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"Inflation in bimetric gravity"

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Inflation in Bimetric Gravity

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What is bimetric gravity?

Introduce massive graviton
→ "massive gravity theory"

When there is general covariance, graviton cannot have mass.

↓

Introduce "reference metric".

↓

It breaks general covariance, then graviton can have mass.

* reference metric is non dynamical in massive gravity

↓

Make the reference metric dynamical

bi(metric)gravity

Bigravity action

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}] + m^2 M_e^2 \int d^4x\sqrt{-g}F_2$$

EH action of $g_{\mu\nu}$ | EH action of $f_{\mu\nu}$ | Interaction term

$g_{\mu\nu}$: physical metric

$f_{\mu\nu}$: reference metric

$$F_2[L^\mu_\nu] = \frac{1}{2}([L]^2 - [L^2])$$

trace

$$L^\mu_\nu = \delta^\mu_\nu - (\sqrt{g^{-1}f})^\mu_\nu$$

m^2 : coupling constant

$$M_e^2 = \left(\frac{1}{M_g^2} + \frac{1}{M_f^2}\right)^{-1}$$

: reduced Plank scale

The motivation of our study

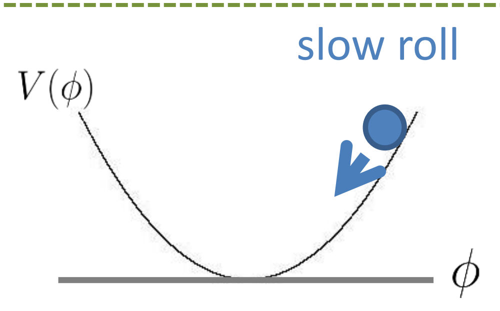
We would like to investigate dynamics of spacetime with matter in bimetric gravity.

As the simple case,
we think of bimetric theory with cosmological constants.

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left(R[g_{\mu\nu}] - 2\Lambda_g\right) + \frac{M_f^2}{2}\int d^4x\sqrt{-f}\left(R[f_{\mu\nu}] - 2\Lambda_f\right) + m^2 M_e^2 \int d^4x\sqrt{-g}F_2[L^\mu_\nu],$$

* We can think of cosmological constants
  as scalar fields in slow roll approximation.

We can also discuss inflation.



de Sitter solution

$$a_g = \frac{M_e^2}{M_g^2}, \quad \xi = \frac{m^2}{M_e^2}, \quad \lambda_g = \frac{\Lambda_g}{3M_e^2}, \quad \lambda_f = \frac{\Lambda_f}{3M_e^2}, \quad {}' = \frac{1}{M_e}\frac{\mathrm{d}}{\mathrm{d}t}.$$

homogeneous metric ansatz

$$ds^2 = -N^2(t)dt^2 + e^{2\alpha}(t)[dx^2 + dy^2 + dz^2]$$

$$ds'^2 = -M^2(t)dt^2 + e^{2\beta}(t)[dx^2 + dy^2 + dz^2]$$

From variational principle of action

$$\epsilon = e^{\beta - \alpha}$$

$$\left(\frac{\alpha'}{N}\right)' - \xi a_g (M - N\epsilon)\left(\frac{3}{2} - \epsilon\right) = 0,$$: EoM of α

$$\left(\frac{\beta'}{M}\right)' + \xi (1 - a_g)\epsilon^{-3}(M - N\epsilon)\left(\frac{3}{2} - \epsilon\right) = 0,$$: EoM of β

$$\left(\frac{\alpha'}{N}\right)^2 = \lambda_g + \xi a_g (2 - \epsilon)(\epsilon - 1),$$: constraint (from variation with respect to N)

$$\left(\frac{\beta'}{M}\right)^2 = \lambda_f + \xi (1 - a_g)\epsilon^{-3}(1 - \epsilon).$$: constraint (from variation with respect to M)

$$\xi\left(\frac{3}{2} - \epsilon\right)\left(\frac{\beta' e^{\beta}}{M} - \frac{\alpha' e^{\alpha}}{N}\right) = 0$$: consistency relation (secondary constraint)

de Sitter solution is represented as positive roots of $g(\epsilon)$: ϵ_0 = const. → $\alpha' = \beta' = H_0$

$$g(\epsilon) = (\lambda_f + \xi a_g)\epsilon^3 - 3\xi a_g \epsilon^2 + [-\lambda_g + 2\xi a_g - \xi(1 - a_g)]\epsilon + \xi(1 - a_g) = 0.$$

de Sitter solution

$$a_g = \frac{M_e^2}{M_g^2}, \quad \xi = \frac{m^2}{M_e^2}, \quad \lambda_g = \frac{\Lambda_g}{3M_e^2}, \quad \lambda_f = \frac{\Lambda_f}{3M_e^2}, \quad {}' = \frac{1}{M_e}\frac{\mathrm{d}}{\mathrm{d}t}.$$

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$$\left(\frac{\beta'}{M}\right)^2 = \lambda_f + \xi(1 - a_g)\epsilon^{-3}(1 - \epsilon).$$: constraint (from variation with respect to M)

$$\xi\left(\frac{3}{2} - \epsilon\right)\left(\frac{\beta' e^\beta}{M} - \frac{\alpha' e^\alpha}{N}\right) = 0$$: con

Expansion rate (Hubble)

$$H_0^2 = \lambda_g + \xi a_g(2 - \epsilon_0)(\epsilon_0 - 1)$$

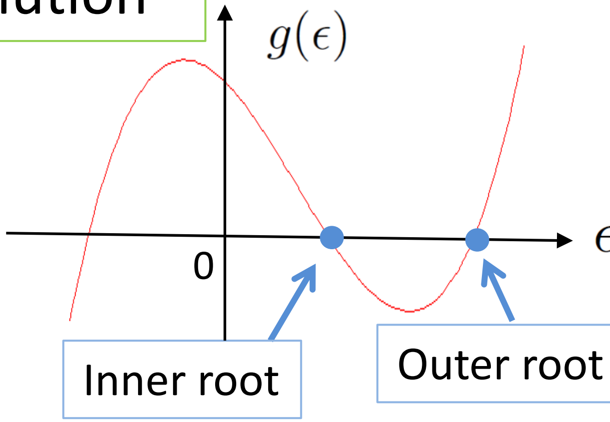
de Sitter solution is represented as positive roots of $g(\epsilon)$: ϵ_0 = const. → $\alpha' = \beta' = H_0$

$$g(\epsilon) = (\lambda_f + \xi a_g)\epsilon^3 - 3\xi a_g \epsilon^2 + [-\lambda_g + 2\xi a_g - \xi(1 - a_g)]\epsilon + \xi(1 - a_g) = 0.$$

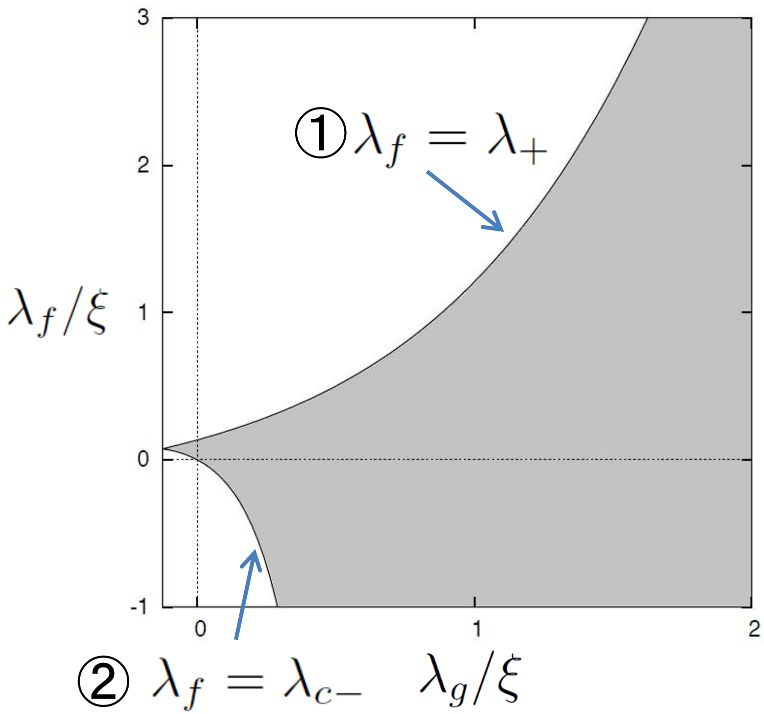
The conditions for the existence of de Sitter solution

Condition① : There exist positive roots of $g(\epsilon)$

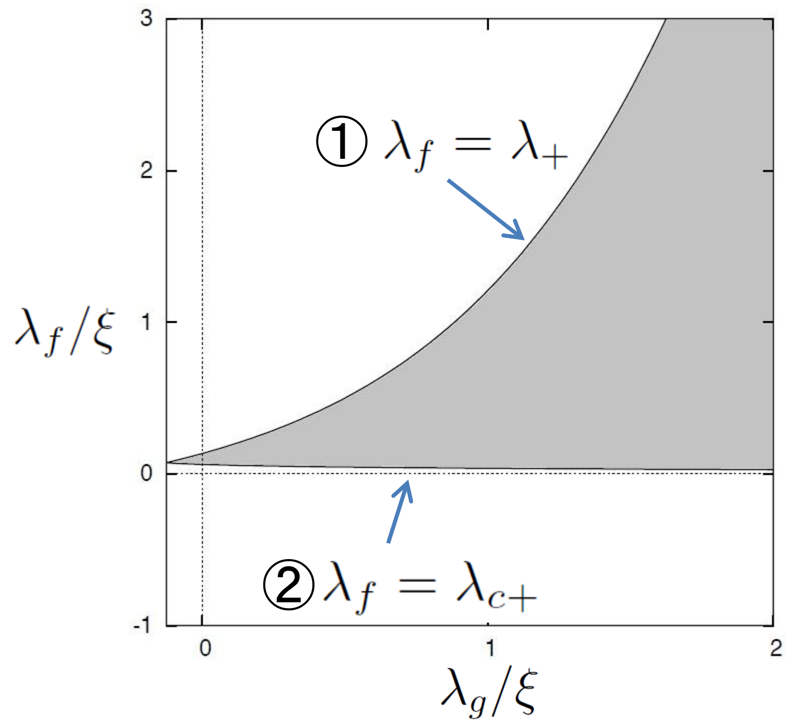
Condition② : The roots satisfy $H_0^2 > 0$



Inner root



Outer root



where $\lambda_+ = \lambda_+(\xi, a_g, \lambda_g)$, $\lambda_{c\pm} = \xi(1-a_g)\left(\frac{1}{2} \pm \sqrt{\frac{\lambda_g}{\xi a_g} + \frac{1}{4}}\right) \Big/ \left(\frac{3}{2} \pm \sqrt{\frac{\lambda_g}{\xi a_g} + \frac{1}{4}}\right)^3$

Anisotropic perturbation

anisotropic ansatz

$$ds^2 = -N^2(t)dt^2 + e^{2\alpha(t)}[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2)],$$

$$ds'^2 = -M^2(t)dt^2 + e^{2\beta(t)}[e^{-4\lambda(t)}dx^2 + e^{2\lambda(t)}(dy^2 + dz^2)],$$

From variational principle of action,

$$\sigma'' + 3H_0\sigma' - \xi a_g \epsilon_0 (3 - 2\epsilon_0) q = 0$$: EoM of σ

$$\lambda'' + 3H_0\lambda' + \xi(1 - a_g)\frac{1}{\epsilon_0}(3 - 2\epsilon_0) q = 0$$: EoM of λ

$$q = \lambda - \sigma$$

From the difference of EoMs,

$$q'' + 3H_0 q' + \xi\left[a_g\epsilon_0 + (1 - a_g)\frac{1}{\epsilon_0}\right](3 - 2\epsilon_0) q = 0$$

$$\omega_0^2 = \xi\left[a_g\epsilon_0 + (1 - a_g)\frac{1}{\epsilon_0}\right](3 - 2\epsilon_0)$$: Effective mass of massive graviton

The stability towards the anisotropic perturbation

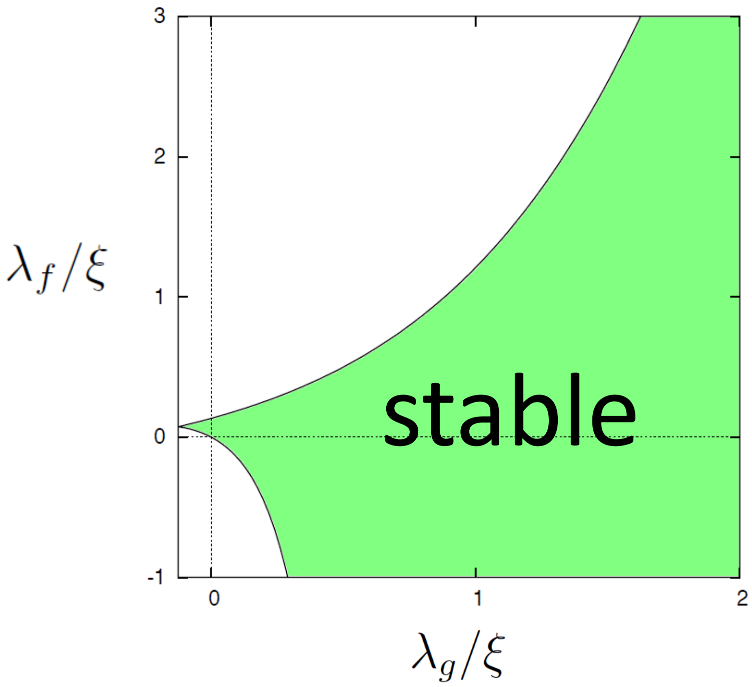
$$\omega_0^2 = \xi\left[a_g\epsilon_0 + (1-a_g)\frac{1}{\epsilon_0}\right](3-2\epsilon_0)$$

positive definite

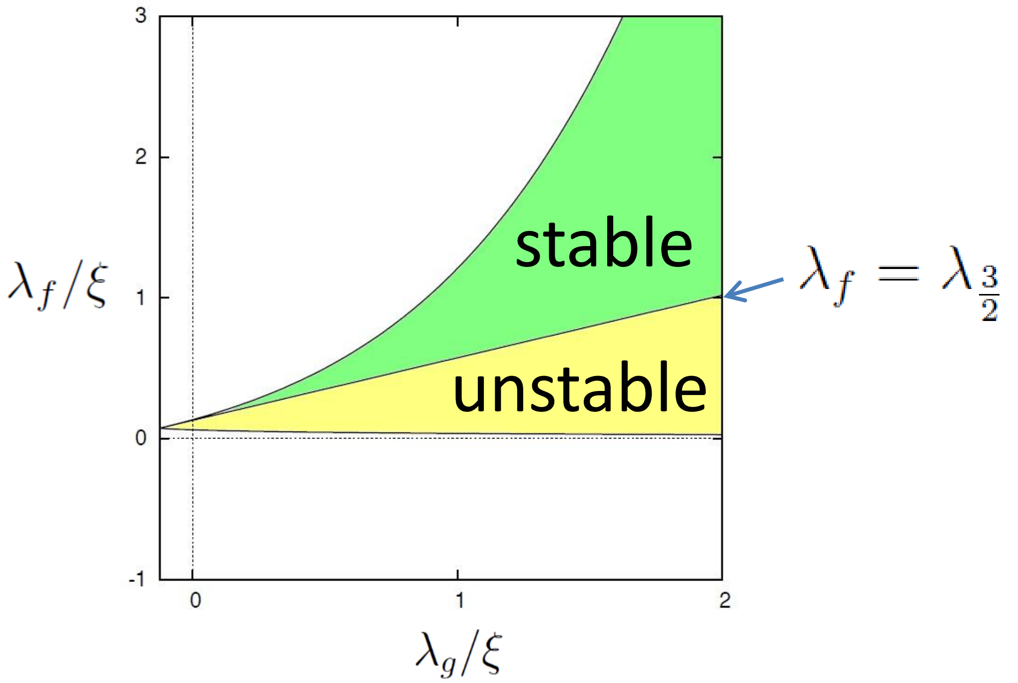
whether ϵ_0 is larger/smaller than $\frac{3}{2}$

The stability is determined by the sign of oscillational term

Inner root



Outer root



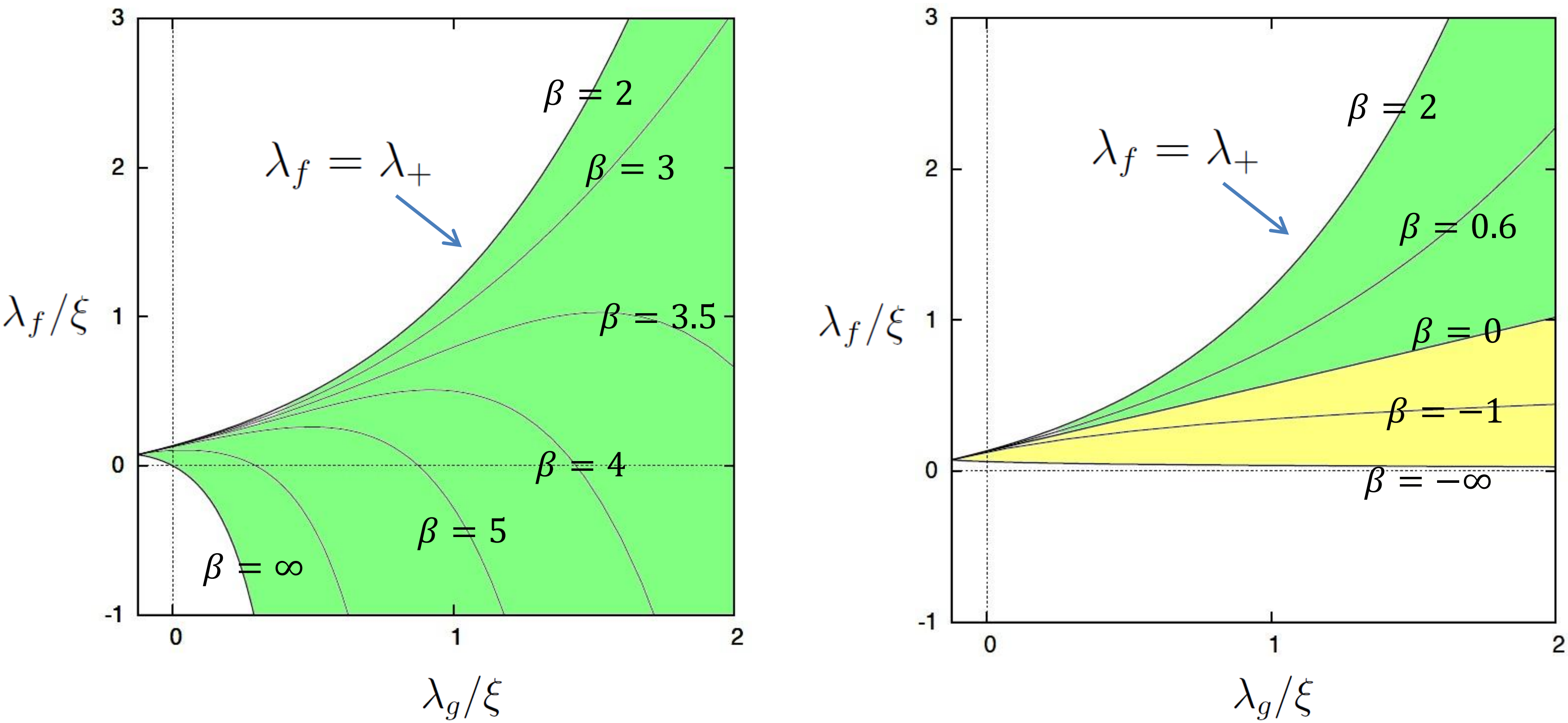
where $\lambda_{\frac{3}{2}} = \frac{4}{27}\left[3\left(\lambda_g + \frac{\xi a_g}{4}\right) + \xi(1-a_g)\right]$

Evaluation of effective mass ω_0^2

$$\beta = \frac{\omega_0^2}{H_0^2} = \frac{\xi[a_g\epsilon_0 + (1-a_g)\frac{1}{\epsilon_0}](3-2\epsilon_0)}{\lambda_g + \xi a_g(2-\epsilon_0)(\epsilon_0 - 1)}$$

: the ratio of effective mass to Hubble scale

Inner root ➡ $\beta > 2$ Outer root ➡ $\beta < 2$



Effective mass exactly equals to $2H_0^2$ on $\lambda_f = \lambda_+$!

Summery

(1) homogeneous isotropic metric ansatz

➡ the condition that de Sitter solution exists

There are two series of solutions: inner root and outer root.

(2) anisotropic perturbation around de Sitter sol.

➡ the stability for the perturbation

- inner root → stable
- outer root → stable for $\lambda_f > \lambda_{\frac{3}{2}}$ and unstable for $\lambda_f < \lambda_{\frac{3}{2}}$

➡ effective mass of massive graviton corresponding to the anisotropy.

- inner root → $\omega_0^2 > 2H_0^2$
- outer root → $\omega_0^2 < 2H_0^2$

For inner root, effective mass is bounded above Hubble scale.
If we consider inflation then the anisotropy decays in inflation time scale.

Future work

When we consider perturbations on de Sitter background in massive gravity, the square of graviton mass should be larger than $2H_0^2$ (Higuchi bound)

ref. A.Higuchi, Nucl,Phys. B 282,397 (1987)

- In our analysis,

Effective mass exactly equals to $2H_0^2$ on $\lambda_f = \lambda_+$!

i.e. the critical condition for the existence of de Sitter solutions

⇕ coincide with

Higuchi bound

- Is there really the relation between them ?
  - explicitly calculating the mass bound of massive graviton in bimetric gravity

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}]$$
$$+m^2M_e^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n[(\sqrt{g^{-1}f})^{\mu}_{\nu}]$$

$$e_n[X^{\mu}_{\nu}] = \frac{(-1)^n}{n!}\sum_{\sigma\in S_n}\mathrm{sig}(\sigma)X^{\mu_{\sigma(1)}}_{\mu_1}X^{\mu_{\sigma(2)}}_{\mu_2}\cdots X^{\mu_{\sigma(n)}}_{\mu_n}$$

equivalent

$$\beta_0 = \alpha_0 - 4\alpha_1 + 6\alpha_2 - 4\alpha_3 + \alpha_4$$ ← cosmological const of physical metric

$$\beta_1 = \alpha_1 - 3\alpha_2 + 3\alpha_3 - \alpha_4$$
$$\beta_2 = \alpha_2 - 2\alpha_3 + \alpha_4$$
$$\beta_3 = \alpha_3 - \alpha_4$$
$$\beta_4 = \alpha_4$$ ← cosmological constant of reference metric

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}]$$
$$+m^2M_e^2\int d^4x\sqrt{-g}\sum_{n=0}^{4}\alpha_n e_n[(1-\sqrt{g^{-1}f})^{\mu}_{\nu}]$$

The condition for the existence of de Sitter solution

* We fix ξ, a_g, λ_g and vary λ_f in the following.
  Then root of $g(\epsilon)$ is function of λ_f . → $\epsilon_0 = \epsilon_0(\lambda_f)$

Condition(1) : There exist positive roots of $g(\epsilon)$

→ $$\lambda_f \le \lambda_+(\xi, a_g, \lambda_g)$$

Condition(2) : The roots satisfy $H_0^2 > 0$

Expansion rate is determined from constraint.

$$H_0^2 = \lambda_g + \xi a_g (2 - \epsilon_0)(\epsilon_0 - 1)$$

→ $$\epsilon_{c-} < \epsilon_0(\lambda_f) < \epsilon_{c+}$$

where $\epsilon_{c\pm} = \frac{3}{2} \pm \sqrt{\frac{\lambda_g}{\xi a_g} + \frac{1}{4}}.$

The behavior of the roots of $g(\epsilon)$

- $\lambda_f = \lambda_+$

$$\frac{d\epsilon_{in}}{d\lambda_f} > 0. \qquad \frac{d\epsilon_{out}}{d\lambda_f} < 0.$$

There exists a positive multiple root .

$$\epsilon_*$$

This root satisfies

$$\epsilon_{c-} < \epsilon_* < \epsilon_{c+}$$

- $\lambda_f < \lambda_+$

There ordinarily exist two positive roots.

ϵ_{in} and ϵ_{out} (inner root and outer root)

As we decrease λ_f, ϵ_{in} decreases. ϵ_{out} increases.

→ $\epsilon_{in} < \epsilon_*$, $\epsilon_{out} > \epsilon_*$

Sometime, ϵ_{in} becomes smaller than ϵ_{c-}; ϵ_{out} becomes larger than ϵ_{c+}

The critical λ_f is λ_{c-} , λ_{c+} , respectively.

we can rewrite the following value as

$$\omega_0^2(\epsilon_0) - 2H_0^2(\epsilon_0) = (\epsilon_* - \epsilon_0) \times (\text{positive definite})$$

$$\omega_0^2 > 2H_0^2 \iff \epsilon_0 < \epsilon_*$$
$$\omega_0^2 = 2H_0^2 \iff \epsilon_0 = \epsilon_*$$
$$\omega_0^2 < 2H_0^2 \iff \epsilon_0 > \epsilon_*$$

From $\begin{array}{l} \epsilon_{\text{in}} < \epsilon_* \\ \epsilon_{\text{out}} > \epsilon_* \end{array}$, inner root satisfies $\omega_0^2 > 2H_0^2$, outer root satisfies $\omega_0^2 < 2H_0^2$

Bigravity action

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g_{\mu\nu}] + \frac{M_f^2}{2}\int d^4x\sqrt{-f}R[f_{\mu\nu}] + m^2 M_e^2 \int d^4x\sqrt{-g}F_2$$

EH action of $g_{\mu\nu}$ — EH action of $f_{\mu\nu}$ — Interaction term

$g_{\mu\nu}$: physical metric

$f_{\mu\nu}$: reference metric

$$F_2[L^\mu_\nu] = \frac{1}{2}([L]^2 - [L^2])$$

trace

$$L^\mu_\nu = \delta^\mu_\nu - (\sqrt{g^{-1}f})^\mu_\nu$$

m^2 : coupling constant

$$M_e^2 = (\frac{1}{M_g^2} + \frac{1}{M_f^2})^{-1}$$

: reduced Plank scale

* when we make the general coordinate transformation at the same time for both metrics, the action is unchanged.
(there are only 4 DOF as general covariance.)