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"Scale-dependent bias with the higher order primordial non-Gaussianity"

RESCEU SYMPOSIUM ON

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Scale-dependent bias with primordial higher order non-Gaussianity

~use of integrated Perturbation Theory~

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arXiv:1210.2495, in prep.

Simplest parameterization

•Focusing on Local type non-Gaussianity

(Komatsu & Spergel(2001), Byrnes, Sasaki & Wands(2006), …)

primordial curvature fluctuations

$$\Phi = \underline{\Phi_{\rm G}} + \underline{f_{\rm NL}} \left(\Phi_{\rm G}^2 - \langle \Phi^2 \rangle\right) + \underline{g_{\rm NL}} \Phi_{\rm G}^3 + \cdots$$

Gaussian fluc. $\sim 10^{-5}$

non-linearity parameters

→ **Non-zero higher order spectra**

(higher order correlation functions)

leadingly, …

- Bispectrum (3-point corr. func.) ⟷ $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle$

$$B_\Phi(k_1, k_2, k_3) \;=\; 2 f_{\rm NL} \left[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_3)P_\Phi(k_1)\right]$$

- Trispectrum (4-point corr. func.) ⟷ $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle_c$

Importance of trispcetrum

•Trispectrum (Byrnes, Sasaki & Wands(2006), Boubekeur & Lyth(2006) …)

$$\langle \Phi_{\mathbf{k}_1}\Phi_{\mathbf{k}_2}\Phi_{\mathbf{k}_3}\Phi_{\mathbf{k}_4}\rangle_c = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

$$T_\Phi(k_1,k_2,k_3,k_4) = 6g_{\rm NL}\,[P_\Phi(k_1)P_\Phi(k_2)+2\text{ perms.}] + \frac{25}{9}\tau_{\rm NL}\{P_\Phi(k_1)\,[P_\Phi(k_{13})+P_\Phi(k_{23})]+10\text{ perms.}\}$$

need two terms (different momentum-dependence)

Suyama and Yamaguchi (2008), …

We can generalize …

e.g.)

$$\Phi = \phi_G + \psi_G + f_{\rm NL}\left(\phi_G^2 - \langle\phi_G^2\rangle\right)$$

$$\langle \phi_G \psi_G\rangle = 0$$

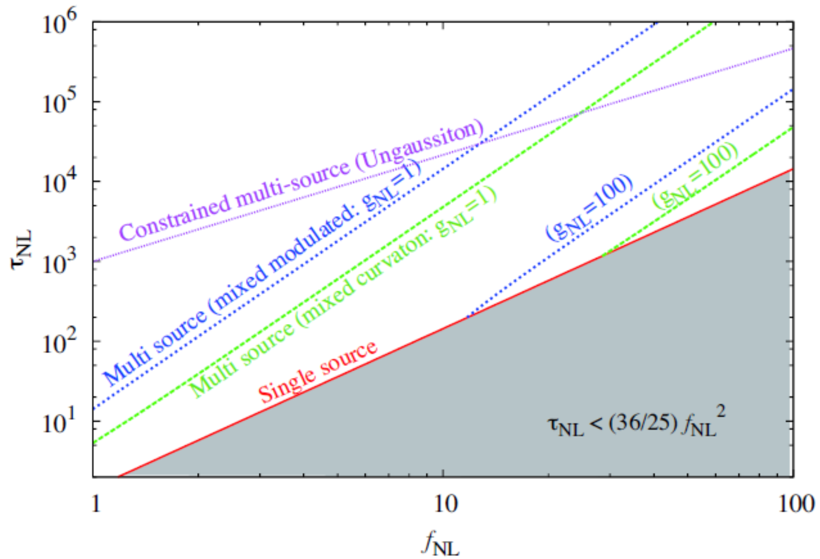
$$R \equiv P_\phi / P_\psi$$

$$\tau_{\rm NL} = \left(\frac{1+R}{R}\right)\left(\frac{6}{5}f_{\rm NL}\right)^2$$

$$\tau_{\rm NL} \geq \left(\frac{6}{5}f_{\rm NL}\right)^2$$

(Suyama, Takahashi, Yamguchi and SY (2010))

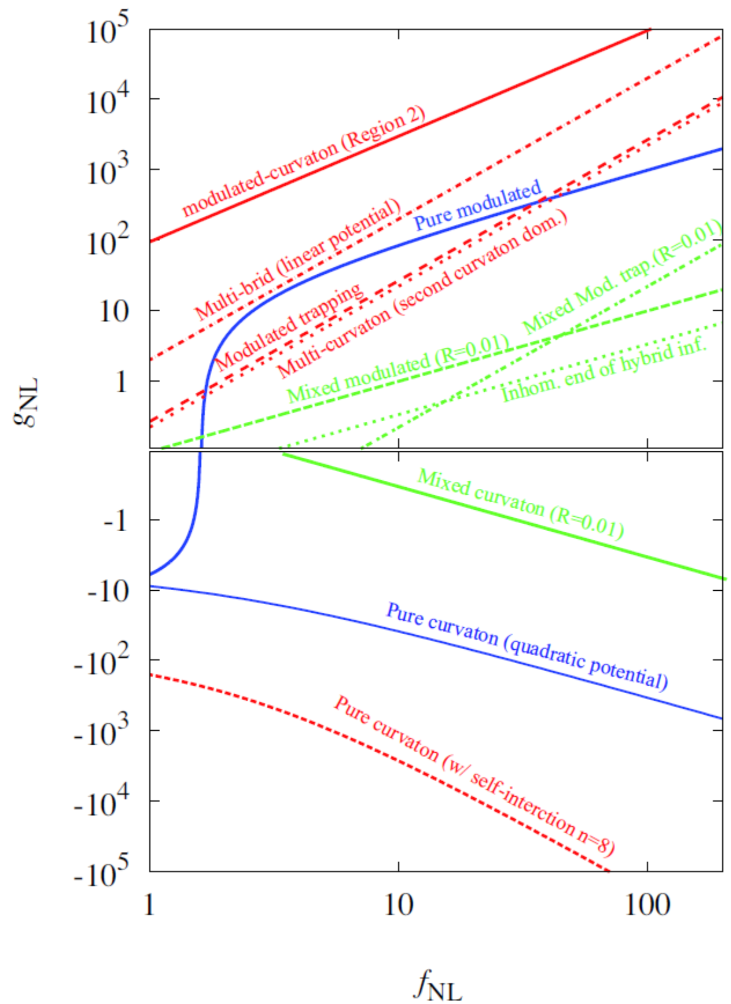
fNL vs tauNL



Different lines represent different models

distinguishing models !!!

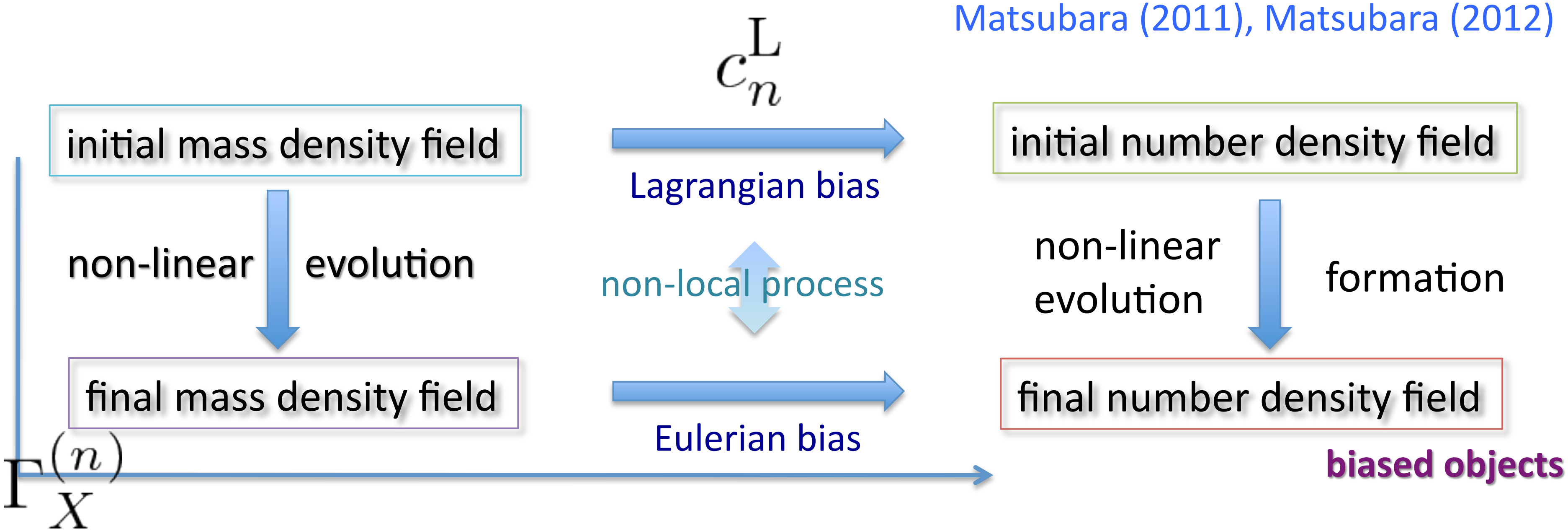
fNL vs gNL



How accuracy can we measure these parameters with using cosmological observations?

Integrated Perturbation Theory (iPT)

Matsubara (2011), Matsubara (2012)



Non-linear perturbation theory integrated with non-local bias, redshift-distorsions, and primordial non-Gaussianity

initial mass density field; δ_{L}

final mass density field; δ_m

initial number density field; δ_X^{L}

final number density field; δ_X

Without high peak limit and peak-background split picture

- Introducing multi-point propagators

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta\delta_L(\mathbf{k}_1)\cdots\delta\delta_L(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta^{(3)}(\mathbf{k}_1+\cdots+\mathbf{k}_n-\mathbf{k})\Gamma_X^{(n)}(\mathbf{k}_1,\cdots,\mathbf{k}_n),$$

- Power spectrum of the biased objects with primordial non-Gaussianity

(without any high peak approximation, peak-backgroud picture, …)

Matsubara(2012)

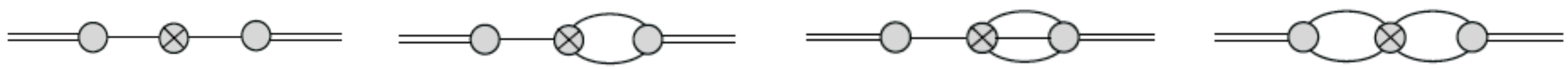
$$\begin{aligned} P_X(k) &= \left[\Gamma_X^{(1)}(\mathbf{k})\right]^2 P_L(k) + \Gamma_X^{(1)}(\mathbf{k})\int \frac{d^3p}{(2\pi)^3}\Gamma_X^{(2)}(\mathbf{p},\mathbf{k}-\mathbf{p})B_L(\mathbf{k},-\mathbf{p},\mathbf{k}+\mathbf{p}) \\ &+\frac{1}{3}\Gamma_X^{(1)}(\mathbf{k})\int \frac{d^3p_1 d^3p_2}{(2\pi)^6}\Gamma_X^{(3)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}-\mathbf{p}_1-\mathbf{p}_2)T_L(\mathbf{k},-\mathbf{p}_1,-\mathbf{p}_2,-\mathbf{k}+\mathbf{p}_1+\mathbf{p}_2) \\ &+\frac{1}{4}\int \frac{d^3p_1 d^3p_2}{(2\pi)^6}\Gamma_X^{(2)}(\mathbf{p}_1,\mathbf{k}-\mathbf{p}_1)\Gamma_X^{(2)}(-\mathbf{p}_2,-\mathbf{k}+\mathbf{p}_2)T_L(\mathbf{p}_1,\mathbf{k}-\mathbf{p}_1,-\mathbf{p}_2,-\mathbf{k}+\mathbf{p}_2) \end{aligned}$$

$$\left\{ \begin{aligned} P_L(k) &= \mathcal{M}(k)^2 P_\Phi(k) \\ B_L(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) &= \mathcal{M}(k_1)\mathcal{M}(k_2)\mathcal{M}(k_3)B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\ T_L(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) &= \mathcal{M}(k_1)\mathcal{M}(k_2)\mathcal{M}(k_3)\mathcal{M}(k_4)T_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) \end{aligned} \right.$$

$$\mathcal{M}(k) = \frac{2}{3}\frac{D(z)}{(1+z_*)D(z_*)}\frac{k^2 T(k)}{H_0{}^2\Omega_{m0}}$$

including growth factor, transfer function, Poisson equation, ..

- Diagrammatically, …



- Introducing renormalized bias functions

$$c_n^{\rm L}(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n) = (2\pi)^{3n} \int \frac{d^3p}{(2\pi)^3} \langle \frac{\delta^n \delta_X^{\rm L}(\mathbf{p})}{\delta\delta_{\rm L}(\mathbf{k}_1)\cdots\delta\delta_{\rm L}(\mathbf{k}_n)} \rangle$$

← depend on the mass function of the biased objects

multiplicity function

$$\Gamma_{\rm X}^{(1)}(\boldsymbol{k}) = 1 + c_1^{\rm L}(k)$$

$$\Gamma_{\rm X}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{{k_1}^2}\right) c_1^{\rm L}(k_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{{k_2}^2}\right) c_1^{\rm L}(k_1) + c_2^{\rm L}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

non-linear evolution
of the matter density field

$$F_n(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n) \to 0$$

$\mathbf{k}_1 + \mathbf{k}_2 + \cdots + \mathbf{k}_n = \mathbf{K} \to 0$ (on large scales)

- Multiplicity function

$$n(M) = -\frac{2\bar{\rho}_0}{M}\frac{f_{\rm MF}(\nu)}{2}\frac{d\ln\sigma_M}{dM}$$

variance of $\delta_M(\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} W(kR)\delta_{\rm L}(\boldsymbol{k})$

number density of the biased objects

multiplicity function

$\nu = \delta_c/\sigma_M$

critical density

Press-Schechter formalism, Sheth-Tormen fitting formula, ..

Then,

$$c_n^{\rm L}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = \frac{A_n(M)}{{\delta_{\rm c}}^n} W(k_1R)\cdots W(k_nR) + \frac{A_{n-1}(M)\,{\sigma_M}^n}{{\delta_{\rm c}}^n}\frac{d}{d\ln\sigma_M}\left[\frac{W(k_1R)\cdots W(k_nR)}{{\sigma_M}^n}\right]$$

$$A_n(M) \equiv \sum_{j=0}^{n} \frac{n!}{j!}{\delta_{\rm c}}^j\, b_j^{\rm L}(M)$$

$$b_n^{\rm L}(M) = (-1/\sigma_M)^n f_{\rm MF}^{(n)}/f_{\rm MF}$$

bias parameter can be evaluated!

Scale-dependent bias

- Bias parameter

$$P_X(k) \equiv b_X(k) P_{\rm L}(k)$$

on large scales $(k \to 0)$, $b_X(k) \approx b_1(M)^2 + \Delta b(k)$

Scale-dependent part;

$$\Delta b(k) \approx 4 f_{\rm NL} \frac{b_1(M)}{\mathcal{M}(k)} \left[b_2^{\rm L}(M) + 2 \frac{1 + \delta_c (b_1(M) - 1)}{\delta_c^2} \right] \sigma_M^2$$

→ k^-2-dependence

$$+ \left(g_{\rm NL} + \frac{25}{27} \tau_{\rm NL} \right) \frac{b_1(M)}{\mathcal{M}(k)}$$

→ k^-2-dependence

$$\times \left[b_3^{\rm L}(M) + \frac{2 + 2\delta_c (b_1(M) - 1) + \delta_c^2 b_2^{\rm L}(M)}{\delta_c^3} \left(4 + \frac{d \ln S_3(M)}{\ln \sigma_M} \right) \right] \sigma_M^4 S_3(M)$$

$$+ \frac{25}{9} \tau_{\rm NL} \frac{1}{\mathcal{M}(k)^2} \left[b_2^{\rm L}(M) + 2 \frac{1 + \delta_c (b_1(M) - 1)}{\delta_c^2} \right]^2 \sigma_M^4,$$

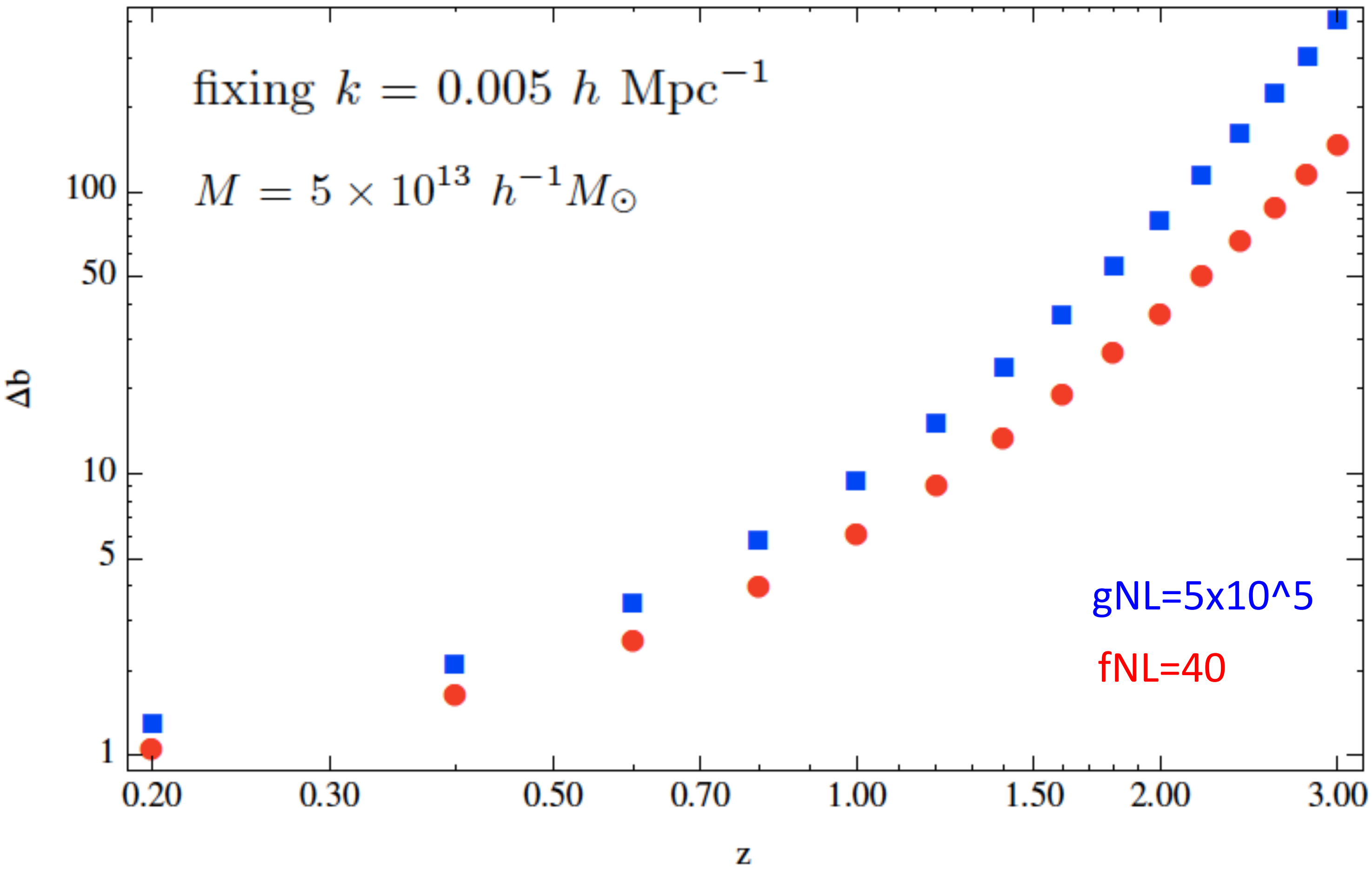
→ K^-4-dependence

$$S_3(M) = \frac{\langle \delta_M^3 \rangle}{\langle \delta_M^2 \rangle^2} \quad \delta_c = 1.68 \quad b_1(M) \equiv 1 + c_1^{\rm L}$$

Matsubara(2012), Yokoyama and Matsubara(2012)

- fNL vs gNL ; same k-dependence…

➔ Different redshift-dependence !



Higher redshift objects ➔ tighter constraints for gNL?

- fNL vs tauNL ; inequality $\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL}\right)^2$

Introducing a stochasticity parameter;

$$r(k) \equiv \frac{P_{\rm m}(k) P_X(k)}{P_{{\rm m}X}(k)^2}.$$

$P_{{\rm m}X}(k)$; matter density field – biased objects cross power spectrum

$P_{\rm m}(k)$; matter density field power spectrum

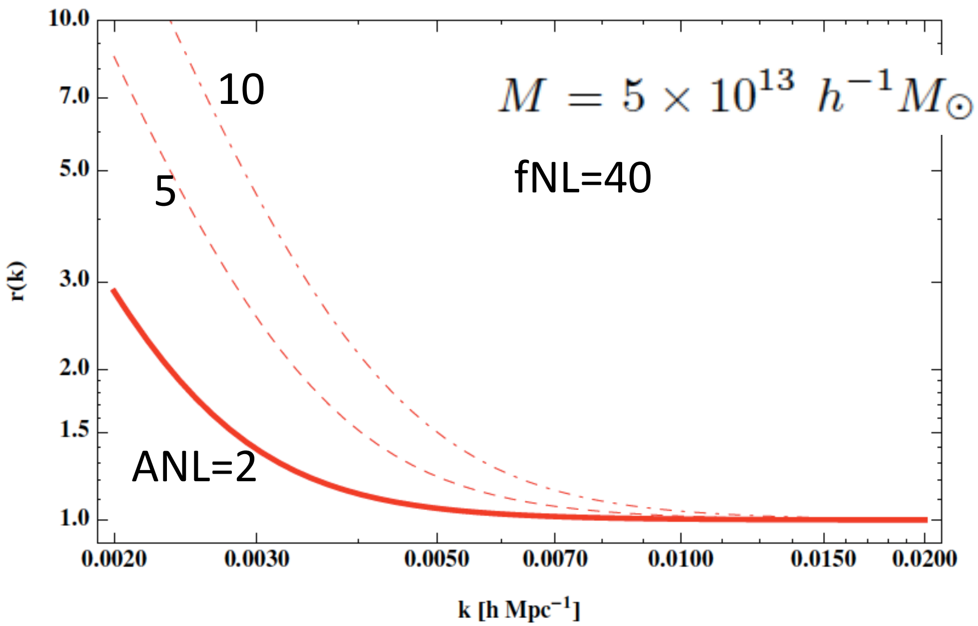
On large scales,

$$r(k) \simeq 1 + \left(\frac{25}{9}\tau_{\rm NL} - 4 f_{\rm NL}^2\right) \frac{1}{b_1(k)^2 \mathcal{M}(k)^2} \left[\int \frac{d^3p}{(2\pi)^3} c_2^{\rm L}(\mathbf{p}, -\mathbf{p}) P_{\rm L}(p)\right]^2$$

Directly dependent on the inequality !

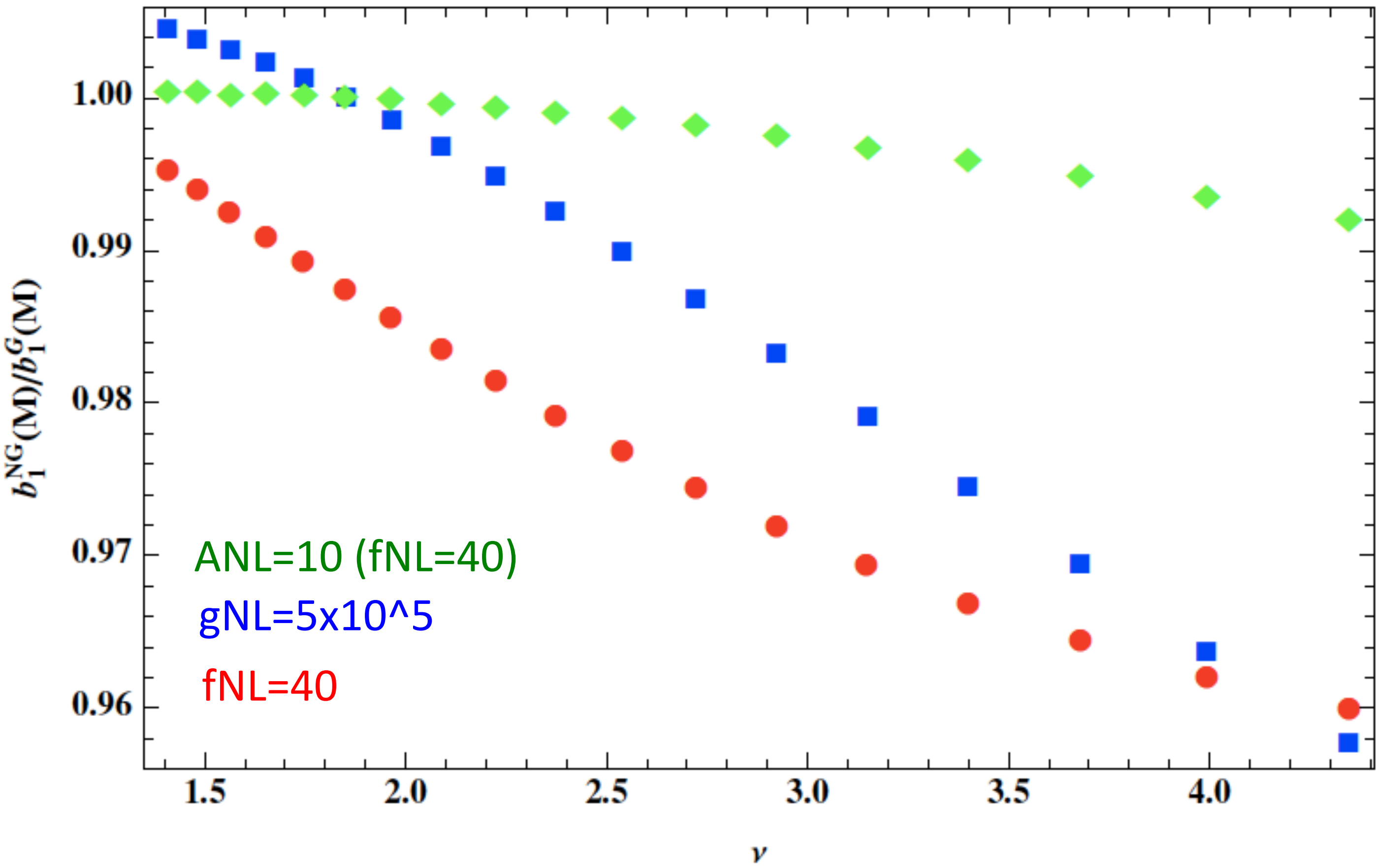
$$A_{\rm NL} = \tau_{\rm NL}/(6 f_{\rm NL}/5)^2$$

r(k) = 0, or < 0, or > 0 ??



- Higher order from non-Gaussian mass function

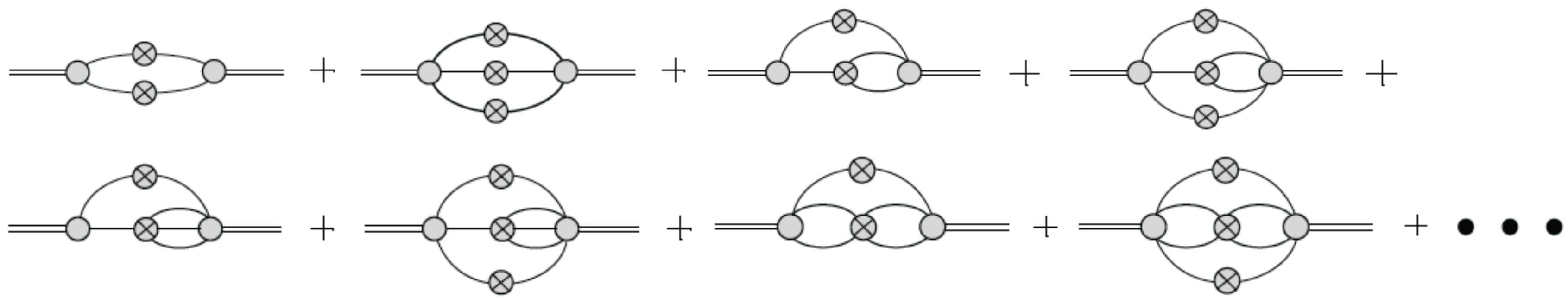
$$f_{\rm MF} = f_{\rm PS(ST)} \times R^{\rm NG}$$ → b ?



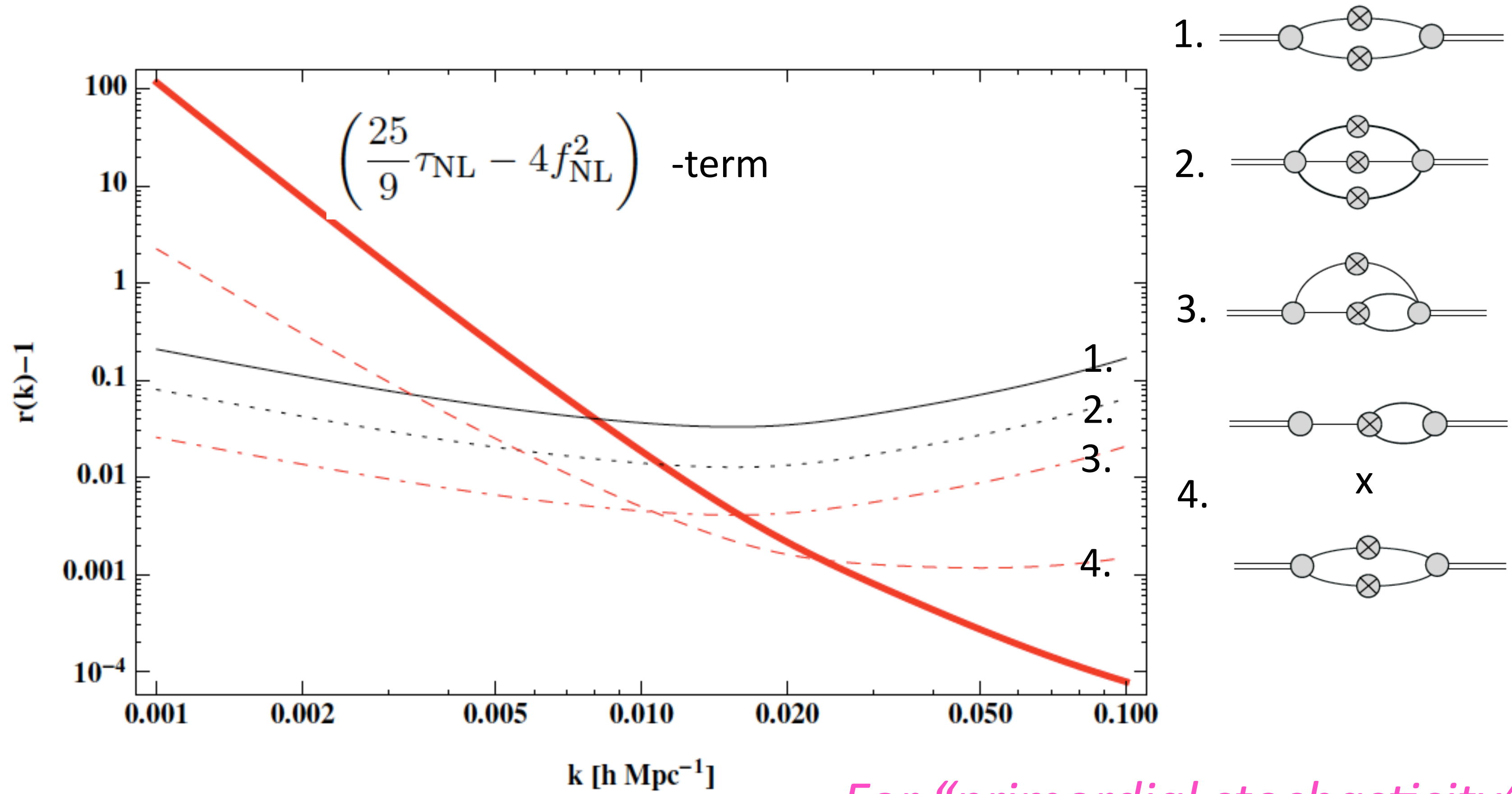
negligible for not so high peak objects..

- Higher order contribution (from b2, b3, …)

$$
\begin{aligned}
P_X(k) \;=\; & \sum_{n=1}^{\infty}\left[\frac{1}{n!}\int\frac{d^3p_1\cdots d^3p_{n-1}}{(2\pi)^{3n}}\Gamma_X^{(n)}(\mathbf{p}_1,\mathbf{p}_2,\cdots,\mathbf{k}-\mathbf{P}_{n-1})^2 P_{\rm L}(p_1)P_{\rm L}(p_2)\cdots P_{\rm L}(|\mathbf{k}-\mathbf{P}_{n-1}|)\right.\\
&+\frac{1}{(n-1)!}\int\frac{d^3p_1\cdots d^3p_n}{(2\pi)^{(3n)}}\Gamma_X^{(n)}(\mathbf{p}_1,\cdots,\mathbf{p}_{n-1},\mathbf{k}-\mathbf{P}_{n-1})\\
&\quad\times\Gamma_X^{(n+1)}(-\mathbf{p}_1,\cdots,-\mathbf{p}_n,-\mathbf{k}+\mathbf{P}_n)P_{\rm L}(p_1)\cdots P_{\rm L}(p_{n-1})B_{\rm L}(\mathbf{k}-\mathbf{P}_{n-1},-\mathbf{p}_n,-\mathbf{k}+\mathbf{P}_n)\\
&+\frac{1}{3(n-1)!}\int\frac{d^3p_1\cdots d^3p_{n+1}}{(2\pi)^{3n+3}}\Gamma_X^{(n)}(\mathbf{p}_1,\cdots,\mathbf{p}_{n-1},\mathbf{k}-\mathbf{P}_{n-1})\\
&\quad\left.\times\Gamma_X^{(n+2)}(-\mathbf{p}_1,\cdots,-\mathbf{p}_{n+1},-\mathbf{k}+\mathbf{P}_{n+1})P_{\rm L}(p_1)\cdots P_{\rm L}(p_{n-1})T_{\rm L}(\mathbf{k}-\mathbf{P}_{n-1},-\mathbf{p}_n,-\mathbf{p}_{n+1},-\mathbf{k}+\mathbf{P}_{n+1})\right]\\
&+\sum_{n=2}^{\infty}\frac{1}{4(n-2)!}\int\frac{d^3p_1\cdots d^3p_{n-1}d^3q}{(2\pi)^{3n}}\Gamma_X^{(n)}(\mathbf{p}_1,\cdots,\mathbf{p}_{n-1},\mathbf{k}-\mathbf{P}_{n-1})\\
&\quad\times\Gamma_X^{(n)}(-\mathbf{p}_1,\cdots,-\mathbf{p}_{n-2},-\mathbf{q},-\mathbf{k}+\mathbf{P}_{n-2}+\mathbf{q})\\
&\qquad\qquad\times P_{\rm L}(p_1)\cdots P_{\rm L}(p_{n-2})T_{\rm L}(\mathbf{p}_{n-1},\mathbf{k}-\mathbf{P}_{n-1},-\mathbf{q},-\mathbf{k}+\mathbf{P}_{n-2}+\mathbf{q}),
\end{aligned}
\tag{33}
$$



- Higher order in stochasticity parameter



$f_{NL} = 40$ and $\tau_{NL} = 5 \times 36 f_{NL}^2/25.$

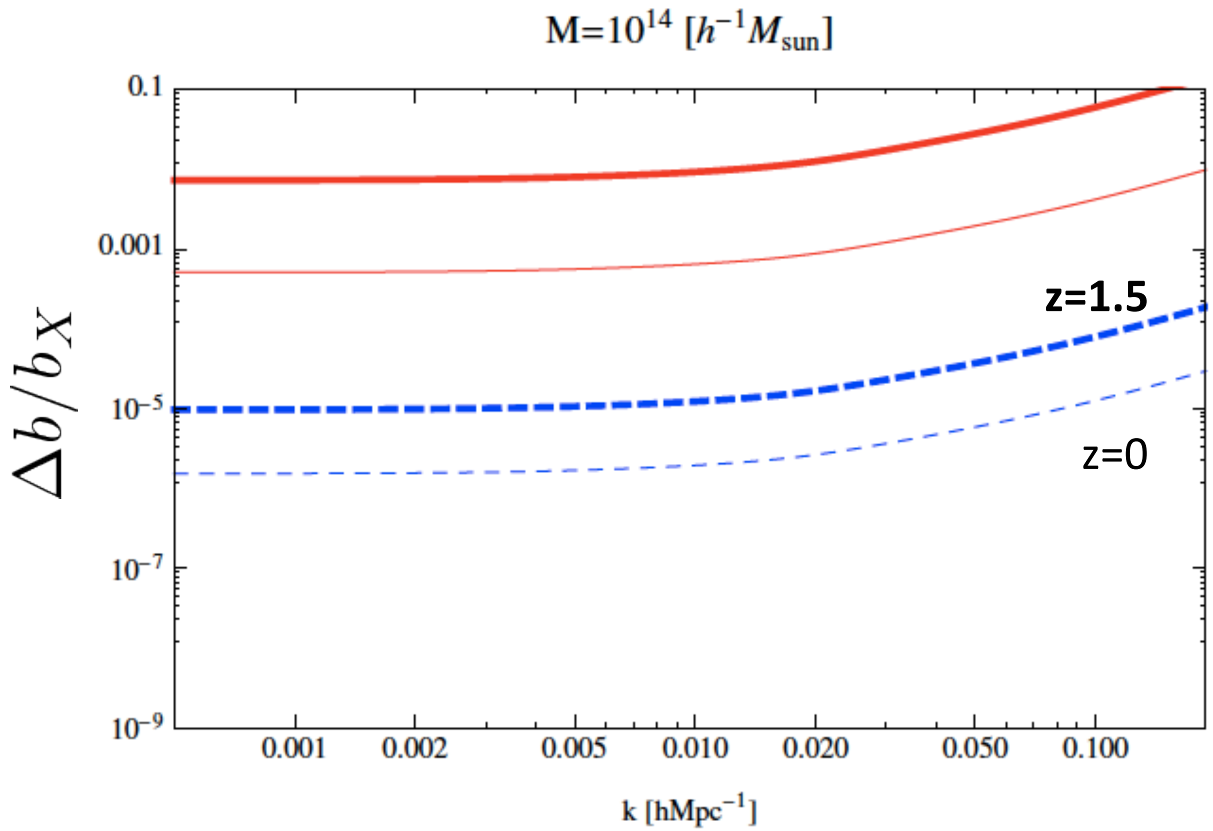
fixing $z = 1$ and $M = 5 \times 10^{13}\ h^{-1} M_\odot$

For "primordial stochasticity",

$$k < O(10^{-2})\ h\mathrm{Mpc}^{-1}$$

is needed ??

- Equilateral ?



fNL^eq = 200

gNL^eq = 4x10^4
(= (fNL^eq)^2)

$$T_\Phi^{\rm eql}(-\mathbf{k},\mathbf{p}_1,\mathbf{p}_2,\mathbf{k}-\mathbf{p}_1-\mathbf{p}_2) \simeq \frac{25}{9}\frac{4^5 g_{\rm NL}^{\rm eql} k^2 P_\Phi(k)\mathcal{P}_\Phi^2}{p_1 p_2 \left|\mathbf{p}_1+\mathbf{p}_2\right| \left(p_1+p_2+\left|\mathbf{p}_1+\mathbf{p}_2\right|\right)^5}$$

See e.g., Mizuno and Koyama (2010)

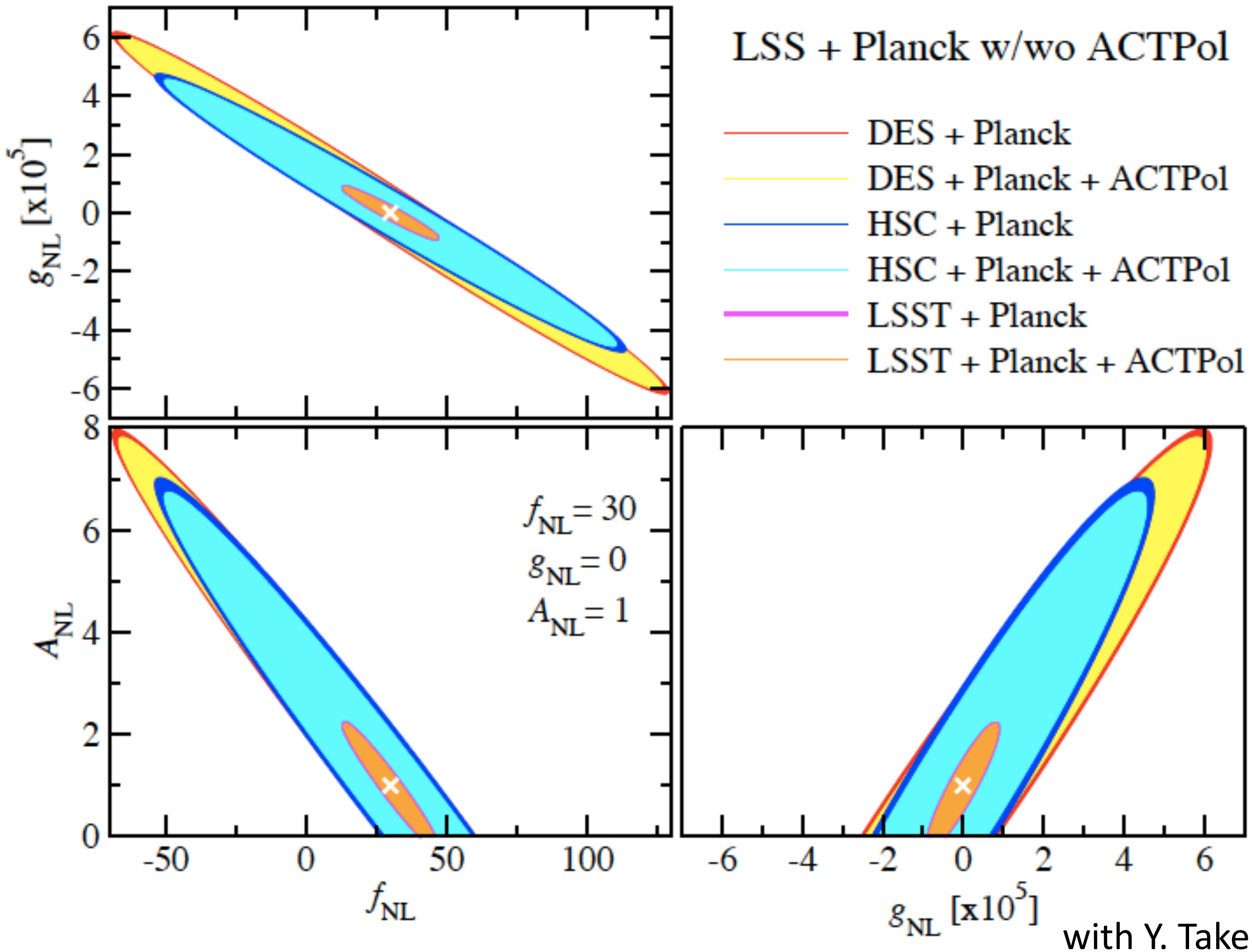
Negligibly small contribution

Summary and Discussion

- Derive an accurate formula for the bias parameter with primordial non-Gaussianity by using integrated Perturbation Theory
- wide (large scales) and deep (redshift-dependence) surveys are needed.

- Galaxy bispectrum?
- Forecast for the constraints on fNL, gNL and tauNL? (HSC, …)

• preliminary



with Y. Takeuchi in prep.

- Bispectrum of the biased objects

Up to the one-loop order in the multi-point propagators, we have

$$\begin{aligned}B_X(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \;=\; & \left[\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)\Gamma_X^{(2)}(-\mathbf{k}_1,-\mathbf{k}_2)P_{\rm L}(k_1)P_{\rm L}(k_2)+2\text{ perms.}\right]\\ & +\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)\Gamma_X^{(1)}(\mathbf{k}_3)B_{\rm L}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\\ & +\frac{1}{2}\left[\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)\int\frac{d^3p}{(2\pi)^3}T_{\rm L}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p},\mathbf{k}_3-\mathbf{p})+2\text{ perms.}\right]\\ & +\left[\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)\int\frac{d^3p}{(2\pi)^3}\Gamma_X^{(3)}(-\mathbf{k}_1,\mathbf{p},-\mathbf{k}_2-\mathbf{p})P_{\rm L}(k_1)B_{\rm L}(\mathbf{k}_2,\mathbf{p},-\mathbf{k}_2-\mathbf{p})+2\text{ perms.}\right]\\ & +\cdots\end{aligned}$$