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New Cosmological Solutions in Massive Gravity

Daisuke Yoshida[1] (a) ,Tsutomu Kobayashia[2] (b) ,Masaru Siinoa[3] (a) ,Masahide Yamaguchia[4] (a)

(a) *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

(b) *Department of Physics, Rikkyo University, Toshima, Tokyo 175-8501, Japan*

Abstract

In massive gravity,we find a series of the metrics for which the effective energy momentum tensor from the graviton mass behave as a cosmological constant. As a result,the metric that is exact solution in general relativity with cosmological constant is a exact solution also in massive gravity. Especially,these solutions include expanding cosmological solutions with flat,open and closed spacial curvature.

1 Introduction

It is very intriguing to explore whether or not the graviton can have a mass. The first attempt to add a mass term to the gravity action was made by Fierz and Pauli [1], who considered the quadratic action for the graviton $h_{\mu\nu}$ in flat space with the mass term

$$m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right). \tag{1}$$

The linear theory with the Fierz-Pauli mass term is ghost-free. However, the theory does not reproduce general relativity in the massless limit $m \to 0$. The extra three degrees of freedom in a massive spin 2 survive even in this limit, and therefore the prediction for light bending is away from that of general relativity, which clearly contradicts solar-system tests. This is called the vDVZ discontinuity [2].

As pointed out by Vainshtein [3], the discontinuity can in fact be cured by going beyond the linear theory. Massive gravity has a new length scale called the Vainshtein radius, below which the nonlinearities of the theory come in and the effect of the extra degrees of freedom is screened safely. The Vainshtein radius becomes larger as m gets smaller, and thereby a smooth massless limit is attained.

However, the very nonlinearities turned out to cause another trouble. Boulware and Deser argued that there appears the sixth scalar degree of freedom at nonlinear order, which has a wrong sign kinetic term, *i.e.,* the sixth mode is a ghost [4]. The presence of the Boulware-Deser (BD) ghost has hindered us from constructing a consistent theory of massive gravity.

Recently, a theoretical breakthrough in this field has been made. Adding higher-order self-interaction terms and tuning appropriately their coefficients, de Rham and collaborators successfully eliminated the dangerous scalar mode from the theory in the decoupling limit [5, 6]. Then, Hassan and Rosen established a complete proof that the theory does not suffer from the BD ghost instability to all orders in perturbations and away from the decoupling limit [7]. Thus, there certainly exists a nonlinear theory of massive gravity that is free of the BD ghost.

2 Action of Ghost Free Massive Gravity

The action of the ghost free massive gravity [6] is following:

$$S = S_{EH} + S_{mass} + S_{matter}, \tag{2}$$

[1]Email address: yoshida"at"th.phys.titech.ac.jp
[2]Email address: tsutomu"at"rikkyo.ac.jp
[3]Email address: msiino"at"th.phys.titech.ac.jp
[4]Email address: gucci"at"phys.titech.ac.jp

here,S_{EH} and S_{matter} is usual Einstein-Hilbert action and matter's action. The additional mass term S_{mass} is

$$S_{mass} = \frac{M_{PL}^2}{2} \int d^4x \sqrt{-g} m^2 \left(\mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4 \right). \tag{3}$$

with graviton mass m and free parameters α_3, α_4. Here,

$$\mathcal{U}_2 \ := \ [\mathcal{K}]^2 - [\mathcal{K}^2], \tag{4}$$

$$\mathcal{U}_3 \ := \ [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \tag{5}$$

$$\mathcal{U}_4 \ := \ [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]. \tag{6}$$

"[ ]" represents trace:

$$[\mathcal{K}] = \mathcal{K}^\mu_\mu, \quad [\mathcal{K}^2] = \mathcal{K}^\mu_\nu \mathcal{K}^\nu_\mu, \quad \cdots, \tag{7}$$

and $\mathcal{K}^\mu_\nu$ is

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\rho} \partial_\rho \phi^a \partial_\nu \phi^b \eta_{ab}}. \tag{8}$$

the Square root of a tensor is defind as follow:

$$\sqrt{A^\mu_\rho} \sqrt{A^\rho_\nu} = A^\mu_\nu. \tag{9}$$

ϕ^a $(a = 0, 1, 2, 3)$ are scalar fields called Stückelberg field.

In the action,Stückelberg fields appear throught the conbination $\partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} \equiv \Sigma_{\mu\nu}$, and , We call this tensor "fiducial metric". In the coordinate

$$x^0 = \phi^0, x^1 = \phi^1, x^2 = \phi^2, x^3 = \phi^3, \tag{10}$$

the fiducial metric becomes the diagonal Minkowski metric:

$$\Sigma_{\mu\nu} = \eta_{\mu\nu}, \tag{11}$$

and,this coordinate called "unitary gauge".

The equation of motion derived from this action is

$$G_{\mu\nu} + m^2 X_{\mu\nu} = \frac{1}{M_{PL}^2} T_{\mu\nu}. \tag{12}$$

Here,$G_{\mu\nu}$is the Einstein tensor,$T_{\mu\nu}$is the energy momentum tensor of the matter,and $m^2 X_{\mu\nu}$ is effective energy momentum tensor from graviton mass:

$$m^2 X_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S_{mass}}{\delta g^{\mu\nu}} \tag{13}$$

Note that if $m^2 X_{\mu\nu} \propto g_{\mu\nu}$, this term behave as a cosmological constant, therefore the equation of motion coincides with the Einstein equation with a cosmological constant.

3 Cosmological Solutions

In massive gravity, a change of coordinate transforms not only $g_{\mu\nu}$ but also $\Sigma_{\mu\nu}$. So, in the coordinate where $g_{\mu\nu}$ is the usual diagonal form:

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right], \tag{14}$$

the fiducial metric $\Sigma_{\mu\nu}$ is a generally complicated form.

The first attempt to find a cosmological solution was made by D'Amico et al [8]. In this analysis, the free parameterα_3, α_4 set to zero,the spacial curvature is flat , and in the familiar diagonal FLRW coordinate of teh physical metric,the fiducial metric is the diagonal Minkowski form ($\phi^\mu = x^\mu$):

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \tag{15}$$

$$\Sigma_{\mu\nu}dx^\mu dx^\nu = \eta_{\mu\nu}dx^\mu dx^\nu, \tag{16}$$

or has same symmetry as the physical metric($\phi^0 = f(t), \phi^i = x^i$):

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{17}$$

$$\Sigma_{\mu\nu}dx^\mu dx^\nu = -\dot{f}^2(t)dt^2 + \delta_{ij}dx^i dx^j. \tag{18}$$

In both cases, equation of motion implies $\dot{a} = 0$, as a result ,our univers can not expand.

To obtain the expanding FLRW solution,we must consider more complicated case. One way is to consider the open FLRW space time [9] ,or more general fiducial metric [8] [10]. But , we take another approach. We choose the coordinate where the fiducial metric is the diagonal Minkowski form(unitary gauge), and the physical metric is not usual diagonal form.

Before our work,the de Sitter solution was already found [11]. In this analysis,the de Sitter space time is described by the Painlevé-Gullstrand (PG) coordinate in the unitary gauge:

$$g_{\mu\nu}dx^\mu dx^\nu = -\kappa^2 dt^2 + \tilde{\alpha}^2\left(dr \pm \sqrt{\frac{\Lambda}{3}}\kappa r dt\right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2 \tag{19}$$

$$\Sigma_{\mu\nu}dx^\mu dx^\nu = \eta_{\mu\nu}dx^\mu dx^\nu. \tag{20}$$

In addition, free parameter α_3, α_4 was choosed as following:

$$\alpha_3 = \frac{1}{3}(\alpha - 1), \alpha_4 = \frac{1}{12}(\alpha^2 - \alpha + 1), \alpha = \frac{\tilde{\alpha}}{1 - \tilde{\alpha}}. \tag{21}$$

In this case,the effective energy momentum tensor from the graviton mass is

$$m^2 X_{\mu\nu} = \frac{m^2}{\alpha}g_{\mu\nu}. \tag{22}$$

Therefore, the equation of motions are Einstein equation with the effective coupling constant m^2/α.So for $\Lambda = m^2/\alpha$,this de Sitter solution is an exact solution in massive gravity.

We extend this analysis to the FLRW space time. The key observation is that ,as the de Sitter space time, the FLRW space time can be described by the PG coordinate[13].So,we impose this form of the physical metric in the unitary gauge:

$$g_{\mu\nu}dx^\mu dx^\nu = -\kappa^2 dt^2 + \frac{\tilde{\alpha}^2}{1 - K\tilde{\alpha}^2 r^2/a^2(t)}\left(dr \pm \frac{\dot{a}}{a}\kappa r dt\right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2 \tag{23}$$

$$\Sigma_{\mu\nu}dx^\mu dx^\nu = \eta_{\mu\nu}dx^\mu dx^\nu. \tag{24}$$

We use same parameters as (21). In the set up ,the effective energy momentum tensor from the graviton mass is same as eq.(22),and,as expected,the equation of motion is the Einstein equation with a cosmological constant. So, as in general relativity,these FLRW space times are solutions in massive gravity.

We can transform the coordinate to the familiar diagonal FLRW coordinate by $r \to R = \tilde{\alpha}r/a(t), t \to T = \kappa t$,

$$g_{\mu\nu}dx^\mu dx^\nu = -dT^2 + a^2(T)\left[\frac{dR^2}{1 - KR^2} + R^2 d\Omega^2\right] \tag{25}$$

$$\Sigma_{\mu\nu}dx^\mu dx^\nu = -\left(\frac{1}{\kappa^2} - \frac{\dot{a}^2 R^2}{\tilde{\alpha}^2}\right)dT^2 + \frac{2a\dot{a}R}{\tilde{\alpha}^2}dT dR + \frac{a^2}{\tilde{\alpha}^2}(dR^2 + R^2 d\Omega^2). \tag{26}$$

In this coordinate,the fiducial metric is inhomogenious form. Thus,the fiducial metric is the form out of ansatz eq.(16),eq.(18).

4 More General Solutions

We can generalize our cosmological solutions. Instead of the PG form FLRW metric (23) ,we use the following general PG form metric:

$$g_{\mu\nu}dx^\mu dx^\nu \quad = \quad -U^2(x^\mu)dt^2 + V^2(x^\mu)\left(dr + f(x^\mu)dt\right)^2 + \tilde{\alpha}^2 r^2 d\Omega^2. \tag{27}$$

Here,U, V, f are arbitrary functions. Also,in this set up,the effective energy momentum tensor is same as (22). As a result,the metric that is an exact solution in general relativity with a cosmological constant ,is a exact solution also in massive gravity.

For example,the Lemaître-Tolman-Bondi (LTB) metric can be described by PG form[13]. Therefore the LTB solution is an exact solution in massive gravity. we can use this solution to study spherical collapse of a perfect fluid in massive gravity.

5 Concludion

We have found new cosmological solutions in massive gravity with flat,open,and closed spatial geometries. Our solutions can be extended to general spherical space time,including LTB space time. In both cases, the key was that general PG form metric (27)gives rise to an effective energy momentum tensor of a cosmological constant m^2/α for the special choice of the parameters (21). This is essential for being able to get analytic solutions in massive gravity easily from the seed solutions in general relativity.

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