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"Wave function in 2+1 dimensional causal dynamical triangulation"

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Wave function in 2+1 dimensional causal dynamical triangulation

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Contents

3D Quantum Gravity defined by CDT (Causal dynamical triangulation)

We consider space-time with a spatial boundary.

We measure the dynamics of spatial volume and whether spatial geometry is homogeneous.

Path integral and Wave function

trajectory（quantum mechanics)

→metric（quantum gravity)

path integral representation of a wave function

$$\Psi(h_{ij}) = \int \mathcal{D}g\, e^{iS^{\mathrm{EH}}[g_{\mu\nu}(x)]}$$

$$S^{\mathrm{EH}}[g_{\mu\nu}(x)] = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{h}K$$

h_{ij} metric of a spatial boundary

The ground state wave function is given by a path integral over all compact Euclidean geometries which have a boundary.

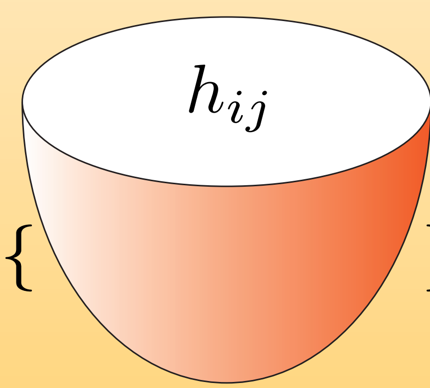
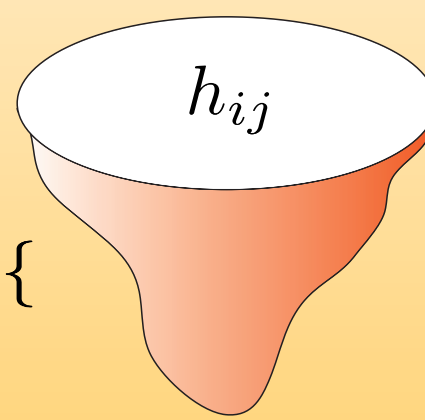
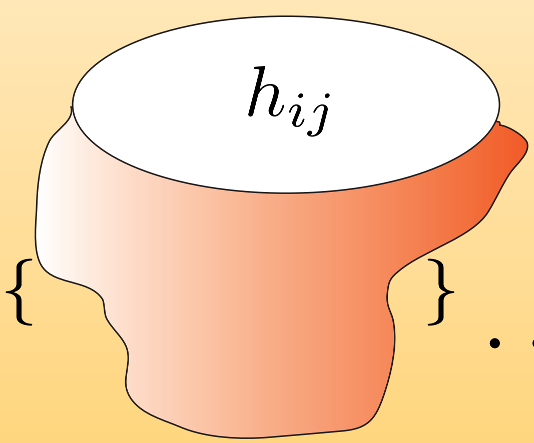
Hartle&Hawking(1983)

Wick rotation

$$\tau_E := it,\ S_E^{\rm EH} := -iS^{\rm EH}\big|_{t=-i\tau_E}$$

Euclidean path integral

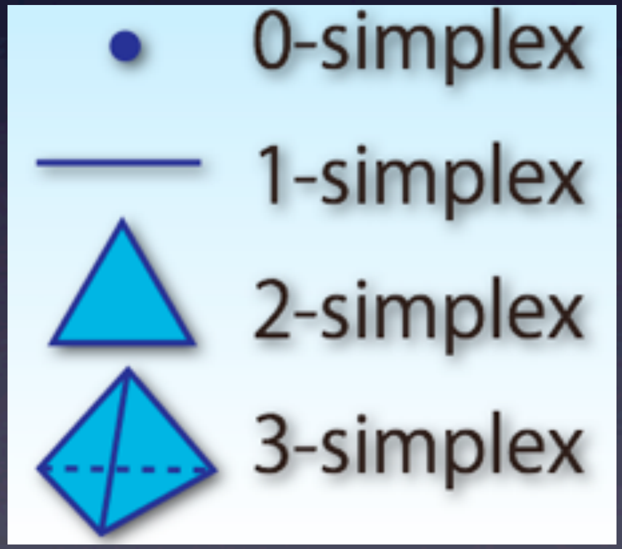
$$\Psi(h_{ij}) = \int \mathcal{D}g\, e^{-S_E^{\rm EH}[g_{\mu\nu}(x)]}$$

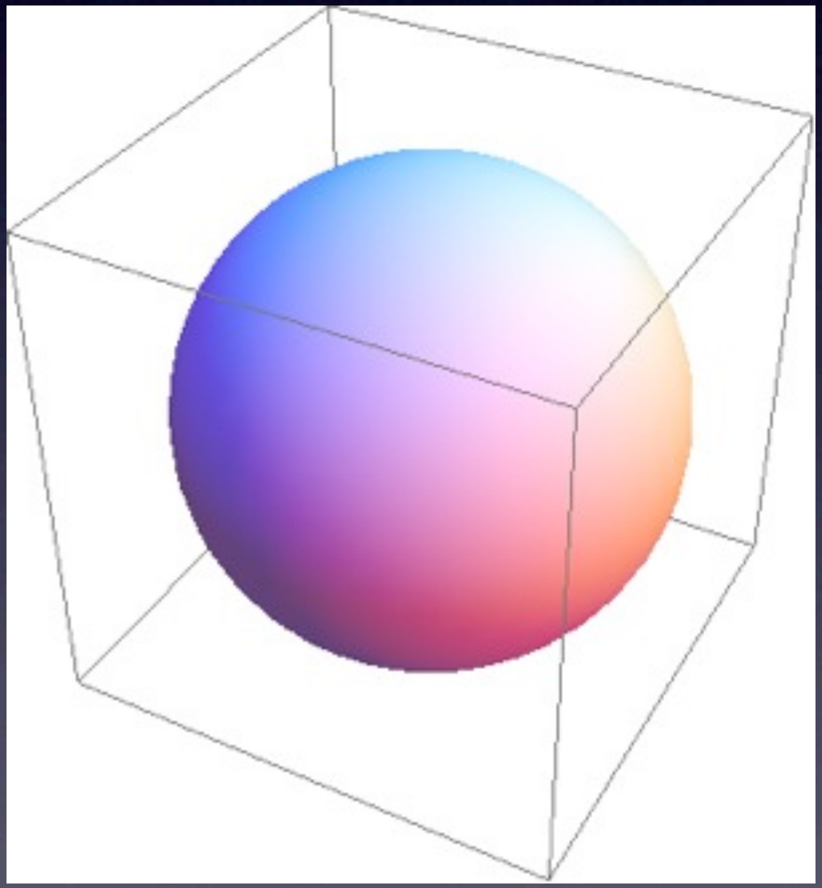
$= e^{-S_E^{\rm EH}}\{$  $\} + e^{-S_E^{\rm EH}}\{$  $\} + e^{-S_E^{\rm EH}}\{$  $\}\ldots$

Causal dynamical triangulation

→ a discrete **regularization method** for gravitational path integral

The space-time is **discretized** by simplices.





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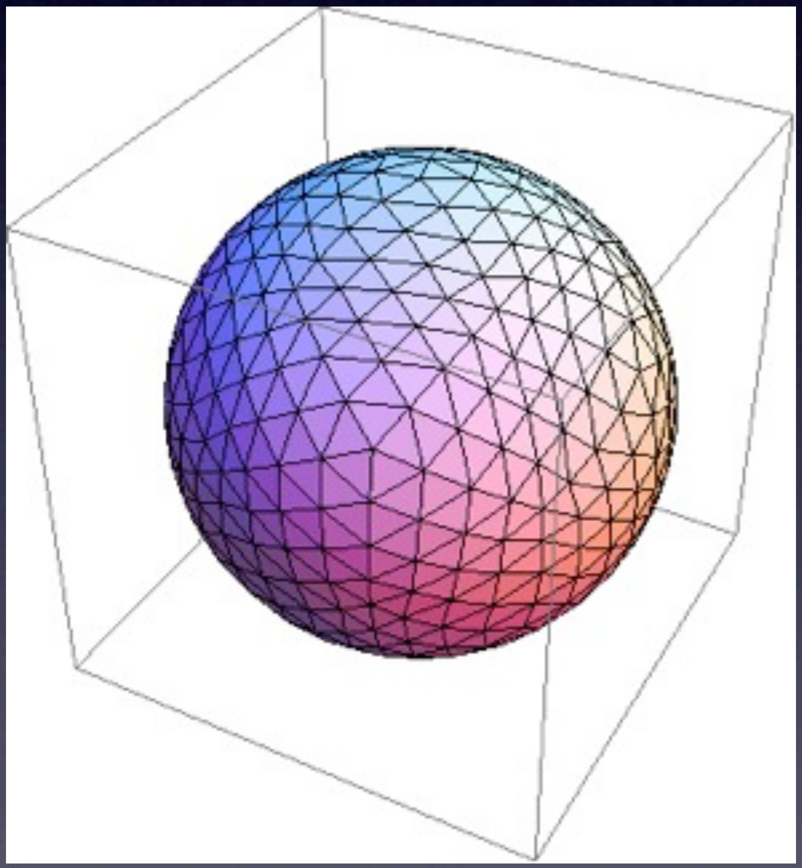
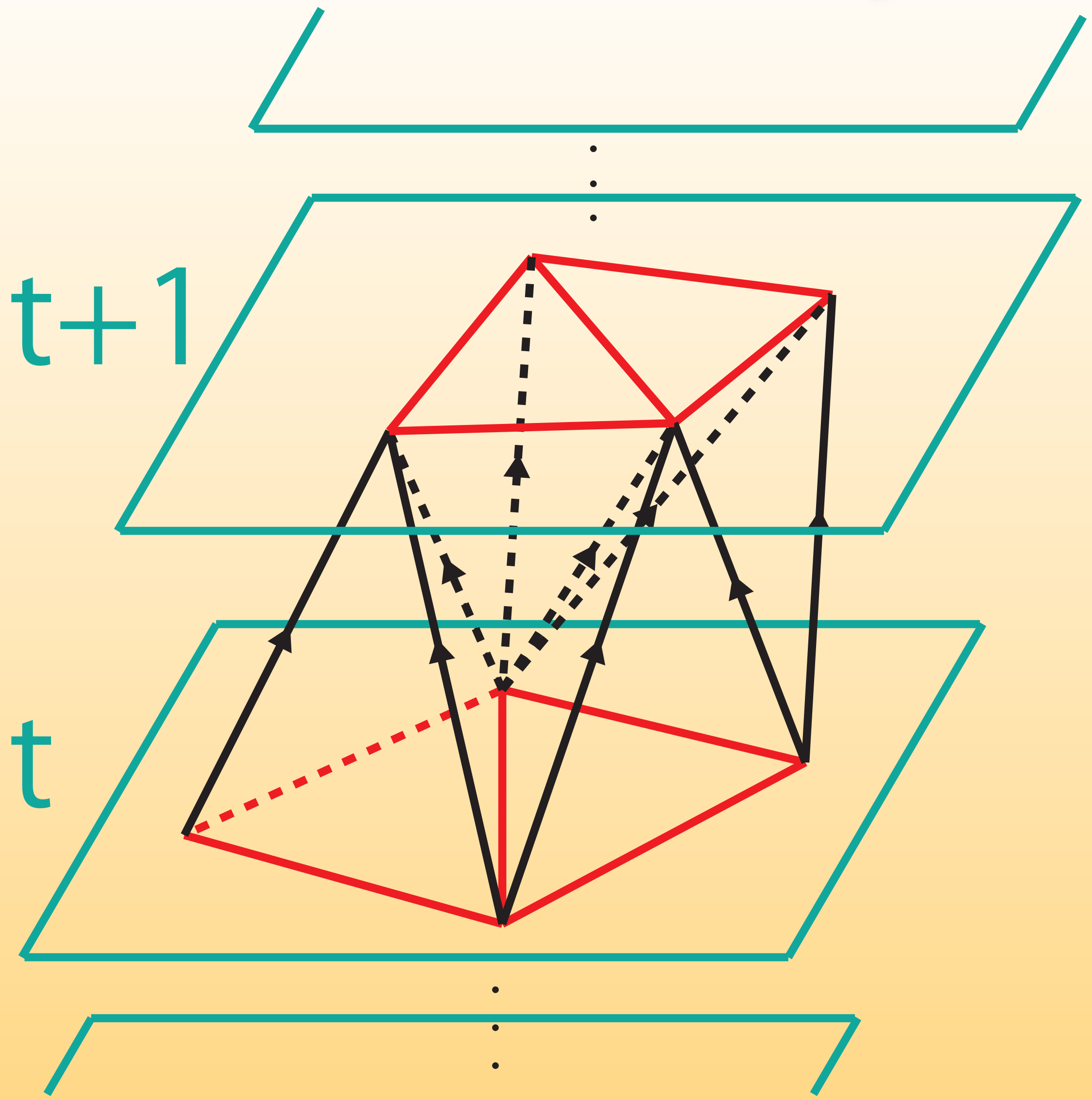


Advantage
- functional integral → **finite summation**
- avoid conformal divergence

Discretization of Lorentzian space-time & Wick rotation in CDT

discretized 3D Lorentzian space-time



squared length of space-like links

$$l_{\mathrm{SL}}^2 = a^2$$

squared length of time-like links

$$l_{\mathrm{TL}}^2 = -\alpha a^2$$

$$\alpha > 0$$

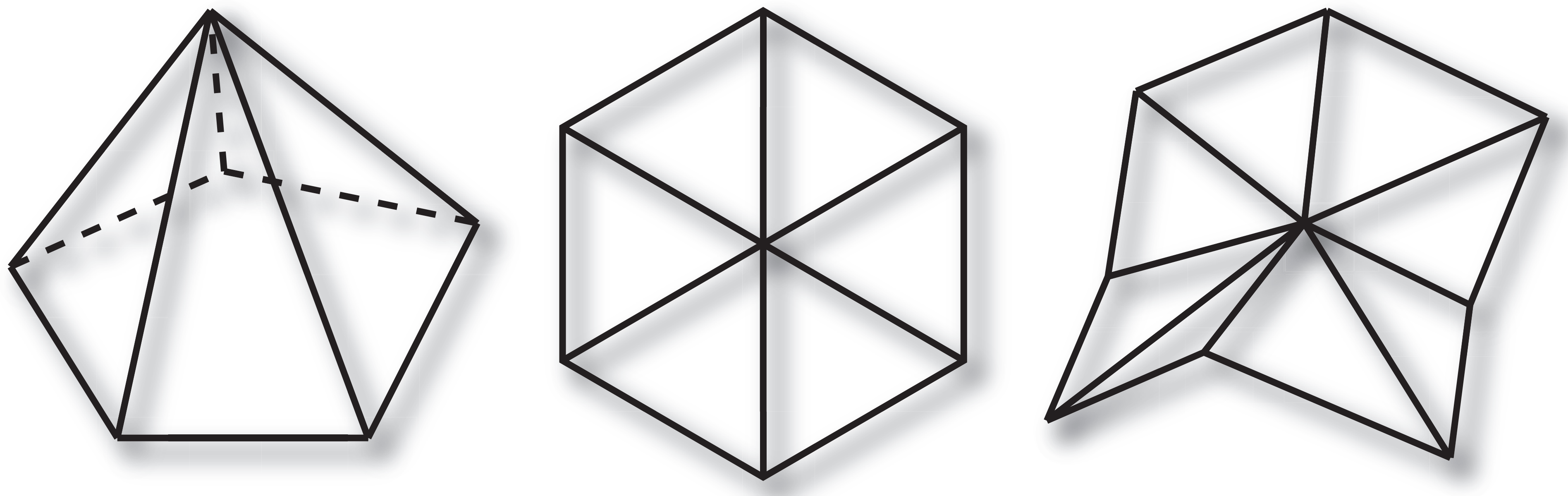
Wick rotation

$$\alpha \to -\alpha$$

(We choose $\alpha = 1$ **)**

Curvature of discretized space-time

e. g. 2D



positive(5) flat(6) negative(8)

coordination number

(number of simplices around a hinge(2d vertex, 3d link))

↔ curvature

flat space coordination number 6(2d), 5.104...(3d)

Wave function in CDT

discretized wave function

$$\Psi(h_{ij}) = \int \mathcal{D}g \, e^{-S_E^{\mathrm{EH}}[g_{\mu\nu}(x)]} \to \sum_{\mathcal{T}} e^{-S_E^{\mathrm{Regge}}[\mathcal{T}]}$$

$\sum_{\mathcal{T}}$ **sum over all discretized space-time**

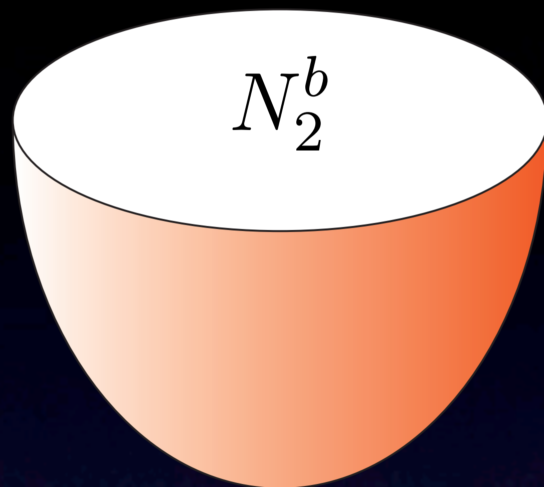
$$S_E^{\mathrm{Regge}} = -\kappa_0 N_0 + \kappa_3 N_3 + \kappa_b N_2^b,$$

$\kappa_0 \cdots$	gravitational constant	$N_0 \cdots$	total number of vertices
$\kappa_3 \cdots$	cosmological constant	$N_3 \cdots$	total number of simplices
$\kappa_b \cdots$	coupling constant of a boundary term	$N_2^b \cdots$	total number of triangles on a boundary

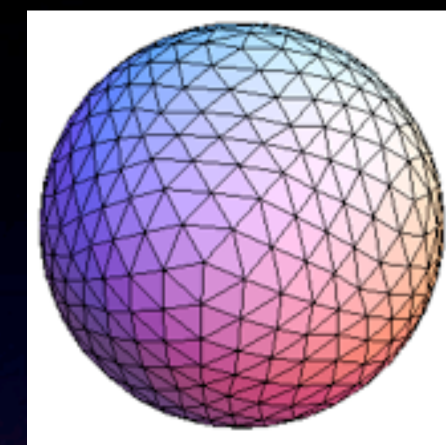
Definition of boundary term in 3d DT: S. Warner, S. Catterall, R. Renken, Phase diagram of three-dimensional dynamical triangulations with a boundary, Phys. Lett. B 442 (1998) 266–272, hep-lat/9808006.

Simulation set-up

- topology $I \times S^2$



spatial slice →



$I \times S^2$ S^2

- pure gravity
- fix volume of the spatial boundary

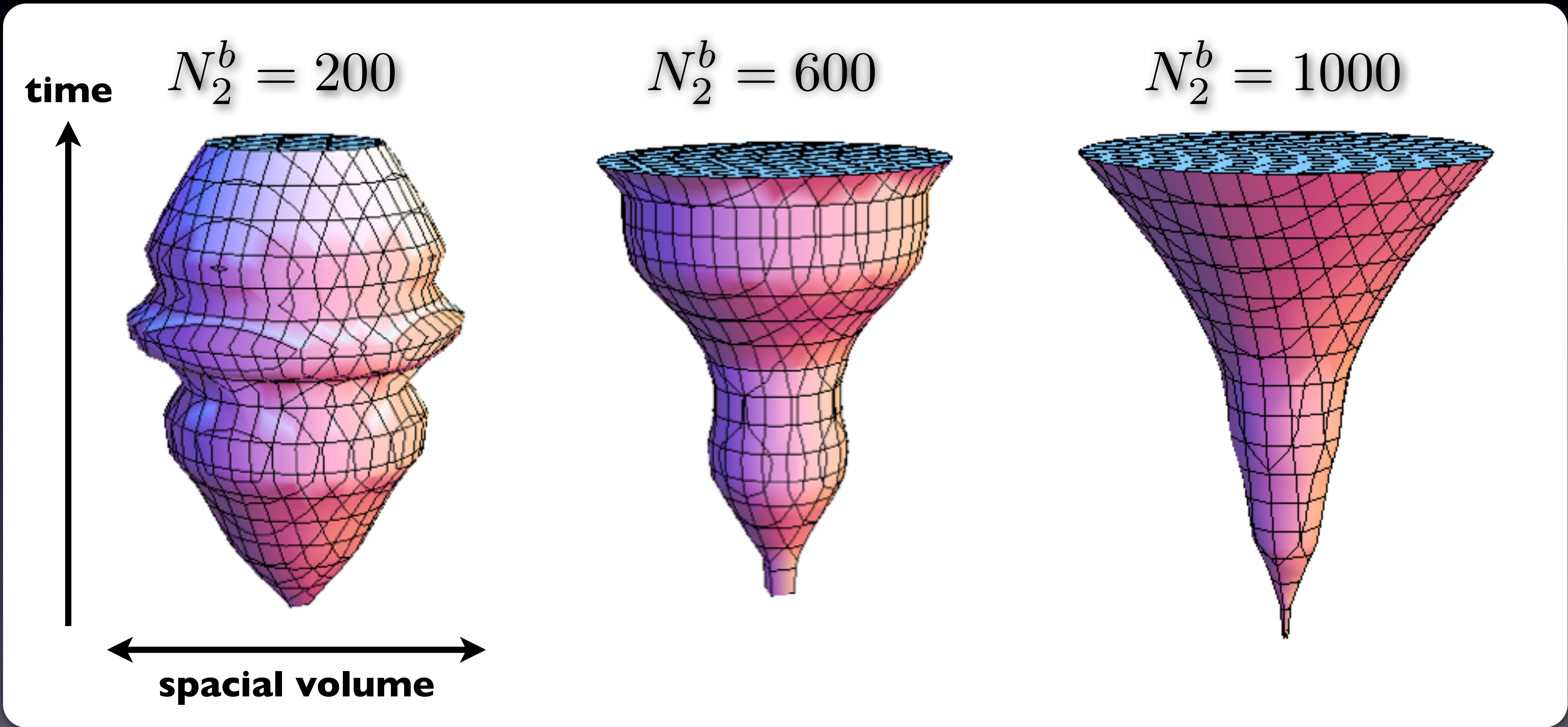
$$N_2^b = 200, 600, 1000$$

- fix space-time volume for technical reason
- We set $N_3 = 10000$
- We set gravitational constant $\kappa_0 = 0.4,\ (3.0)$

We perform Markov chain Monte Carlo simulation.

Results

Typical configuration



$$N_3 = 10000,\ \kappa_0 = 0.4$$

Spatial volume dynamics

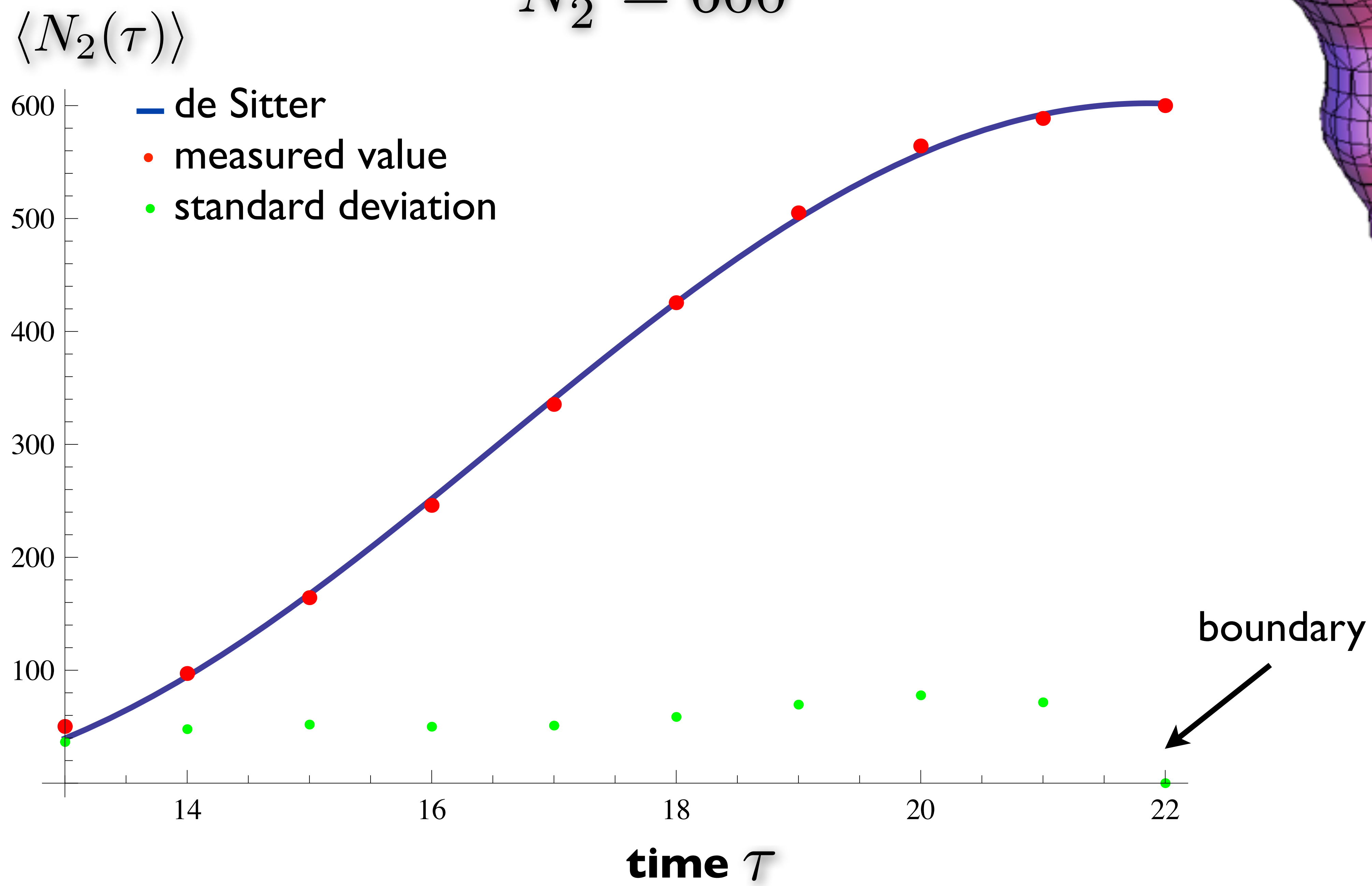
In DT classical space-time didn't emerge.

In CDT the averaged spacial volume $\langle N_2(\tau)\rangle$ at Euclidean time τ can be described by de Sitter instanton.

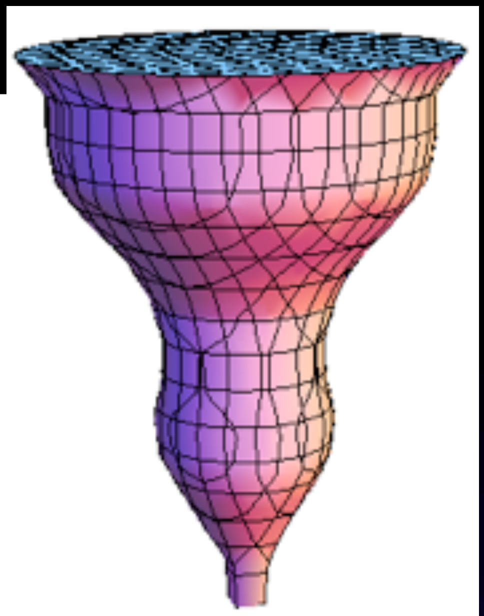
$$\langle N_2(\tau)\rangle = A\cos^2(\frac{\tau}{B})$$

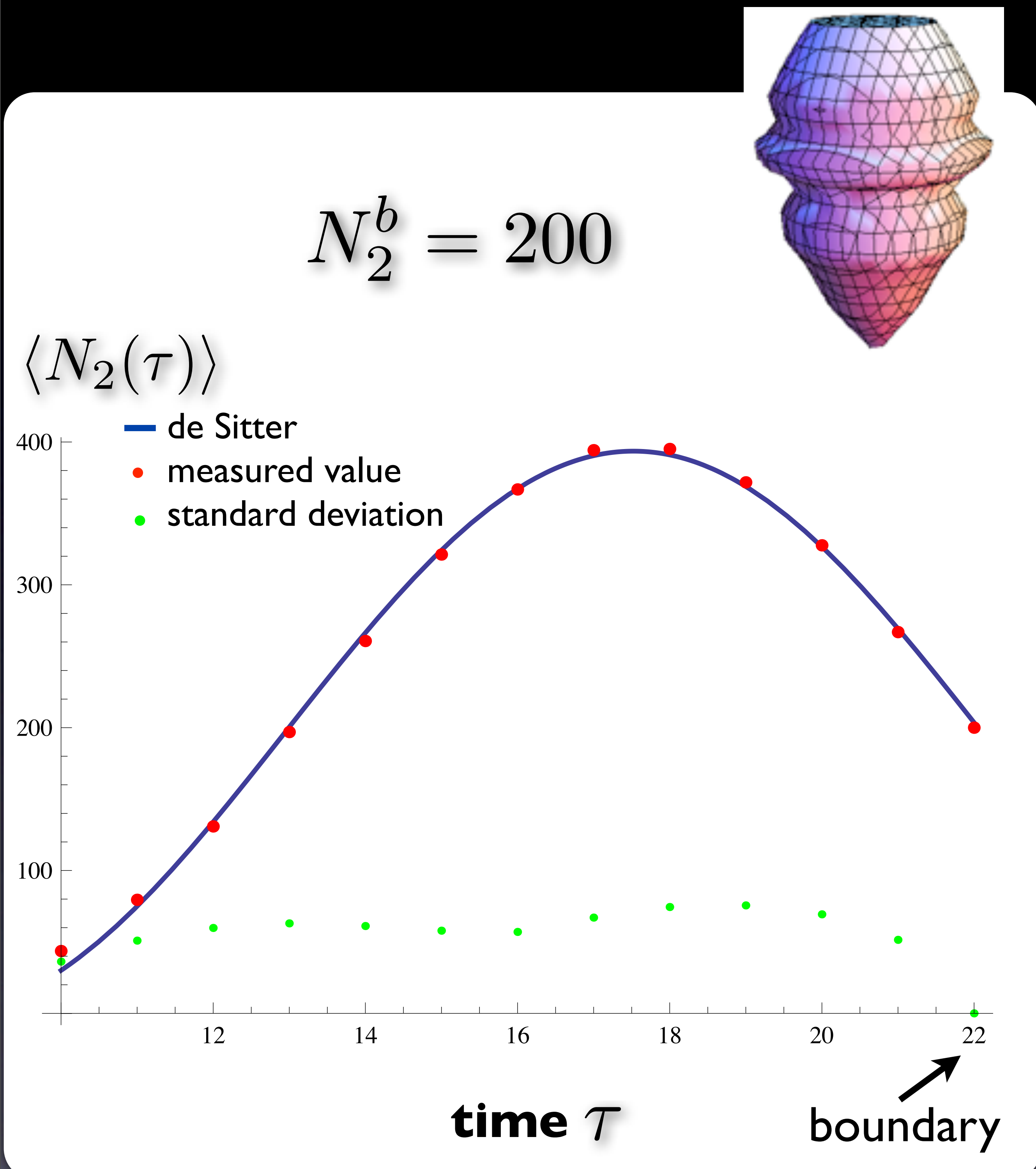
A, B constant

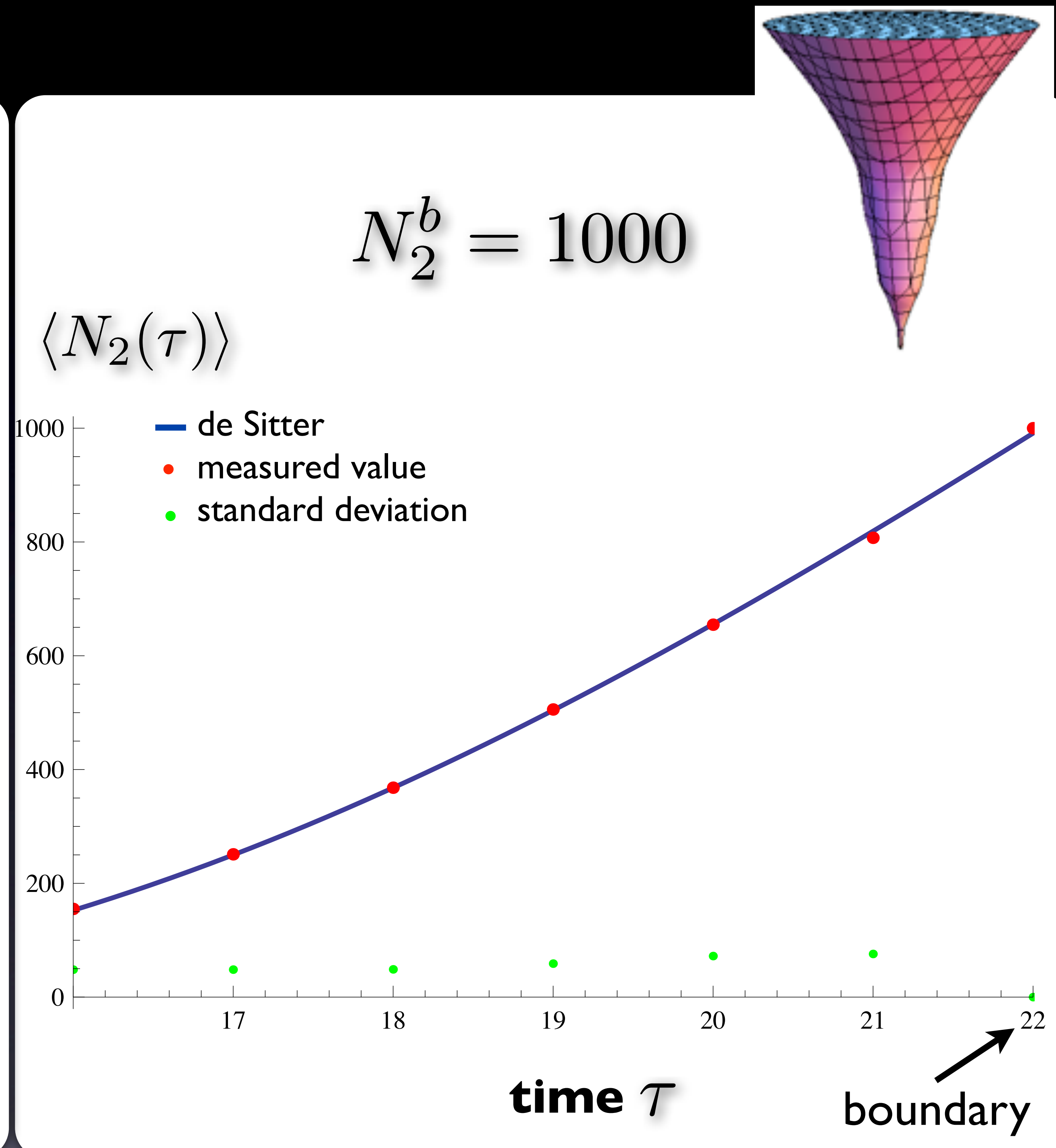
$$N_2^b = 600$$



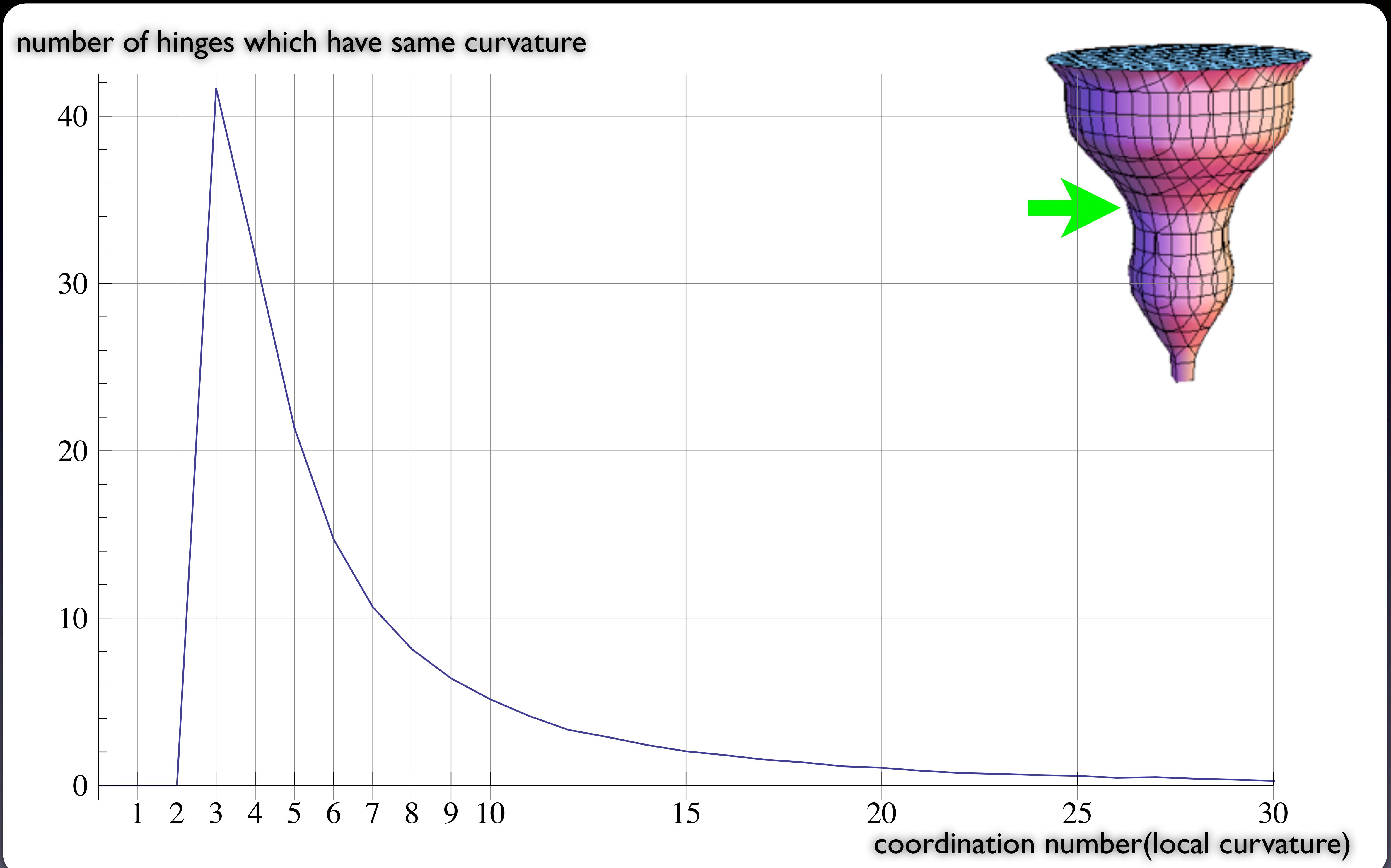
$\langle N_2(\tau) \rangle$
600
500
400
300
200
100
de Sitter
measured value
standard deviation
boundary
14
16
18
20
22
time τ




$N_2^b = 200$
$\langle N_2(\tau) \rangle$
de Sitter
measured value
standard deviation
400
300
200
100
12
14
16
18
20
22
time τ
boundary

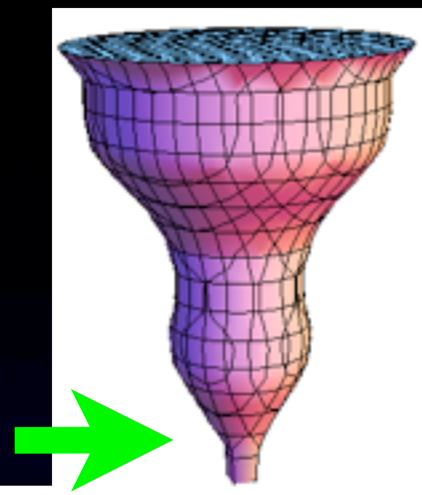

$N_2^b = 1000$
$\langle N_2(\tau) \rangle$
de Sitter
measured value
standard deviation
1000
800
600
400
200
0
17
18
19
20
21
22
time τ
boundary

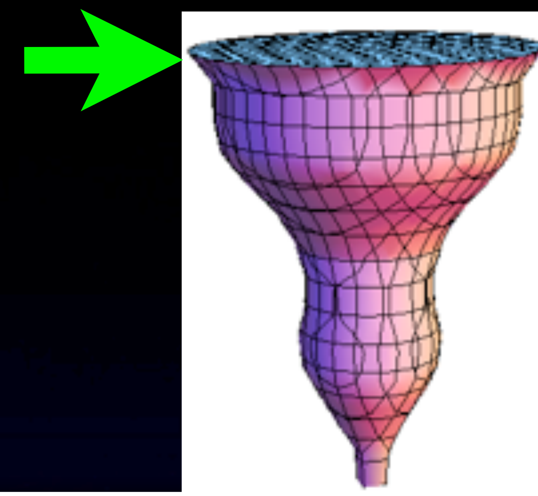
Homogeneity of spacial slice

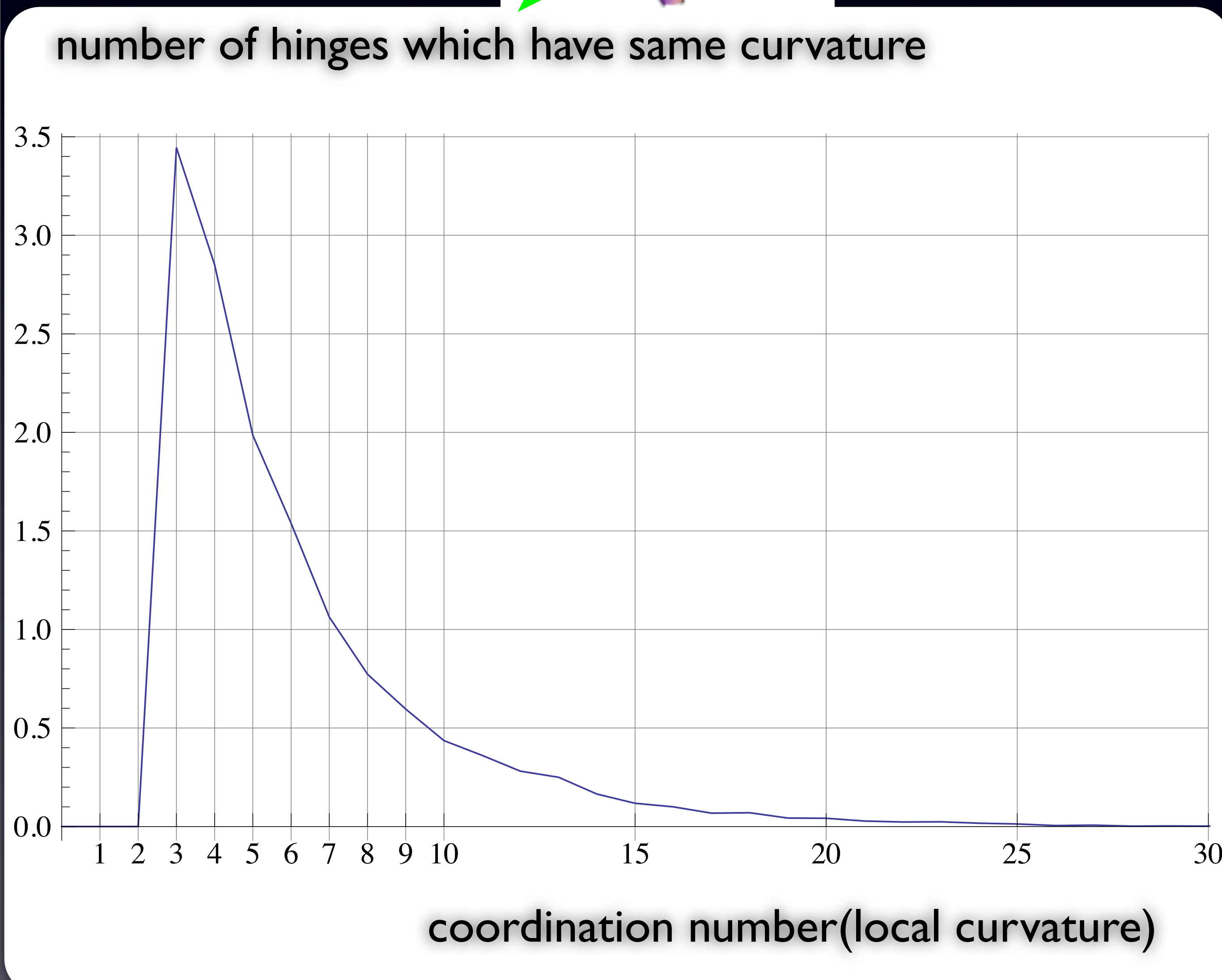


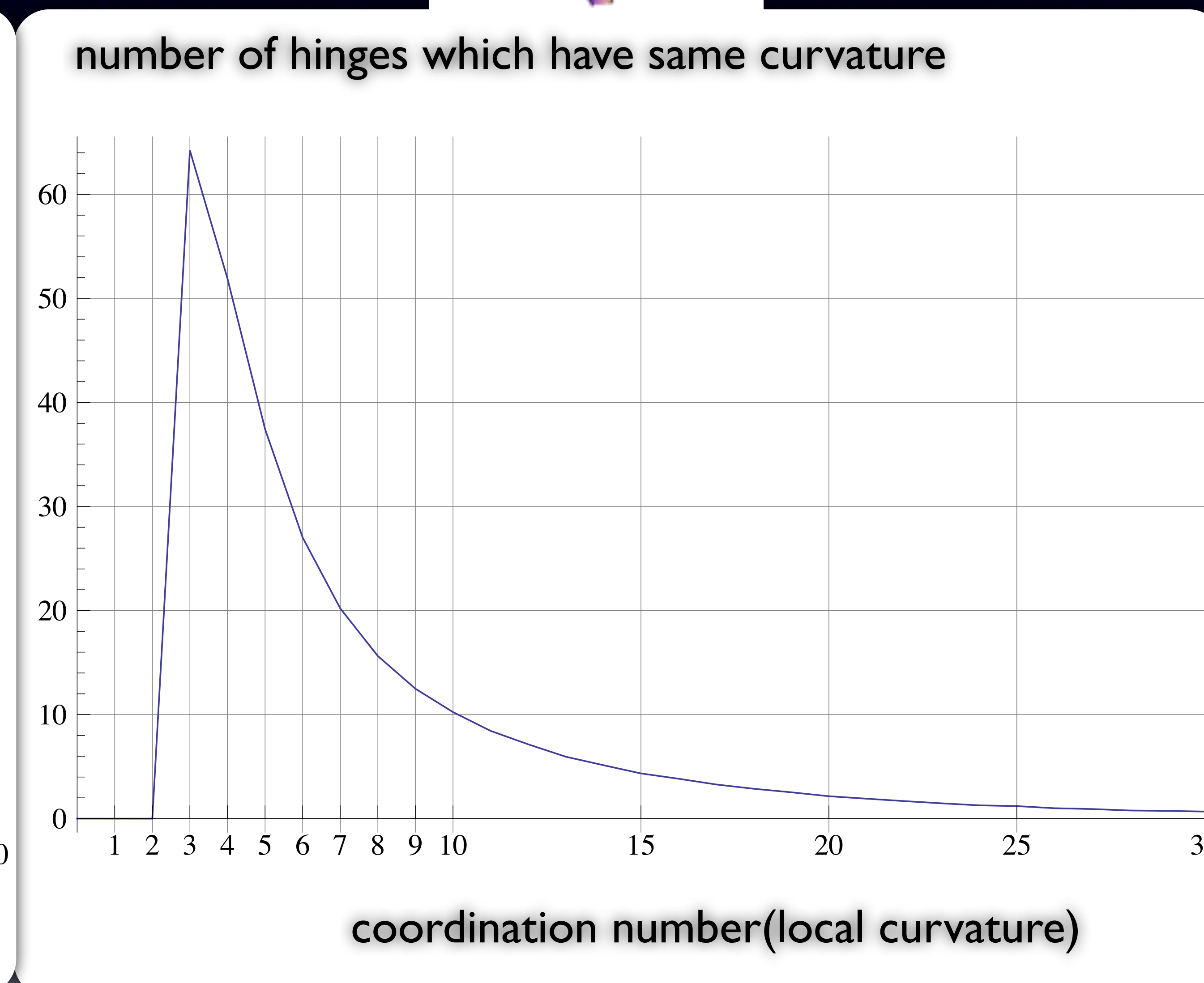
$$N_3 = 10000,\ N_2^b = 600,\ \kappa_0 = 0.4$$

Homogeneity of spacial slice







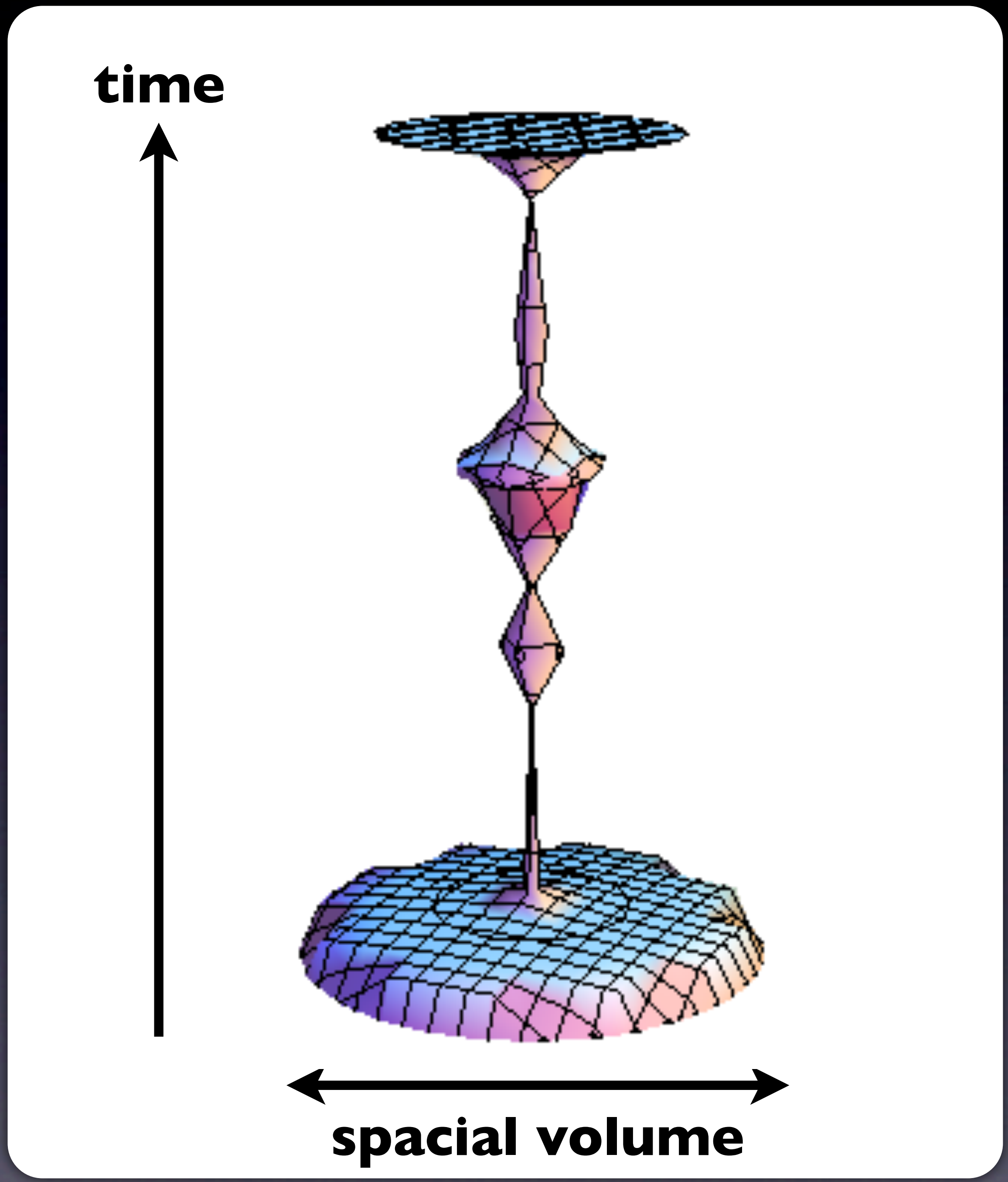


$$N_3 = 10000, \ N_2^b = 600, \ \kappa_0 = 0.4$$

Summary

- CDT is a discrete regularization method for gravitational path integral.
- The emergent space-time in $\kappa_0 = 0.4$ corresponds to de Sitter instanon and the spatial slices are homogenous.

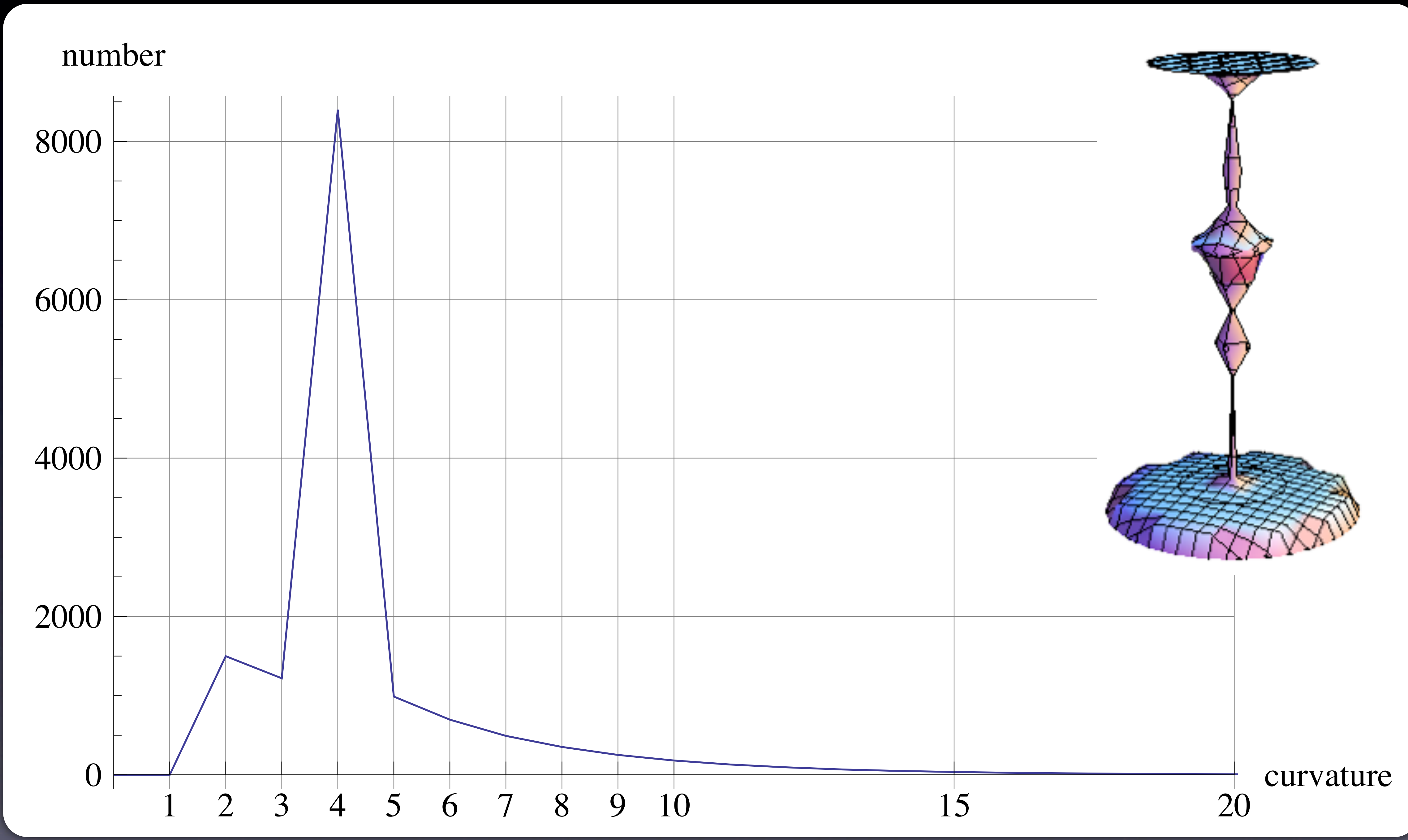
Typical configuration



$$N_3 = 10000,\ N_2^b = 1000,\ \kappa_0 = 3.0$$

2012年11月12日月曜日

Homogeneity of space-time



$$N_3 = 10000,\ N_2^b = 1000,\ \kappa_0 = 3.0$$