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"Theory for observational verification of black hole existence"

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Theory for observational verification of black hole

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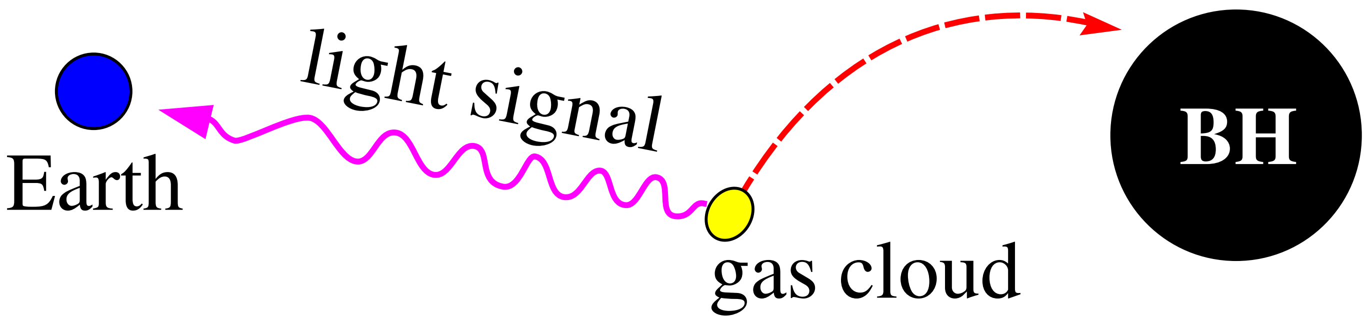


JGRG22, 2012.11.12–16 at Tokyo Univ.

1. Intro.

- I want to see black hole : **Directly observe the black hole horizon**

  $\rightarrow$ We suggest,

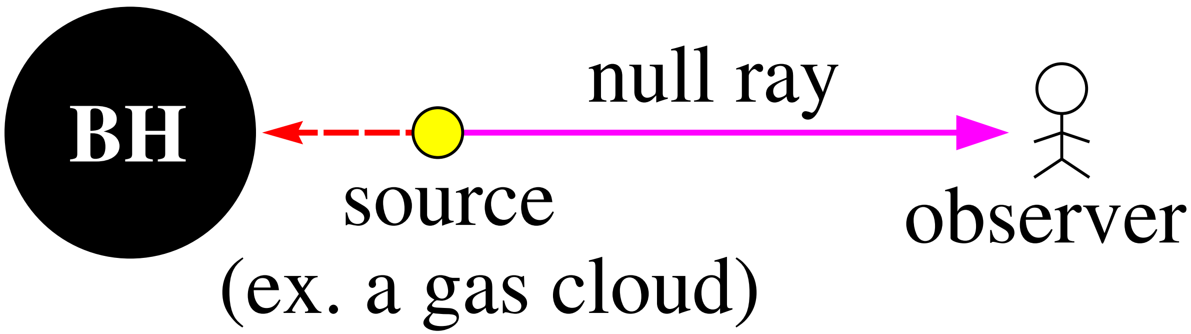
  - $\diamond$ **The observable verifying the existence of BH horizon**
  - $\diamond$ **The observable measuring the surface gravity of BH horizon**

- Note: **Our observable is realized in any wave propagating on BH spacetime (GW, EM waves, and so on), and independent of details of BH environments.**

2. Set up a situation : a simple example

- Consider the radially alinged situation

$$\begin{cases} \text{Black hole} & : \text{Schwarzschild} \rightarrow \text{radius } R_{\mathrm{BH}} \\ \text{Source} & : \text{radial free fall (time like geodesic)} \\ \text{Wave} & : \text{propagate on a radial null geodesic} \\ \text{Observer} & : \text{rest at a distant point} \end{cases}$$



◇ Note: It is found observationally that, in next summer 2013, a gas cloud is going to fall into the BH candidate at the center of our galaxy.

3. The observable verifying the existence of BH horizon

- The source is falling into BH

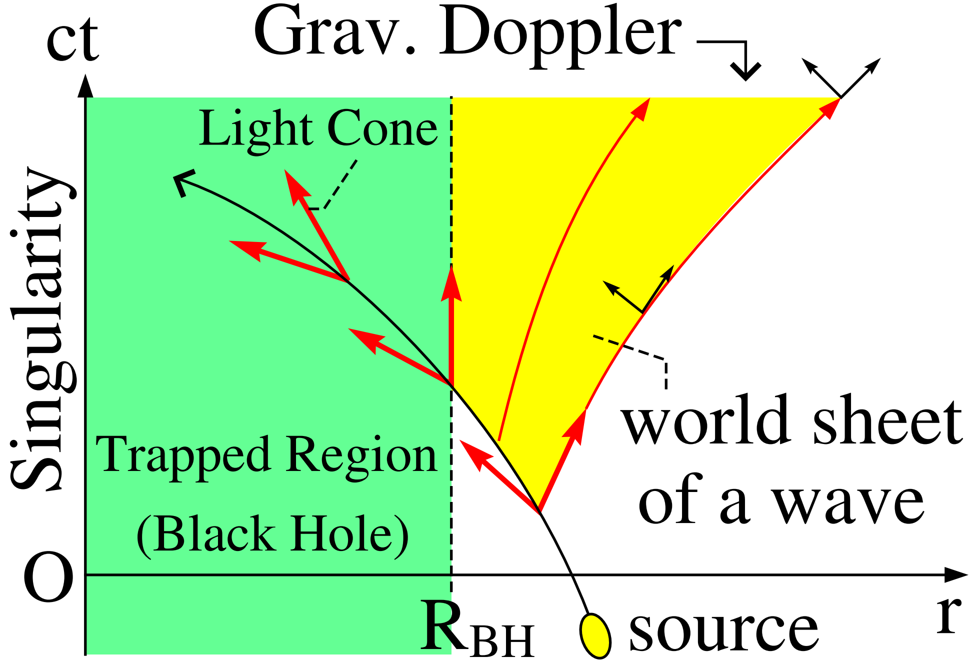
→ The wave length of wave emitted by this source is prolonged infinitely by the Grav. Doppler (redshift) due to BH horizon.

→ The infinite prolongation of wave length can verify the existence of BH horizon.

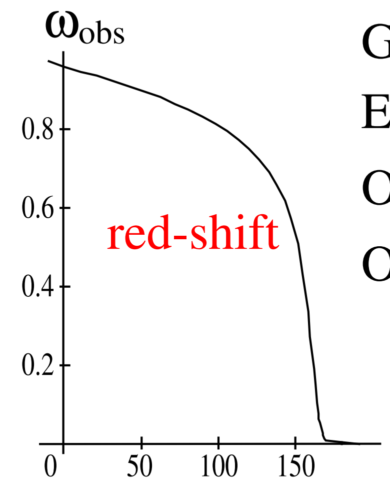
⇓

The freezing of wave should be observed.

(Numerical example is shown next)



- Ex. Freezing oscillation of observed wave (↓ "Gas's" = "Source's" ↓)



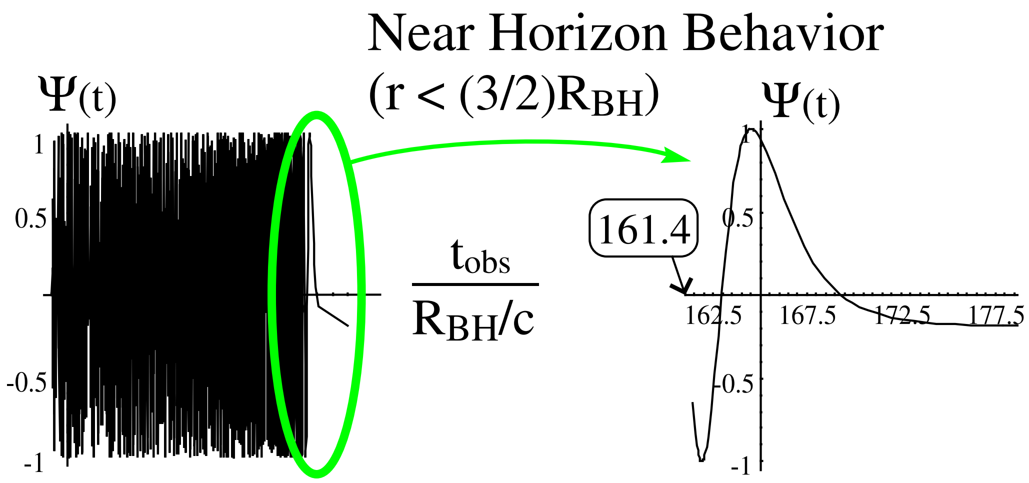
Gas's initial place $r_0 = 20\ R_{BH}$

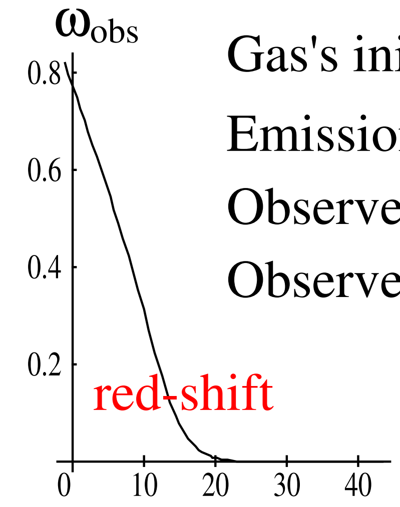
Emission Freq. $\omega_{emit} = 1$

Observed red-shift ω_{obs}

Observed wave (model)

$$\Psi(t) = \mathrm{Sin}[\int \omega_{obs}\, d(t_{obs})]$$





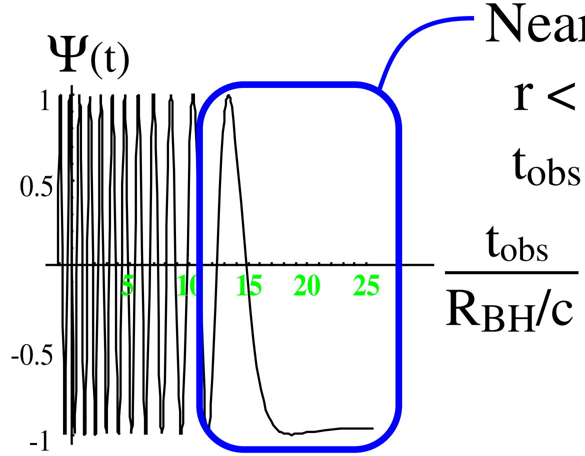
Gas's initial place $r_0 = 3\ R_{BH}$ (ISCO)

Emission Freq. $\omega_{emit} = 1$

Observed red-shift ω_{obs}

Observed wave (model)

$$\Psi(t) = \mathrm{Sin}[\int \omega_{obs}\, d(t_{obs})]$$



→ **The freezing oscillation appears in observation,**

when the source approaches BH horizon ($r \sim 3M$).

4. The observable measuring the mass and angular momentum of BH horizon

- Look precisely at the freezing ocsillation of observed wave:

Time eovlution of the phase of observed oscillation (wave)

$$\Theta(t_{\rm obs}) = \int \omega_{\rm obs}\, dt_{\rm obs} \sim \omega_0 \exp\left[-\frac{c\, t_{\rm obs}}{2R_{\rm BH}}\right] \,, \quad \begin{cases} \omega_0 = \text{const.} \\ t_{\rm obs} = \text{observer's time} \end{cases}$$

$\rightarrow$ Power spectrum of this time evolution $\propto$ **Planckian distribution**

$$\begin{cases} \text{Typical oscillation} : \Psi(t_{\rm obs}) = A(\omega_0) \exp[\, i\, \Theta(t_{\rm obs})\,] \\ \text{Fourier trans.} : F(\Omega, \omega_0) = \int_{-\infty}^{\infty} e^{-i\,\Omega\, t_{\rm obs}}\, \Psi(t_{\rm obs})\, dt_{\rm obs} \end{cases}$$

$$\therefore \quad P(\Omega) := \left|F(\Omega, \omega_0)\right|^2 \sim \frac{4\pi R_{\rm BH} |A|^2}{c} \frac{1}{\Omega} \frac{1}{e^{4\pi R_{\rm BH}\Omega/c} - 1}$$

$\rightarrow$ [**"Temperature"** $\dfrac{\hbar\, c}{4\pi R_{\rm BH}}$ **shown in time evolution**]

$=$ [**Hawking temperature** $\dfrac{\kappa}{2\pi}$] (κ : surface gravity of BH)

for Kerr BH, $\kappa = \dfrac{\sqrt{M^2 - a^2}}{2\left(M^2 + M\sqrt{M^2 - a^2}\right)}$

$\rightarrow$ **This gives a relation of M and a**

indepndent of details of BH environment.

$\rightarrow$ Combining with the other observation,

M and a will be determined observationally.

- But there is a point of notice w.r.t. observation $\cdots$

Real observation detects the wave as an oscillation of "real number". ($\Psi(t_{\rm obs})$ should be a wave in real number.)

$\cdots$ continued to next page

→ Wave in real number: $\Psi_{\rm R}(t_{\rm obs}) \;:=\; A(\omega_{\rm emit})\,\cos\Theta(t_{\rm obs})$

→ Fourier trans.: $F_{\rm R}(\Omega,\omega_{\rm emit}) \;=\; \int_{-\infty}^{\infty} e^{-i\,\Omega\,t_{\rm obs}}\,\Psi_{\rm R}(t_{\rm obs})\;{\rm d}t_{\rm obs}$

→ Power spectrum of time variation:

$$P_{\rm R}(\Omega) \;:=\; \left|F_{\rm R}(\Omega,\omega_{\rm emit})\right|^2 \;\sim\; \frac{\pi R_{\rm BH} A(\omega_{\rm emit})^2}{c}\,\frac{h(\Omega)}{\Omega}\,\frac{1}{e^{4\pi R_{\rm BH}\Omega/c}-1}$$

$$\text{where}\;\begin{cases} h(\Omega) \;=\; e^{4\pi R_{\rm BH}\Omega/c} + 2\left[\cos\Theta_\infty\right] e^{2\pi R_{\rm BH}\Omega/c} + 1 \\ \Theta_\infty \;=\; \Theta(t_{\rm obs}\to\infty) \;\;\textbf{:}\;\; \text{“Frozen” Phase} \end{cases}$$

Note 1: In real observation, the frozen phase Θ_∞ is required in odrde to describe the curve fitting to observed data.

Note 2: $A(\omega_{\rm emit})$ is a constant depending only on the source, not on the mass and angular momentum of BH.

5. Summary

- The observable verifying the existence of BH horizon

  → **The freezing oscillation found in time evolution of observed wave**

- The observable measuring the surface gravity of BH horizon

  → **The planckian distribution found in the power spectrum of time evolution of freezing oscillation**

  (Combining with the other observations, M and a will be determined.)

- Next issue (under consideration):
  - ◇ Application to gravitational collapse
  - ◇ Observation time required to obtain the Planckian distribution with good precision