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Equilibrium states of magnetized disc-central compact object systems

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Abstract

We have obtained equilibrium states of magnetized disc-central compact object systems. Under the assumption of stationary and axisymmetry, we have derived generalized Grad-Shafranov equation (GS equation). We have succeed in solving GS equation and including not only poloidal but also toroidal magnetic fields under the ideal MHD approximation. We have obtained a toroid which sustains the extreme strong poloidal magnetic field near the central compact object.

1 Introduction

Recent GR simulations show that a few-solar-mass black hole and a highly dense toroid whose maximum density can reach $10^{10} - 10^{11}\mathrm{g/cm^3}$ around the black hole can be formed after the merging of binary neutron stars [1], the merging of a neutron and BH binary [2] and the collapsing of a massive star [3]. Therefore, dense toroids and central compact objects could be formed after collapsing or merging of compact objects. If the compact objects have strong magnetic fields such as a magneter, the toroid can also sustain the strong magnetic fields. Although, in order to understand the origin and dynamical formation processes of these systems, we must take into account many realistic physics and compute stationary configurations with magnetic fields in GR, nobody has yet succeeded in solving stationary states both with poloidal and toroidal magnetic fields in GR at present. Therefore, we explore such stationary states of axisymmetric magnetized barotropic systems in the framework of Newtonian gravity. Although there were many papers to obtain magnetized stationary states of discs/toroids only with poloidal magnetic fields (e.g. [4]) and discs/toroids only with toroidal magnetic fields inside the disks/toroids (e.g. [5]), no solutions both with poloidal and toroidal components of magnetic fields have been obtained yet. This is because it has been difficult to solve the Grad-Shafranov equation as well as the equations of motion consistently by some means.

Recently, Otani et al.[6] have obtained magnetized self-gravitating equilibrium states both with poloidal and toroidal magnetic fields self-consistently in the framework of Newtonian gravity. In this study we have extended the method employed in Otani's work. We calculate the equilibrium states of magnetized disc-central compact object systems with very strong magnetic field. We study what kind of magnetic configurations can sustain larger magnetic field energy.

2 Formulation and settings

Our formulation is essentially the same as that of Otani's work [6]. In order to obtain magnetized equilibrium states self-consistently, we solve the Euler equation,

$$\frac{1}{\rho}\nabla p = -\nabla \phi_g + \nabla\left(\frac{GM_c}{r}\right) + R\Omega^2 \mathbf{e}_R + \frac{1}{\rho}\left(\frac{\boldsymbol{j}}{c}\times\boldsymbol{B}\right) \tag{1}$$

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and GS equation,

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\right)+\frac{\partial^2\Psi}{\partial z^2}=-4\pi R\frac{j_\varphi(\Psi)}{c}, \tag{2}$$

self-consistently. Where, ρ, p, ϕ_g, M_c, Ω, $\boldsymbol{B}$, $\boldsymbol{j}$, G and c are mass density, pressure, gravitational potential of a toroid, mass of the central compact object, angular velocity of a toroid, magnetic field, current density, the gravitational constant and the speed of light. Ψ is a magnetic flux function defined as below:

$$\boldsymbol{B}_{\rm p}=\nabla\Psi\times\nabla\varphi. \tag{3}$$

$\boldsymbol{B}_p$ denotes a poloidal magnetic field. We solve GS equation by using Green function as below

$$\frac{\Psi(\boldsymbol{r})\sin\varphi}{R}=\frac{1}{c}\int_V\frac{j_\varphi(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}dV'. \tag{4}$$

The φ component of current density is expressed as an arbitrary function of flux function in our formulation as below:

$$\frac{j_\varphi}{c}=\frac{\kappa'(\Psi)\kappa(\Psi)}{4\pi R}+\rho R\mu(\Psi), \tag{5}$$

where $\kappa(\Psi)$ and $\mu(\Psi)$ are arbitrary functions of flux function Ψ. We can change the magnetic field structure by changing these arbitrary function forms [7]. We can integrate the Euler equation by using these arbitrary functions and obtain the first integral as below:

$$\int\frac{dp}{\rho}=-\phi_g+\frac{GM_c}{r}+\frac{1}{2}R^2\Omega^2(\Psi)+\int_\Psi\mu(\Psi)\ d\Psi+C. \tag{6}$$

C is a integral constant. In order to obtain the magnetized equilibrium state self-consistently, we fix one functional form and solve Eq.(4) and Eq.(6) iteratively until the system becomes convergence. Then, we obtain one magnetized equilibrium states after this iteration process.

In order to explore the magnetic field structure which sustains the large magnetic field energy, we calculate the critical equilibrium states of such systems beyond which no equilibrium states are allowed to exist when we fix one arbitrary functional forms set [6]. The critical model can sustain the largest magnetic filed energy with one functional form. A toroid can reach the central compact the most closely when the system is critical equilibrium state.

We assume that the mass of the central compact object, the toroid and the maximum density as below:

$$M_c=5.0M_\odot, \tag{7}$$

$$M_t=0.1M_\odot, \tag{8}$$

$$\rho_c=10^{11}{\rm g/cm^3}. \tag{9}$$

These values of physical quantities are typical ones obtained by the recent GR simulations [3].

3 Numerical Result

We show two critical configurations by changing the functional form of $\mu(\Psi)$ [7]. We set $\Omega(\Psi)=0$ in these calculations for simplicity. Therefore, these toroids are sustained by the magnetic pressure. mainly. Fig.1 shows the structures of the toroid (blue and red thick curves), the poloidal magnetic field lines (black curves) and the distributions of $\log_{10}|\boldsymbol{B}\ |$ (G) (color maps). The blue toroid (the left panel in Fig.1) has the largest magnetic fields among critical models in our parameter space, while the red has the smallest magnetic fields. Fig.2 shows the density distribution of each toroid in critical configuration. Both toroids have cusp structures at the inner edge. The blue toroid is stretched inward by the strong gravitational force of the central compact object.

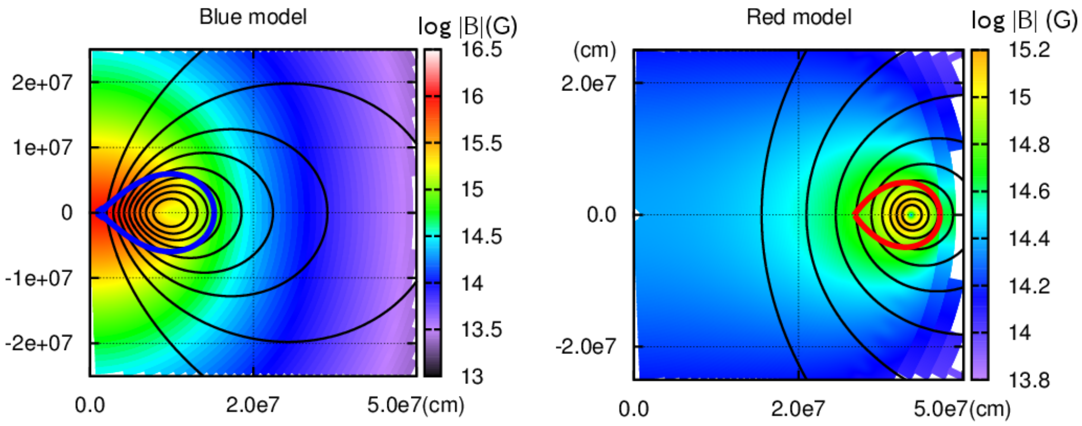

Figure 1: The magnetic field configuration and its magnitude ($\log_{10}|B|$ G). The curves denote magnetic field lines. The blue toroid (left) can approach near the central object because the magnetic pressure is larger than that of the red toroid (right).

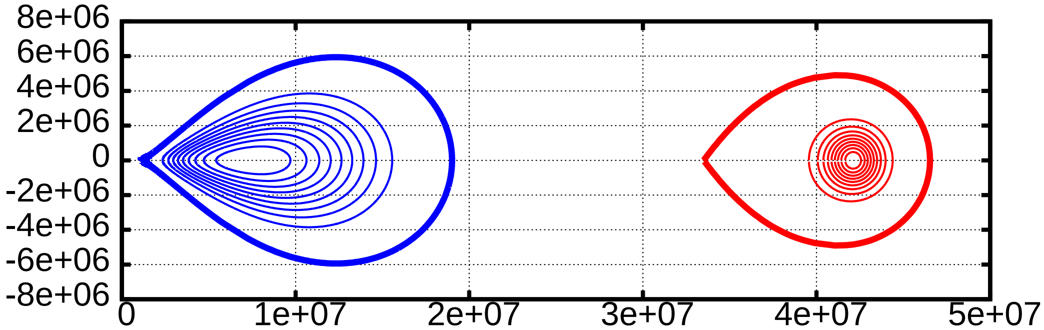

Figure 2: The density contours of toroids in the critical configuration. The blue toroid is stretched inward by the strong gravitational force of the central compact object.

4 Discussion and Conclusion

These toroids are sustained by the extreme strong magnetic pressure. As we have seen Fig.1 and Fig.2, the blue model can approach near the central compact object, because the strong magnetic fields pressure inside the toroid can balance the stronger gravity force of the central object. The distributions of magnetic fields of these models are different from each other. As seem from Fig.1, the magnetic field becomes maximum near the inner edge in the blue toroid model, while in the red toroid model, the magnetic fields become maximum near the center of the toroid. The magnitude of the magnetic fields of the blue model can reach about 10^{16} G near the inner edge of the toroid. The total magnetic field energy of the blue toroid is about three times as large as red toroid. Therefore, we conclude that the larger magnetic energy can be sustained in the toroid whose magnetic field is highly localized around the inner edge of the toroids and which locate closer to the central compact object.

Our model is, however, formulated in Newtonian gravity. The inner edge of the red toroid is located near the innermost stable circular orbit of the 5.0 $M_{\odot}$ central compact object. Since the Newtonian approximation near the region is invalid, we must treat this system by general relativistic formulation. We will extend our model to the general relativistic framework in the future.

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