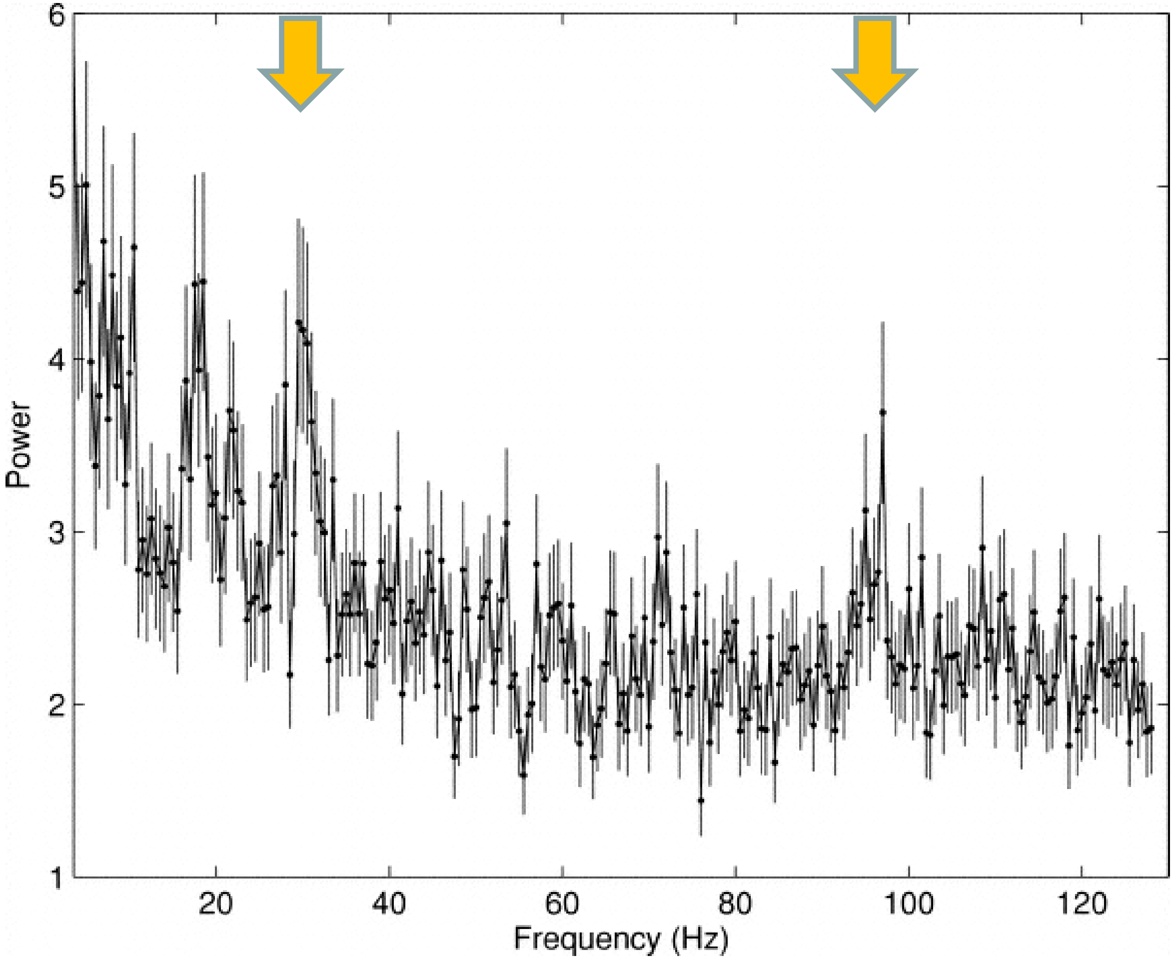
+Magnetic fields

Torsional shear mode in Magnetar

SGR1806-20 Israel+ 2005

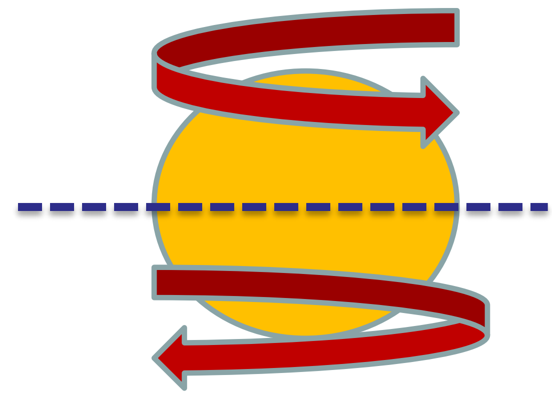


SGR1806-20 Israel+ 2005



Peaks at 20, 30, 95Hz during 200-300s

シア（捩れ）振動　軸性方向



変動　(l,n)　$T_{(l,n)} = 1/\nu_{(l,n)}$

$\nu_{(l,n)} \uparrow$　$(l\uparrow, \quad n\uparrow)$

典型的な大きさ $\mu \approx (Ze)^2 n_N^{4/3}$

$$C \approx (\mu/\rho)^{1/2} \approx 10^9 cm/s$$

$$T \approx 2\pi R/[(l(l+1))^{1/2} C] \approx 10ms$$

$$100Hz$$

磁場の影響　$C \approx C_A \propto (B/10^{15}G)$

マグネタでの強磁場では重大

（磁場の配置？）

QPOs in Flares

振動論的解釈　　QPO振動数 Hz(l,n)

〇2004 SGR1806-20

30Hz(2,0)　92Hz(6,0)　150Hz(10,0),….

are observed in hard X-rays (<200kev)

〇1998 SGR1900+14

28Hz(2,0)　53Hz(3,0)　84Hz(6,0) ,….

〇1979 SGR0526-66

QPOs in Magnetar Giant Flare

- SGR1806-20(Galactic 6-15kpc) Event in 2004

  P=7.5s, B=2.1x10^15G

  E(EM)~10^46 ergs

  [>>Brigtening AXP E~10^41 erg]

$$E_g \approx 10^{53}, E_R \approx 10^{45}, E_B \approx 10^{48}, E_T \approx 10^{43}, \ldots$$

✓ Magnetic Energy Source ~ 10^15 G

✓QPO observed in hard X-rays (<200kev)

30Hz(2,0) 92Hz(6,0) 150Hz(10,0),…

Torsional Shear Oscillation in Crust (l,n)

Giant Flare

Energetics

- SGR1806-20(Galactic 6-15kpc)

  P=7.5s, B=2.1x10^15G

  Giant Flare in 2004 E(EM)~10^46 ergs

$$E_g \approx 10^{53}, E_R \approx 10^{45}, \underline{E_B \approx 10^{48}}, E_T \approx 10^{43}, \ldots$$

[Brigtening AXP E~10^41 erg]

✓ Upper limit of GW (LIGO PR07)

$$h_{rms} \leq 4.5 \times 10^{-22} Hz^{-1/2}, \Delta E \leq 7.7 \times 10^{46} erg$$

$$\Delta E \approx (c^5 / G)(\nu d / c)^2 (h_{rms})^2$$

Summary

- NS Oscillations

  -> Excitation?
- NSs in LMXB emitting GWs

  (Continuous GW from Pulsars)

  -> Stronger astrophysical limit?
- Magnetar Giant Flare

  -> Rare chance?

Observation is very important to …