

Andrei Frolov, JGRG 22(2012)111301

“Primordial non-gaussianity from preheating”

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## RESCEU SYMPOSIUM ON GENERAL RELATIVITY AND GRAVITATION

# JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan



# PRIMORDIAL NON-GAUSSIANITY FROM PREHEATING

arXiv:0903.3407 (BFHK) ; arXiv:1004.3559 (REVIEW)



Andrei Frolov

Department of Physics  
Simon Fraser University



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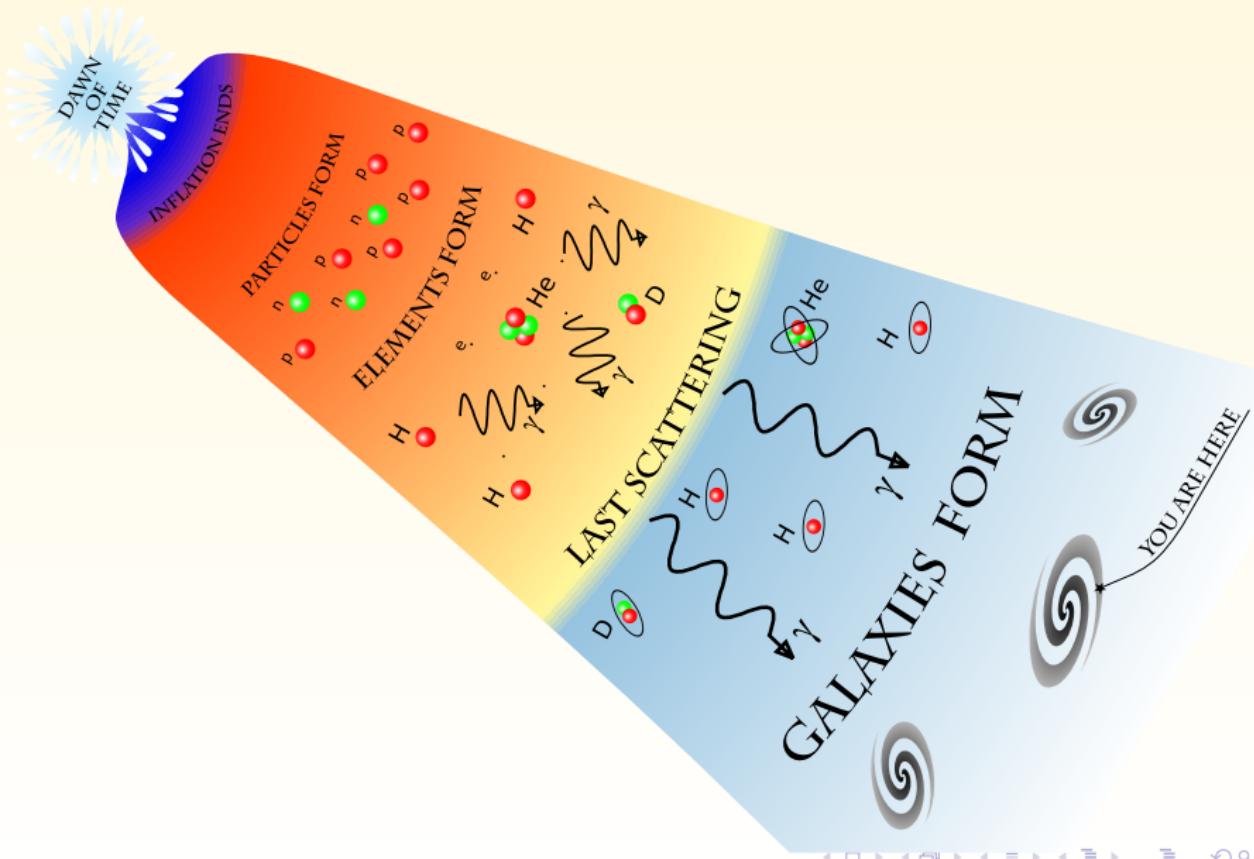
Lev Kofman, Dick Bond, Jonathan Braden, Zhiqi Huang (CITA)

*RESCEU Symposium on General Relativity and Gravitation*

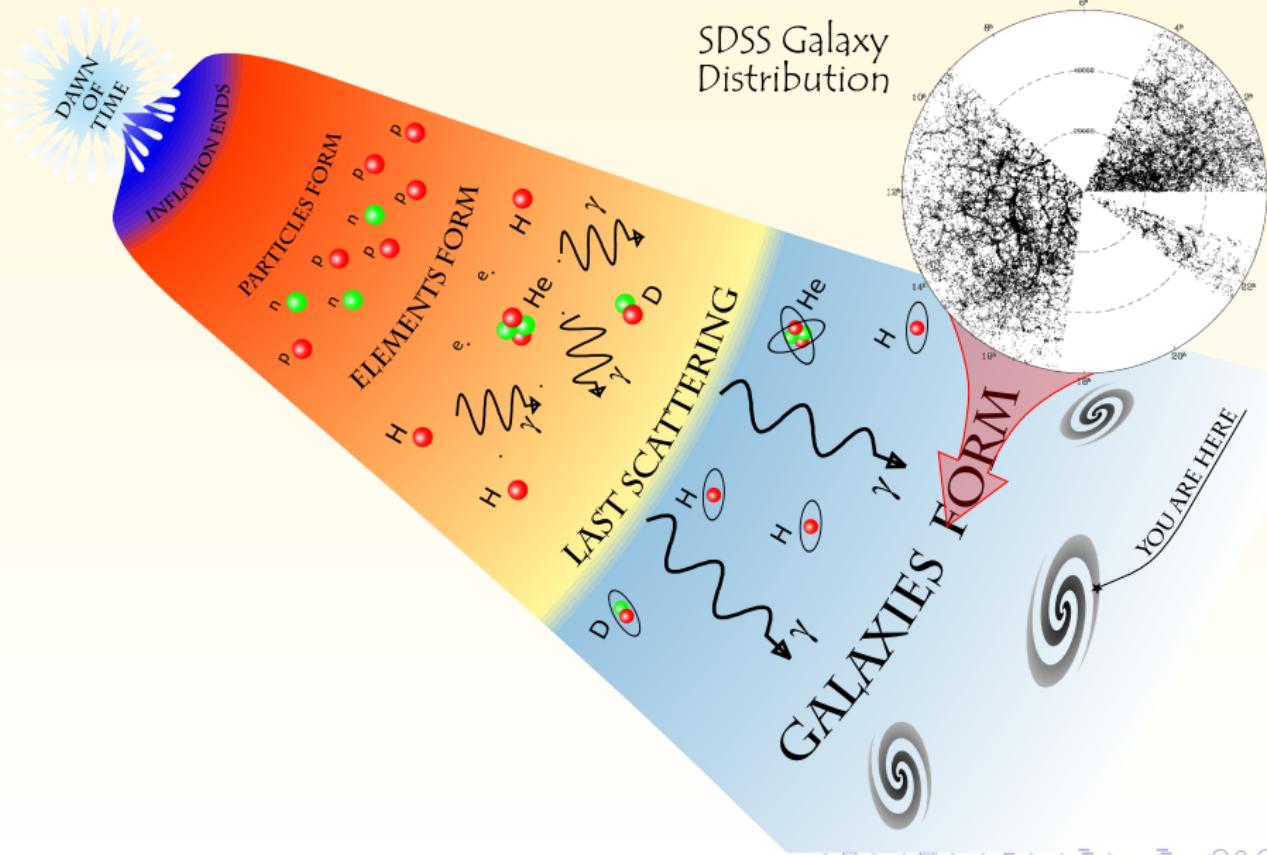
*University of Tokyo, Tokyo, Japan*

*13 November 2012*

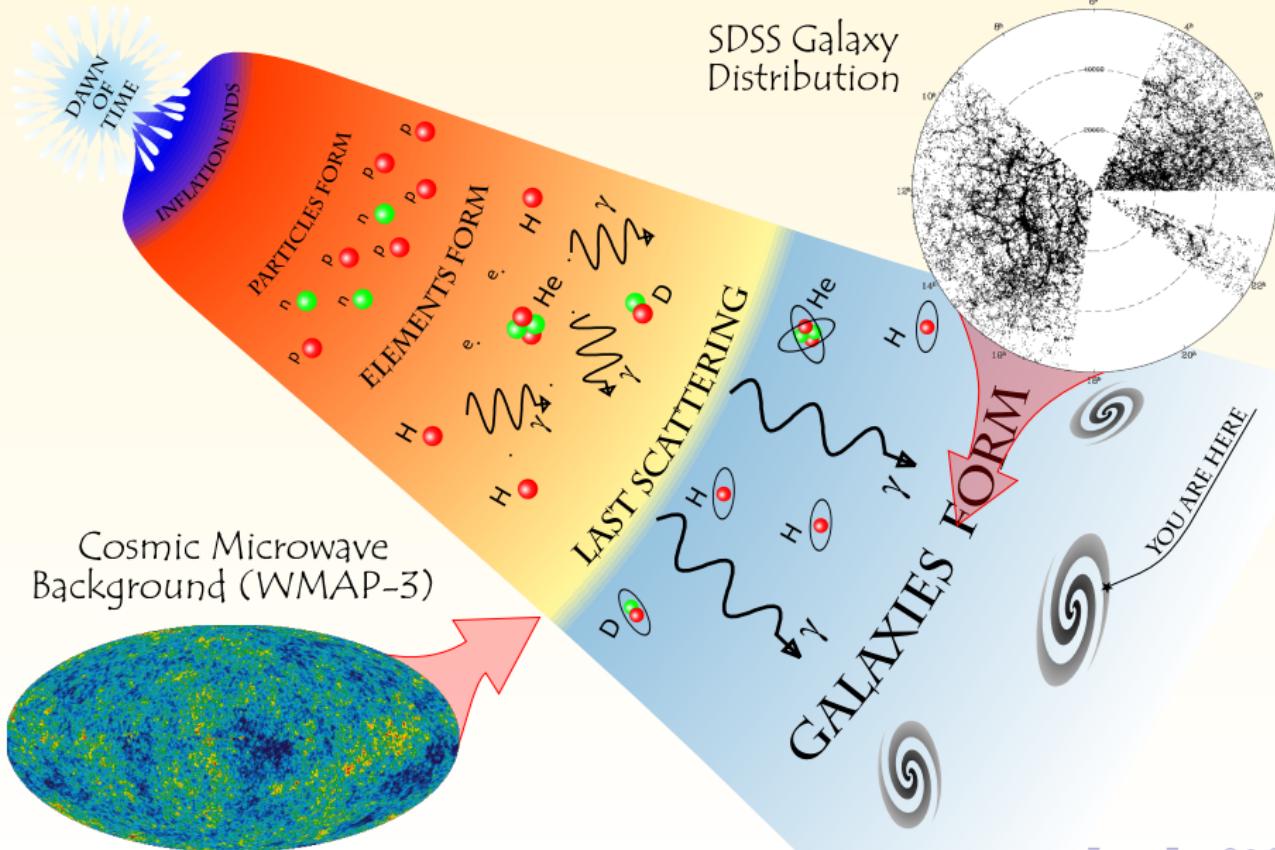
# BRIEF HISTORY OF THE UNIVERSE



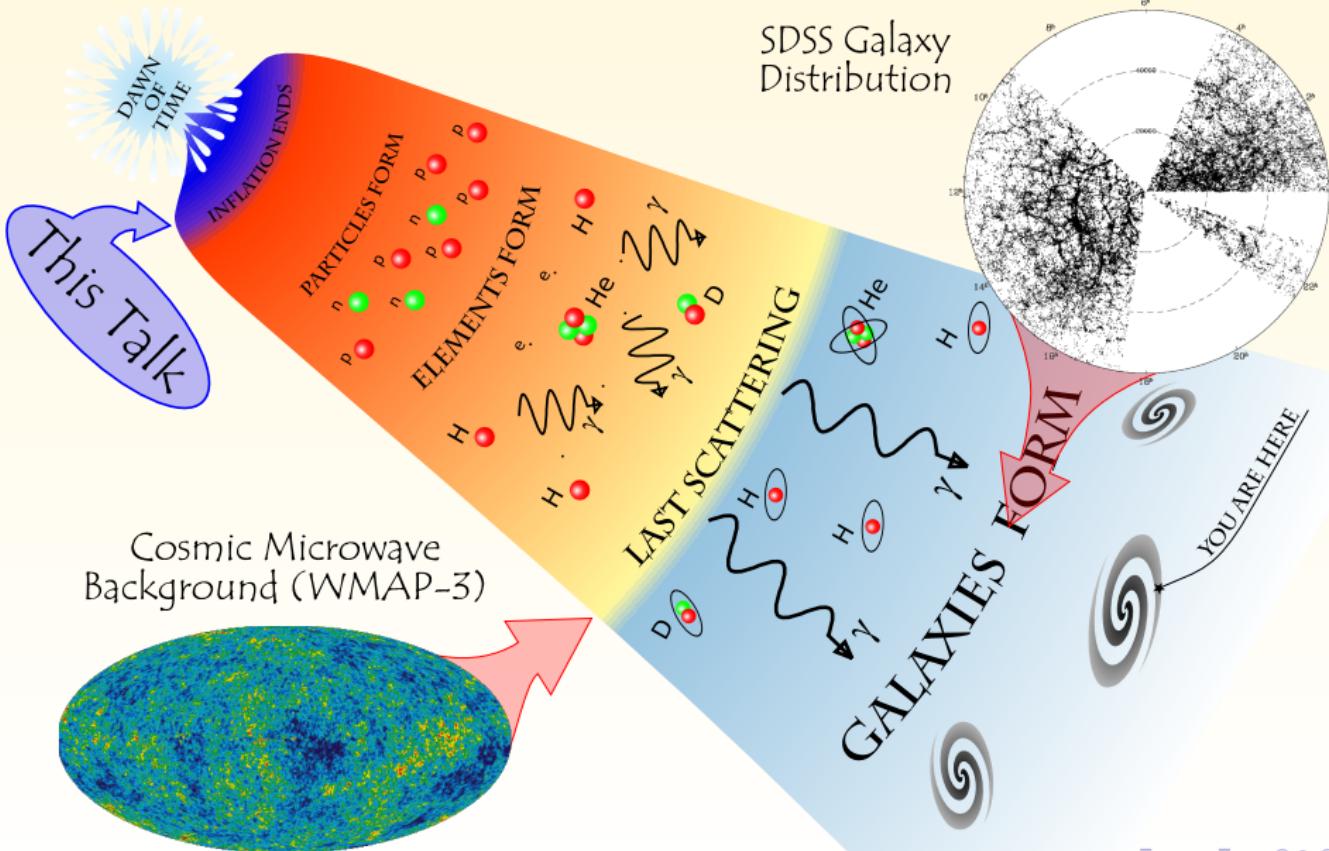
# BRIEF HISTORY OF THE UNIVERSE



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# A SIMPLE MODEL OF INFLATION...

Model with potential

$$V(\phi, \chi) = \frac{\lambda}{4} \phi^4 + \frac{g^2}{2} \phi^2 \chi^2$$

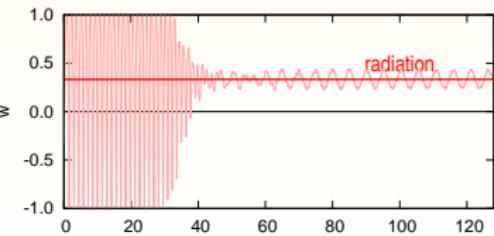
is invariant under

$$g_{\mu\nu} \mapsto a^{-2} g_{\mu\nu},$$

$$\phi \mapsto a\phi,$$

$$\chi \mapsto a\chi$$

equation of state is  $1/3$



# ... ENDING VIA BROAD PARAMETRIC RESONANCE

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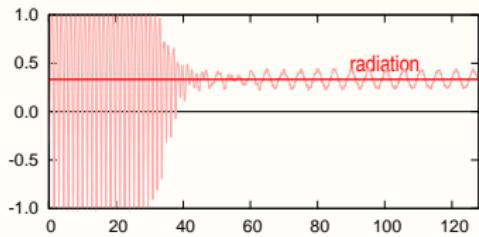
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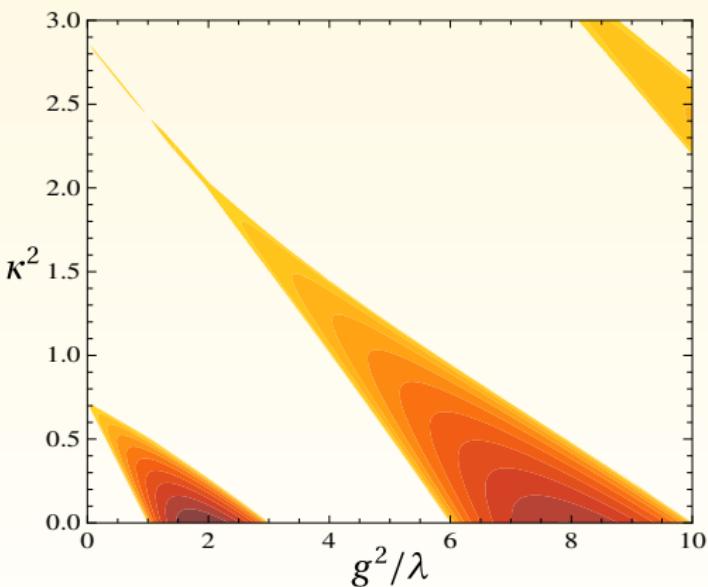
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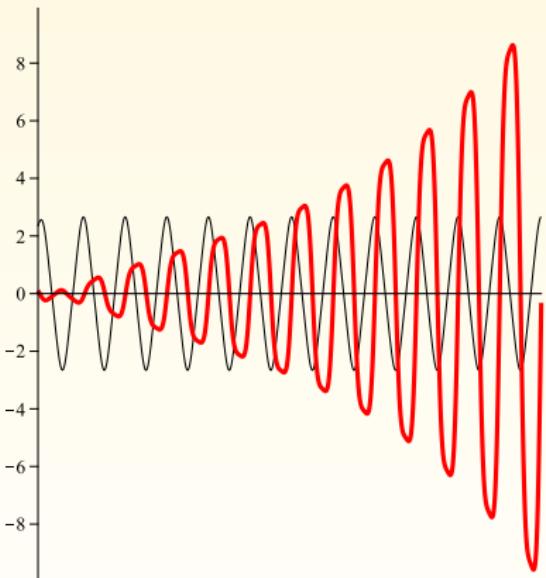


Stability of Lame equation:

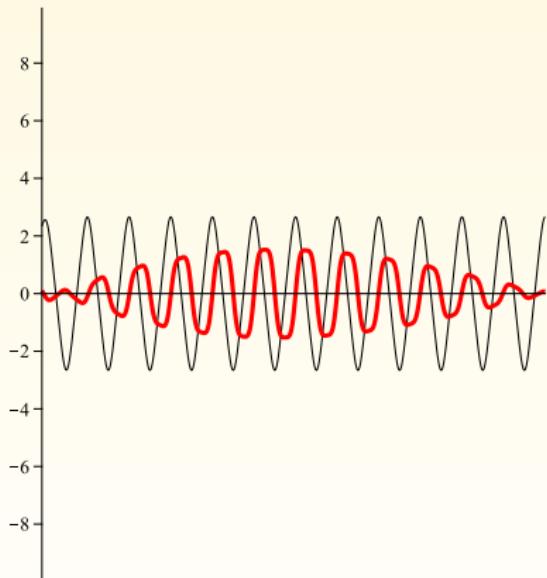


$$(a\chi_k)'' + \left[ \kappa^2 + \frac{g^2}{\lambda} \text{cn}^2(x, 2^{-1/2}) \right] (a\chi_k) = 0$$

# DEVELOPMENT OF LINEAR INSTABILITY



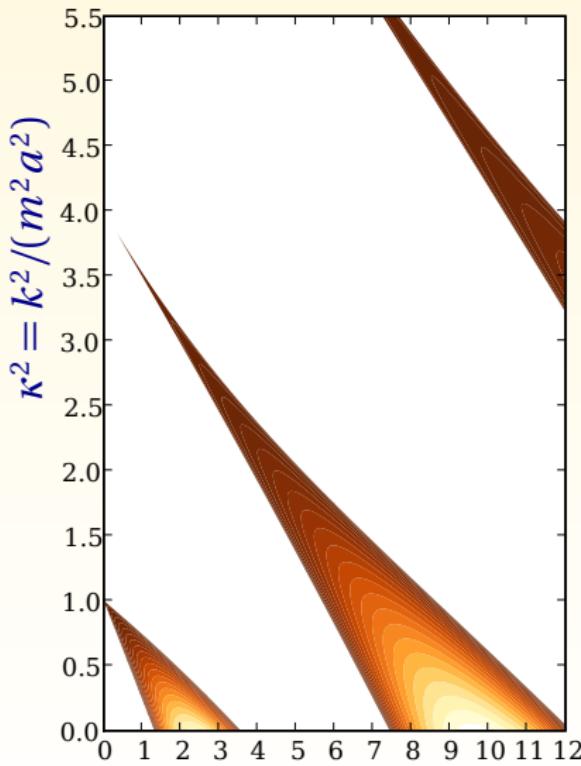
$$g^2/\lambda = 2.99, \kappa^2 = 0$$



$$g^2/\lambda = 3.01, \kappa^2 = 0$$

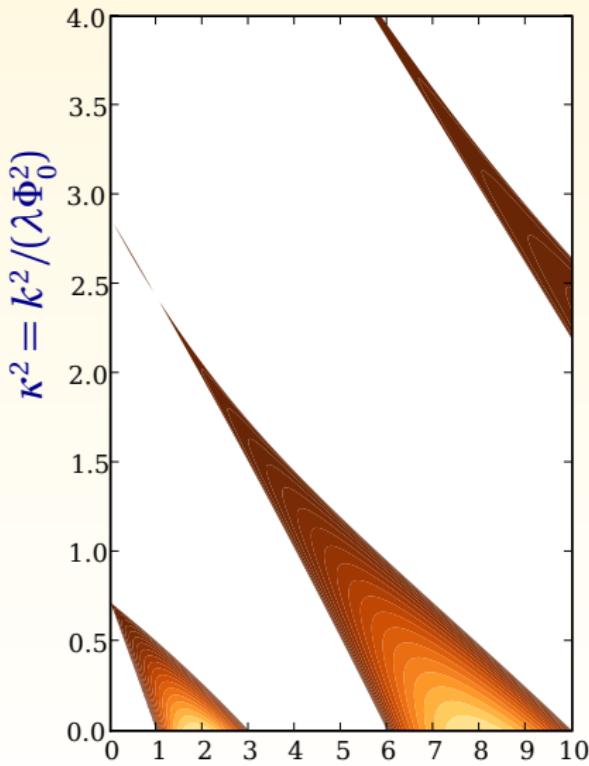
# PARAMETRIC RESONANCE IS A GENERIC FEATURE

$$\chi_k'' + [\kappa^2 + q \cos^2(\tau)] \chi_k = 0$$

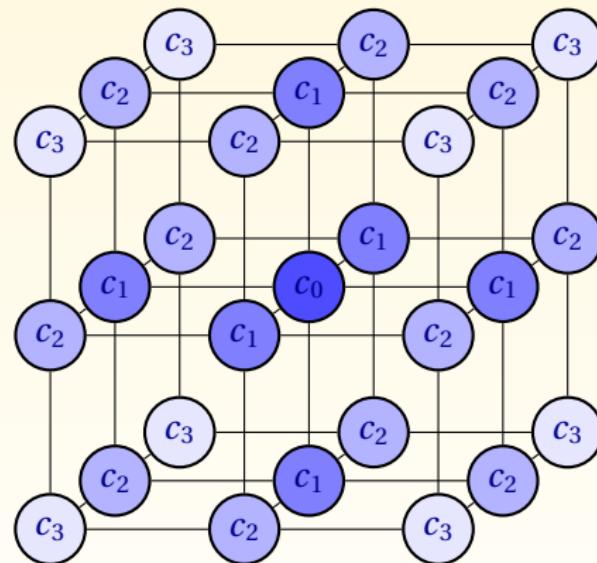


$$q = g^2 \Phi_0^2 / (m^2 a^3)$$

$$\chi_k'' + [\kappa^2 + q \operatorname{cn}^2(\tau, 2^{-1/2})] \chi_k = 0$$



# DEFROST: A NEW 3D NUMERICAL SOLVER



| coefficient | $c_3$          | $c_2$          | $c_1$          | $-c_0$          |
|-------------|----------------|----------------|----------------|-----------------|
| degeneracy  | 8              | 12             | 6              | 1               |
| standard    | 0              | 0              | 1              | 6               |
| isotropic A | 0              | $\frac{1}{6}$  | $\frac{1}{3}$  | 4               |
| isotropic B | $\frac{1}{12}$ | 0              | $\frac{2}{3}$  | $\frac{14}{3}$  |
| isotropic C | $\frac{1}{30}$ | $\frac{1}{10}$ | $\frac{7}{15}$ | $\frac{64}{15}$ |

<http://www.sfu.ca/physics/cosmology/defrost>

[Fortran-90, 600 lines, very fast, instrumented for 3D]

# FIELD EVOLUTION AS A CHAOTIC BILLIARD

# DENSITY EVOLUTION

$$[V(\phi, \chi) = \frac{1}{4} \lambda \phi^4 + \frac{1}{2} g^2 \phi^2 \chi^2]$$

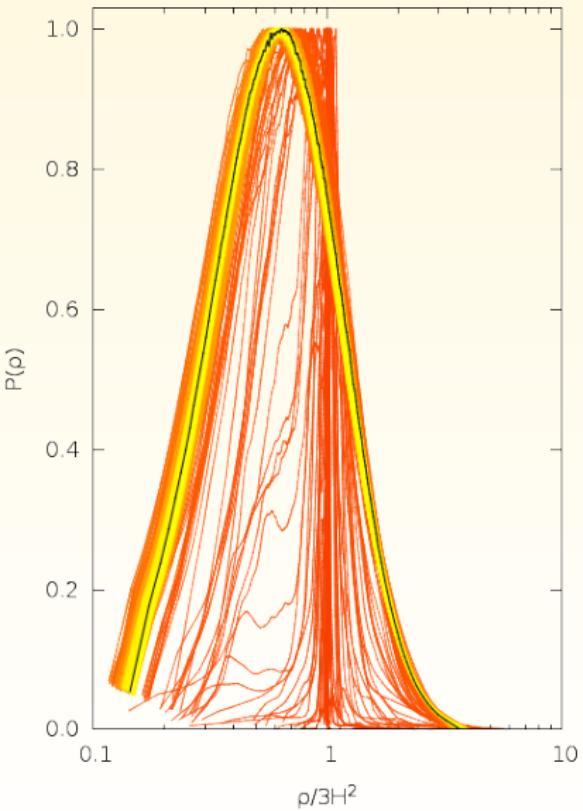
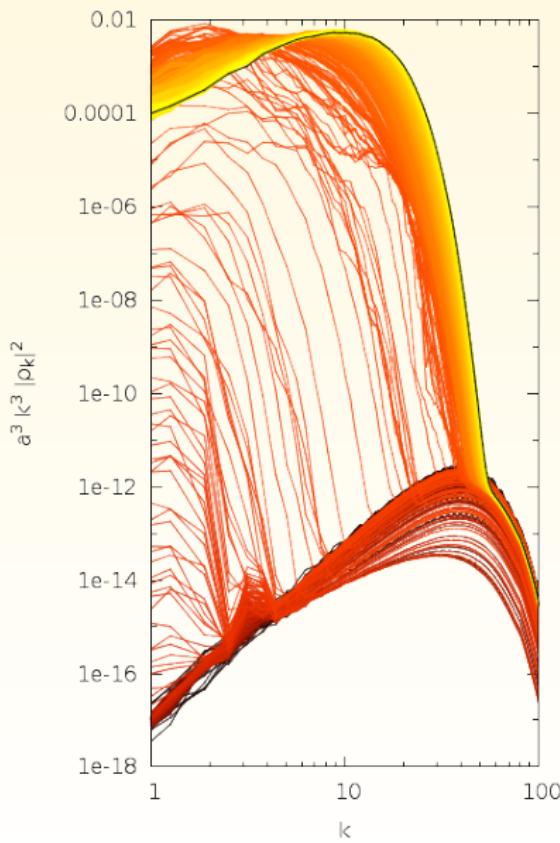
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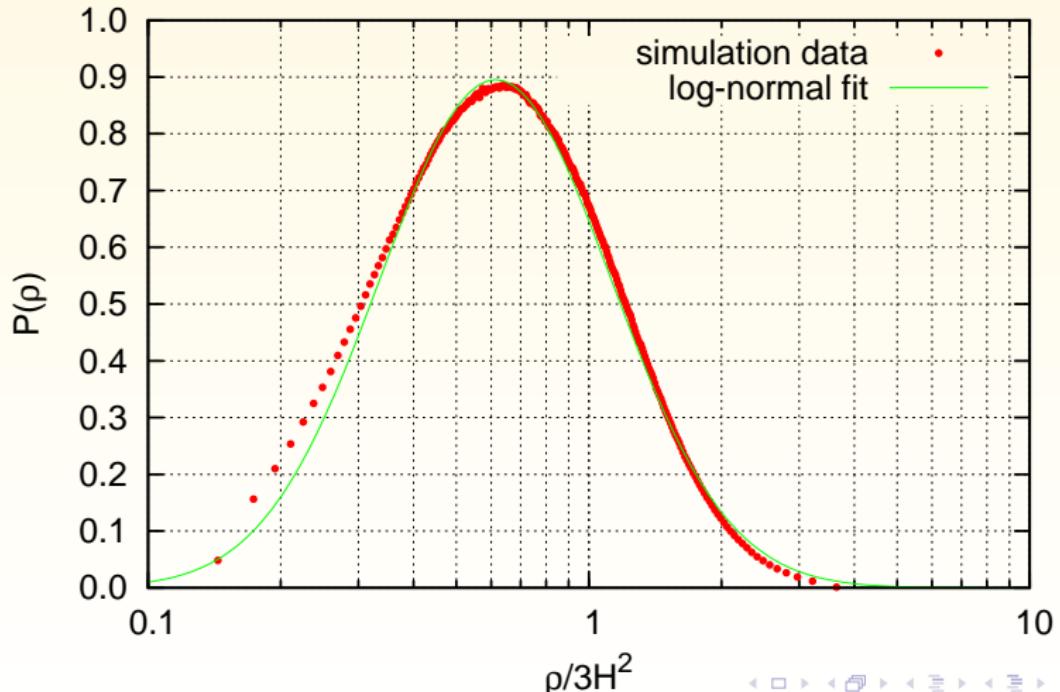
$$[V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \sigma \phi \chi^2 + \frac{1}{4} \lambda \chi^4]$$

# HERE IS HOW INSTABILITY DEVELOPS!



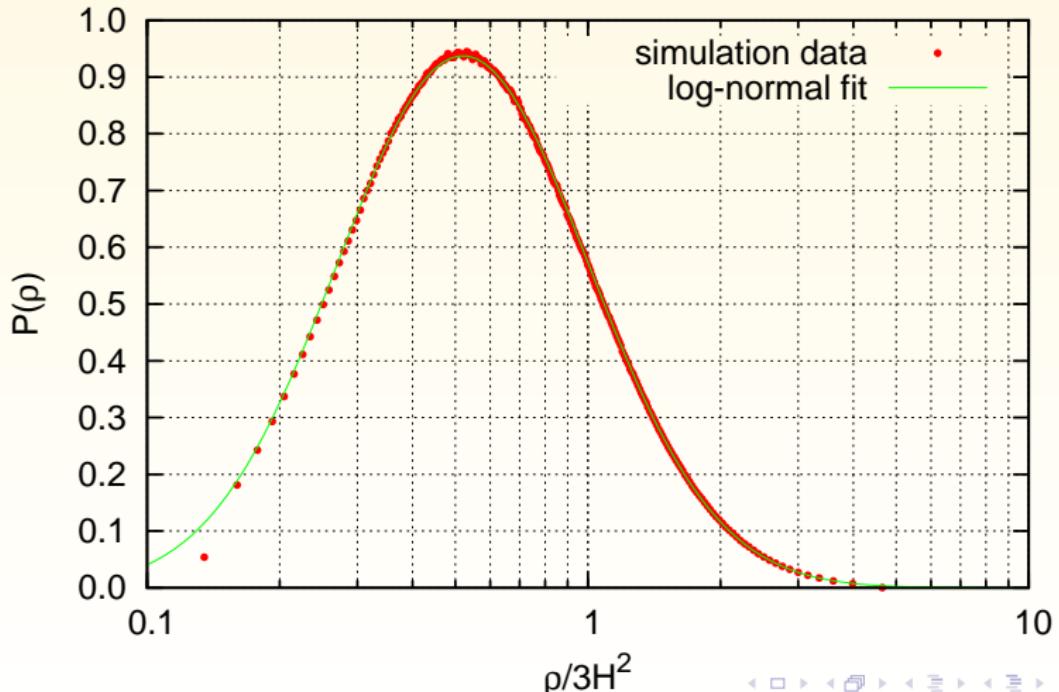
# IN FACT, IT'S QUITE LOG-NORMAL!

$$V(\phi, \chi) = \frac{1}{4} \lambda \phi^4 + \frac{1}{2} g^2 \phi^2 \chi^2$$



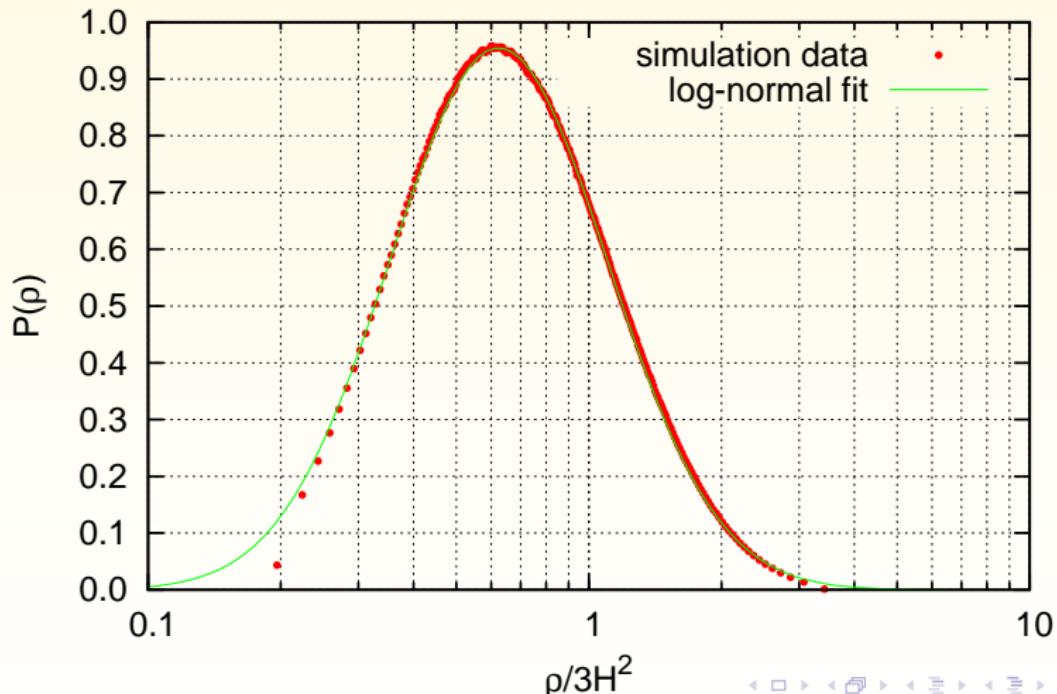
# IN FACT, IT'S UNIVERSALLY LOG-NORMAL!

$$V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2$$



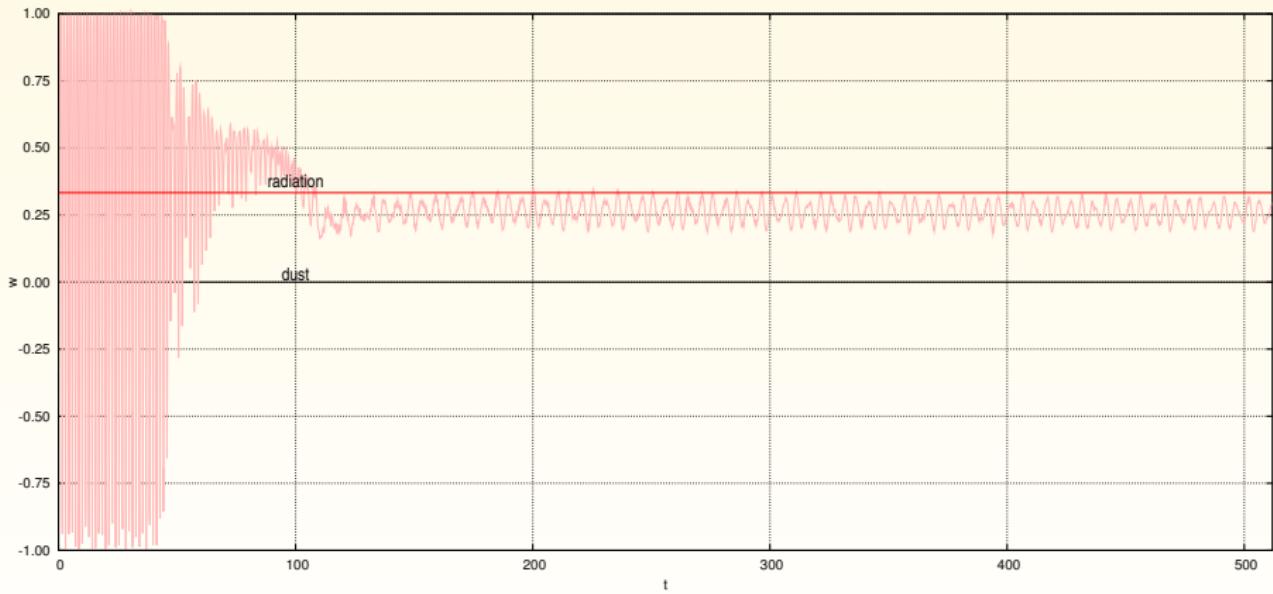
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$$V(\phi, \psi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \sigma \phi \chi^2 + \frac{1}{4} \lambda \chi^4$$



# (NON-THERMAL) SCALAR FIELD FIXED POINT?

$$V(\phi, \chi) = \frac{1}{4} \lambda (\phi^2 + \chi^2 - v^2)^2$$



THIS SIMPLY CAN'T BE JUST A COINCIDENCE...

A universal characteristic of  
random scalar field evolution?!

What's Going On Here?

Very tempting to blame  
scalar field turbulence!..

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## Thermal Bath Wipes Out Everything?! Scales Too Small?!

WE SHOULD LOOK FOR THINGS THAT CAN SURVIVE THERMALIZATION:

- ① stable relics (primordial black holes)
- ② decoupled fields (gravitational waves)
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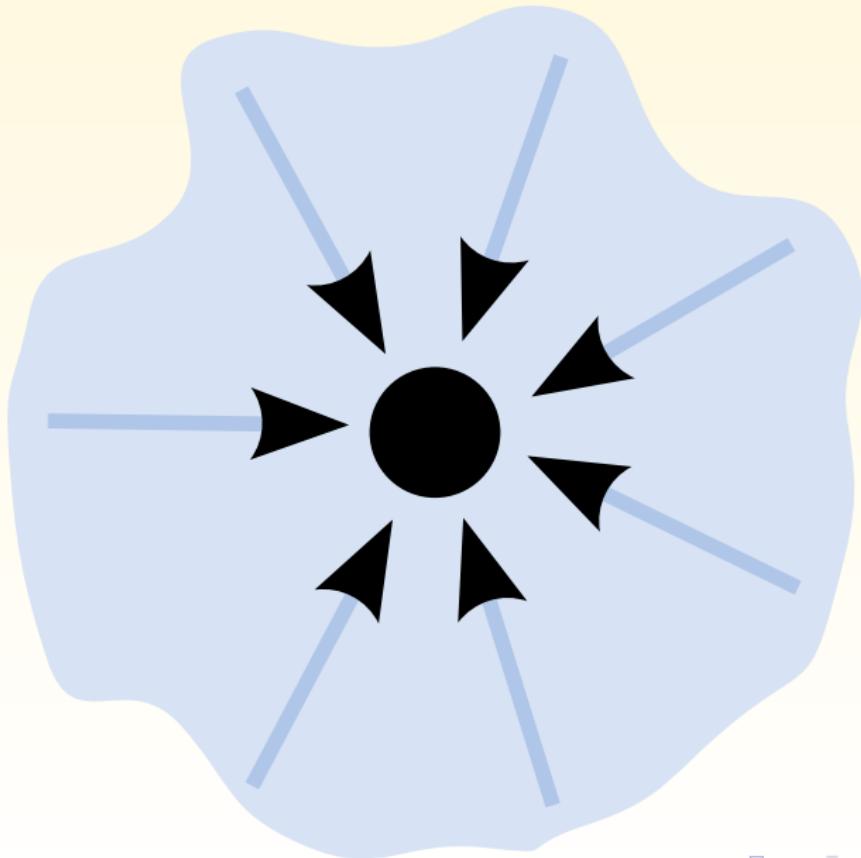
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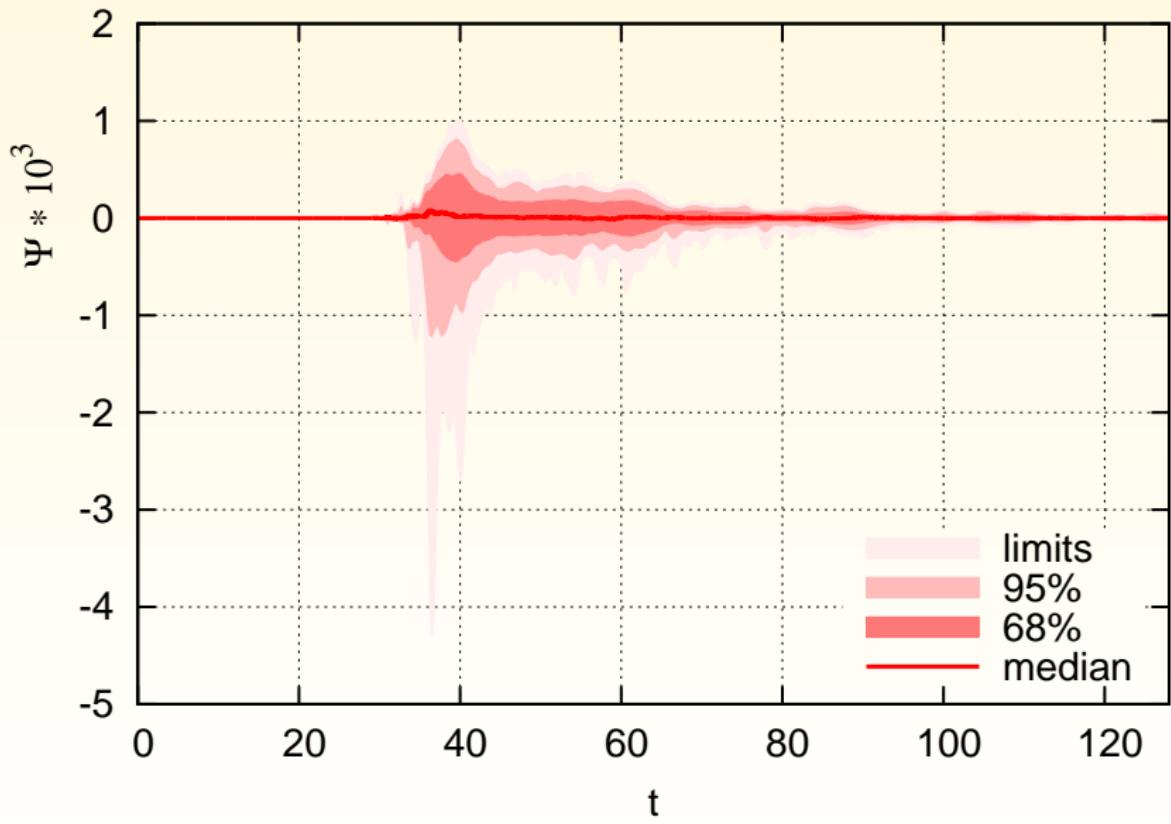
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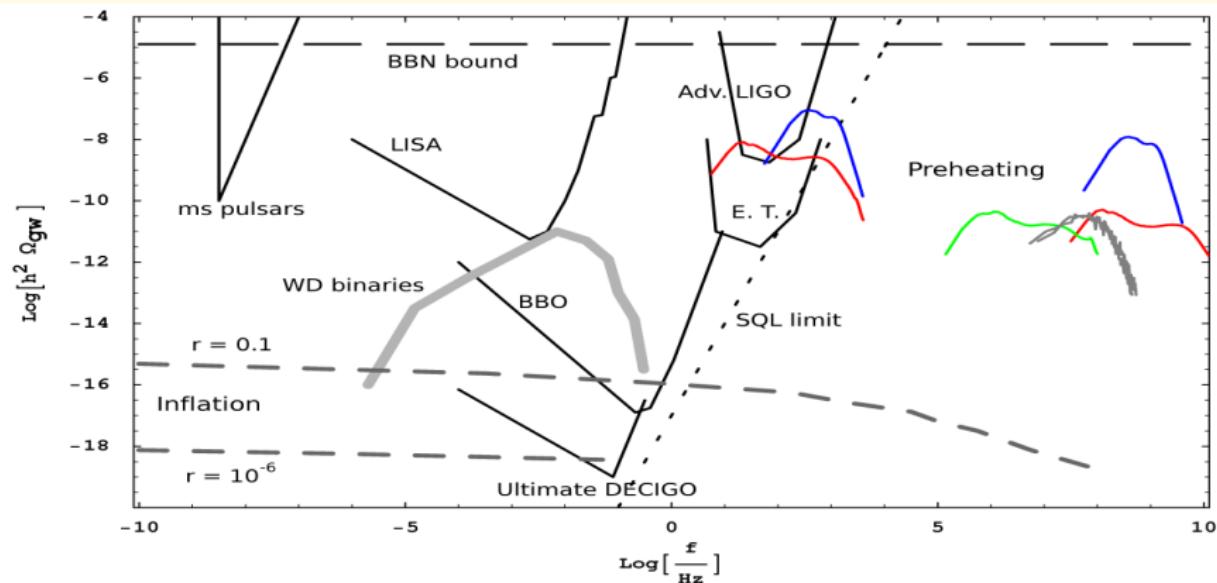
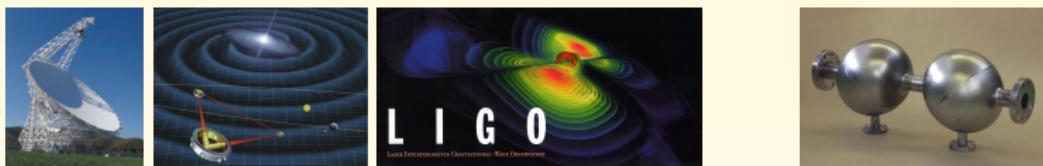
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# GRAVITATIONAL WAVES FROM PREHEATING?



Dufaux, Felder, Kofman & Navros (2009)

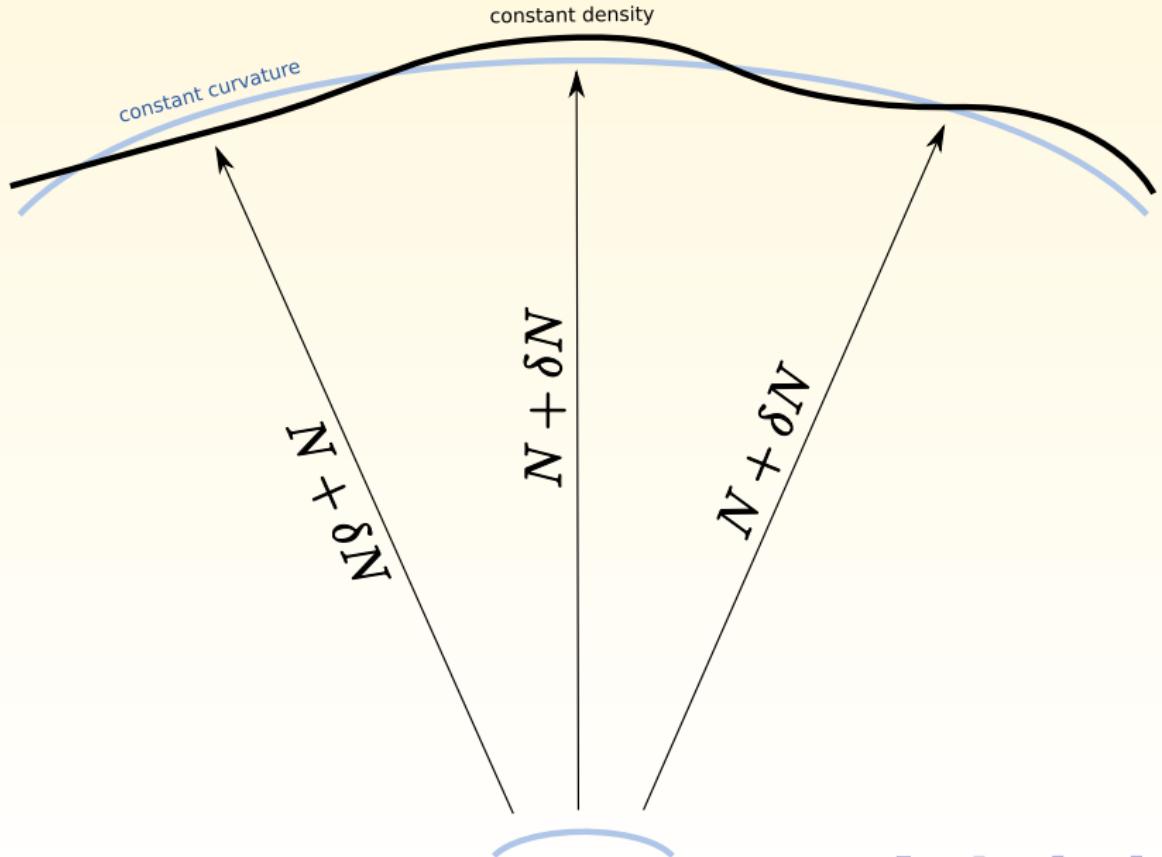
# NON-GAUSSIANITY FROM PREHEATING?

CMB and Reheating scales different by 50+ e-folds!

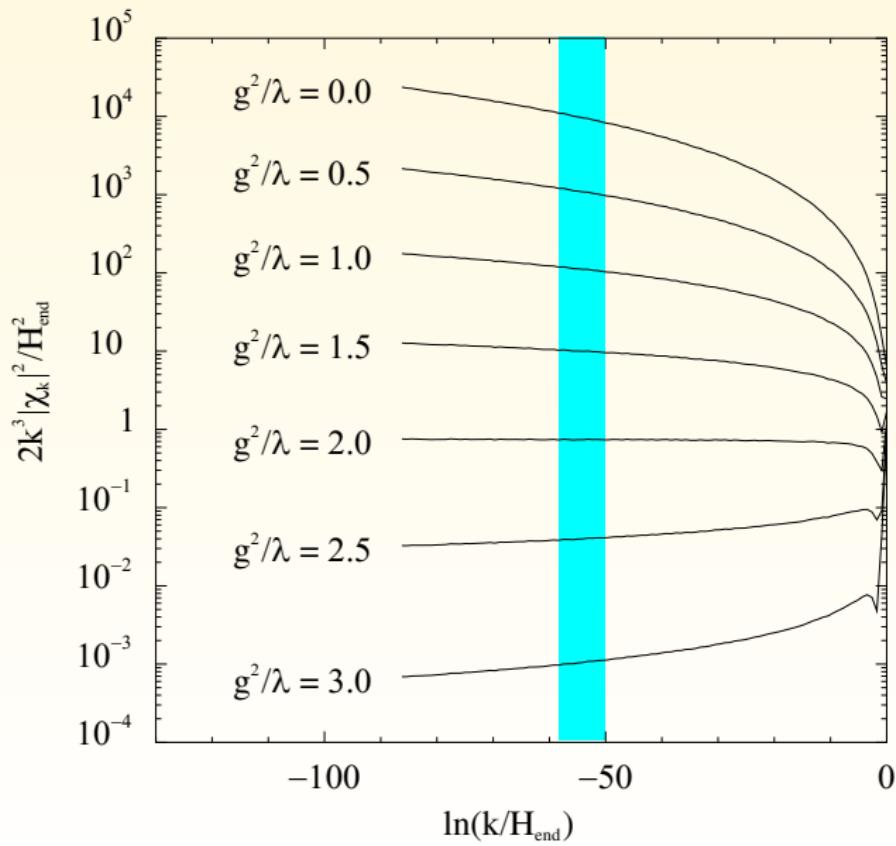
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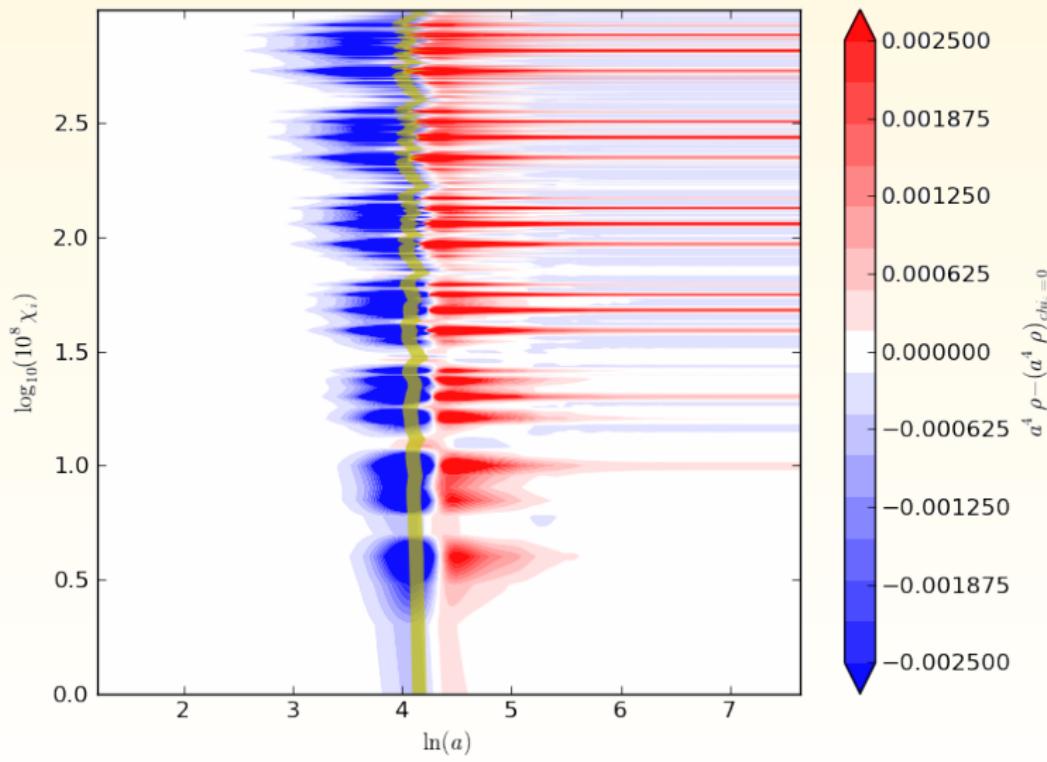
# DIFFERENCES IN EXPANSION CAN MODULATE CMB



# MODULATION COMES FROM ISOCURVATURE MODE



# EVOLUTION DEPENDS ON INITIAL ISOCON VALUE

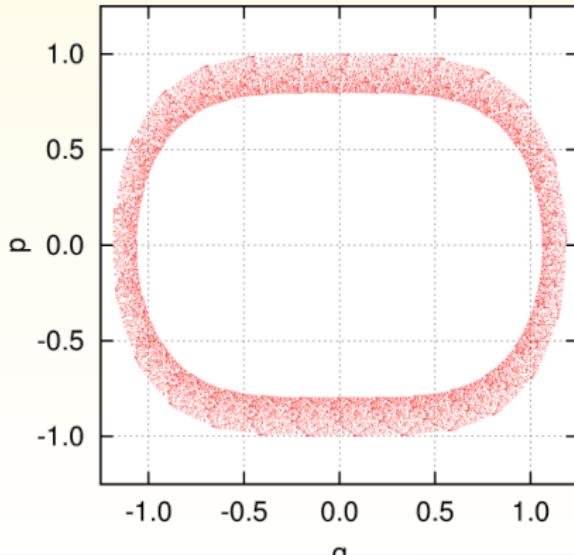


## Problem:

long-term oscillator evolution

$$H = \frac{p^2}{2} + \frac{q^4}{4}$$

4<sup>th</sup> order Runge-Kutta

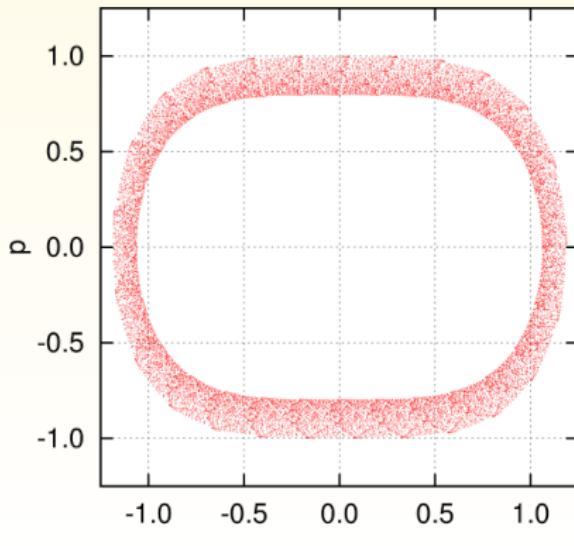


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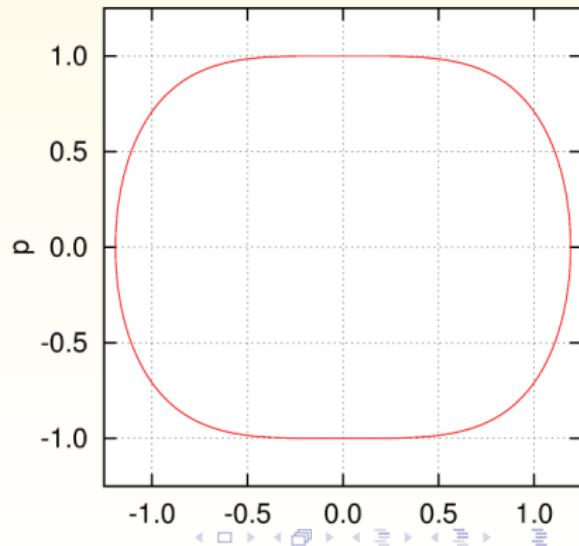


## Solution:

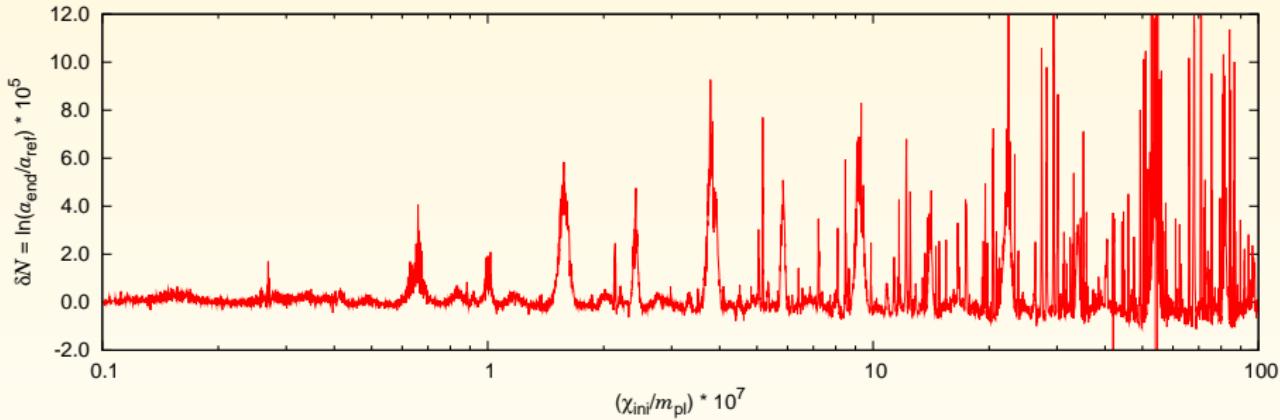
enforce energy conservation

$$e^{At/2} e^{Bt} e^{At/2} = e^{(A+B)t+O(t^3)}$$

4<sup>th</sup> order Symplectic

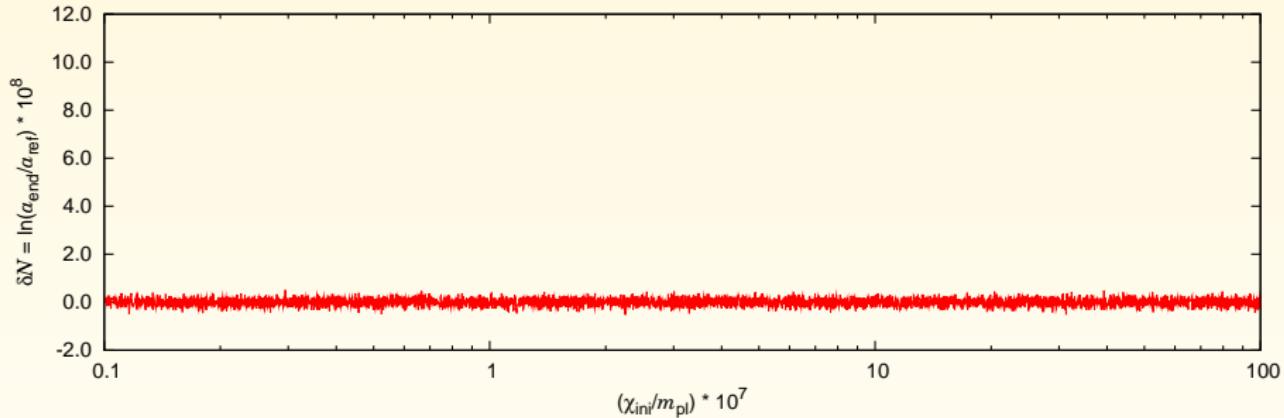


# ISOCURVATURE MODE CONVERTS TO CURVATURE



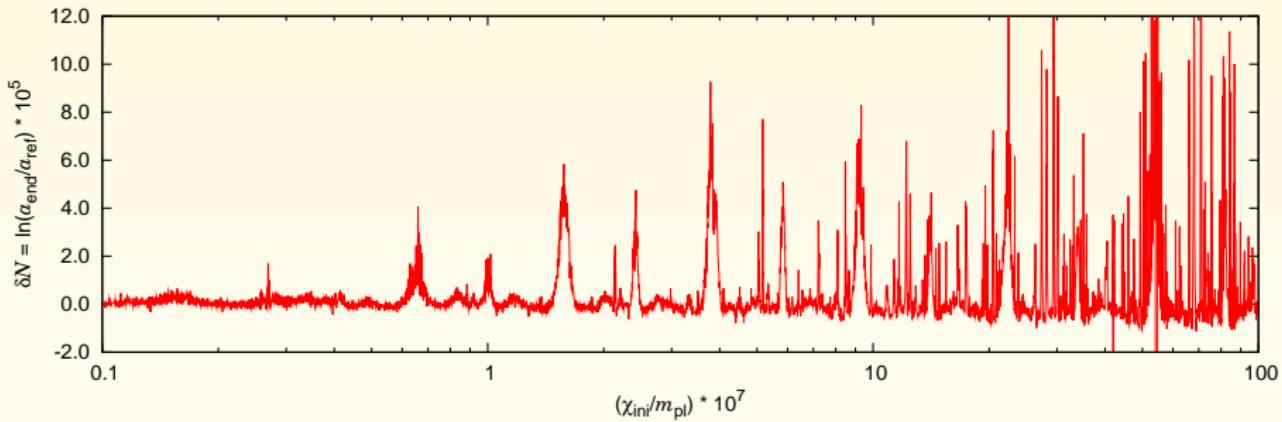
Are these peaks Real? Yes...

# ISOCURVATURE MODE CONVERTS TO CURVATURE



Are these peaks Real? Yes...

# PRIMORDIAL NON-GAUSSIANITY IS PRODUCED!

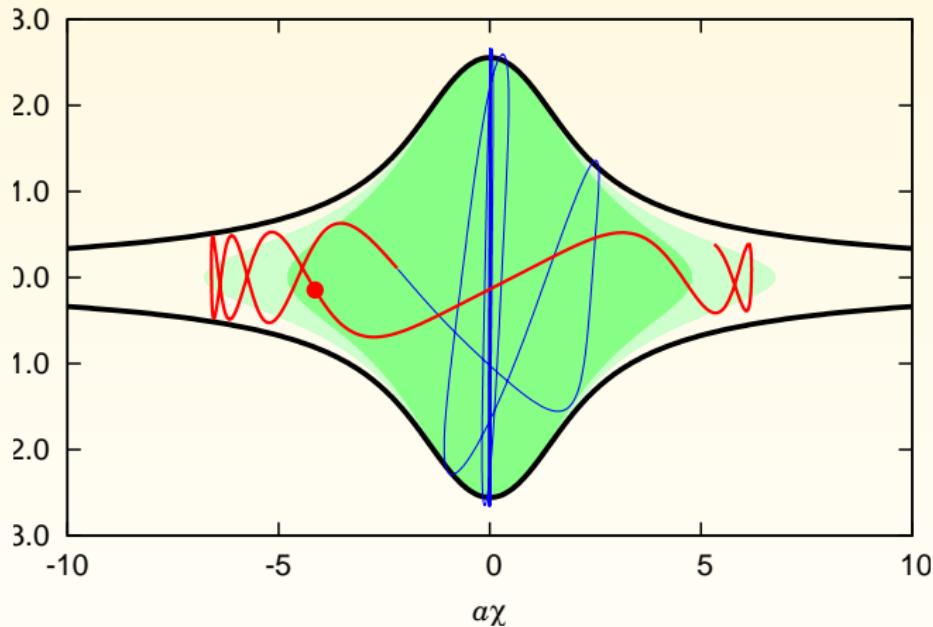


$$\Phi(\vec{x}) = \Phi_G(\vec{x}) + F_{NL}(\chi_G)$$

very different from  $f_{NL}$  parametrization!

# HERE IS A TRAJECTORY FROM THE PEAK

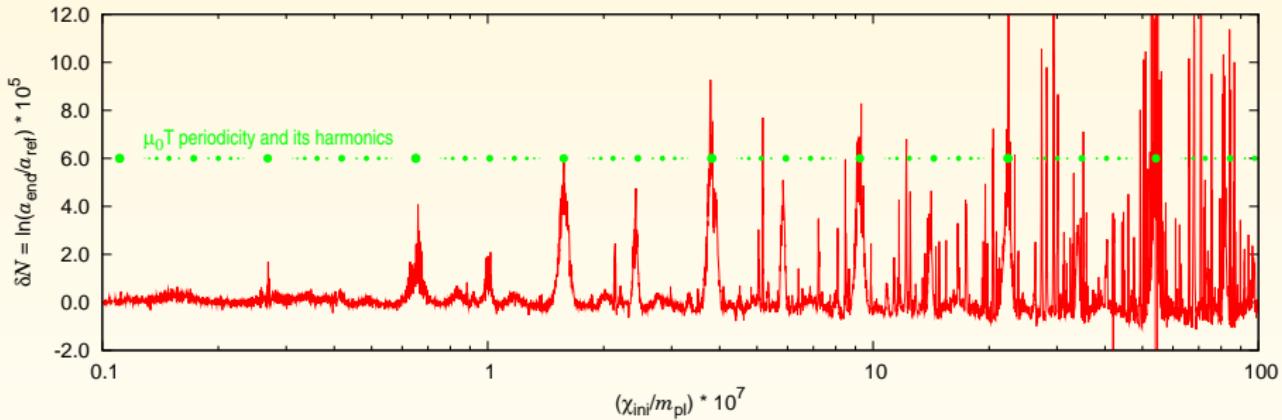
# A SIMPLE ANALYTICAL MODEL



$$V_{\text{eff}}(\phi, \chi) = \frac{\lambda}{4} \phi^4 + \frac{g^2}{2} (\phi^2 + \langle \delta \phi^2 \rangle) \chi^2$$

# TIME EVOLUTION OF EFFECTIVE POTENTIAL

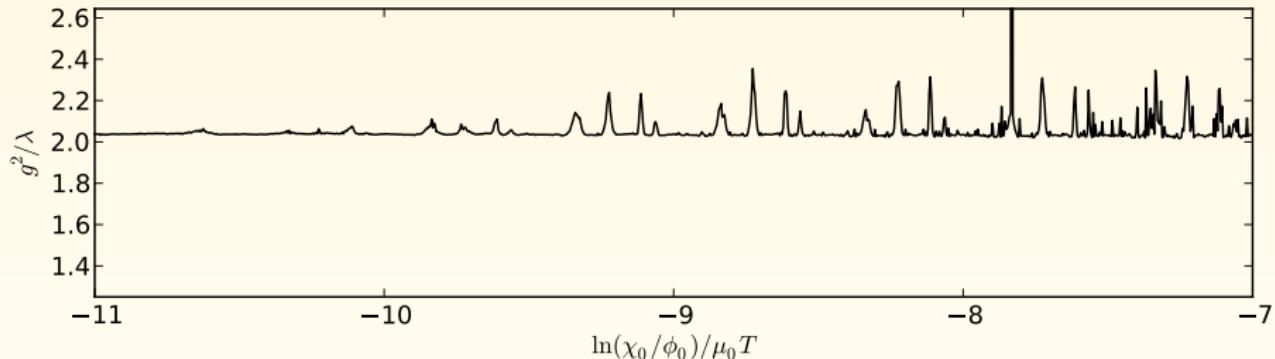
# THIS EXPLAINS WHY PEAKS ARE LOG-PERIODIC!



$$\phi(t + T) = \phi(t)$$

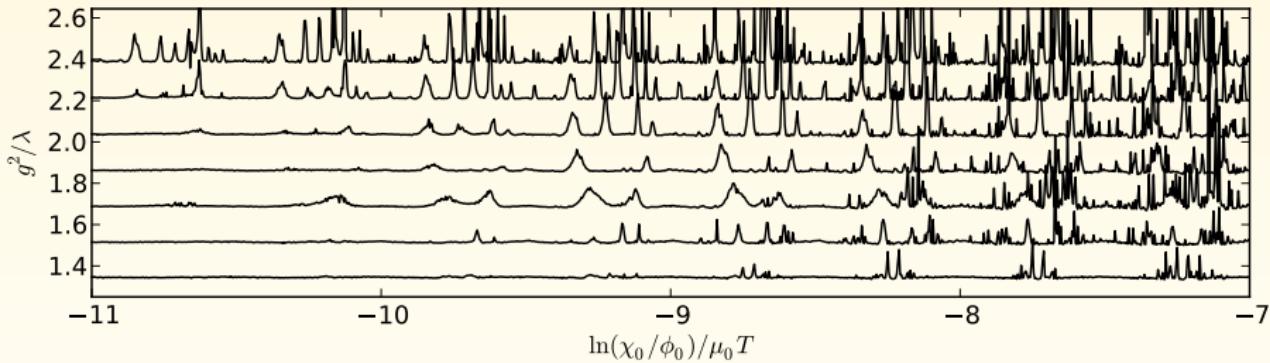
$$\chi(t + T) = \chi(t)e^{\mu_0 T}$$

# LET'S EXPLORE THE PARAMETER SPACE...



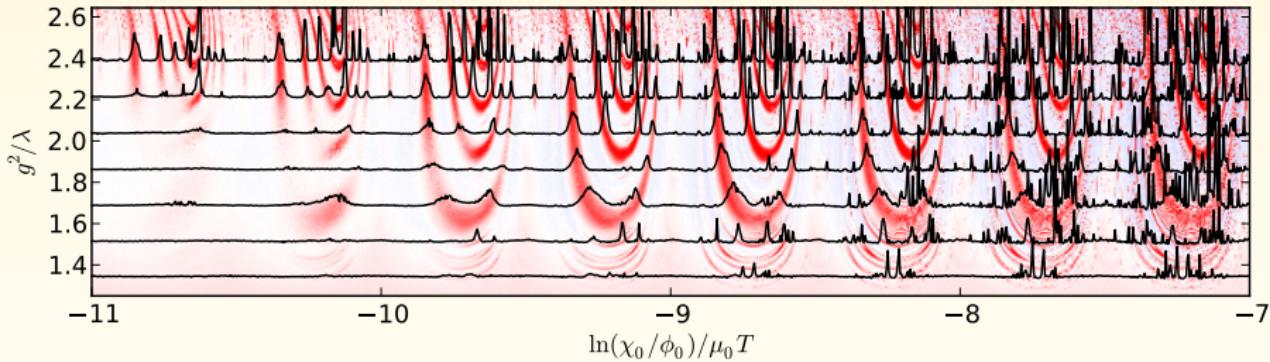
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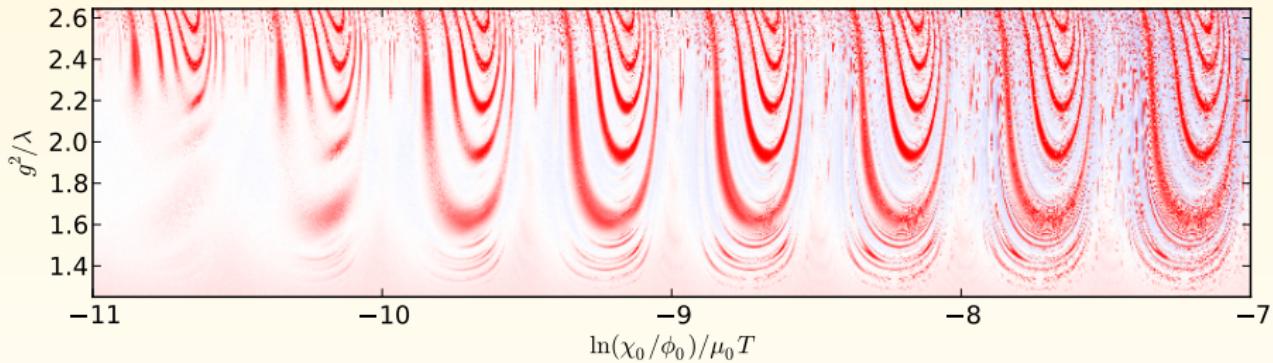
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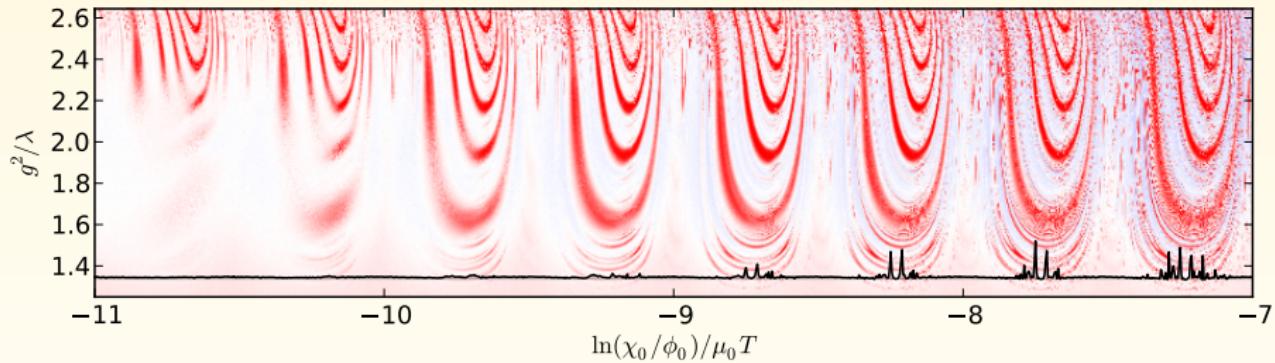
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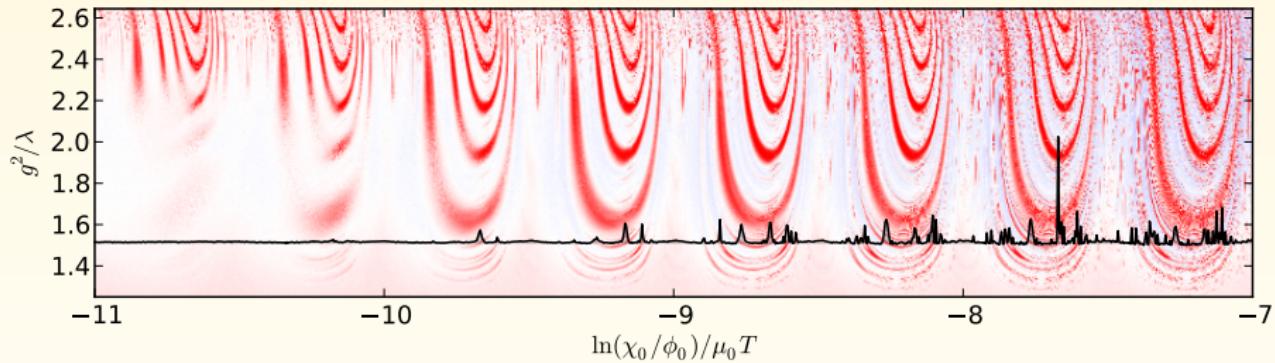
# ... AND HERE'S HOW PEAKS SPLIT & MULTIPLY!



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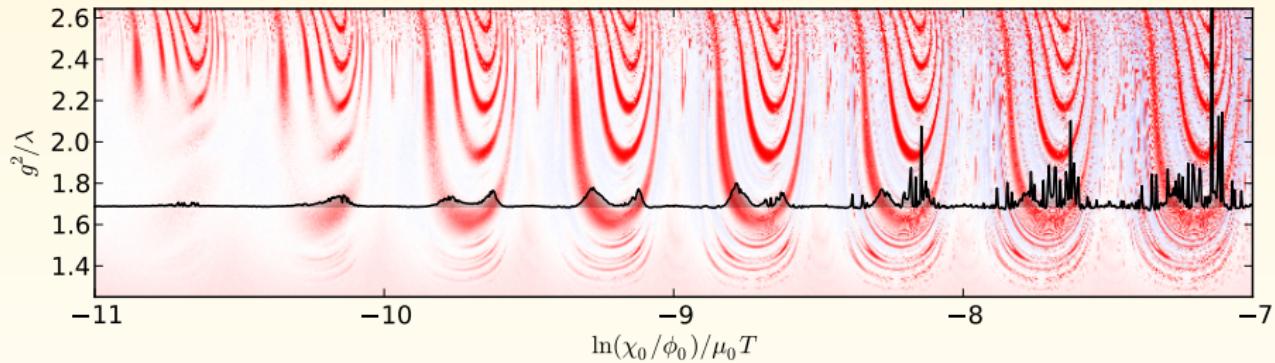
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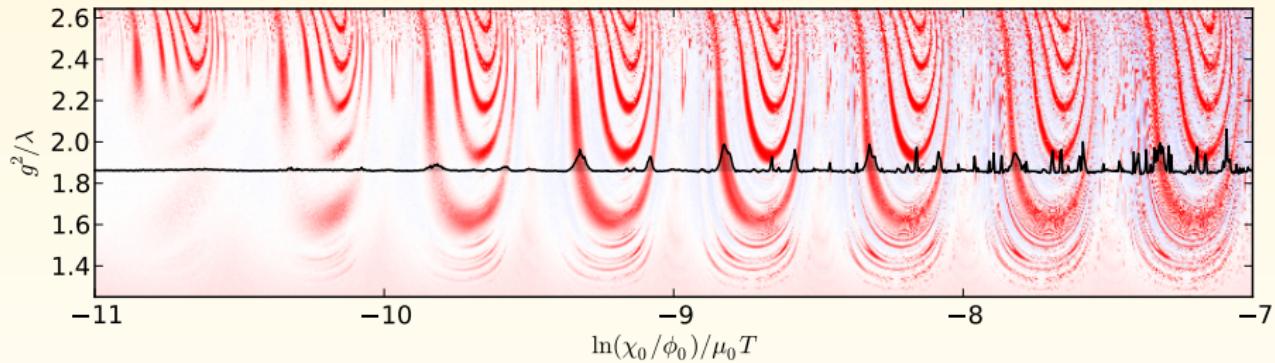
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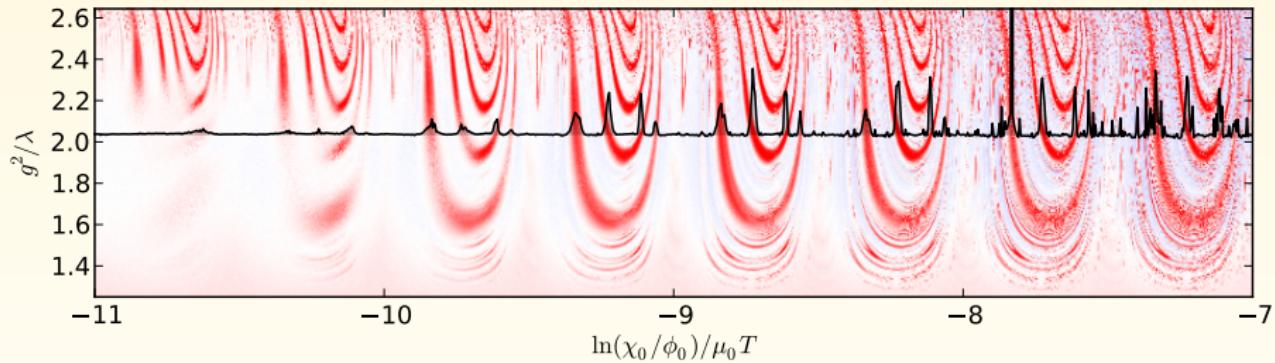
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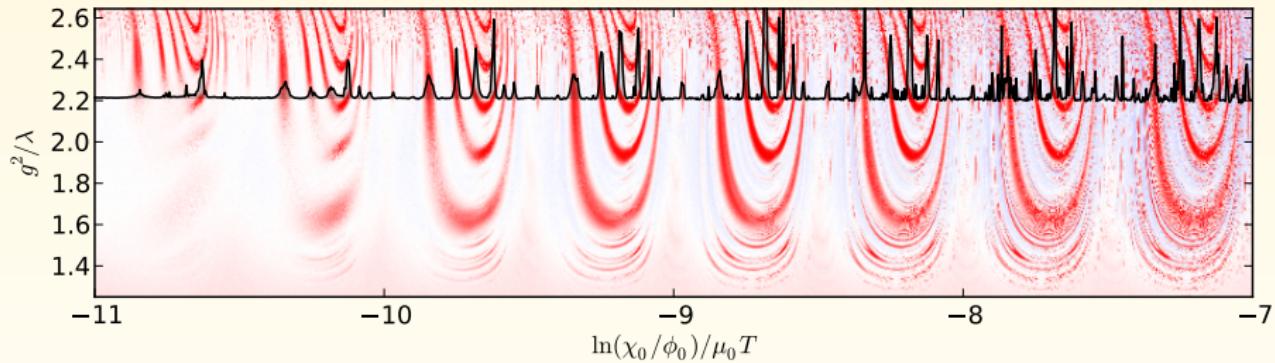
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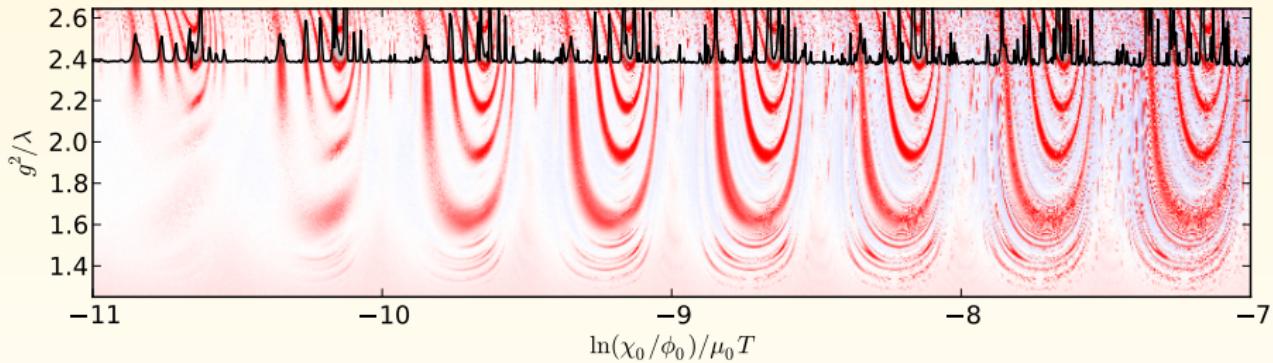
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# PREHEATING CAN LEAVE A SIGNATURE IN CMB SKY!

the *way* inflation ends  
can lead to a new signal:

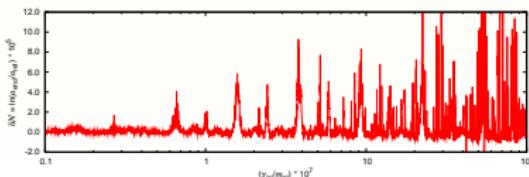
usual non-Gaussianity:

$$\Phi(\vec{x}) = \Phi_G(\vec{x}) + f_{\text{NL}} \Phi_G^2(\vec{x})$$

new from preheating:

$$\Phi(\vec{x}) = \Phi_G(\vec{x}) + F_{\text{NL}}(\chi_G)$$

$F_{\text{NL}}$  can be a *very*  
non-trivial function:



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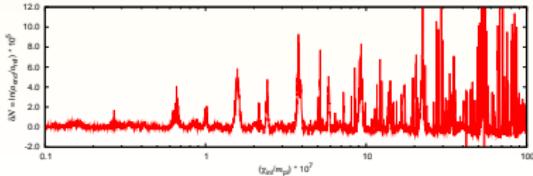
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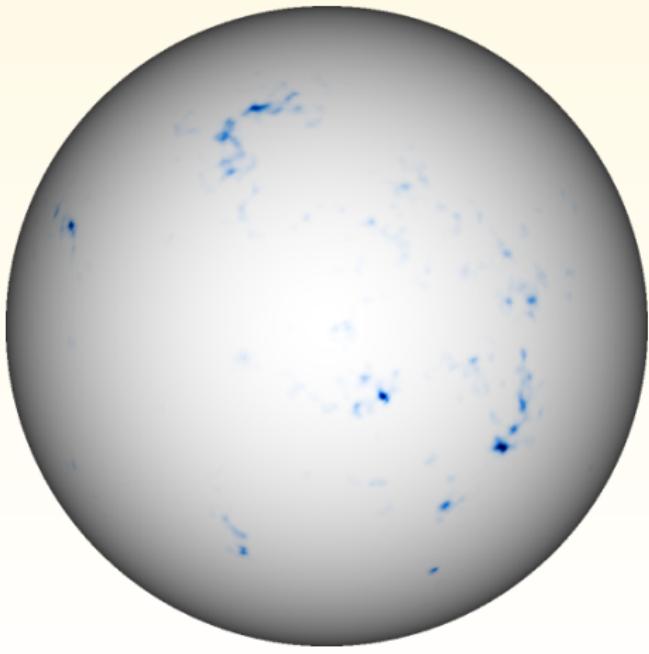
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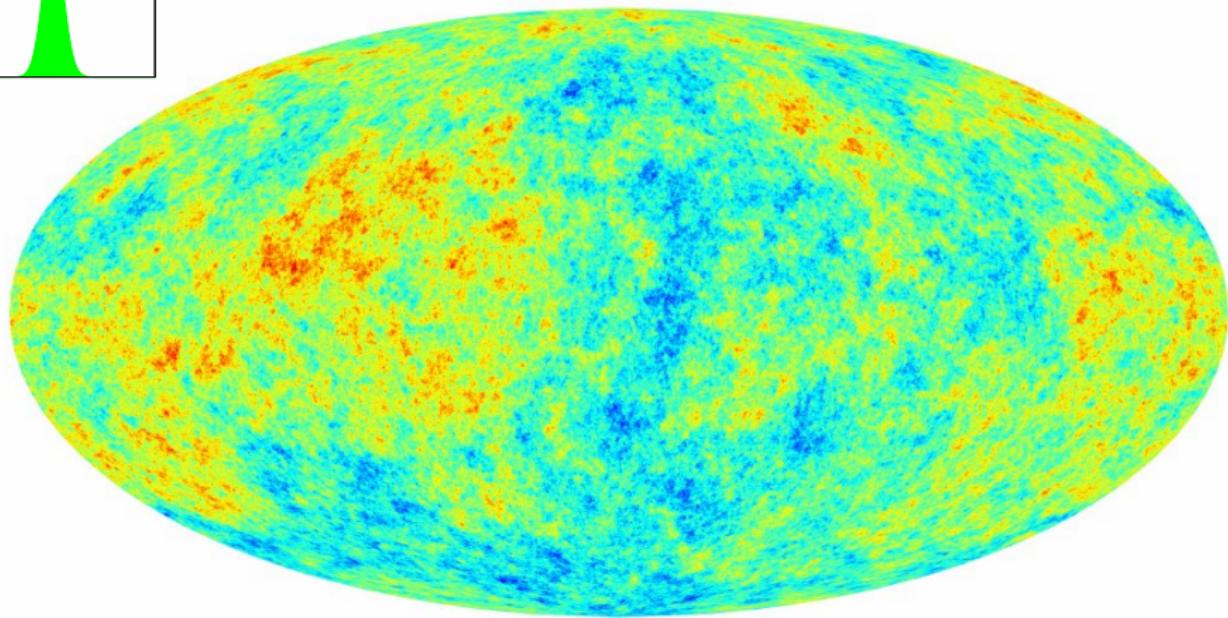
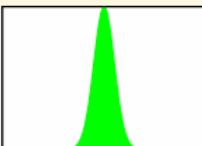
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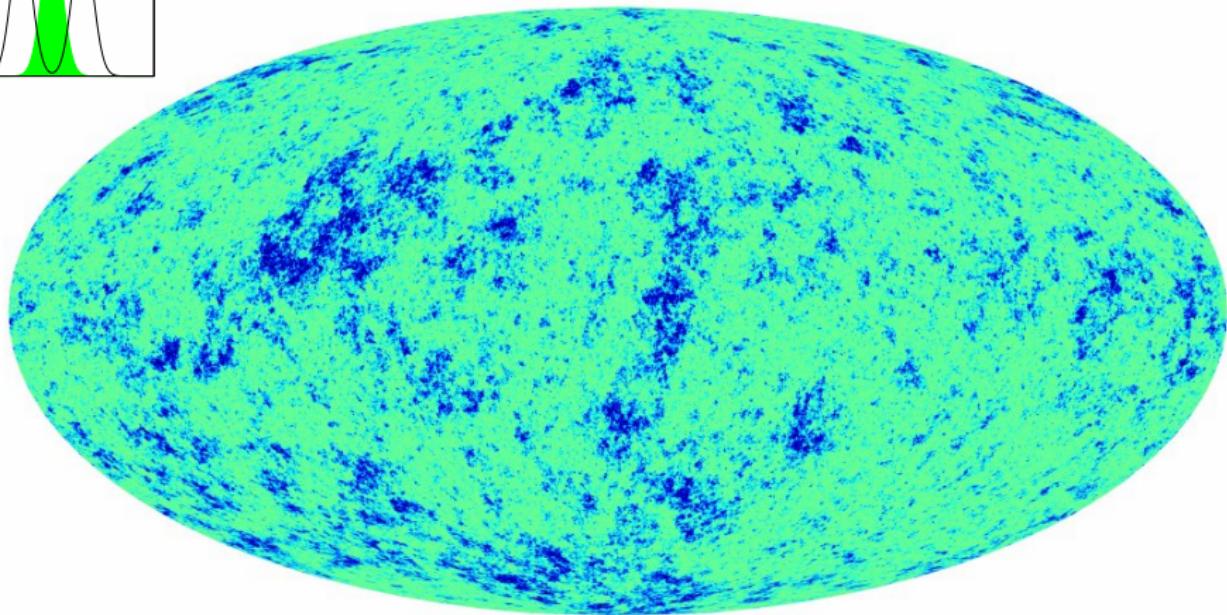
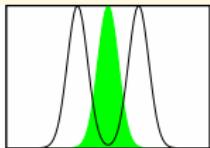
realization of *excursion set*  
can naturally give cold spots!



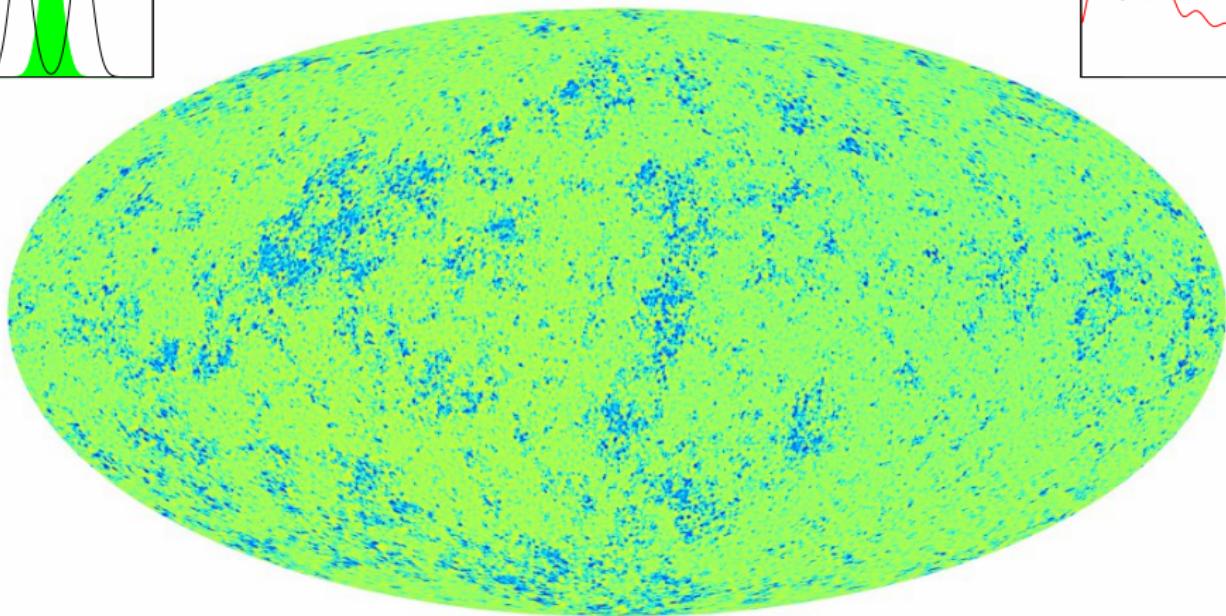
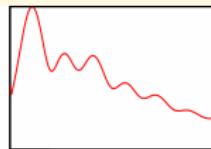
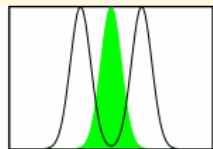
# How WOULD IT LOOK LIKE ON THE SKY?



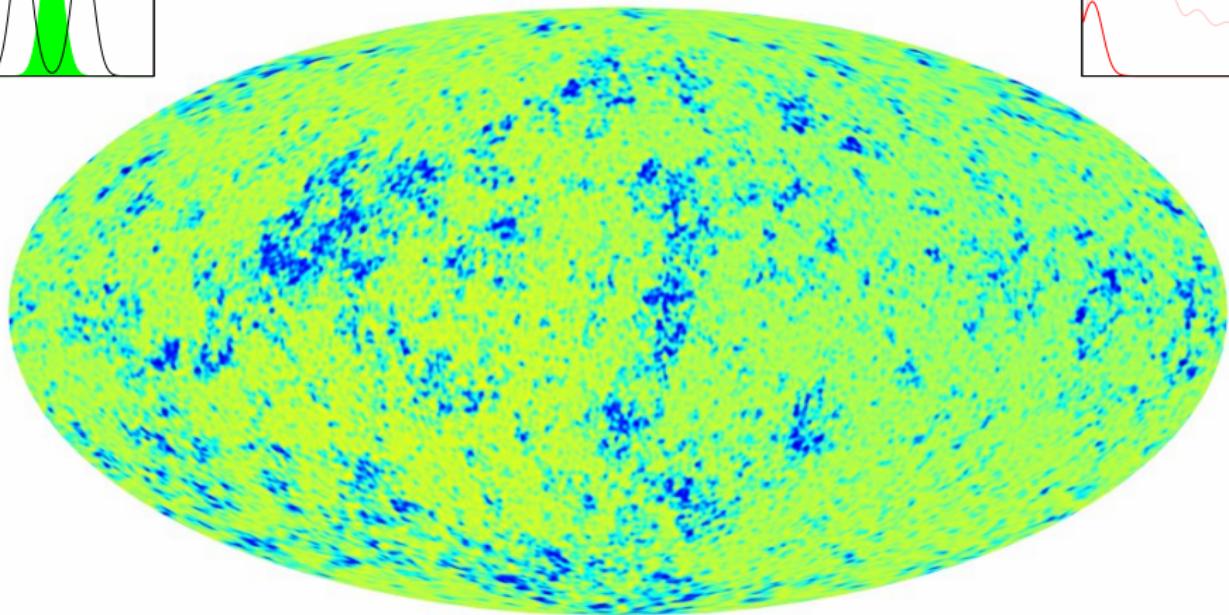
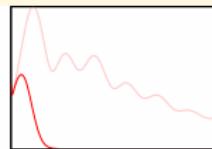
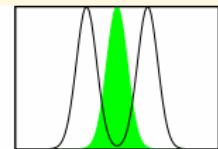
# How WOULD IT LOOK LIKE ON THE SKY?



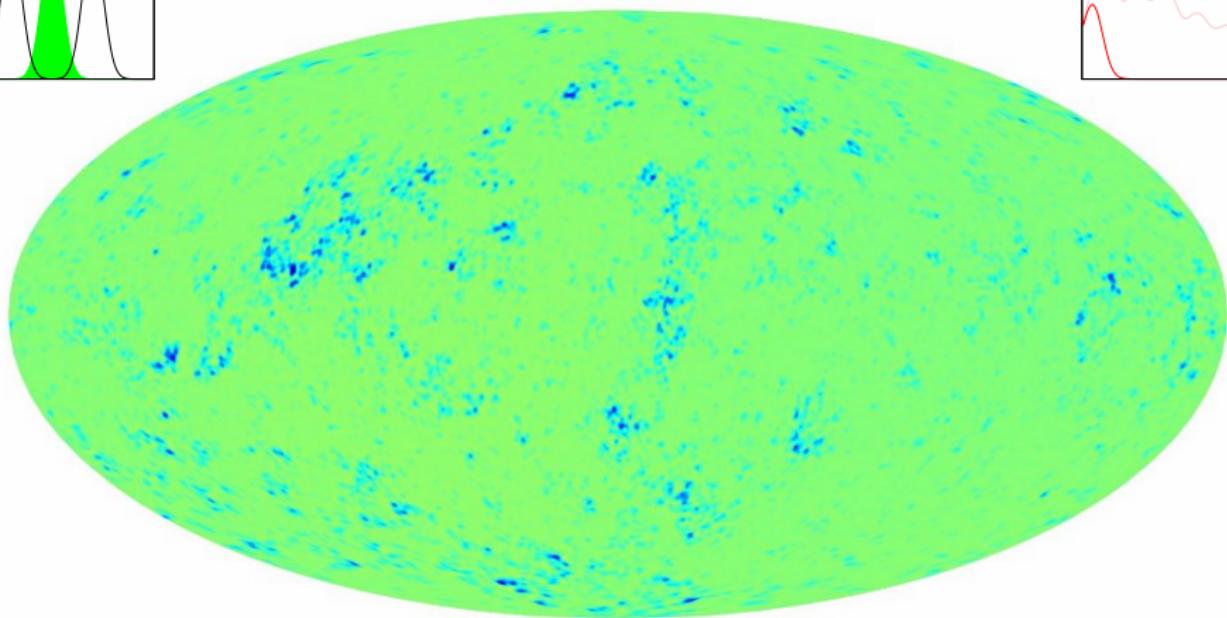
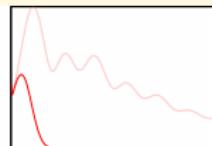
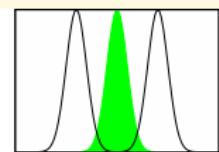
# How WOULD IT LOOK LIKE ON THE SKY?



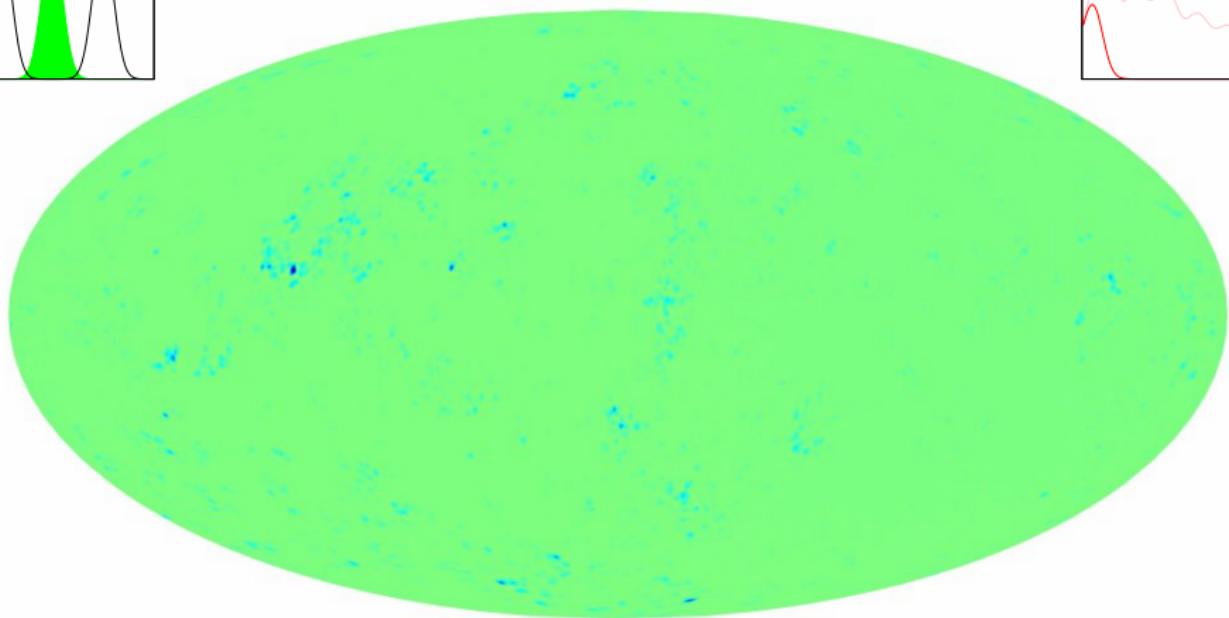
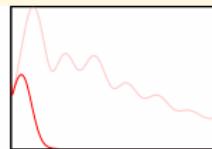
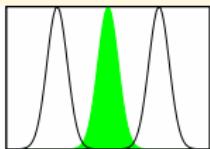
# How WOULD IT LOOK LIKE ON THE SKY?



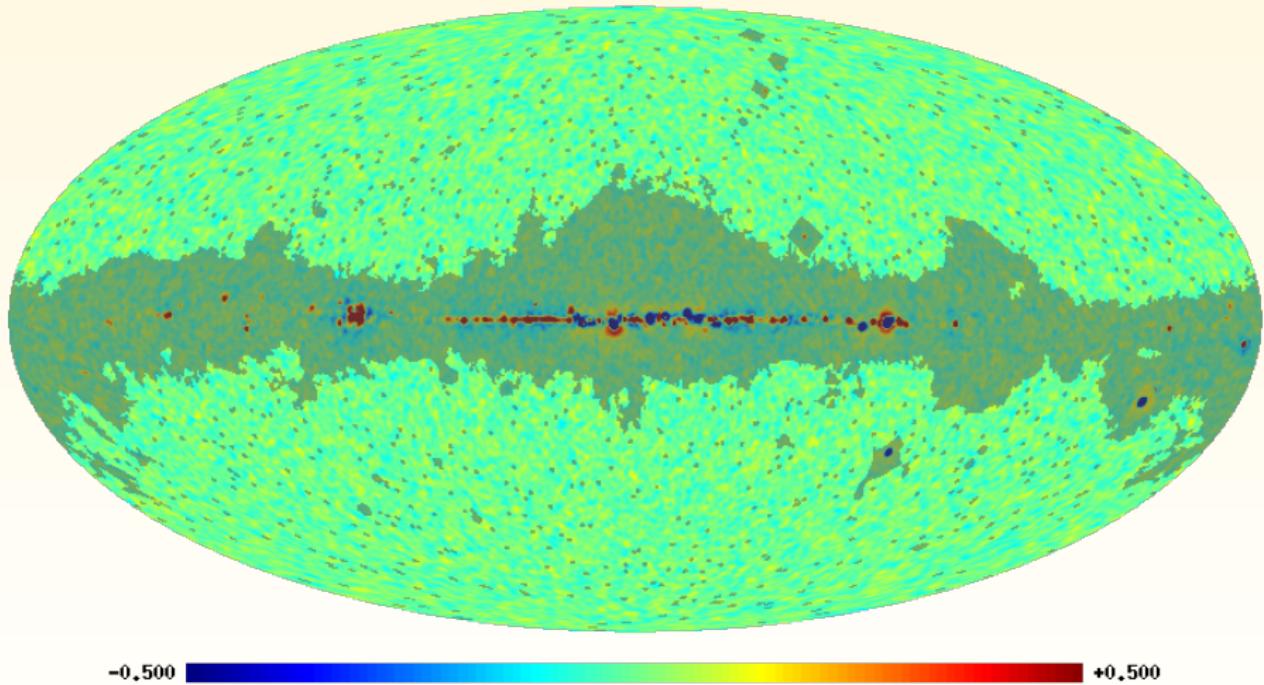
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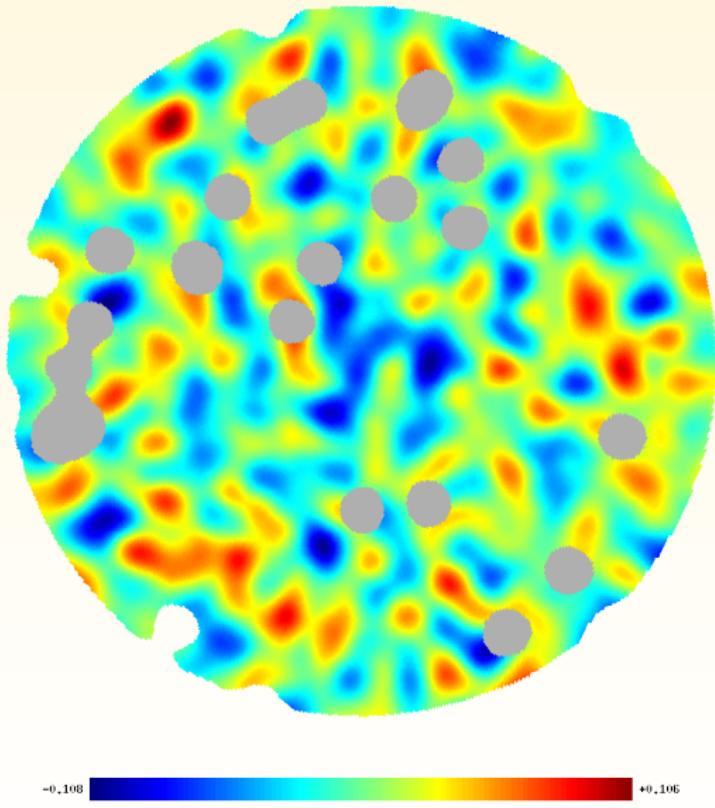
# ERR, THERE IS A COLD SPOT ON CMB SKY!..



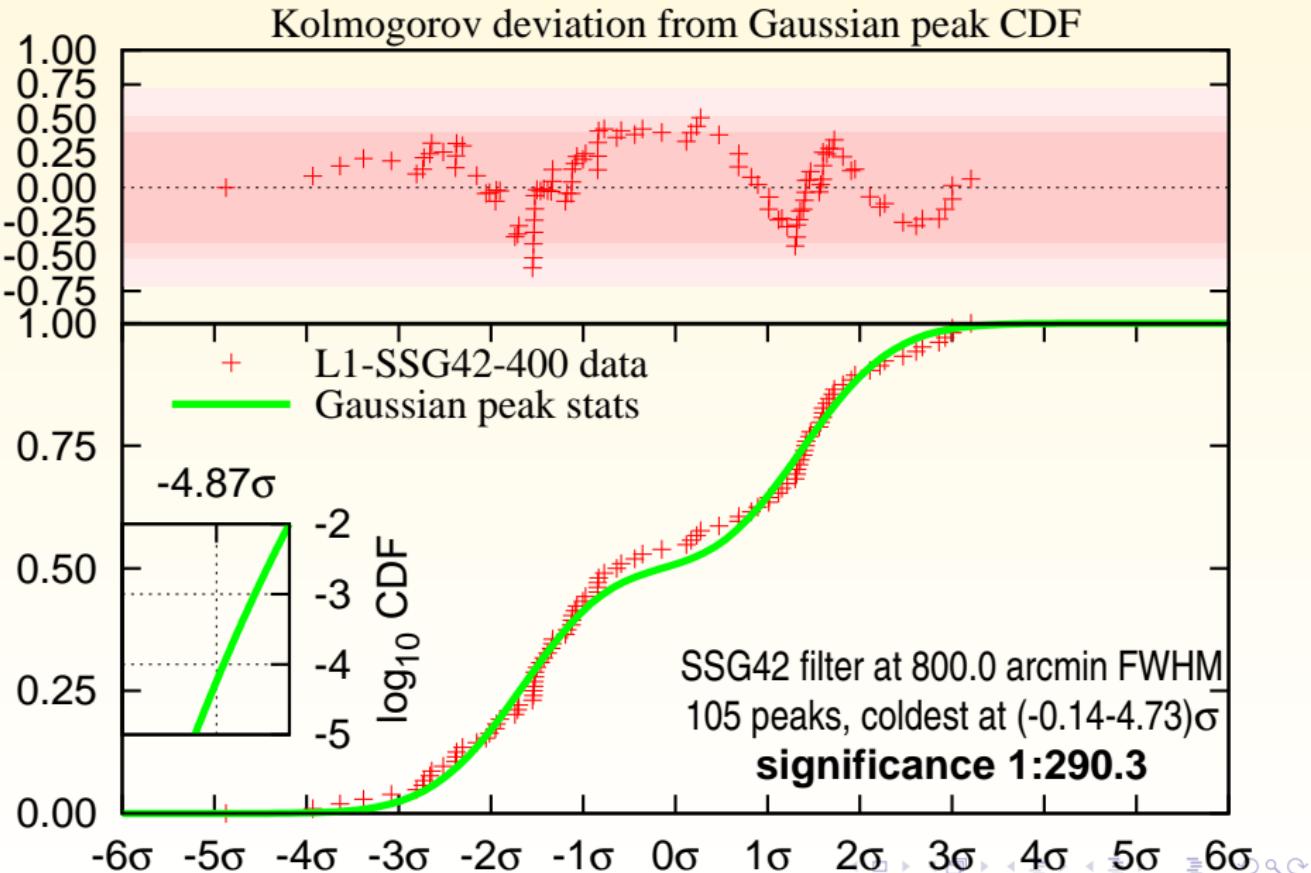
-0.500

+0.500

# ERR, THERE IS A COLD SPOT ON CMB SKY!..



# ERR, THERE IS A COLD SPOT ON CMB SKY!..



# POLARIZATION WILL TELL IF IT'S PRIMORDIAL!

*Komatsu et. al. (2010)*

