Jerome Martin, JGRG 22(2012)111501

“The cosmological constant problem”

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

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The Cosmological Constant Problem

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"Joyeux anniversaire et meilleurs voeux aux professeurs Futamase, Kodama & Sasaki!"

RESCEU SYMPOSIUM ON GENERAL RELATIVITY AND GRAVITATION
JGRG22 November 12 - 16 2012

Outline

1- Introduction: the cosmological constant in the Einstein equations.

2- Observational constraints on the CC.

3- Regularization (or renormalization) of the vacuum energy density.

4- Possible loopholes in our approach to the CC problem.

5- General conclusions.

Outline

Based on

"Everything you always wanted to know about the Cosmological constant problem (but were afraid to ask)"

Comptes Rendus Physique 13 (2012) 566-665

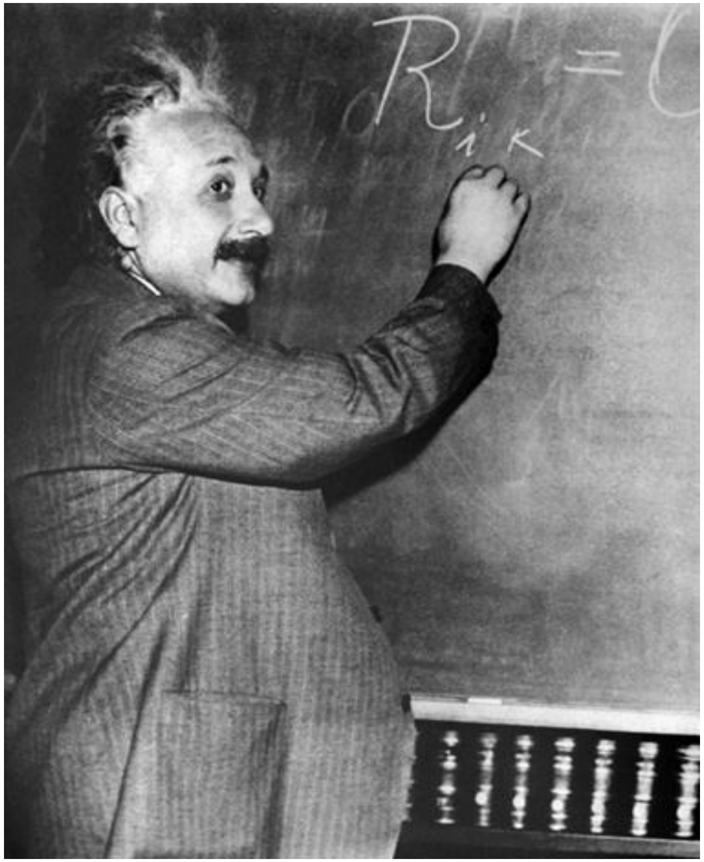
arXiv:1205.3365

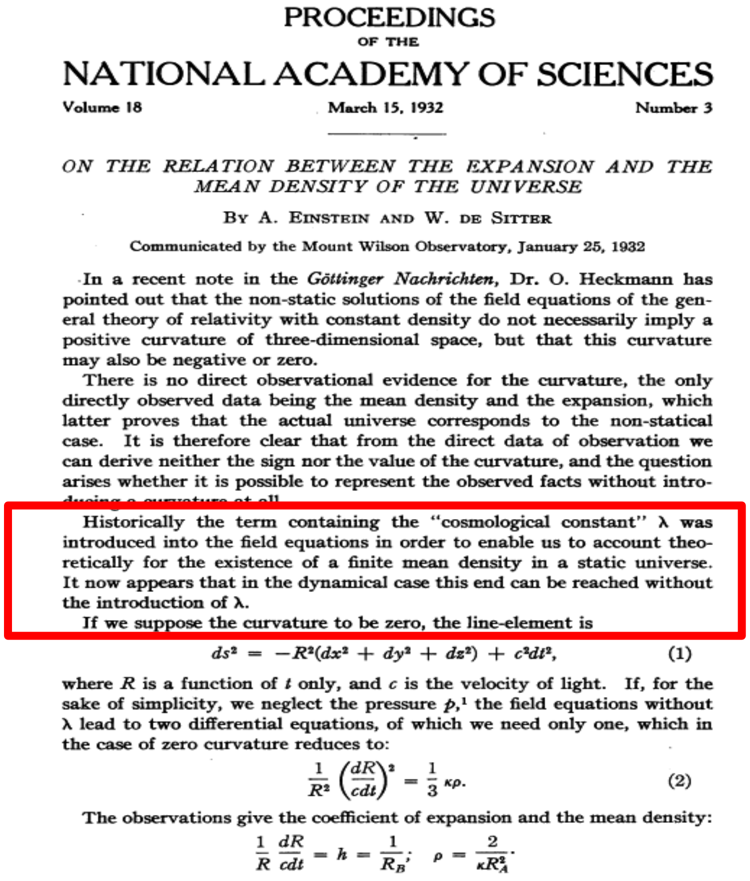
See also:

- S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)
- V. Sahni & A. Starobinsky, astro-ph/9904398
- T. Padmanabhan, hep-th/0212290
- J. Yokoyama, gr-qc/0305068
- J. Polchinsky, hep-th/0603249
- M. Li, X. Li, S. Wang & Y. Wang, arXiv:1103.5870

The cosmological constant (CC): introduction

Historically introduced by Einstein to find a static cosmological solution in General Relativity (GR) [see N. Straumann, gr-qc/0208027]





PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 18 March 15, 1932 Number 3

ON THE RELATION BETWEEN THE EXPANSION AND THE MEAN DENSITY OF THE UNIVERSE

By A. Einstein and W. de Sitter

Communicated by the Mount Wilson Observatory, January 25, 1932

In a recent note in the *Göttinger Nachrichten*, Dr. O. Heckmann has pointed out that the non-static solutions of the field equations of the general theory of relativity with constant density do not necessarily imply a positive curvature of three-dimensional space, but that this curvature may also be negative or zero.

There is no direct observational evidence for the curvature, the only directly observed data being the mean density and the expansion, which latter proves that the actual universe corresponds to the non-statical case. It is therefore clear that from the direct data of observation we can derive neither the sign nor the value of the curvature, and the question arises whether it is possible to represent the observed facts without introducing a curvature at all.

Historically the term containing the "cosmological constant" λ was introduced into the field equations in order to enable us to account theoretically for the existence of a finite mean density in a static universe. It now appears that in the dynamical case this end can be reached without the introduction of λ.

If we suppose the curvature to be zero, the line-element is

$$ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2dt^2, \tag{1}$$

where R is a function of t only, and c is the velocity of light. If, for the sake of simplicity, we neglect the pressure p,[1] the field equations without λ lead to two differential equations, of which we need only one, which in the case of zero curvature reduces to:

$$\frac{1}{R^2}\left(\frac{dR}{cdt}\right)^2 = \frac{1}{3}\kappa\rho. \tag{2}$$

The observations give the coefficient of expansion and the mean density:

$$\frac{1}{R}\frac{dR}{cdt} = h = \frac{1}{R_B}; \quad \rho = \frac{2}{\kappa R_A^2}.$$

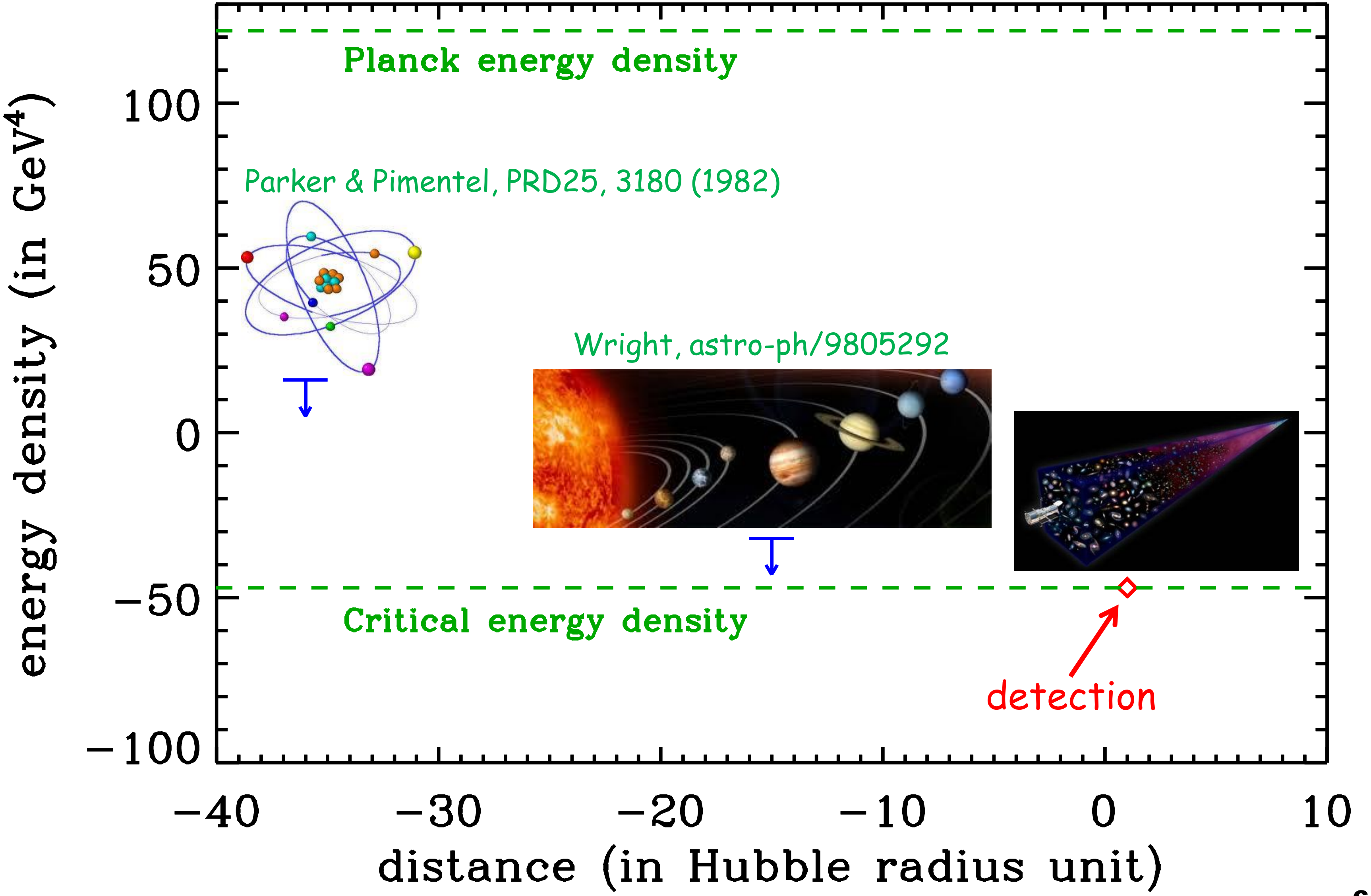
The cosmological constant (CC): introduction

In presence of a Cosmological Constant, the Einstein field equations read

$$\underbrace{R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}}_{\text{geometry}} + \underbrace{\Lambda_{\mathrm{B}} g_{\mu\nu}}_{\text{CC}} = \underbrace{\kappa T_{\mu\nu}}_{\text{matter}}$$

- Preserves covariance
- Covariant derivative vanishes hence compatible with a conserved energy momentum tensor
- Dimension length^ (-2)
- The CC can always been seen as an extra source of matter: $T_{\mu\nu} = -\frac{\Lambda_{\mathrm{B}}}{\kappa} g_{\mu\nu}$
- The equation of state of the CC is: $w = \frac{p}{\rho} = -1$. The effective pressure is negative.

The cosmological constant: constraints



In 1998, two groups measure the expansion of the Universe and claim detection of a non-vanishing CC.

The Astronomical Journal, 116:1009–1038, 1998 September

OBSERVATIONAL EVIDENCE FROM SUPERNOVAE FOR AN ACCELERATING UNIVERSE AND A COSMOLOGICAL CONSTANT

Adam G. Riess,[1] Alexei V. Filippenko,[1] Peter Challis,[2] Alejandro Clocchiatti,[3] Alan Diercks,[4]
Peter M. Garnavich,[2] Ron L. Gilliland,[5] Craig J. Hogan,[4] Saurabh Jha,[2] Robert P. Kirshner,[2]
B. Leibundgut,[6] M. M. Phillips,[7] David Reiss,[4] Brian P. Schmidt,[8,9] Robert A. Schommer,[7]
R. Chris Smith,[7,10] J. Spyromilio,[6] Christopher Stubbs,[4]
Nicholas B. Suntzeff,[7] and John Tonry[11]

Received 1998 March 13; revised 1998 May 6

ABSTRACT

We present spectral and photometric observations of 10 Type Ia supernovae (S
range $0.16 \leq z \leq 0.62$. The luminosity distances of these objects are determined by
relations between SN Ia luminosity and light curve shape. Combined with pre
High-z Supernova Search Team and recent results by Riess et al., this expanded s
supernovae and a set of 34 nearby supernovae are used to place constraints on
logical parameters: the Hubble constant (H_0), the mass density (Ω_M), the cosmolog
vacuum energy density, Ω_Λ), the deceleration parameter (q_0), and the dynamical ag
The distances of the high-redshift SNe Ia are, on average, 10%–15% farther than ex
density ($\Omega_M = 0.2$) universe without a cosmological constant. Different light curve f
subsamples, and prior constraints unanimously favor eternally expanding models
logical constant (i.e., $\Omega_\Lambda > 0$) and a current acceleration of the expansion (i.e., q
constraint on mass density other than $\Omega_M \geq 0$, the spectroscopically confirmed S
consistent with $q_0 < 0$ at the 2.8 σ and 3.9 σ confidence levels, and with $\Omega_\Lambda > 0$ a
confidence levels, for two different fitting methods, respectively. Fixing a "minimal"
0.2, results in the weakest detection, $\Omega_\Lambda > 0$ at the 3.0 σ confidence level from one
For a flat universe prior ($\Omega_M + \Omega_\Lambda = 1$), the spectroscopically confirmed SNe Ia
and 9 σ formal statistical significance for the two different fitting methods. A univer
matter (i.e., $\Omega_M = 1$) is formally ruled out at the 7 σ to 8 σ confidence level for th
methods. We estimate the dynamical age of the universe to be 14.2 ± 1.7 Gyr includ
tainties in the current Cepheid distance scale. We estimate the likely effect of several sources of systematic error, including progenitor and metallicity evolution, extinction, sample selection bias, local perturbations in the expansion rate, gravitational lensing, and sample contamination. Presently, none of these effects appear to reconcile the data with $\Omega_\Lambda = 0$ and $q_0 \geq 0$.

Key words: cosmology: observations — supernovae: general

1. INTRODUCTION

This paper reports observations of 10 new high-redshift Type Ia supernovae (SNe Ia) and the values of the cosmological parameters derived from them. Together with the four high-redshift supernovae previously reported by our High-z Supernova Search Team (Schmidt et al. 1998; Garnavich et al. 1998a) and two others (Riess et al. 1998b), the sample of 16 is now large enough to yield interesting cosmological results of high statistical significance. Confidence in these results depends not on increasing the sample size but on improving our understanding of systematic uncertainties.

The time evolution of the cosmic scale factor depends on the composition of mass-energy in the universe. While the universe is known to contain a significant amount of ordinary matter, Ω_M, which decelerates the expansion, its dynamics may also be significantly affected by more exotic forms of energy. Preeminent among these is a possible energy of the vacuum (Ω_Λ), Einstein's "cosmological con-

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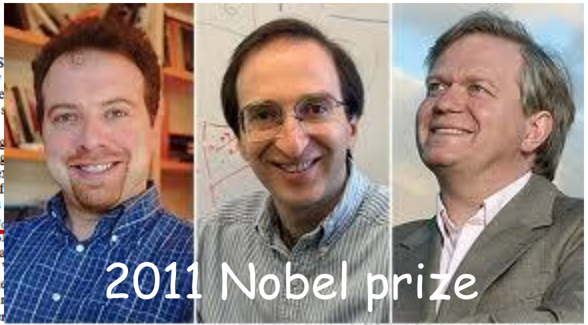

Johns Hopkins University; University Of California At Berkeley; Australian National University

From left, Adam Riess, Saul Perlmutter and Brian Schmidt shared the Nobel Prize in physics awarded Tuesday.

The Astrophysical Journal, 517:565–586, 1999 June 1

MEASUREMENTS OF Ω AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE

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(The Supernova Cosmology Project)

Received 1998 September 8; accepted 1998 December 17

ABSTRACT

We report measurements of the mass density, Ω_M, and cosmological-constant energy density, Ω_Λ, of the universe based on the analysis of 42 type Ia supernovae discovered by the Supernova Cosmology Project. The magnitude-redshift data for these supernovae, at redshifts between 0.18 and 0.83, are fitted jointly with a set of supernovae from the Calán/Tololo Supernova Survey, at redshifts below 0.1, to yield values for the cosmological parameters. All supernova peak magnitudes are standardized using a SN Ia light-curve width-luminosity relation. The measurement yields a joint probability distribution of the cosmological parameters that is approximated by the relation $0.8\Omega_M - 0.6\Omega_\Lambda \approx -0.2 \pm 0.1$ in the region of interest ($\Omega_M \lesssim 1.5$). For a flat ($\Omega_M + \Omega_\Lambda = 1$) cosmology we find $\Omega_M^{\rm flat} = 0.28^{+0.09}_{-0.08}$ (1 σ statistical) $^{+0.05}_{-0.04}$ (identified systematics). The data are strongly inconsistent with a $\Lambda = 0$ flat cosmology, the simplest inflationary universe model. An open, $\Lambda = 0$ cosmology also does not fit the data well: the data indicate that the cosmological constant is nonzero and positive, with a confidence of $P(\Lambda > 0) = 99\%$, including the identified systematic uncertainties. The best-fit age of the universe relative to the Hubble time is $t_0^{\rm flat} = 14.9^{+1.4}_{-1.1}(0.63/h)$ Gyr for a flat cosmology. The size of our sample allows us to perform a variety of statistical tests to check for possible systematic errors and biases. We find no significant differences in either the host reddening distribution or Malmquist bias between the low-redshift Calán/Tololo sample and our high-redshift sample. Excluding those few supernovae that are outliers in color excess or fit residual does not significantly change the results. The conclusions are also robust whether or not a width-luminosity relation is used to standardize the supernova peak magnitudes. We discuss and constrain, where possible, hypothetical alternatives to a cosmological constant.

Subject headings: cosmology: observations — distance scale — supernovae: general

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The cosmological constant in cosmology

- The hard fact is that the following equation does not fit well the data

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho_{\rm CDM}$$

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- If the Universe is homogeneous and isotropic and if gravity is described by GR and if there is no other exotic fluid then the CC is non-vanishing.

$$\begin{aligned} \rho_\Lambda &\sim 10^{-47}{\rm GeV}^4 \sim \rho_{\rm cri} \\ &\sim \left(10^{-3}{\rm eV}\right)^4 \end{aligned}$$

The cosmological constant in cosmology

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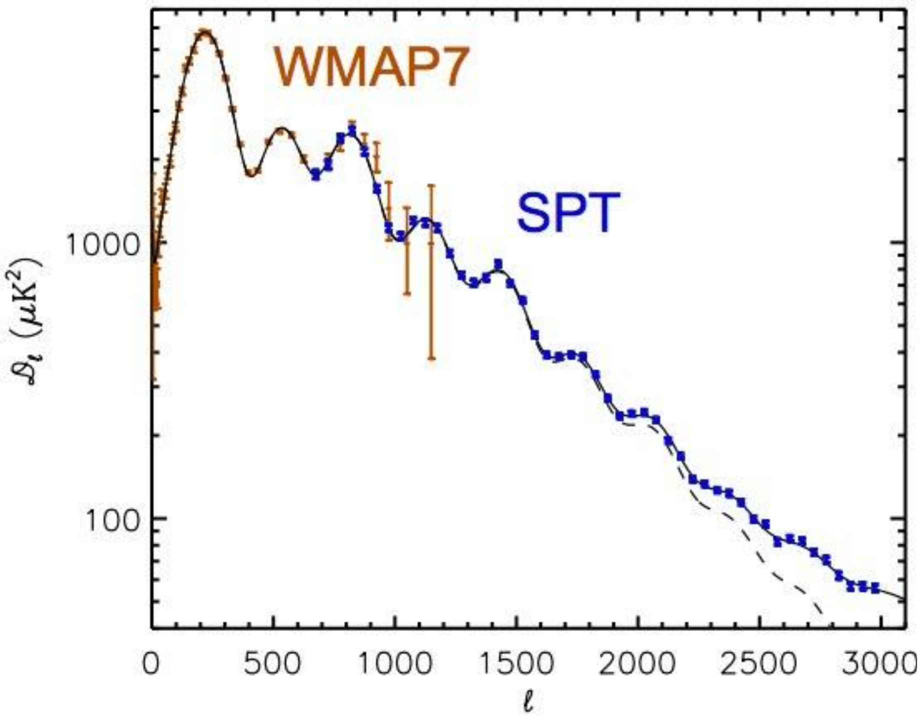
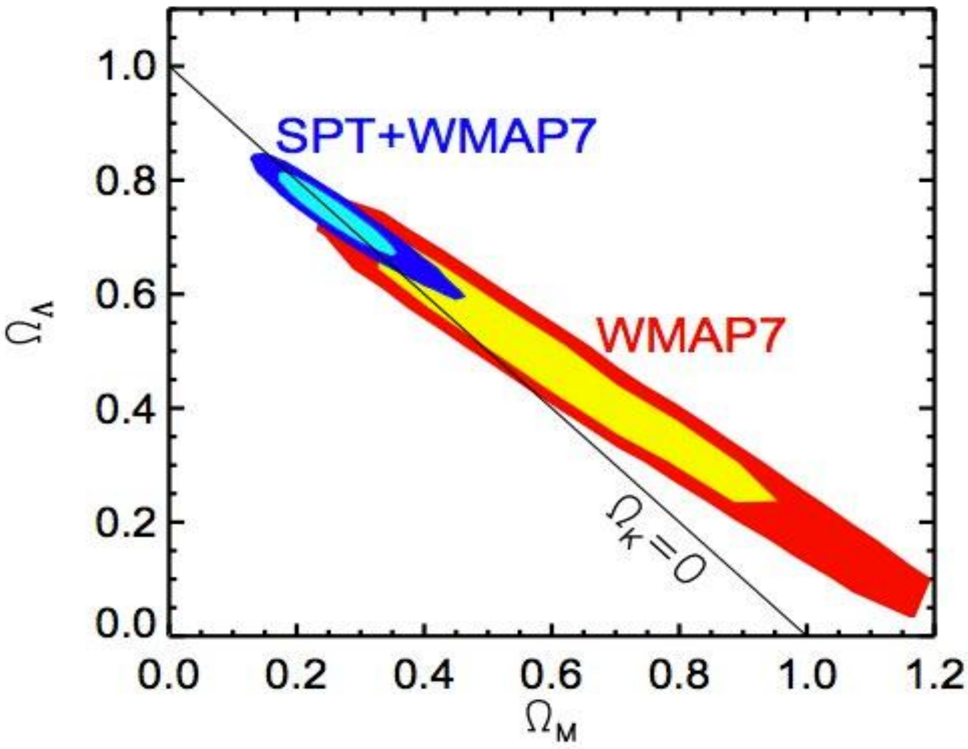
- 2012: there is now a bunch of different and independent measurements pointing towards this conclusion (age of the universe, SNIa, clusters abundance, lensing etc …)

The cosmological constant in cosmology

Example: using the CMB only, a vanishing CC now seems to be ruled out at more than 5 sigma …





SPT data, arXiv:1210.7231

The cosmological constant

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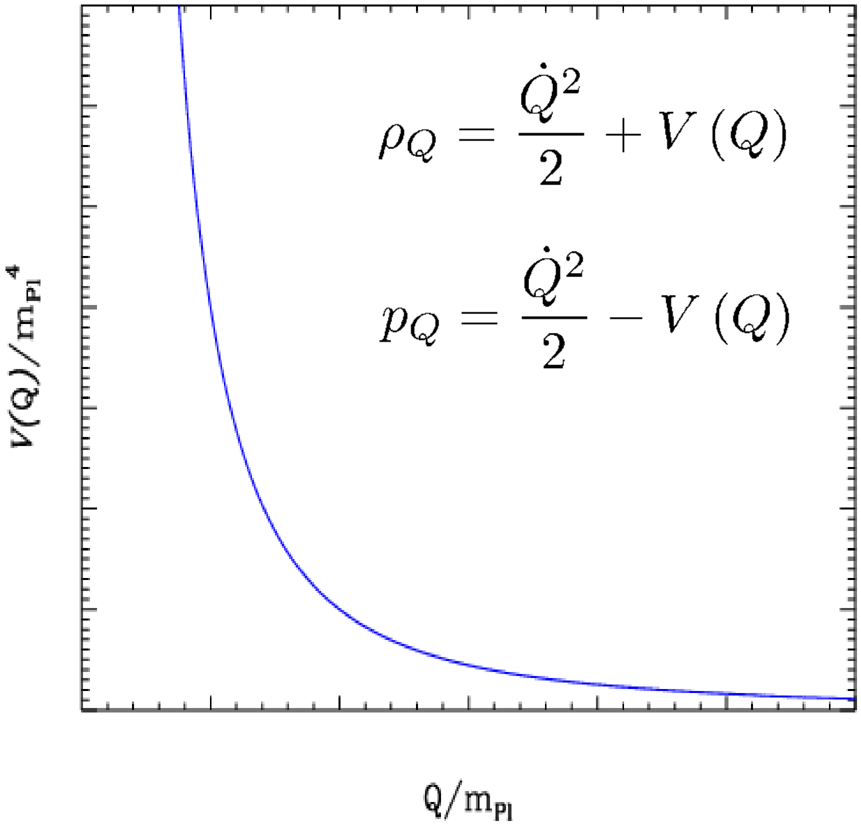
- The other alternatives (in-homogeneous universe, modified gravity, quintessence etc …) have their own problems.

Quintessence

A possible alternative is that there is no CC but a scalar field ("quintessence") playing the role of a "dark energy".

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\rho_{\rm CDM} + \rho_{\rm DE}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm Pl}^2}\left(\rho_{\rm CDM} + \underbrace{\rho_{\rm DE} + 3p_{\rm DE}}_{\text{must be <0}}\right)$$



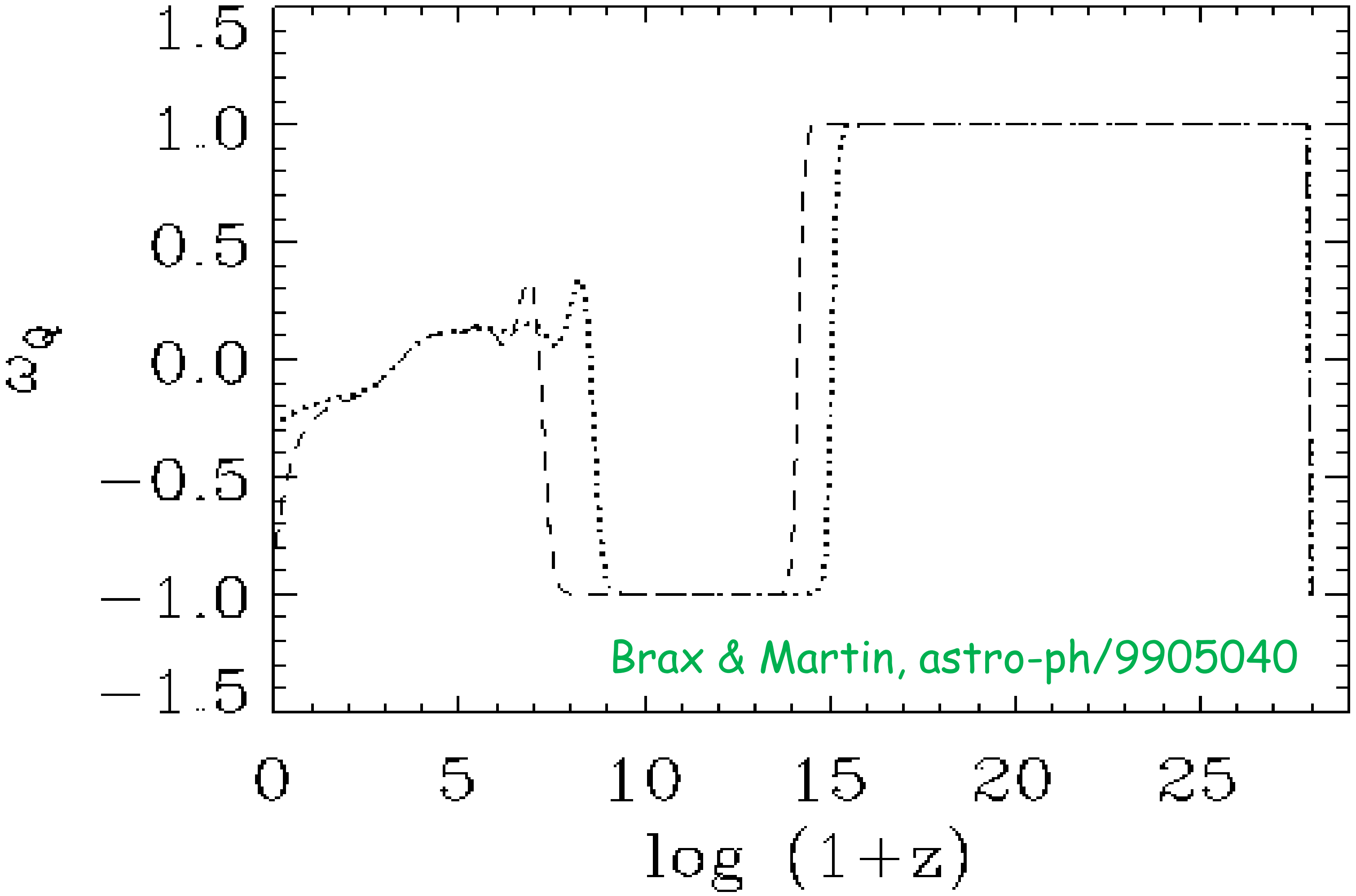
$$\rho_Q = \frac{\dot{Q}^2}{2} + V(Q)$$

$$p_Q = \frac{\dot{Q}^2}{2} - V(Q)$$

Ratra & Peebles, PRD37 3406 (1988)

Quintessence

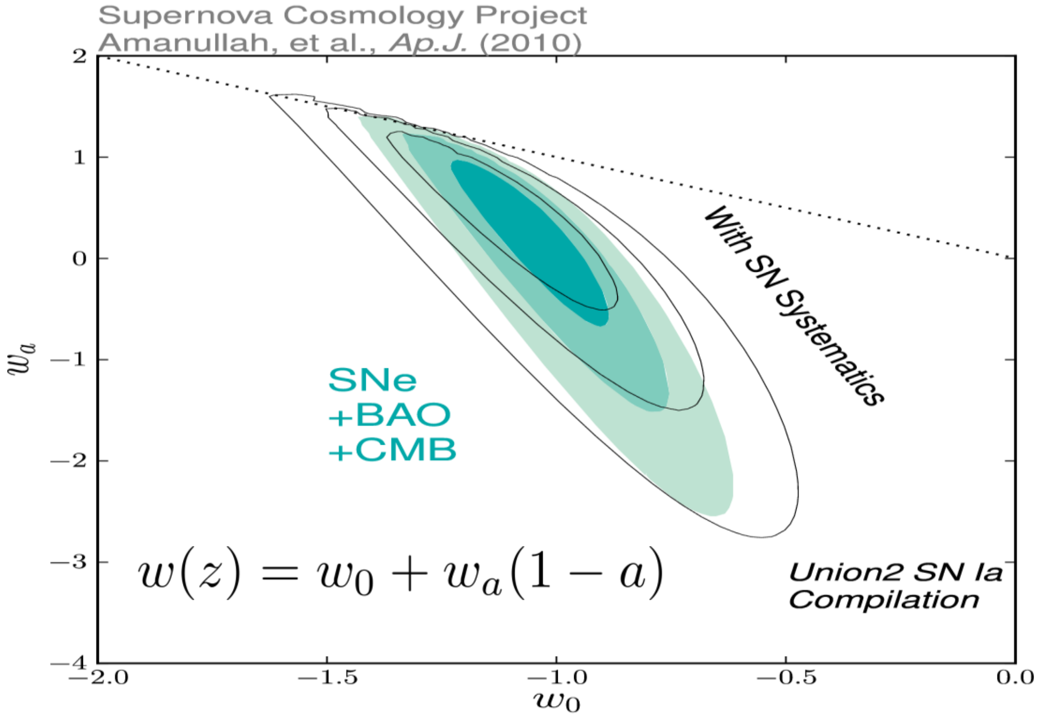
In these models, dark energy is dynamical and the equation of state is a time-dependent quantity. Falsifiable since different from the CC



Brax & Martin, astro-ph/9905040

Quintessence

- Hard to find good models of particle physics which lead to the correct potentials
- Hard to control the interactions of quintessence with the other fields
- Hard not to destroy the flatness of the potential by quantum corrections
- Everything seems to indicate that w=-1 …



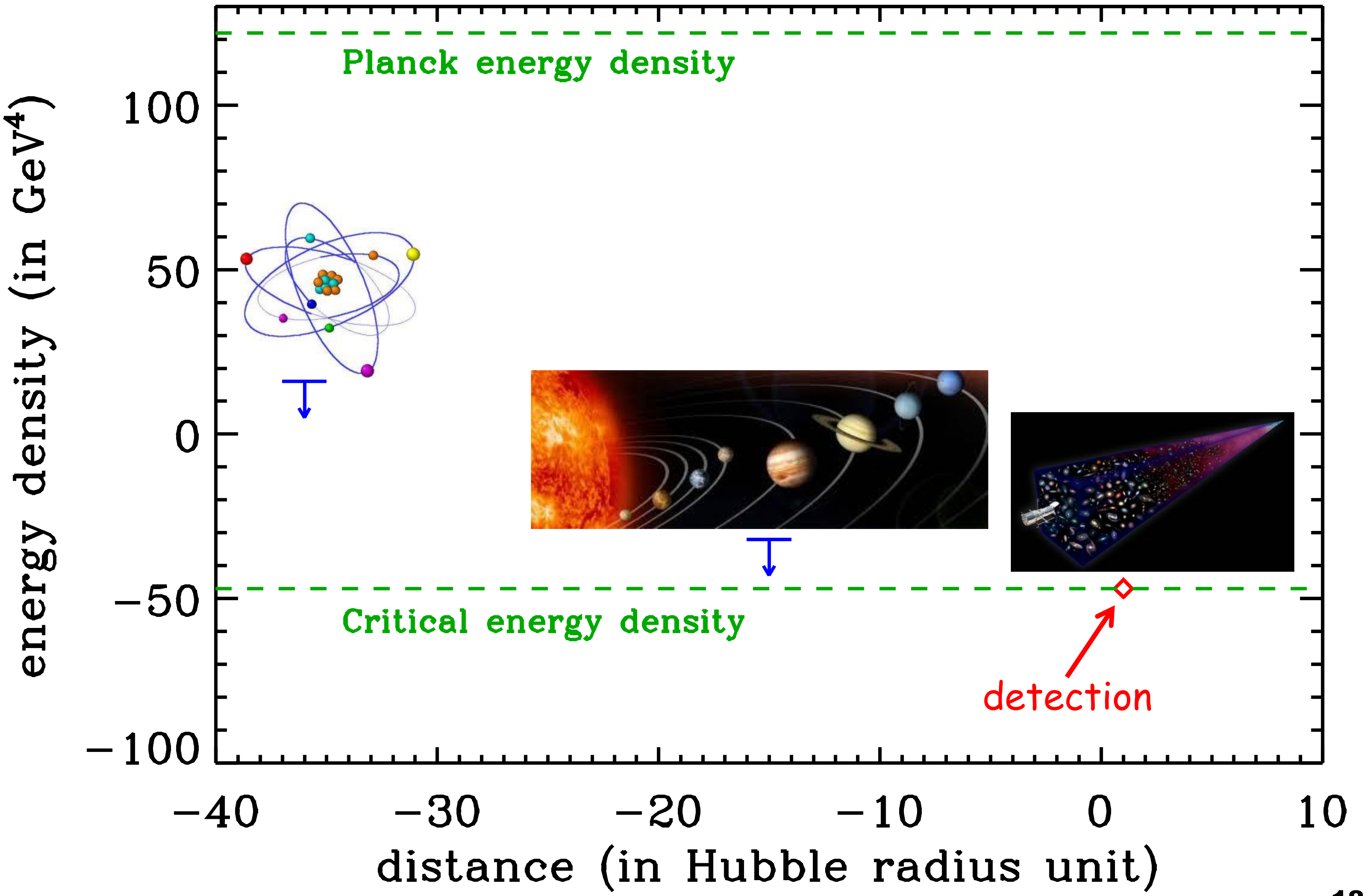
The cosmological constant

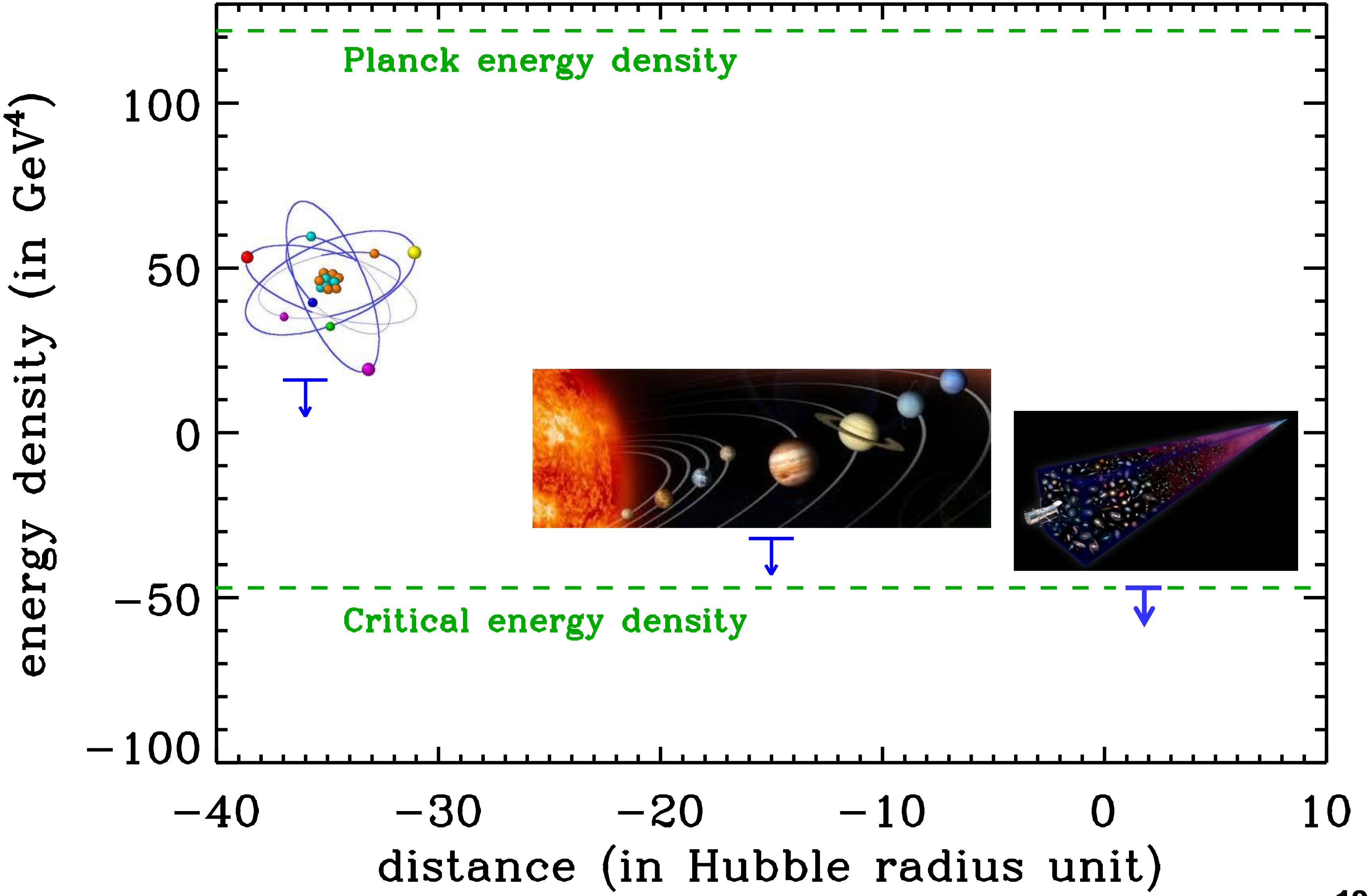
- The hard fact is that the following equation does not fit well the data

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho_{\rm CDM}$$

- If the Universe is homogeneous and isotropic and if gravity is described by GR and if there is no other exotic fluid then the CC is non-vanishing.
- In this framework, the Universe is accelerating.
- 2012: there is now a bunch of different and independent measurements pointing towards this conclusion. (age of the universe, SNIa, clusters abundance, lensing etc …)
- The other alternatives (in-homogeneous universe, modified gravity, quintessence etc …) have their own problems.
- Even if what we see in cosmology is not the CC, this implies a new upper limit on the CC energy density

The cosmological constant




Planck energy density
Critical energy density
100
50
0
−50
−100
−40
−30
−20
−10
0
10
energy density (in GeV^4)
distance (in Hubble radius unit)

The cosmological constant: summary of the classical discussion

- Therefore, the CC remains the simplest explanation of the different cosmological measurements

- There is no sign in the observations that we need a dark energy different from the CC

- At this (classical) level, we have a theory with a new fundamental constant and its value has been determined by the measurements to be

$$\Lambda_{\mathrm{B}} \simeq 10^{-52} m^{-2}$$

- The CC is such that it is very difficult to check this value elsewhere than in cosmology … always a negligible effect.

The cosmological constant: the quantum side

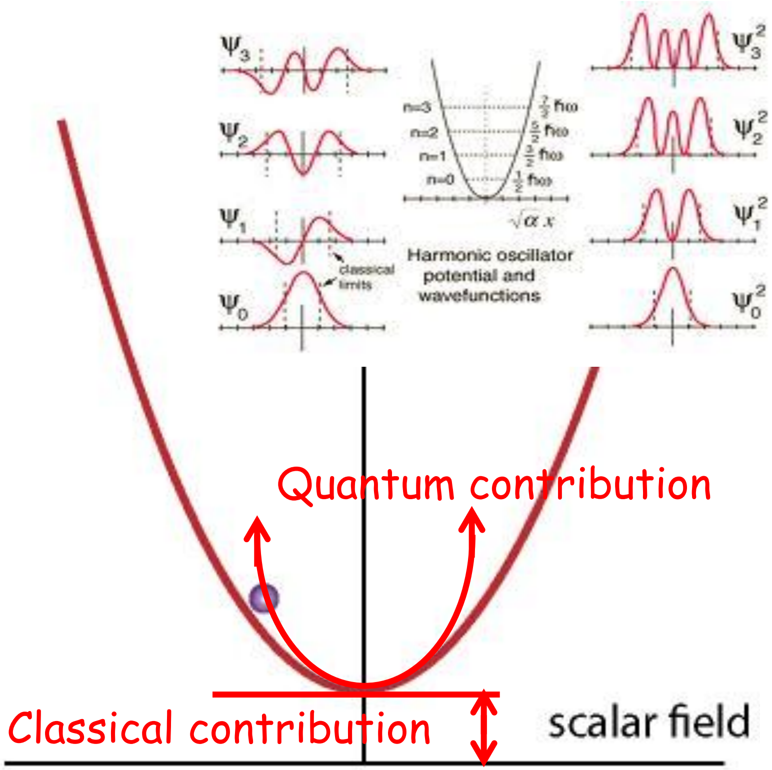
When QM and QFT are taken into account, the nature of the discussion is however drastically modified [A. Sakharov, Sov. Phys. Dokl. 12, 1040 (1968)]

- The vacuum state has the following stress-energy tensor

$$\langle T_{\mu\nu}\rangle = -\left[V\left(\phi_{\rm min}\right)+\rho_{\rm vac}\right]g_{\mu\nu}$$

$$\downarrow$$

$$\Lambda_{\rm eff}=\Lambda_{\rm B}+\frac{1}{M_{\rm Pl}^2}\left[V\left(\phi_{\rm min}\right)+\rho_{\rm vac}\right]$$

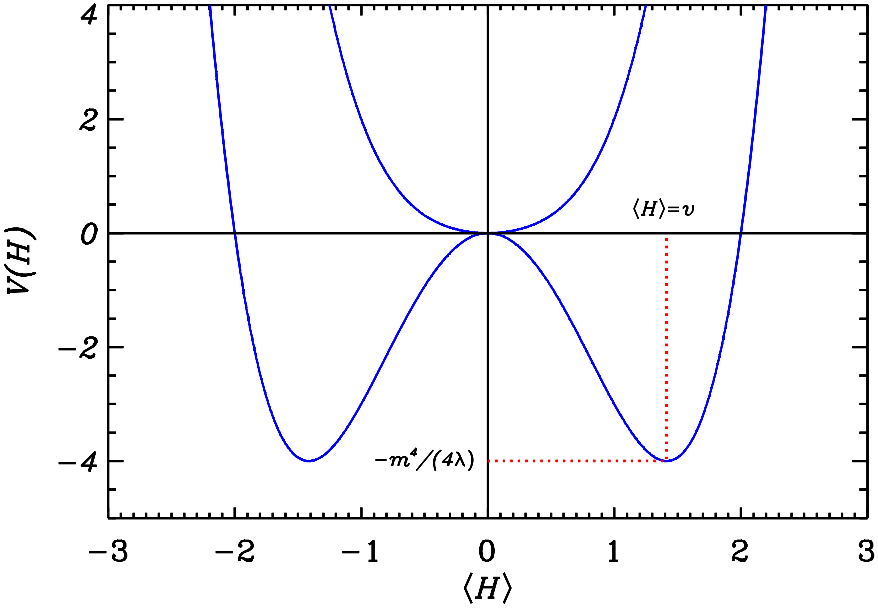


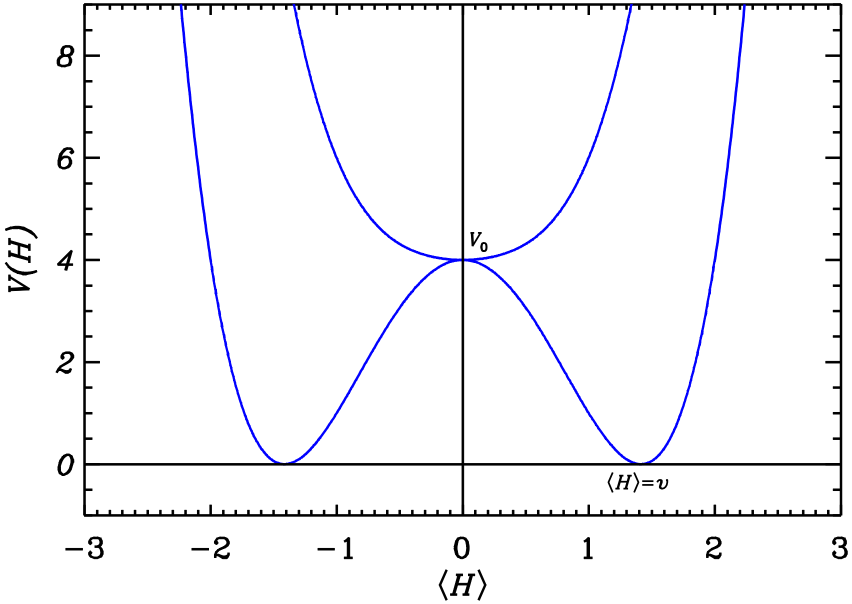
- In flat spacetime, only differences of energy are measurable so not important … In curved spacetime, the absolute value is important.

- A priori, the vacuum fluctuations gravitate as any other form of energy

The weigh of the vacuum

An example is the Electro-Weak transition





$$V\left(\phi_{\min}\right)=-\frac{1}{4}m_{\mathrm{H}}^{2}v^{2}=-\frac{\sqrt{2}}{16}\frac{m_{\mathrm{H}}^{2}}{G_{\mathrm{F}}^{2}}\simeq-1.2\times10^{8}\,\mathrm{GeV}^{4}$$

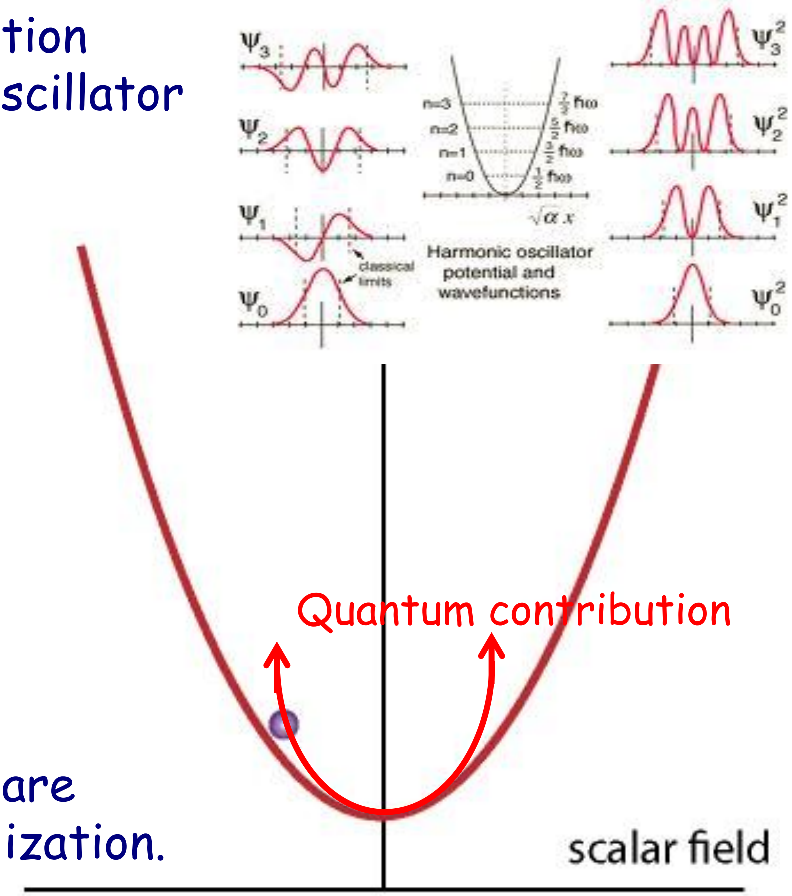
The cosmological constant: the quantum side

- Because of Heisenberg principle the position and the velocity of a quantum harmonic oscillator cannot vanish at the same time

$$\langle H \rangle = \frac{\hbar\omega}{2}$$

- A quantum field=infinite collections of quantum oscillators

$$\langle H \rangle = \sum \frac{\hbar\omega}{2} = \infty$$



- This should not cause any panic since we are used to tame infinities in QFT: renormalization.

- However, this particular type of infinity is usually not renormalized but ignored on the basis that, in flat spacetime, only differences of energies are measurable.

The weigh of the vacuum

The first attempt to estimate the gravitational impact of vacuum fluctuations was done by W. Pauli [see "Die allgemeinen Principein des Wellenmechanik"]

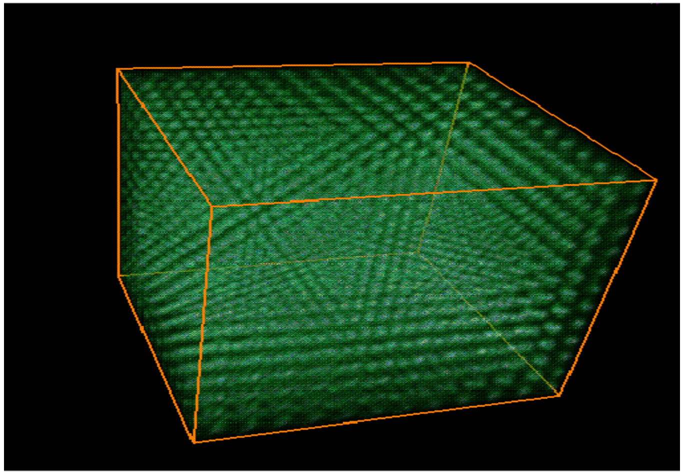
$$H^2 + \frac{k}{a^2} = \frac{1}{3M_{\rm Pl}^2}\rho + \frac{\Lambda}{3}$$

$$-\left(2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right) = \frac{1}{M_{\rm Pl}^2}p - \Lambda$$

Einstein static universe

$$a^2 = \frac{2M_{\rm Pl}^2}{\rho} \qquad a \simeq 31 \text{ kms}$$

$$\rho = \frac{1}{8\pi^2}\omega_{\rm max}^4$$



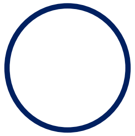
Radiation field in a box

"it could not even reach to the moon"

The cosmological constant & QFT

In a modern language, the main issue is how to renormalize the vacuum energy density

$$\Lambda_{\rm eff} = \Lambda_{\rm B} + \frac{1}{M_{\rm Pl}^2}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\,\frac{1}{2}\omega(\mathbf{k}) = \Lambda_{\rm B} - \frac{m^2}{4M_{\rm Pl}^2}$$

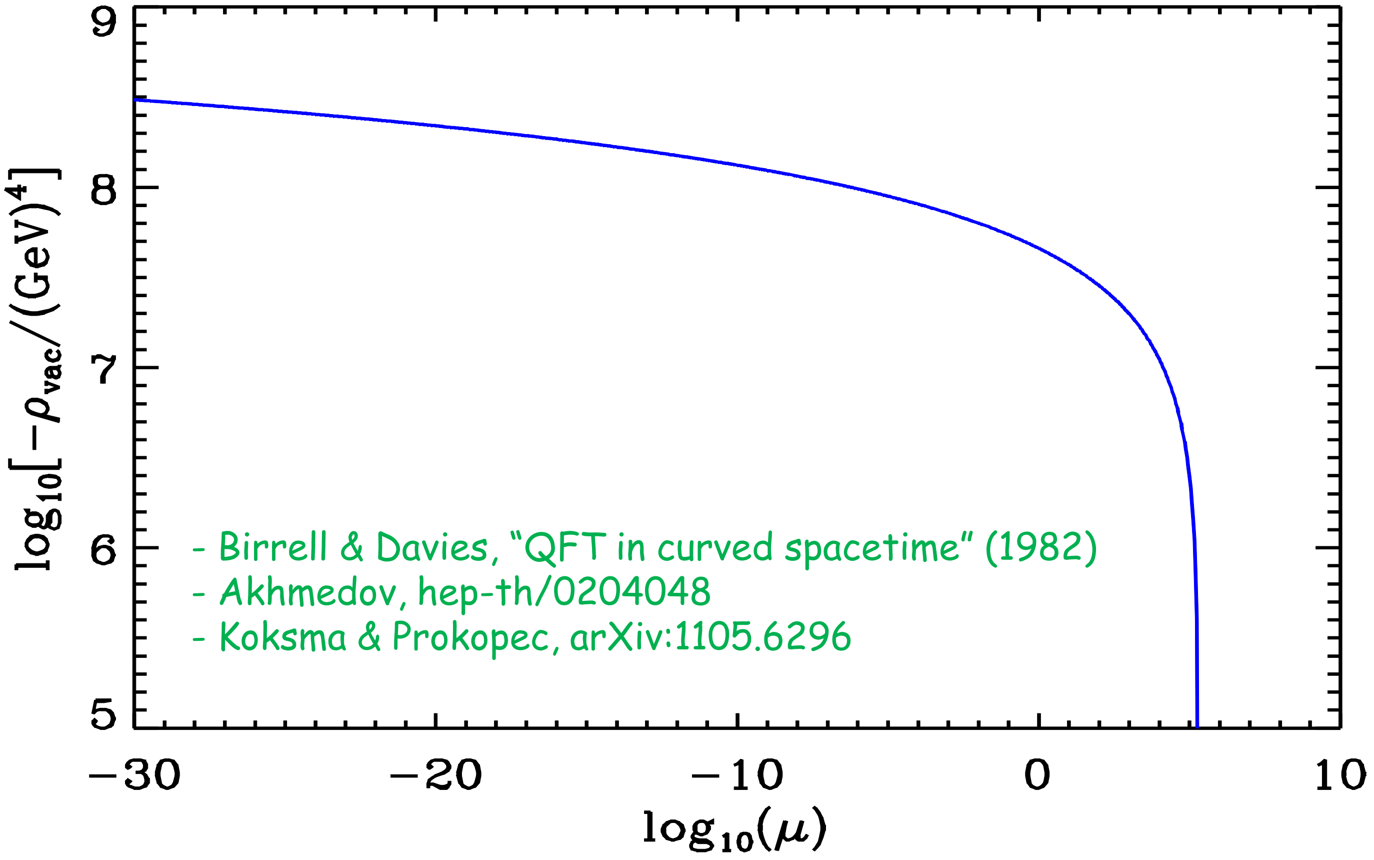


- The vacuum contribution is expressed in terms of Feynman bubble diagrams, ie diagrams with no external leg.
- These diagrams have bad convergence properties, worst than ordinary loop diagrams: they remain infinite even in the QM limit.
- In non-gravitational physics, these graphs always cancel out.
- When gravity is taken into account, one must regularize them.

Regularizing the cosmological constant

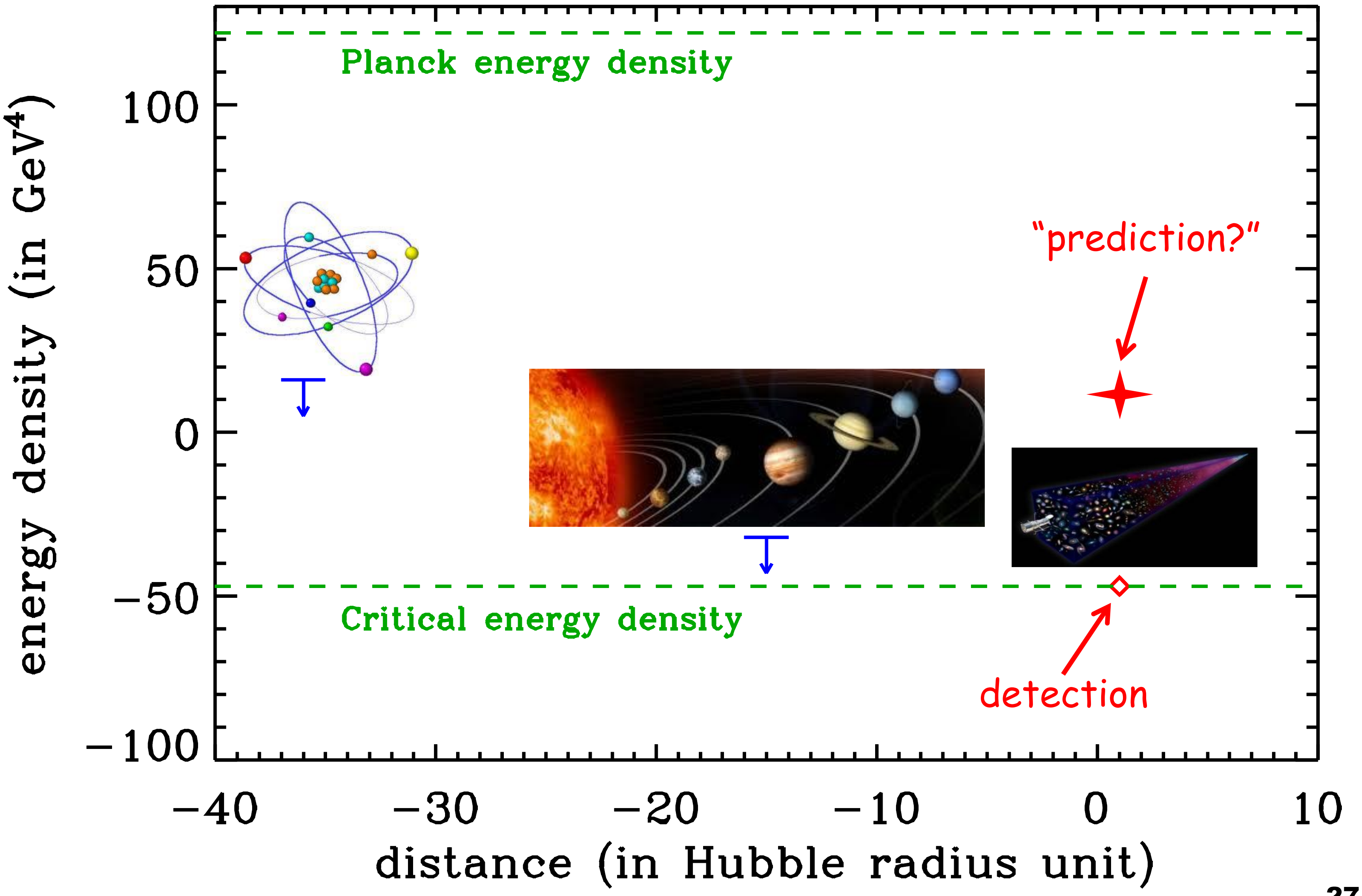
Renormalization leads to the following expression for the CC

$$\rho_{\rm vac} = \rho_{\rm B} + \rho_{\rm vac}^{\rm EW} + \rho_{\rm vac}^{\rm QCD} + \sum_i n_i \frac{m_i^4}{64\pi^2} \ln\left(\frac{m_i^2}{\mu^2}\right)$$



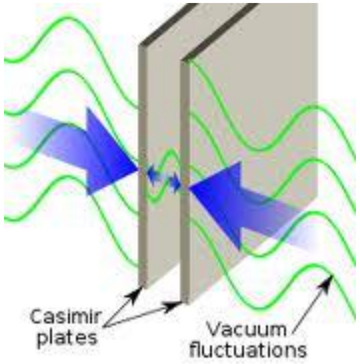
- Birrell & Davies, "QFT in curved spacetime" (1982)
- Akhmedov, hep-th/0204048
- Koksma & Prokopec, arXiv:1105.6296

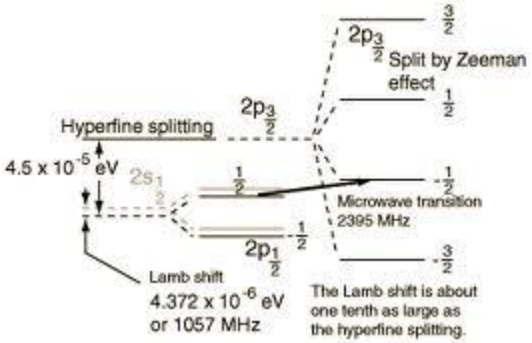
The value of the cosmological constant



The cosmological constant: possible loopholes

- A possible loophole is that vacuum fluctuations are just an artifact of QFT. However, we observe their influence in the Casimir effect or in the Lamb shift effect.





- Maybe vacuum fluctuations have abnormal gravitational properties?? But vacuum fluctuations participate for a non-negligible amount to the mass of nuclei … and they are observed to obey the UFF (WEP).

- What about the EP (UFF) in the quantum regime??

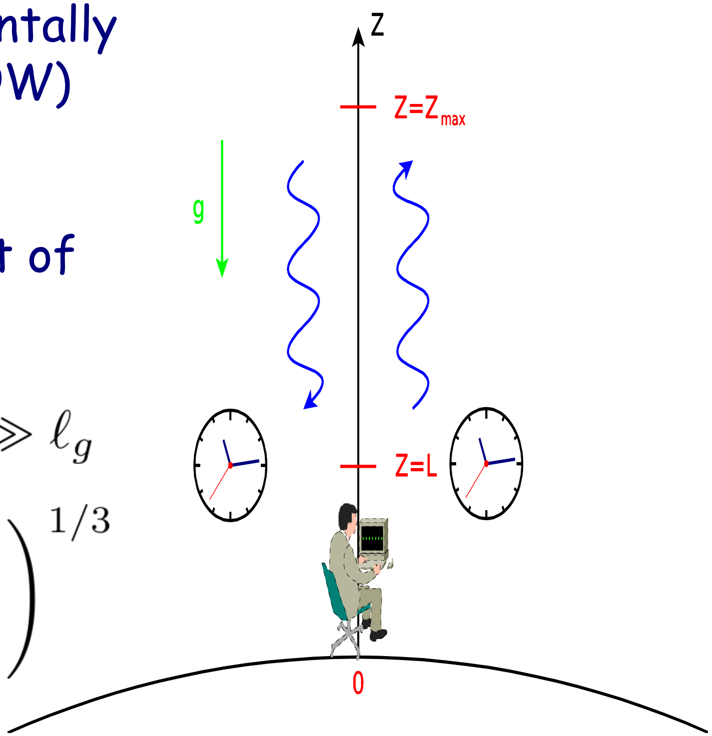
Gravitational coupling in the QM regime

The UFF in QM is described by the following Schrodinger equation

$$i\hbar\frac{\partial\Psi(t,z)}{\partial t} = -\frac{\hbar^2}{2m_{\rm ini}}\frac{\partial^2\Psi(t,z)}{\partial z^2} + m_{\rm grav}gz\Psi(t,z)$$

- The validity of this equation has been experimentally checked by the Collela Overhausser Werner (COW) experiment and by atomic interferometry.
- UFF can be checked by measuring times of flight of quantum particles.
- The classical result is recovered if $|L - z_{\rm max}| \gg \ell_g$

$$\ell_g = \left(\frac{\hbar^2}{2m_{\rm ini}m_{\rm grav}g}\right)^{1/3} = \left(\frac{\hbar^2 r_\oplus^2}{2m_{\rm ini}m_{\rm grav}GM_\oplus}\right)^{1/3}$$



One gram particle: $\ell_g = 10^{-10}\,\mathrm{m}$

Neutron: $\ell_g = 1.5\,\mathrm{mm}$

P. Davies, CQG 21 5677 (2004)

Conclusions:

- The cosmological constant problem is the impossibility to reconcile the renormalized value of vacuum energy with its observed value in cosmology and/or with the upper contraints obtained in others experimental situations.

- It is then natural to question the assumptions made to arrive at this result: failure of our renormalization technique, vacuum fluctuations=fake , abnormal gravitational properties of the vacuum etc …

- However, investigating these issues does not seem to reveal any inconsistencies (at the theoretical/observational level).

- It is frustrating that cosmology be the only situation where one can measure (and not only constrain) the CC!

- The CC problem is a deep problem since it lies at the crossroads between gravity and QM. In brief, the question is: what are the gravitational properties of the quantum vacuum?