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“Constraints on general second-order scalar-tensor models from gravitational Cherenkov radiation”

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Constraints on general second-order scalar-tensor models from gravitational Cherenkov radiation

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Based on

RK and Kazuhiro Yamamoto, JCAP 07 (2012) 050

Introduction

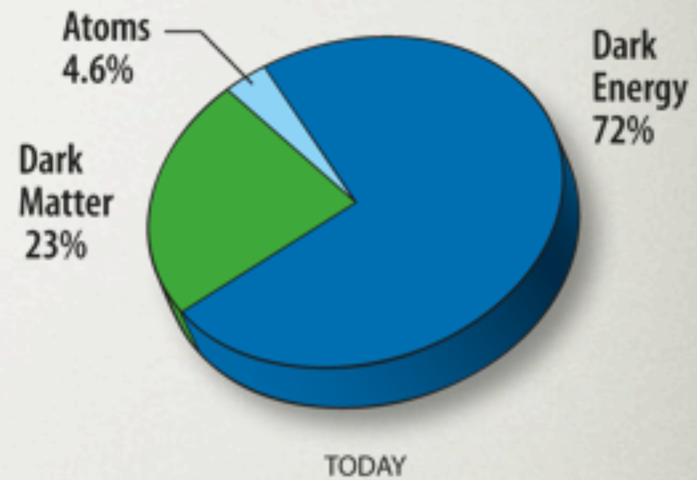
- Accelerating universe
  - Implication of cosmological constant?

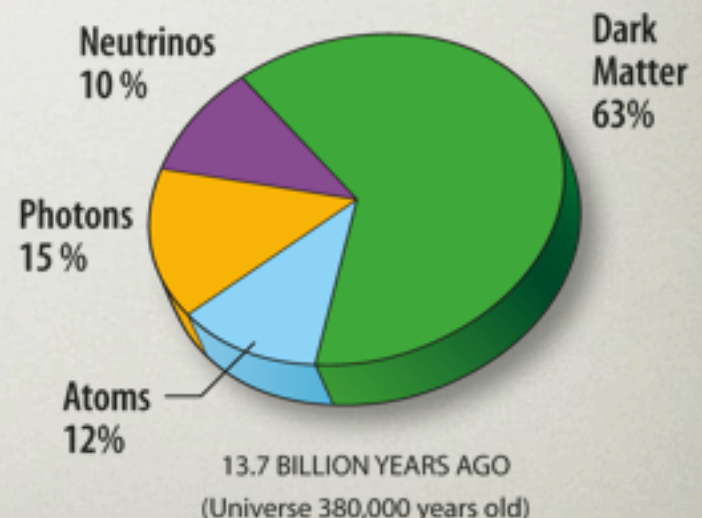
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

  - Observationally, fine !!
  - **Cosmological constant problem**

121 orders of magnitude differences

Can **modification of gravity** solve this puzzle ???





(from WMAP website)

Galileon theory

✓ Galileon (Nicolis et al. '09)

$$\Box = \nabla_\mu \nabla^\mu$$

$$\mathcal{L} \supset (\partial\varphi)^2 \,\Box\varphi$$

second derivative with respect to space-time

✓ Galileon term contains the second derivative term, but ...

coupling between scalar and curvature

$$\mathrm{EOM} \;\supset\; (\Box\varphi)^2 - (\nabla_\mu\nabla_\nu\varphi)^2 - R_{\mu\nu}\nabla^\mu\varphi\nabla^\nu\varphi$$

No higher-order derivative terms in EOM !!

Most general second-order scalar-tensor theory (MGST)

✓ Horndeski found the most general Lagrangian whose EOM is second-order differential equation for ϕ and $g_{\mu\nu}$ (also known as **Generalized galileon**)

Horndeski, Int. J. Theor. Phys. 10,363 (1974) , Deffayet, Gao, Steer (2011)

$$\mathcal{L}_2 = K(\phi, X)$$

→ **K-essence term** $\mathcal{L}_2 \supset (\partial\phi)^2,\ V(\phi)$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

→ **Cubic galileon term** $\mathcal{L}_3 \supset (\partial\phi)^2\Box\phi$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)]$$

→ **Einstein-Hilbert term** $\mathcal{L}_4 \supset (M_{\rm Pl}^2/2)R$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\Big]$$

→ **Non-minimal derivative coupling** $\mathcal{L}_5 \supset G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

(Germani et al. 2011; Gubitosi, Linder 2011)

$X = -(\partial\phi)^2/2, \quad G_{iX} = \partial G_i/\partial X$

WHY GALILEON??

- **Self-accelerating solution**
- **Free of ghost-instabilities**
- **Vainshtein mechanism** (Vainshtein 1972)
  - Scalar field is effectively weakly coupled to matter in a high density region
  - Reduce general relativity at small scales
- Relation with decoupling limit in **massive gravity**

(de Rham, Gabadadze, Tolley, 2010)

Cosmological observations

RK, Kazuhiro Yamamoto, JCAP 04 (2011) 025
RK, Tsutomu Kobayashi, Kazuhiro Yamamoto, Physical Review D 85 (2012) 123503

- Standard rulers (supernovae + CMB shift parameter)
  - **Not powerful tools** to constrain model parameters in modified gravity theories, but **useful tools** to determine cosmological parameters
- Galaxy distribution (SDSS LRG sample)
  - **The error bar is still large** to constrain model parameters
- Cross correlation between LSS and ISW
  - **Excellent tool** to constrain modified gravity
  - Indicates that the effective gravitational coupling **G_{eff} has to be smaller than ~1.2 G_N**, otherwise CCF becomes **negative** which contradicts with observations

Other signatures ??

Sound speed of graviton in MGST

- Quadratic action for **a tensor mode** in the most general scalar-tensor theory

Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011),

$$S_T^{(2)} = \frac{1}{8}\int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

- Sound speed of graviton

$$c_T^2 \equiv \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

$$\mathcal{F}_T \equiv 2\left[G_4 - X\left(\ddot{\phi} G_{5X} + G_{5\phi}\right)\right]$$

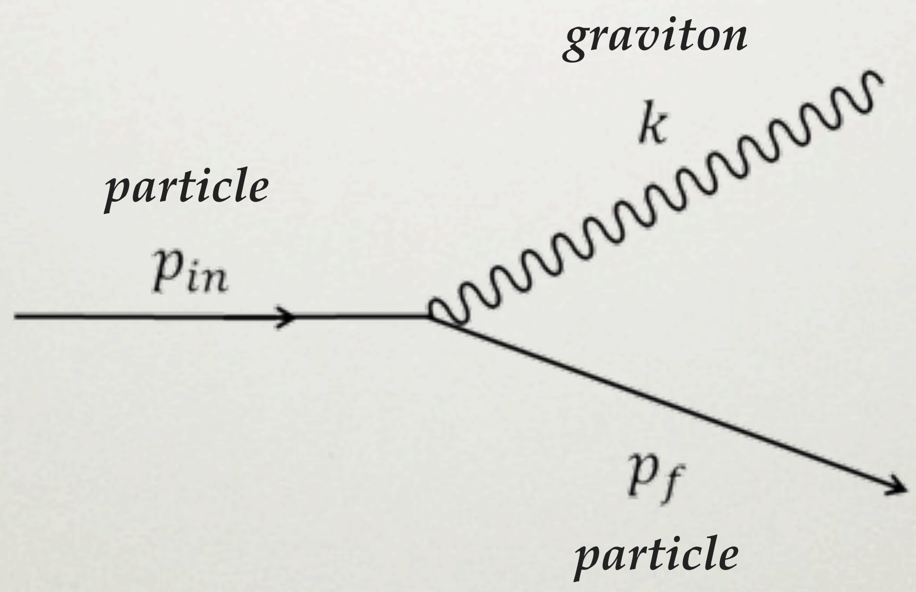
$$\mathcal{G}_T \equiv 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi} G_{5X} - G_{5\phi}\right)\right]$$

Sound speed of graviton could be **different from speed of light** !!!

Gravitational Cherenkov Radiation

- If the sound speed of graviton is smaller than the speed of light, **particle should emit graviton** through the similar process to Cherenkov radiation

Moore and Nelson (2001)



- Highest energy cosmic ray (p ~ 3×10^{11} GeV) can provide us **the lower bound on the sound speed of graviton**

Gravitational Cherenkov Radiation

- Consider the complex scalar in **a FRW background**

$$S_m = \int d^4x\sqrt{-g}\left[-g^{\mu\nu}\partial_\mu\Psi^*\partial_\nu\Psi - m^2\Psi^*\Psi - \xi R\Psi^*\Psi\right]$$

- Quantize the complex scalar and tensor field as

$$\hat{\Psi}(\eta,\mathbf{x}) = \frac{1}{a}\int\frac{d^3p}{(2\pi)^{3/2}}\left[\hat{b}_{\mathbf{p}}\psi_p(\eta)e^{i\mathbf{p}\cdot\mathbf{x}} + \hat{c}^\dagger_{\mathbf{p}}\psi^*_p(\eta)e^{-i\mathbf{p}\cdot\mathbf{x}}\right]$$

$$\hat{h}_{\mu\nu} = \frac{1}{a}\sqrt{\frac{2}{\mathcal{G}_T}}\sum_\lambda\int\frac{d^3k}{(2\pi)^{3/2}}\left[\varepsilon^{(\lambda)}_{\mu\nu}\hat{a}_{\mathbf{k}}h_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \varepsilon^{(\lambda)}_{\mu\nu}\hat{a}^\dagger_{\mathbf{k}}h^*_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$[\hat{b}_{\mathbf{p}}, \hat{b}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$$

$$[\hat{c}_{\mathbf{p}}, \hat{c}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$$

- Mode functions satisfy

$$\left(\frac{d^2}{d\eta^2} + p^2 + m^2a^2\right)\psi_p(\eta) = 0$$

$$\left(\frac{d^2}{d\eta^2} + c_T^2k^2 - \frac{a''}{a}\right)h_k(\eta) = 0$$

Gravitational Cherenkov Radiation

- The total radiation energy from the complex scalar field

$$E = \sum_{\lambda}\sum_{\mathbf{k}} (\omega_k/a)\langle \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}} \rangle$$

where

$$\left\langle \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}} \right\rangle = 2\Re \int_{t_{\rm in}}^{t} dt_2 \int_{t_{\rm in}}^{t_2} dt_1 \left\langle H_I(t_1) \hat{a}^{\dagger(\lambda)}_{\mathbf{k}} \hat{a}^{(\lambda)}_{\mathbf{k}} H_I(t_2) \right\rangle$$

$$H_I = a \int d^3x h_{ij} \partial_i \Psi \partial_j \Psi^*$$

- Graviton emission rate (using sub-horizon approximation)

$$\frac{dE}{dt} \simeq \frac{G_N\, p_{\rm in}^4}{a^4} \frac{4(1-c_T)^2}{3(1+c_T)^2}$$

- A particle with momentum p **cannot** possibly have been traveling for **longer than**

$$t \sim \frac{a^4}{G_N} \frac{(1+c_T)^2}{4(1-c_T)^2} \frac{1}{p^3}$$

Gravitational Cherenkov Radiation

- Observations from cosmic rays tells

Time scale that cosmic ray turn into radiation energy of graviton $>$ **Time scale that cosmic ray travels from origin to us**

- The highest energy cosmic ray
  - Energy $E_{\text{highest}} \sim 10^{11}\,\text{GeV}$
  - Distance $c\,t \sim 10\,\text{kpc}$

- **Constraint on the sound speed of graviton**

$$1 - c_T \lesssim 2 \times 10^{-15}$$

Toy Model 1

- Gubitosi and Linder model (Gubitosi and Linder 2011)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + X + \frac{\lambda}{M_{\rm Pl}^2}G^{\mu\nu}\nabla_\mu\phi\nabla_\mu\phi + \mathcal{L}_m[g_{\mu\nu},\psi]\right]$$

$K = X$

$G_3 = 0$

$G_4 = M_{\rm Pl}^2/2$

$G_5 = -\lambda\phi/M_{\rm Pl}^2$

- Condition for existence of self-accelerating solution and avoiding the ghost-instability

$$-\frac{1}{18}\frac{M_{\rm Pl}^2}{H_0^2} < \lambda < -\frac{1}{30}\frac{M_{\rm Pl}^2}{H_0^2}$$

λ is always negative

- Sound speed of graviton

$$c_T^2 = \frac{M_{\rm Pl}^2 + 2\lambda\dot{\phi}^2/M_{\rm Pl}^2}{M_{\rm Pl}^2 - 2\lambda\dot{\phi}^2/M_{\rm Pl}^2} < 1$$

Inconsistent with the constraint from the gravitational Cherenkov radiation...

Toy Model 2

$c_T > 1 - \epsilon$

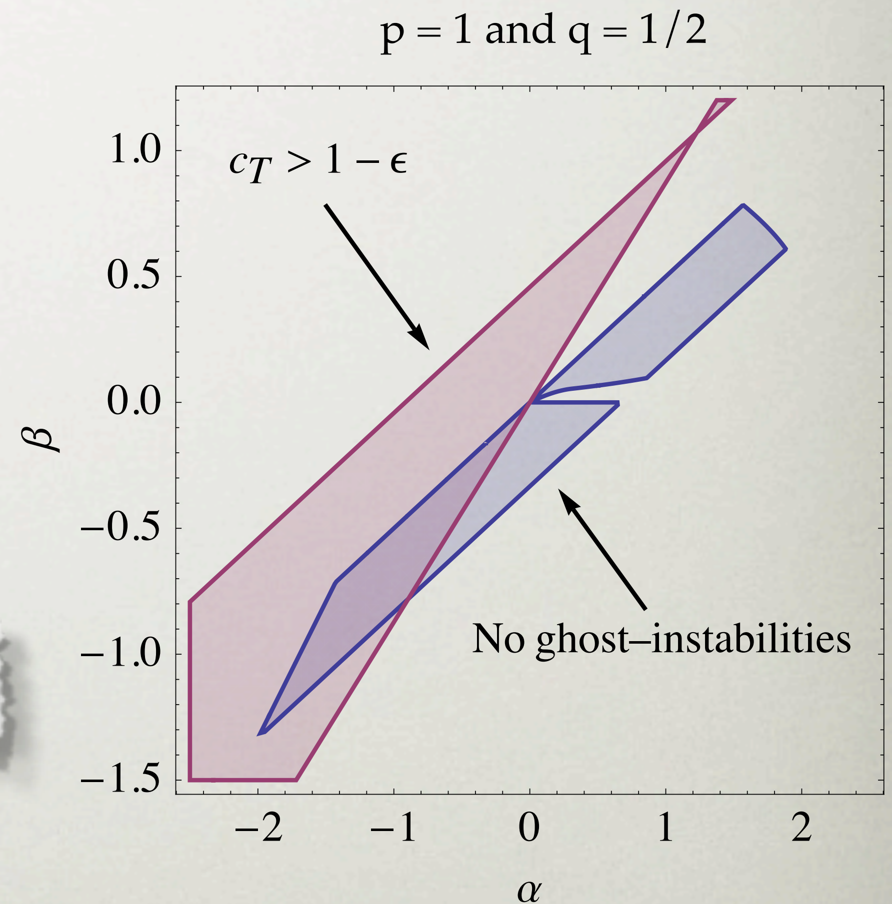
$\epsilon = 2 \times 10^{-15}$

- Extended galileon model (De Felice and Tsujikawa 2011)

$$
\begin{aligned}
K &= -c_2 M_2^{4(1-p_2)} X^{p_2}, \\
G_3 &= c_3 M_3^{1-4p_3} X^{p_3}, \\
G_4 &= \frac{1}{2} M_{\rm pl}^2 - c_4 M_4^{2-4p_4} X^{p_4}, \\
G_5 &= 3 c_5 M_5^{-(1+4p_5)} X^{p_5},
\end{aligned}
$$

$$
\begin{aligned}
p_2 &= p \\
p_3 &= p + (2q - 1/2) \\
p_4 &= p + 2q \\
p_5 &= p + (6q - 1)/2
\end{aligned}
$$

4 model parameters p, q, α(c_3), β (c_4)



Strong constraints for the model parameters α and β

Summary

- In the most general scalar-tensor theory, sound speed of graviton could be different from speed of light
- The constraints from **gravitational Cherenkov radiation** would be a powerful probe.
- **Gravitational Cherenkov radiation** could be a criteria for the construction of modification of gravity

Gravitational Cherenkov radiation for massive graviton

- Quadratic action for **a tensor mode** for massive graviton

Gumrukcuoglu, Kuroyanagi, Lin, Mukohyama, Tanahashi (2012)

$$I^{(2)}_{tensor} = \frac{M_{Pl}^2}{8}\int dtd^3x\, Na^3\sqrt{\Omega}\left[\frac{1}{N^2}\dot{\gamma}^{ij}\dot{\gamma}_{ij} + \frac{c_g^2(t)}{a^2}\gamma^{ij}\left(\triangle - 2K\right)\gamma_{ij} - M_{GW}^2(t)\gamma^{ij}\gamma_{ij}\right]$$

- Dispersion relation

$$\omega_k^2 = c_g^2 k^2 + a^2 M_{GW}^2$$

For c_g=c, there is **no gravitational Cherenkov radiation** even if m≠0

Currently, checking the case c_g≠c and m≠0...