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Instability of Charged Lovelock Black Holes

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Abstract

We study stability of charged black hole solutions in Lovelock theory that is a natural higher dimensional extension of 4-dimensional Einstein theory. In the Lovelock-Maxwell system, there exist black hole solutions with two parameters, i.e., mass and charge. We examine the linear stability of this solution and construct master equations. We present the criterion for instability using these equations. By checking these criterion numerically, we show that nearly extreme black holes show instability in high frequency modes.

1 Introduction

In higher dimensional gravitational theory, when we adopt brane world scenarios, black holes might be created at colliders [1]. Thus it is important to examine the properties of higher dimensional black holes. There exist many properties, but linear stability is one of the most important property. This is because black holes with instability must not be created.

When we consider higher dimension, we must extend 4-dimensional Einstein theory into higher dimensional gravitational theory. In 4-dimensions, Einstein theory is constructed on the basis of two assumptions: the covariance and no higher derivative term. Thus, it is natural to extend the 4-dimensional gravitational theory keeping these assumptions. When we extend like this, the most general gravitational theory is not Einstein theory. It is Lovelock theory [2]. Therefore, it is important to examine stability of black hole solutions in Lovelock theory.

In Lovelock theory, there exist black hole solution with mass and stability of this solution has been already examined in [4]. However, it is supposed that black holes in colliders have charge because such objects result from proton-proton collisions. Thus we must regard Maxwell charge in examining stability. In the Lovelock-Maxwell system, there exist black holes with mass and charge, namely charged Lovelock black hole solutions. Here, we examine these solutions.

The organization is as follows. In section 2, we present charged Lovelock black hole solutions. In section 3, we examine the stability. The background solution has the spherical symmetry, so we can discuss tensor-type, vector-type and scalar-type perturbations separately. Thus we present the criteria for instability separately. Furthermore, we present a numerical result for 5-dimensional case. In section 4, we summarize the discussions.

2 Charged Lovelock Black Holes

In this section, we present charged Lovelock black hole as the solution of the Lovelock-Maxwell system. This system is governed by the action

$$\int d^D x \sqrt{-g}\left[\mathcal{L}_{\text{Lovelock}} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] \tag{1}$$

where $F_{\mu\nu}$ is the field strength of Maxwell field A_μ and $\mathcal{L}_{\text{Lovelock}}$ is

$$\mathcal{L}_{\text{Lovelock}} = \beta_1 R + \sum_{m=2}^{k} \frac{\beta_m (2m)!}{2^m m \prod_{p=1}^{2m-2}(n-p)} \delta^{[\lambda_1}_{\kappa_1}\delta^{\sigma_1}_{\rho_1}\delta^{\lambda_2}_{\kappa_2}\cdots\delta^{\sigma_m]}_{\rho_m} R_{\lambda_1\sigma_1}{}^{\kappa_1\rho_1}\cdots R_{\lambda_m\sigma_m}{}^{\kappa_m\rho_m} . \tag{2}$$

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Here n is related to the dimension D as $n = D - 2$ and β_ms are arbitrary constants which are called Lovelock coefficients. And k corresponds to the maximum order defined as $k \equiv [(D-1)/2]$.

In this system, there exist spherically symmetric solutions. We here start from the ansatz

$$ds^2 = -f(r)dt^2 + 1/f(r)dr^2 + r^2\gamma_{ij}dx^i dx^j, \tag{3}$$
$$F^{tr} = E(r), \quad \text{other components} = 0 \ . \tag{4}$$

Substituting these into the equations of motion derived from the above action, we obtain the following solution:

$$\begin{aligned} f(r) &= 1 - r^2\psi(r) \ , \\ \mathcal{P}[\psi] &\equiv \sum_{m=2}^{k}\left[\frac{\beta_m}{m}\psi^m\right] + \psi = \frac{\mathcal{M}}{r^{n+1}} - \frac{\mathcal{Q}^2}{r^{2n}} \ , \end{aligned} \tag{5}$$

and

$$E(r) = \sqrt{n(n-1)}\mathcal{Q}/r^n \ . \tag{6}$$

Here $\mathcal{M}$ is a mass parameter of a black hole and $\mathcal{Q}$ corresponds to charge. In order to gain the explicit form, we must solve the k-th order polynomial equation (5). This equation has at most k roots, but there must only an asymptotically flat root. Here we call this asymptotic flat root charged Lovelock black hole.

3 Stability Analysis

In this section, we check the stability of this charged Lovelock black hole solution [3]. This solution has the spherical symmetry, so we can classify the first order perturbations into tensor-type, vector-type and scalar-type and we can treat these separately.

3.1 Tensor-type Perturbations

Firstly, we discuss the tensor-type perturbations. The Maxwell field A_μ has no tensorial perturbations and then we only consider the gravitational perturbations

$$\delta g_{tt} = \delta g_{tr} = \delta g_{ti} = \delta g_{rr} = \delta g_{ri} = 0, \ \delta g_{ij} = r^2\phi(r)e^{i\omega t}\mathcal{T}_{ij}. \tag{7}$$

Here, $\mathcal{T}_{ij}$ is tensor harmonics which is characterized by $\mathcal{T}_{ij}{}^{|k}{}_{|k} = -(\ell(\ell+n-1)-2)\mathcal{T}_{ij}$. Using this metric perturbation, we can obtain

$$\left(-\partial_{r^*}^2 + V_g(r)\right)\Psi = \omega^2\Psi \ , \tag{8}$$

where

$$\begin{aligned} \Psi &= \phi(r)r\sqrt{T'(r)}, \ dr^*/dr = 1/f \ , \ T(r) = r^{n-1}\partial_\psi\mathcal{P} \ , \\ V_g(r) &= \frac{\ell(\ell+n-1)f}{(n-2)r}\frac{T''}{T'} + \frac{1}{r\sqrt{T'}}\partial_{r^*}^2(r\sqrt{T'}) \ . \end{aligned} \tag{9}$$

This is a Schrödinger-type equation and its eigenvalue is ω^2 and we decompose like $e^{i\omega t}$. Thus, if there exist negative eigenvalue states, charged Lovelock black holes are unstable. The effective potential for this equation is characterized by the function $T(r)$ and the constant ℓ. Then , for instance, negative eigenvalue states may exist if $T(r)$ behaves peculiar. Indeed, as is shown In [5], we can show that the Schrödinger equation for large ℓ modes has negative eigenvalue states if T' or T'' have negative regions. Thus what we have to do next is checking these criteria and we will do in subsection 3.4.

3.2 Vector-type Perturbations

For vector-type perturbations, we take the Regge-Wheeler gauge in which perturbative variables are expressed as

$$\begin{aligned}&\delta A_t = \delta A_r = 0,\ \delta A_i = C e^{i\omega t}\mathcal{V}_i\\&\delta g_{ti} = w e^{i\omega t}\mathcal{V}_i,\ \delta g_{ri} = v e^{i\omega t}\mathcal{V}_i,\ \text{otherwise} = 0\ ,\end{aligned} \tag{10}$$

where $\mathcal{V}_i$ is the vector harmonics which is characterized by ${\mathcal{V}_i}^{|l}{}_{|l} = -\kappa_v \mathcal{V}_i$. Substituting these into equation of motion and using suitable variables, we can gain the following master equation:

$$\left(-\partial_{r^*}^2 + \begin{pmatrix} V_g(r) & V_c(r) \\ V_c(r) & V_{em}(r) \end{pmatrix}\right)\begin{pmatrix} \Psi \\ \zeta \end{pmatrix} = \omega^2 \begin{pmatrix} \Psi \\ \zeta \end{pmatrix}\ , \tag{11}$$

where

$$\begin{aligned}
V_g(r) &= \frac{\kappa_v - (n-1)}{n-1}\frac{fT'}{rT} + r\sqrt{T'}\partial_{r^*}^2 \frac{1}{r\sqrt{T'}}\ ,\\
V_c(r) &= -2(Er^n)\sqrt{\frac{\kappa_v - (n-1)}{2(n-1)}}\frac{f\sqrt{T'}}{r^{(n+2)/2}T}\ ,\\
V_{em}(r) &= (\kappa_v + (n-1))\frac{f}{r^2} + 2(Er^n)^2\frac{f}{r^{n+1}T} + r^{-(n-2)/2}\partial_{r^*}^2 r^{(n-2)/2}\ .
\end{aligned} \tag{12}$$

This is a Schrödinger equation with two components and ω^2 is its eigenvalue. In the same way as tensor-type, if the above Schrödinger equation has negative eigenstates, black holes are unstable. However, as is shown in [5], the Schrödinger operator is positive-definite. Therefore, there is no negative eigenstate in the above system.

3.3 Scalar-type Perturbations

Finally, we consider the scalar-type perturbations. In the Zerilli gauge, perturbative variables are expressed as

$$\begin{aligned}&\delta g_{tt} = fH_0\mathcal{Y},\ \delta g_{tr} = H_1\mathcal{Y},\ \delta g_{rr} = H\mathcal{Y}/f,\ \delta g_{ij} = r^2 K\mathcal{Y}\gamma_{ij},\\&\delta F_{tr} = X\mathcal{Y},\ \delta F_{ti} = Y\mathcal{Y}_{|i},\ \delta F_{ri} = Z\mathcal{Y}_{|i},\ \text{otherwise} = 0\ ,\end{aligned} \tag{13}$$

where $\mathcal{Y}$ is the scalar harmonics which is characterized by ${\mathcal{Y}_{|l}}^{|l} = -\kappa_s\mathcal{Y}$. Substituting these into equation of motion and using suitable variables which is discussed in [5], the perturbative equation can be summarized as

$$\left(-\partial_{r^*}^2 + \begin{pmatrix} V_g(r) & V_c(r) \\ V_c(r) & V_{em}(r) \end{pmatrix}\right)\begin{pmatrix} \Psi \\ \zeta \end{pmatrix} = \omega^2 \begin{pmatrix} \Psi \\ \zeta \end{pmatrix}\ , \tag{14}$$

where

$$\begin{aligned}
V_g(r) &= \kappa_s\frac{f}{nr}\left(4(\kappa_s - n)\frac{T'}{\mathcal{A}T} - \frac{T''}{T'}\right)\\
&\quad + \frac{2n(r^nE)^2f^2}{\mathcal{A}Tr^n}\left(\ln\left(\frac{fT'}{r^{n-2}(\mathcal{A}T)^2}\right)\right)' + \frac{\mathcal{A}T}{r\sqrt{T'}}f\partial_r\left(f\partial_r\frac{r\sqrt{T'}}{\mathcal{A}T}\right)\ ,\\
V_c(r) &= \sqrt{\frac{\kappa_s - n}{n}}\frac{\sqrt{T'}}{r^{n/2}\mathcal{A}}\left[-\kappa_s\frac{4(r^nE)f}{rT} + \frac{2n(r^nE)f^2}{T}\left(\ln\left(\frac{fT'}{r^{n-2}(\mathcal{A}T)^2}\right)\right)'\right]\ ,\\
V_{em}(r) &= fr^{(n-2)/2}\partial_r(f\partial_r\frac{1}{r^{(n-2)/2}})\\
&\quad + \kappa_s\frac{f}{r^2}\left(1 + \frac{4(Er^n)^2}{\mathcal{A}Tr^{n-1}}\right) + \frac{2n(r^nE)^2f^2}{r^n\mathcal{A}T}\left(\ln\left(\frac{r^{2n-2}(\mathcal{A}T)^2}{f}\right)\right)'\ ,\\
\mathcal{A} &= 2\kappa_s + nrf' - 2nf\ .
\end{aligned} \tag{15}$$

The above equation is Schrödinger equation. Thus, in the same way as tensor-type perturbations, whether negative energy states exist or not determines the stability of the background solutions. For this, as is shown in [5], we can prove that "if T' or $2T'^2 - TT''$ have negative regions, black holes show instability". Thus what we have to do next is checking the behaviors of the above functions and we examine these in the next subsection.

3.4 Numerical Result in 5-dimensions

Here, we check the behaviors of three function T', T'' and $2T'^2 - TT''$ numerically. For instance, we present the result for the 5-dimensional case. Fig.1 corresponds to this result.

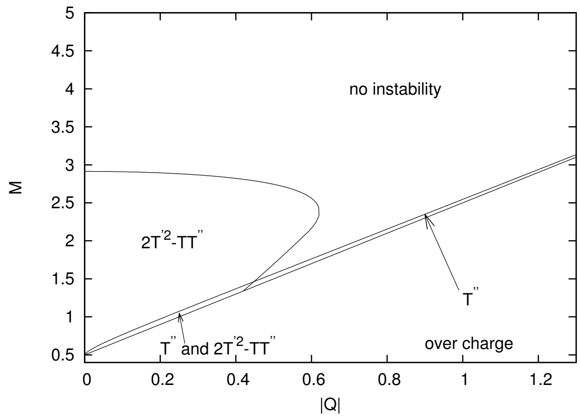

Figure 1: We show the unstable parameters in 5-dimensions. The horizontal axis correspond charge and the vertical one is mass. From this figure, there exist instability for nearly extreme black holes.

From this figure, we can easily see that nearly extreme black hole show instability. Note that this property is common to the other dimensions. Therefore, we can say that nearly extreme black holes are unstable in the Lovelock-Maxwell system.

4 Conclusion

We have studied the stability of charged black holes in Lovelock black holes. we have presented the master equation for each type of perturbations and shown the criteria for instability. We have also examined the criteria numerically and shown that nearly extreme black holes are unstable. These result indicates that black hole with nearly extreme mass are not created and very high energy is needed to produce black holes in colliders if Lovelock theory is true.

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