

Maresuke Shiraishi, JGRG 22(2012)111426

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**RESCEU SYMPOSIUM ON
GENERAL RELATIVITY AND GRAVITATION**

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan



Parity violation in the CMB bispectrum

Maresuke Shiraishi^{1(a)}

^(a)*Department of Physics and Astrophysics, Nagoya University, Nagoya, Aichi, 464-8602, Japan*

Abstract

We analyze the response of the parity violation in the primordial non-Gaussianity to the CMB bispectrum. We find that the parity-violating non-Gaussianity creates *III*, *IIE*, *IEE*, *IBB*, *EEE*, *EBB* spectra obeying $\sum_{n=1}^3 \ell_n = \text{odd}$ and *IIB*, *IEB*, *EEB*, *BBB* spectra satisfying $\sum_{n=1}^3 \ell_n = \text{even}$. We confirm these features through the case where the parity-violating Weyl cubic action or primordial helical magnetic field exists.

1 Introduction

Non-Gaussian features in the cosmological perturbations include detailed information on the nature of the early Universe, and there have been many works that attempt to extract them from the bispectrum (three-point function) of the cosmic microwave background (CMB) anisotropies (e.g., Ref. [1]). However, most of these discussions are limited in the cases that the scalar-mode contribution dominates in the non-Gaussianity and also are based on the assumption of rotational invariance and parity conservation.

In contrast, there are several studies on the non-Gaussianities of not only the scalar-mode perturbations but also the vector- and tensor-mode perturbations [2]. These sources produce the additional signals on the CMB bispectrum [3] and can give a dominant contribution by considering such highly non-Gaussian sources as the stochastic magnetic fields [4]. Furthermore, even in the CMB bispectrum induced from the scalar-mode non-Gaussianity, if the rotational invariance is violated in the non-Gaussianity, the characteristic signals appear [5]. Thus, it is very important to clarify these less-noted signals to understand the precise picture of the early Universe.

This paper discusses how the parity violation in the primordial non-Gaussianity affects in the CMB bispectrum. The effects on the cosmic microwave background (CMB) have been well-studied and the cosmological parity violation has been verified by analyzing the non-vanishing cross-correlated power spectra between the intensity (*I*) and *B*-mode polarization (*B*) anisotropies and those between *E*-mode (*E*) and *B*-mode polarization anisotropies. Furthermore, beyond the linear-order effects, the impacts of the parity violation on the graviton non-Gaussianities have recently been discussed [6, 7]. According to the concept of this literature, in Ref. [8], we have formulated the CMB bispectrum generated from the parity-violating graviton non-Gaussianities and find special signatures of primordial parity violation. Moreover, Ref. [9] have evaluated the detectability of the parity-violating non-Gaussianity by using the CMB *III* bispectrum under an assumption of the existence of large-scale helical magnetic fields.

This paper corresponds to a review of our papers [8, 9]. Detailed organization is as follows. In the next section, we summarize the effects of the parity in the primordial non-Gaussianity on the CMB bispectrum. In Secs. 3 and 4, we present the CMB bispectra sourced from the non-Gaussianities due to the parity-violating Weyl gravity and primordial helical magnetic fields. The final section is devoted to summary and discussion.

2 Response of parity in non-Gaussianity to CMB bispectrum

The CMB fluctuation on any direction, $\hat{\mathbf{n}}$, is quantified in multipole space as

$$\frac{\Delta X(\hat{\mathbf{n}})}{X} = \sum_{\ell m} a_{X,\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \quad (1)$$

¹Email address: mare@nagoya-u.jp

where X denotes the intensity (I) and two polarization fields (E, B). If keeping the rotational invariance of the primordial non-Gaussianity, the CMB bispectrum is also defined in multipole space as

$$B_{X_1 X_2 X_3, \ell_1 \ell_2 \ell_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle \prod_{n=1}^3 a_{X_n, \ell_n m_n} \right\rangle \quad (2)$$

The multipole configuration of the CMB bispectrum can be easily understood only if one consider the parity transformation of the CMB intensity and polarization fields in real space. III, IIE, IEE, IBB, EEE and EBB spectra from parity-conserving non-Gaussianity, and $IIB, IEB, EEB,$ and BBB spectra from parity-violating non-Gaussianity have even parity, namely,

$$\left\langle \prod_{i=1}^3 \frac{\Delta X_i(\hat{\mathbf{n}}_i)}{X_i} \right\rangle = \left\langle \prod_{i=1}^3 \frac{\Delta X_i(-\hat{\mathbf{n}}_i)}{X_i} \right\rangle. \quad (3)$$

Then, from the multipole expansion (1) and its parity flip version as

$$\frac{\Delta X(-\hat{\mathbf{n}})}{X} = \sum_{\ell m} a_{X, \ell m} Y_{\ell m}(-\hat{\mathbf{n}}) = \sum_{\ell m} (-1)^\ell a_{X, \ell m} Y_{\ell m}(\hat{\mathbf{n}}), \quad (4)$$

one can notice that $\sum_{n=1}^3 \ell_n = \text{even}$ must be satisfied. On the other hand, since $IIB, IEB, EEB,$ and BBB spectra from parity-conserving non-Gaussianity, and $III, IIE, IEE, IBB, EEE,$ and EBB spectra from parity-violating non-Gaussianity have odd parity, namely,

$$\left\langle \prod_{i=1}^3 \frac{\Delta X_i(\hat{\mathbf{n}}_i)}{X_i} \right\rangle = - \left\langle \prod_{i=1}^3 \frac{\Delta X_i(-\hat{\mathbf{n}}_i)}{X_i} \right\rangle, \quad (5)$$

one can obtain $\sum_{n=1}^3 \ell_n = \text{odd}$.

3 CMB signatures from parity-violating Weyl gravity

Primordial non-Gaussianity from parity-odd (parity-violating) Weyl cubic action has been firstly reported by Ref.[6] in the context of ultraviolet modification of gravity. After that, careful treatments by Ref. [7] have shown that resultant parity-violating non-Gaussianity of the primordial gravitational waves emerges not in the exact de Sitter space-time but in the quasi de Sitter space-time, and hence, its amplitude is proportional to a slow-roll parameter. In these studies, the authors have assumed that the coupling constant of the Weyl cubic terms is independent of time.

In this section, we focus on the graviton non-Gaussianity generated from the parity-violating Weyl gravity with the running coupling constant as a function of a conformal time, $f(\tau)$, whose action is given by

$$S = \int d\tau d^3x \frac{f(\tau)}{\Lambda^2} \epsilon^{\alpha\beta\mu\nu} W_{\mu\nu\gamma\delta} W^{\gamma\delta}{}_{\sigma\rho} W^{\sigma\rho}{}_{\alpha\beta}. \quad (6)$$

Here, $W^{\alpha\beta}{}_{\gamma\delta}$ denotes the Weyl tensor, $\epsilon^{\alpha\beta\mu\nu}$ is a 4D Levi-Civita tensor normalized as $\epsilon^{0123} = 1$, and Λ is a scale that sets the value of the higher derivative corrections [6]. The coupling constant is assumed as the power-law type: $f(\tau) = (\tau/\tau_*)^A$, where τ is a conformal time. Here, we have set $f(\tau_*) = 1$. Such a coupling can be readily realized by considering a dilaton-like coupling in the slow-roll inflation. In this case, depending on time dependence of coupling, we can realize finite non-Gaussianity of gravitational waves even in the exact de Sitter space time. Note that primordial tensor bispectrum from such non-Gaussianity is categorized as the equilateral type.

Figure. 1 shows an example of the CMB III and BBB bispectra from such primordial tensor bispectrum. Here, we plot the reduced bispectrum defined by

$$b_{X_1 X_2 X_3, \ell_1 \ell_2 \ell_3} \equiv (G_{\ell_1 \ell_2 \ell_3})^{-1} B_{X_1 X_2 X_3, \ell_1 \ell_2 \ell_3} \quad (7)$$

with

$$G_{\ell_1 \ell_2 \ell_3} \equiv \frac{2\sqrt{\ell_3(\ell_3+1)\ell_2(\ell_2+1)}}{\ell_1(\ell_1+1) - \ell_2(\ell_2+1) - \ell_3(\ell_3+1)} \sqrt{\frac{\prod_{n=1}^3 (2\ell_n+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & -1 & 1 \end{pmatrix}. \quad (8)$$

It is observed that *III* and *BBB* spectra obey $\sum_{n=1}^3 \ell_n = \text{odd}$ and $\sum_{n=1}^3 \ell_n = \text{even}$, respectively. These are consistent with the discussion in the previous section and characteristic imprints of the parity-violating non-Gaussianity.

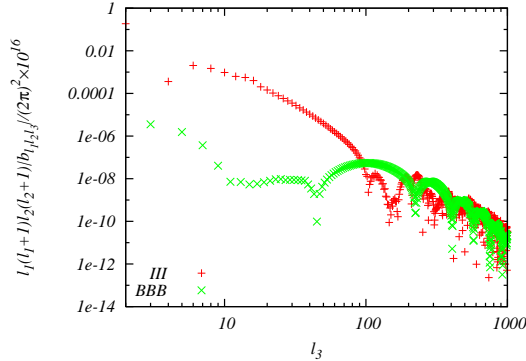


Figure 1: Absolute values of the CMB *III* and *BBB* spectra from parity-violating graviton non-Gaussianity. Here, three multipoles are fixed as $\ell_1 - 2 = \ell_2 - 1 = \ell_3$. Here, we fix the parameters as $\Lambda = 3 \times 10^6 \text{ GeV}$, $A = 1$ and $\tau_* = -k_*^{-1} = -14 \text{ Gpc}$, and other cosmological parameters are fixed as the mean values limited from the WMAP 7-yr data [10].

4 CMB signatures from helical magnetic fields

If there exists the primordial magnetic field (PMF), which is a favored candidate for the seed field of microgauss-level magnetic fields in galaxies and cluster of galaxies, their power spectrum may involve the parity-violating component. Assuming Gaussianity of such PMF, beneficial signals are generated in the CMB bispectra due to the quadratic dependence of the CMB fluctuation on the PMF. In the case where only parity-conserving component exists, the contributions of PMFs to the primordial non-Gaussianities and the CMB bispectra have been deeply investigated in e.g., Ref. [4].

In this section, let us consider the effects of PMFs including the parity violation on the CMB bispectrum. We shall start from the convention of the parametrization for the PMF as

$$\langle B_a(\mathbf{k}) B_b(\mathbf{k}') \rangle = \frac{(2\pi)^3}{2} \left[P_B(k) P_{ab}(\hat{\mathbf{k}}) + i\eta_{abc} \hat{k}_c P_B(k) \right] \delta(\mathbf{k} + \mathbf{k}'), \quad (9)$$

where $\hat{\mathbf{k}}$ is a unit vector, η_{abc} is the 3D Levi-Civita tensor normalized by $\eta_{123} = 1$, and $P_{ab}(\hat{\mathbf{k}}) \equiv \delta_{ab} - \hat{k}_a \hat{k}_b$ is a projection tensor coming from the divergence free nature of PMFs. The first and second terms in the bracket of r.h.s. represent non-helical and helical contributions, respectively. The magnetic anisotropic stress, which depends quadratically on the PMF, acts as a source of metric perturbations prior to neutrino decoupling. Then, due to Gaussianity of the PMF, resulting curvature perturbation and gravitational wave become highly non-Gaussian fields. Such non-Gaussianities result in finite CMB bispectra of scalar and tensor modes.

Figure 2 describes the signal-to-noise ratio from the parity-violating components of the CMB *III* bispectrum ($\sum_{n=1}^3 \ell_n = \text{odd}$) in the cosmic variance limit. As shown in this figure, at large scales, tensor mode dominates over scalar one because of the ISW amplification. Note that the auto-correlated bispectrum of scalar modes is exactly zero because the scalar mode has no spin dependence and cannot hold the information on parity. The parity-odd *III* bispectrum depends on $P_B^2 P_B$. Introducing the PMF strengths smoothed on 1Mpc, namely, $B_{1\text{Mpc}} (\propto P_B^{1/2})$ and $\mathcal{B}_{1\text{Mpc}} (\propto P_B^{1/2})$, we can conclude that the signal-to-noise ratio from parity-odd *III* bispectrum exceeds unity if $B_{1\text{Mpc}}^{2/3} \mathcal{B}_{1\text{Mpc}}^{1/3} > 2.3 \text{ nG}$.

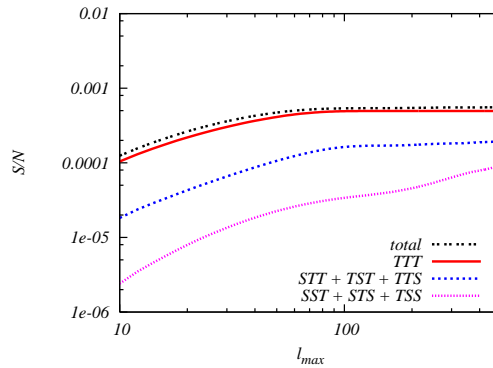


Figure 2: Signal-to-noise ratios from the parity-odd CMB III bispectra coming from $\sum_{n=1}^3 \ell_n = \text{odd}$, respectively, in the cosmic variance limit. The “total” line denotes S/N obtained from the total spectrum of $TTT, SST, TST, TTS, SST, STS$ and TSS modes, and the others correspond to S/N 's coming from each mode. Here, we fix the PMF parameters as $B_{1\text{Mpc}} = 1.0\text{nG}, \mathcal{B}_{1\text{Mpc}} = 0.287\text{nG}$, and assume the PMF generation at the GUT scale and nearly scale invariance of P_B and $P_{\mathcal{B}}$. The other parameters are identical to the mean values obtained from the WMAP-7yr data [10].

5 Summary and discussion

In this paper, we have summarized the impacts of parity-violating non-Gaussianities on the CMB bispectra. Non-Gaussian level parity violation limits the multipole configurations of the CMB bispectra. That is, $III, IIE, IEE, IBB, EEE, EBB$ spectra obey $\sum_{n=1}^3 \ell_n = \text{odd}$, and IIB, IEB, EEB, BBB spectra arise if satisfying $\sum_{n=1}^3 \ell_n = \text{even}$. Actually, these special properties associated with parity-violating non-Gaussianity can be predicted in the several early Universe models, such as the Weyl gravity and inflation with the electromagnetic field [8, 9]. Constraining these models from the CMB observation remains as a future issue.

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