Tomohiro Harada, JGRG 22(2012)111410

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RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan

Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole

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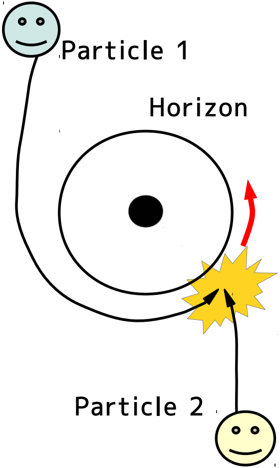
12-16/11/2012 JGRG22@RESCEU, U Tokyo

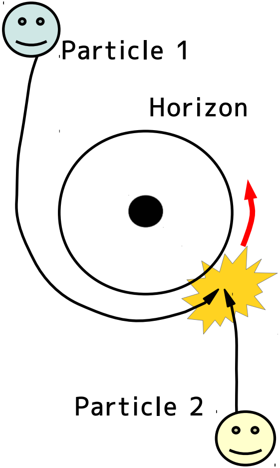
Introduction

- "Kerr BHs as particle accelerators" (Bañados, Silk & West 2009): Collision with an arbitrarily high centre-of-mass (CM) energy near the horizon of a maximally rotating BH. Implication to DM particles pair annihilation.
- Critical comments: Berti et al. 2009, Jacobson & Sotirou 2010
- Astrophysical relevance: Harada & Kimura 2011abc



Can we observe new physics?

- Particle collision with extremely high CM energy might produce an exotic particle. Can we observe it?
- If a high-energy and/or super-heavy particle is to be emitted from the collision of ordinary particles, we need energy extraction from the BH.
- This is possible in general for a rotating BH, as is well known.



Collisional Penrose Process

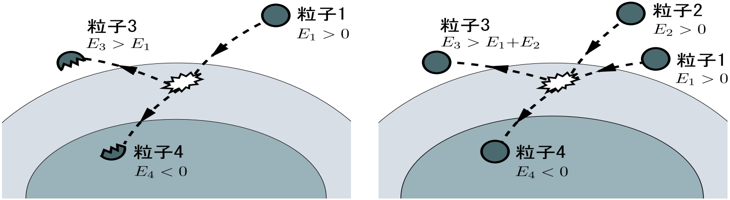


Figure: Left: Penrose process, right: Collisional Penrose process. The light and deep shaded regions denote the ergoregions and BHs, respectively.

- Energy can be extracted from a rotating BH due to the negative energy orbit in the ergoregion.
- Collisional Penrose process (Piran, Shaham & Katz 1975)
- Jacobson & Sotiriou (2010) argue that no energy extraction occurs through the BSW collision.

Maximally rotating BH

- Maximally rotating Kerr BH
  - Boyer-Lindquist coordinates: (t, r, θ, ϕ)
  - $a = M$: $r_H = M$, $\Omega_H = 1/(2M)$, $\kappa_H = 0$
  - Ergoregion: $M < r < M(1 + \sin\theta)$
- Geodesic motion in the equatorial plane
  - 1D potential problem

$$\frac{1}{2}(p^r)^2 + V(r) = 0, \text{ or } p^r = \sigma\sqrt{-2V(r)}, \text{ where } p^r = \frac{dr}{d\lambda},$$

    where λ is the affine parameter,

$$V(r) = -\frac{Mm^2}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{E^2 - m^2}{2},$$

    and E and L are conserved.
  - Forward-in-time condition: $p^t = dt/d\lambda > 0$
  - This implies $2E - \tilde{L} \geq 0$ in the limit $r \to r_H$, where $\tilde{L} = L/M$. We define a critical particle as a particle satisfying $2E - \tilde{L} = 0$.

Collision and reaction

- Collision and reaction: $1 + 2 \to 3 + 4$
  - CM energy: $E_{\rm cm}^2 = -(p_1^a + p_2^a)(p_{1a} + p_{2a}) = -(p_3^a + p_4^a)(p_{3a} + p_{4a})$
  - Conservation: $E_1 + E_2 = E_3 + E_4$ and $\tilde{L}_1 + \tilde{L}_2 = \tilde{L}_3 + \tilde{L}_4$
  - Radial momentum conservation : $p_1^r + p_2^r = p_3^r + p_4^r$
- BSW collision: particle 1 is critical ($2E_1 - \tilde{L}_1 = 0$), while particle 2 is subcritical ($2E_2 - \tilde{L}_2 > 0$). If the two particles collide at $r = M/(1-\epsilon)$ $(0 < \epsilon \ll 1)$ with $p^r < 0$,

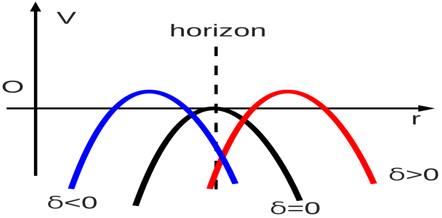
$$E_{\rm cm} \approx \sqrt{\frac{2(2E_1 - \sqrt{3E_1^2 - m_1^2})(2E_2 - \tilde{L}_2)}{\epsilon}}.$$

$E_{\rm cm} \to \infty$ as $\epsilon \to 0$.

Particle motion near the horizon

- Let $\tilde{L} = 2E(1 + \delta)$, $\delta = \delta_{(1)}\epsilon + \delta_{(2)}\epsilon^2 + O(\epsilon^3)$.
- The forward-in-time condition at $r = M/(1 - \epsilon)$ yields $\delta < \epsilon + O(\epsilon^2)$.
- Turning points of the potential

$$r_{t,\pm}(e) = M\left(1 + \frac{2e}{2e \mp \sqrt{e^2+1}}\delta_{(1)}\epsilon\right) + O(\epsilon^2), \quad \text{where} \quad e = E/m.$$



- To escape to infinity from $r = M/(1 - \epsilon)$, we need $e \geq 1$ and
  - (a) $\delta_{(1)} < 0$ and $\sigma = 1$
  - (b) $\delta_{(1)} > 0$ and $r \geq r_{t,+}(e)$ or $0 < \delta_{(1)} \leq \delta_{(1),\max} = (2e - \sqrt{e^2+1})/(2e)$.

Collision and reaction near the horizon

- Let us consider a collision at $r = M/(1-\epsilon)$.
- Let $\tilde{L}_3 = 2E_3(1+\delta)$, $\sigma_3 = \pm 1$ and $\sigma_4 = -1$.
- The forward-in-time condition is taken into account.
- The radial momentum conservation: $p_1^r + p_2^r = p_3^r + p_4^r$.
  - Expand p_i^r $(i = 1, 2, 3, 4)$ in terms of ϵ.
  - The radial momentum conservation implies at $O(\epsilon)$

$$(2E_1 - \sqrt{3E_1^2 - m_1^2}) + 2E_3(\delta_{(1)} - 1) = \sigma_3 \sqrt{E_3^2(3 - 8\delta_{(1)} + 4\delta_{(1)}^2) - m_3^2}.$$

  - It implies at $O(\epsilon^2)$ an equation including m_4. With this equation, we can check whether $m_4^2 \geq 0$ is satisfied or not.

The energy of the escaping particle

- The radial momentum conservation implies at $O(\epsilon)$

$$(2E_1 - \sqrt{3E_1^2 - m_1^2}) + 2E_3(\delta_{(1)} - 1) = \sigma_3 \sqrt{E_3^2(3 - 8\delta_{(1)} + 4\delta_{(1)}^2) - m_3^2}. \quad (1)$$

- Squaring the both sides of Eq. (1) yields the following quadratic equation for E_3.

$$4A_1E_3(1 - \delta_{(1)}) = A_1^2 + (E_3^2 + m_3^2), \quad (2)$$

where $A_1 = 2E_1 - \sqrt{3E_1^2 - m_1^2} > 0$.

- Solving Eq. (2) for $\delta_{(1)}$ and substituting it into Eq. (1) yields

$$A_1^2 - (E_3^2 + m_3^2) = 2\sigma_3 A_1 \sqrt{E_3^2(3 - 8\delta_{(1)} + 4\delta_{(1)}^2) - m_3^2}. \quad (3)$$

Upper limits of the emitted particle's energy

- We assume $E_1 \geq m_1$ so that particle 1 is initially at infinity.

(i) $\sigma_3 = 1$: Eq. (3) immediately implies $E_3 \leq \sqrt{A_1^2 - m_3^2} < E_1$, i.e., no energy extraction.

(ii) $\sigma_3 = -1$ and $0 < \delta_{(1)} \leq \delta_{(1),\max}$: $E_3 = 2.186E_1$ is possible.

- Eq. (2) immediately implies $\lambda_- \leq E_3 \leq \lambda_+$, where $\lambda_\pm = 2A_1 \pm \sqrt{3A_1^2 - m_3^2}$ and the equality holds for $\delta_{(1)} = 0$.
- This implies that E_3/E_1 takes a maximum $(2 - \sqrt{2})/(2 - \sqrt{3}) \simeq 2.186$ for $E_1 = m_1$, $m_3 = 0$ and $\delta_{(1)} = +0$.

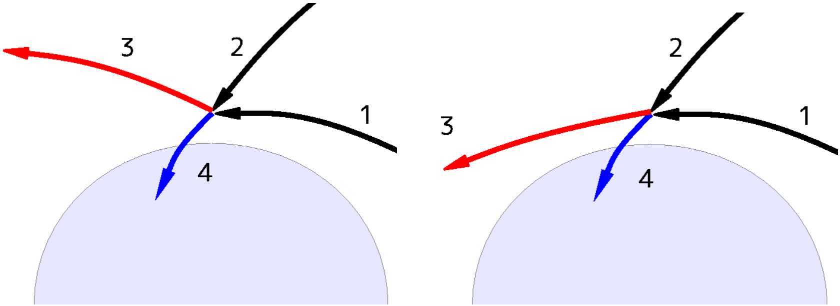


Figure: Left: escape without bounce ($\sigma = 1$), right: escape with bounce ($\sigma = -1$).

- Energy extraction is possible only with bounce ($\sigma_3 = -1$).

Energy gain efficiency

- The upper limit of the energy gain efficiency $\eta = E_3/(E_1 + E_2)$ can be further studied based on $O(\epsilon^2)$ equation.
- The upper limit of the efficiency for $E_3 = E_B$ is given by 146.6 % for any BSW collision.
- The upper limits are 117.6 % for perfectly elastic collision, 137.2 % for inverse Compton scattering and 109.3 % for pair annihilation.
- Our result agrees with a numerical work by Bejger, Piran, Abramowicz & Hakanson (2012) and contradicts a simplistic argument by Jacobson & Sotiriou (2010).
- On the other hand, the efficiency is not very high but modest at most.

Summary

- The rotational energy of a maximally rotating BH can be extracted through a BSW collision, whereas the emitted particle cannot be highly energetic.
- Note, however, that the BSW collision may open a new reaction channel because of high CM energy, which can leave its features on the gamma-ray spectrum (cf. Cannoni, Gomez, Perez-Garcia & Vergados 2012).