



Taro Kunimitsu, JGRG 22(2012)111211

“Higgs Condensation in the inflationary universe”

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**RESCEU SYMPOSIUM ON  
GENERAL RELATIVITY AND GRAVITATION**

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# Higgs Condensation in the Inflationary Universe

Research Center for the Early Universe (RESCEU)

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with Jun'ichi Yokoyama

PRD 86, 083541 (2012)

July 4, 2012



ATLAS Collaboration

$$m_H = 126.0 \pm 0.4 \pm 0.4 \text{ GeV}$$

Phys.Lett. B 716, 1 (2012)

CMS Collaboration

$$m_H = 125.3 \pm 0.4 \pm 0.5 \text{ GeV}$$

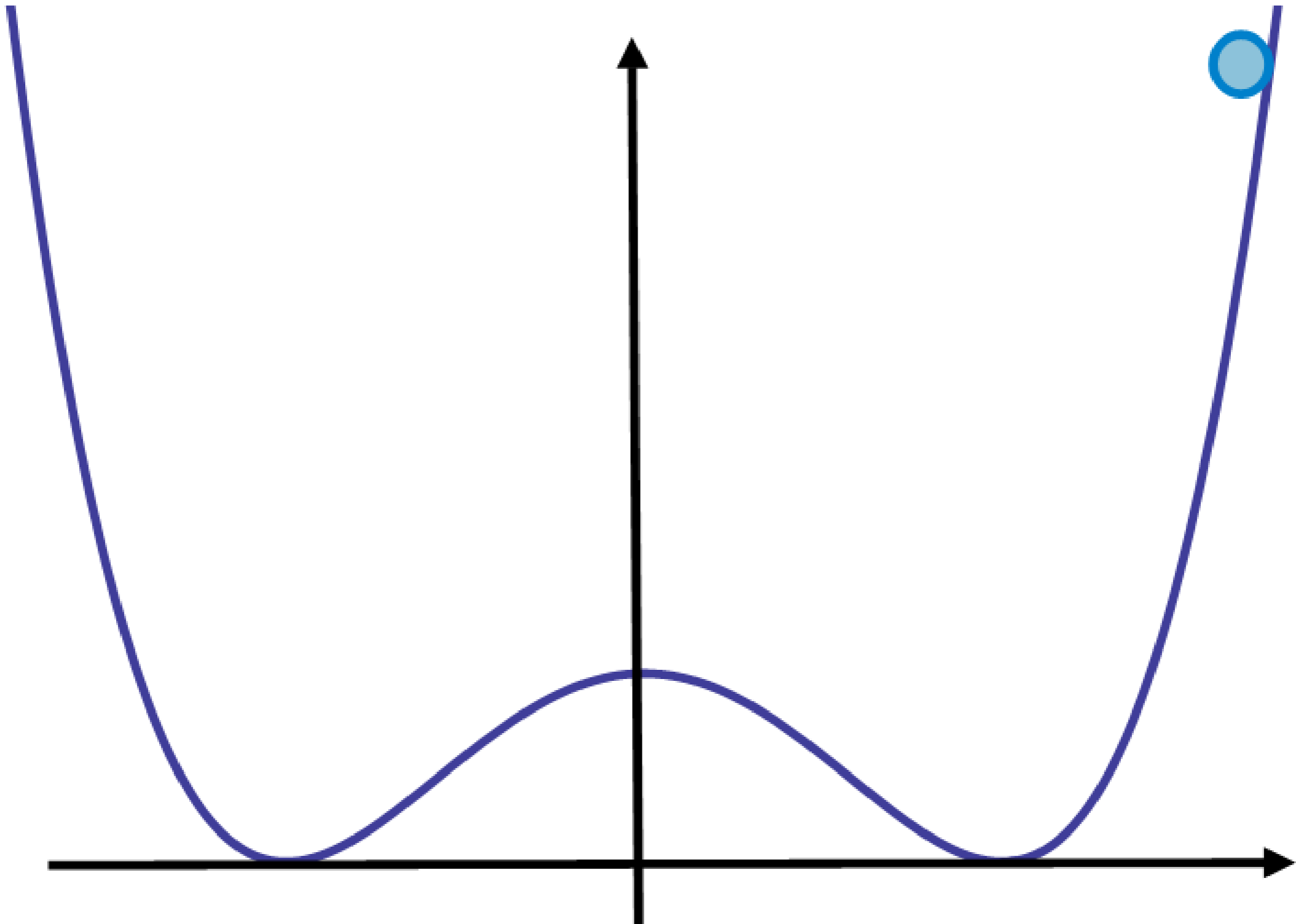
Phys.Lett. B 716, 30 (2012)

**Could the Higgs field  
have dominated the  
Universe ?**

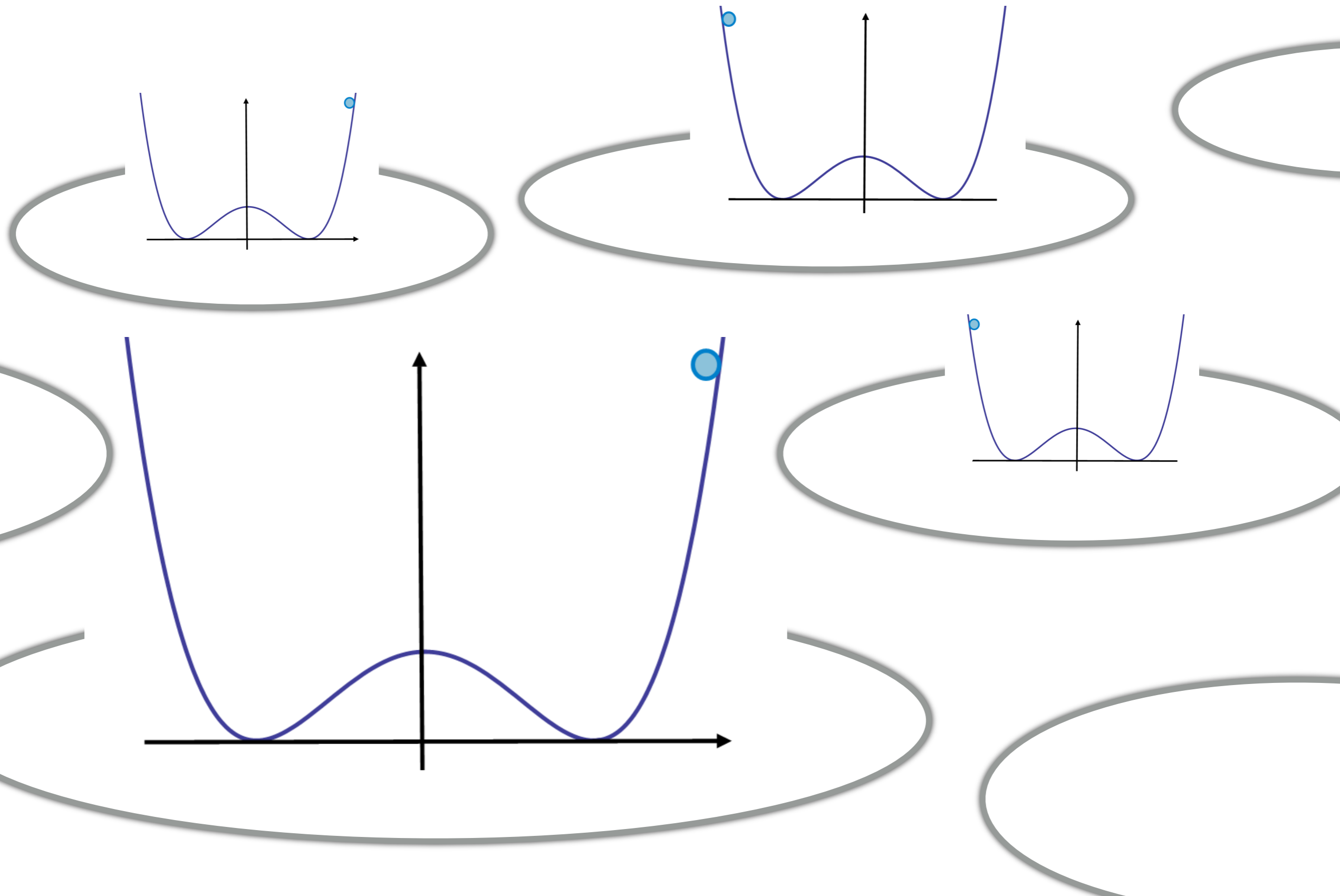
**Could the Higgs field  
have dominated the  
Universe ?**

**(within the SM + Inflaton framework)**

# Higgs Field during inflation



# Higgs Field during inflation



A close-up photograph of a blue plastic water bottle covered in condensation droplets. The bottle is the central focus, with numerous small, clear water droplets of varying sizes clinging to its surface. The background is blurred, showing a wooden table and some greenery. The text "Higgs Condensation" is overlaid on the lower left portion of the image in a white, sans-serif font.

**“Higgs  
Condensation”**



# Stochastic Inflation

Starobinsky (1984, 1986)

$$\begin{aligned} \varphi(\mathbf{x}, t) = & \bar{\varphi}(\mathbf{x}, t) \quad \swarrow \text{long-wave part} \\ & + \int \frac{d^3 k}{(2\pi)^{\frac{3}{2}}} \theta(k - \epsilon a(t) H) \\ & \times \left[ a_{\mathbf{k}} \varphi_{\mathbf{k}}(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^\dagger \varphi_{\mathbf{k}}^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right] \\ & \quad \quad \quad \nwarrow \text{short-wave part} \end{aligned}$$

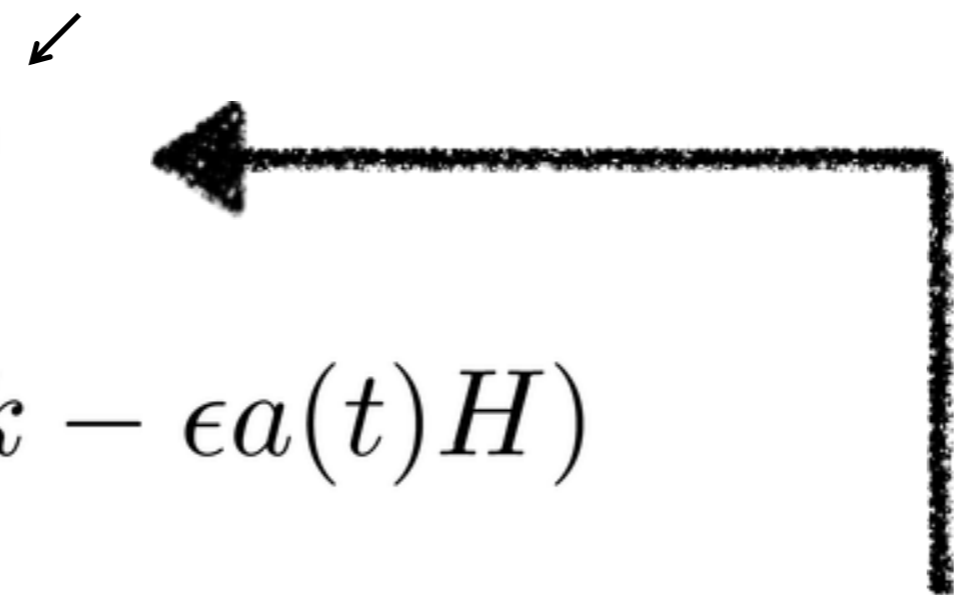
# Stochastic Inflation

Starobinsky (1984, 1986)

$$\varphi(\mathbf{x}, t) = \bar{\varphi}(\mathbf{x}, t) + \int \frac{d^3 k}{(2\pi)^{\frac{3}{2}}} \theta(k - \epsilon a(t) H) \times \left[ a_{\mathbf{k}} \varphi_{\mathbf{k}}(t) e^{-i\mathbf{k} \cdot \mathbf{x}} + a_{\mathbf{k}}^\dagger \varphi_{\mathbf{k}}^*(t) e^{i\mathbf{k} \cdot \mathbf{x}} \right]$$

long-wave part

short-wave part



# Fokker-Planck equation

$$\frac{\partial \rho_1[\varphi(\mathbf{x}, t)]}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \varphi} \{V'[\varphi(\mathbf{x}, t)] \rho_1[\varphi(\mathbf{x}, t)]\} + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho_1[\varphi(\mathbf{x}, t)]}{\partial \varphi^2}$$

# Fokker-Planck equation

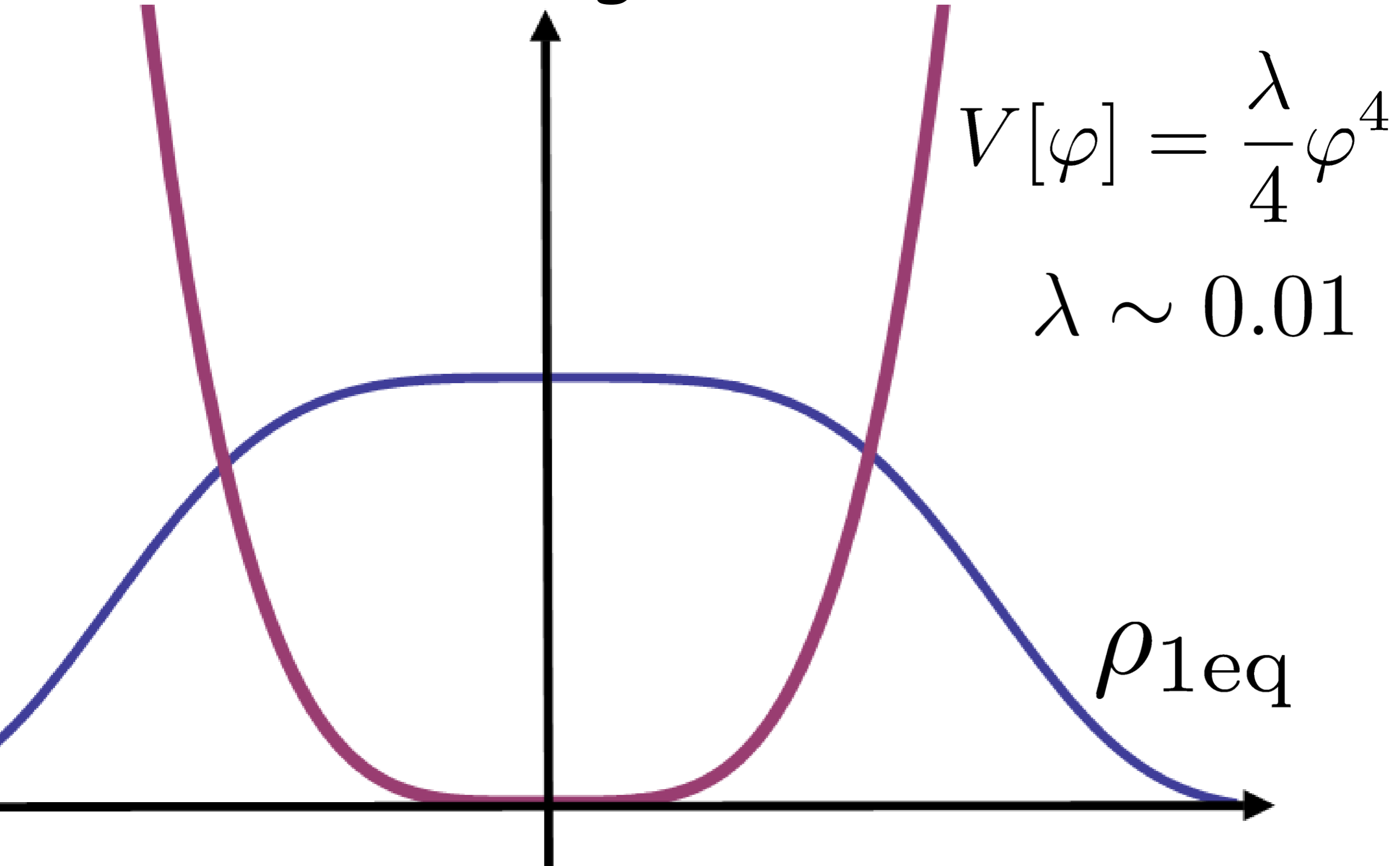
$$\frac{\partial \rho_1[\varphi(\mathbf{x}, t)]}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \varphi} \{V'[\varphi(\mathbf{x}, t)] \rho_1[\varphi(\mathbf{x}, t)]\} + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho_1[\varphi(\mathbf{x}, t)]}{\partial \varphi^2}$$

→ An equilibrium state exists

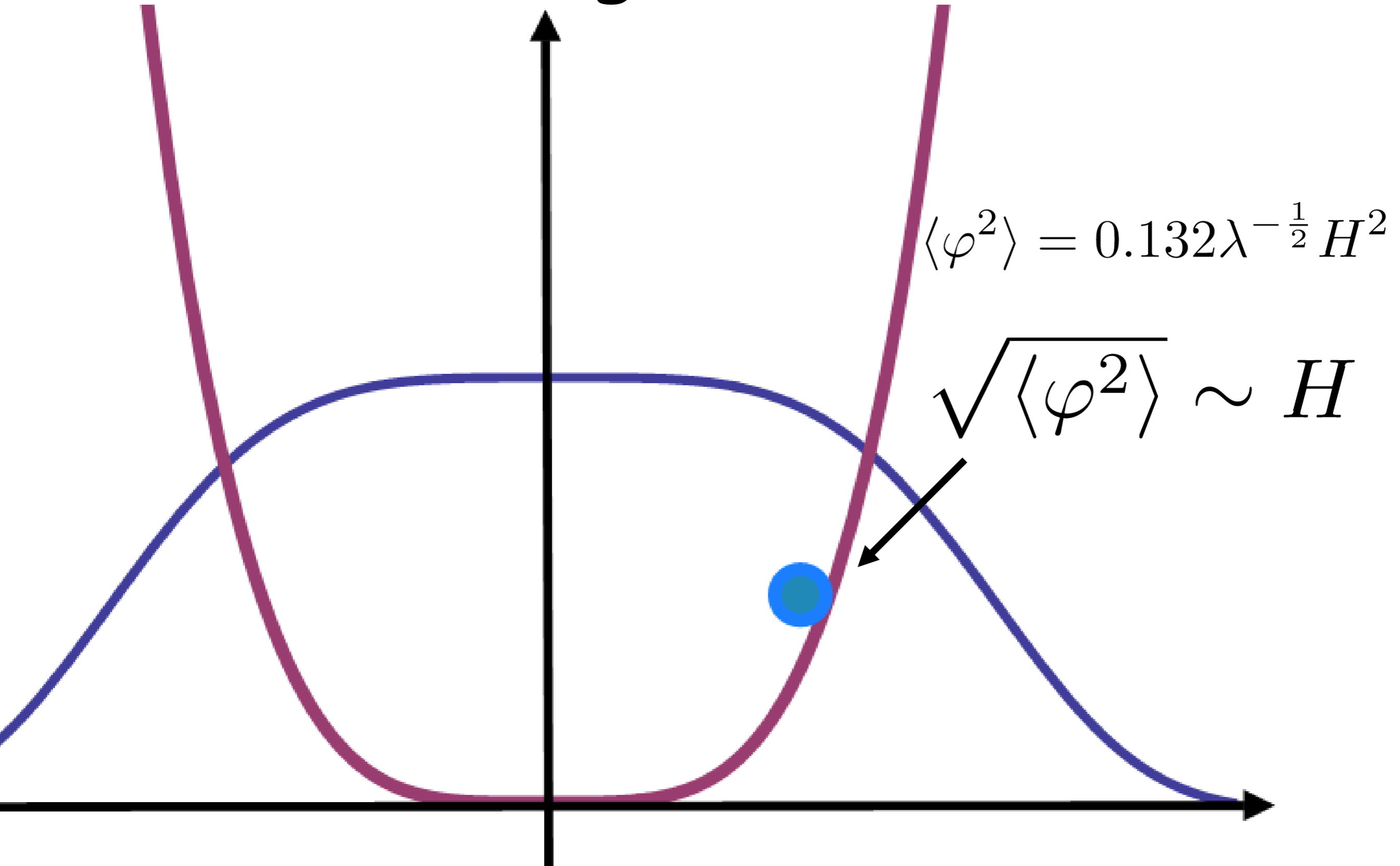
$$\text{for } V[\varphi] = \frac{\lambda}{4} \varphi^4$$

$$\rho_{1\text{eq}}(\varphi) = \left( \frac{32\pi^2 \lambda}{3} \right)^{\frac{1}{4}} \frac{1}{\Gamma(\frac{1}{4})H} \exp\left( -\frac{3\pi^2 \lambda \varphi^4}{3H^4} \right)$$

# Probability distribution



# Probability distribution

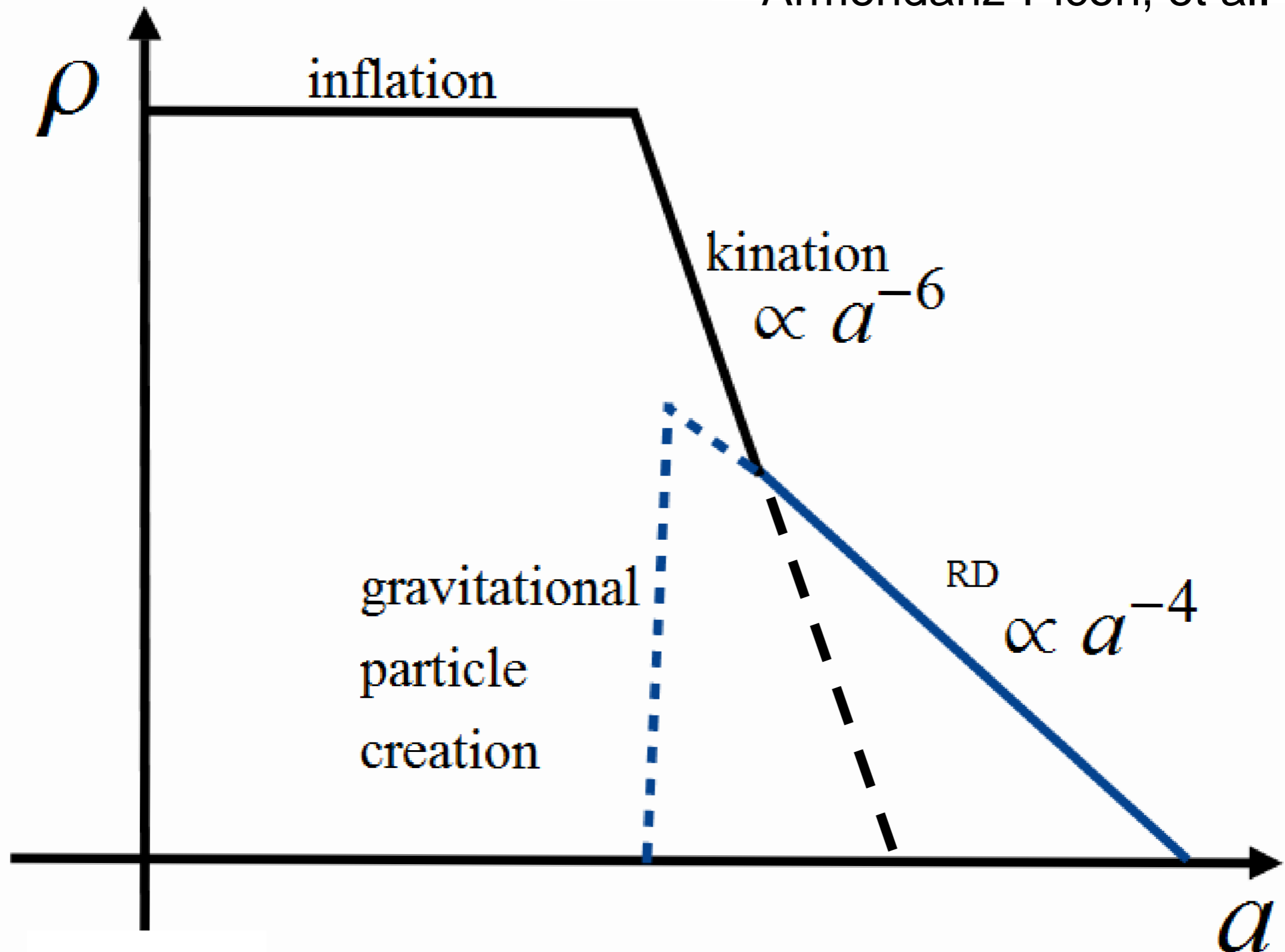


A close-up photograph of a blue plastic water bottle with a pink and blue top. The bottle is covered in numerous small, clear water droplets, indicating it is cold. The background is blurred, showing a wooden surface and some greenery. The text "Inflation Model" is overlaid in white, bold font in the center of the image.

# Inflation Model

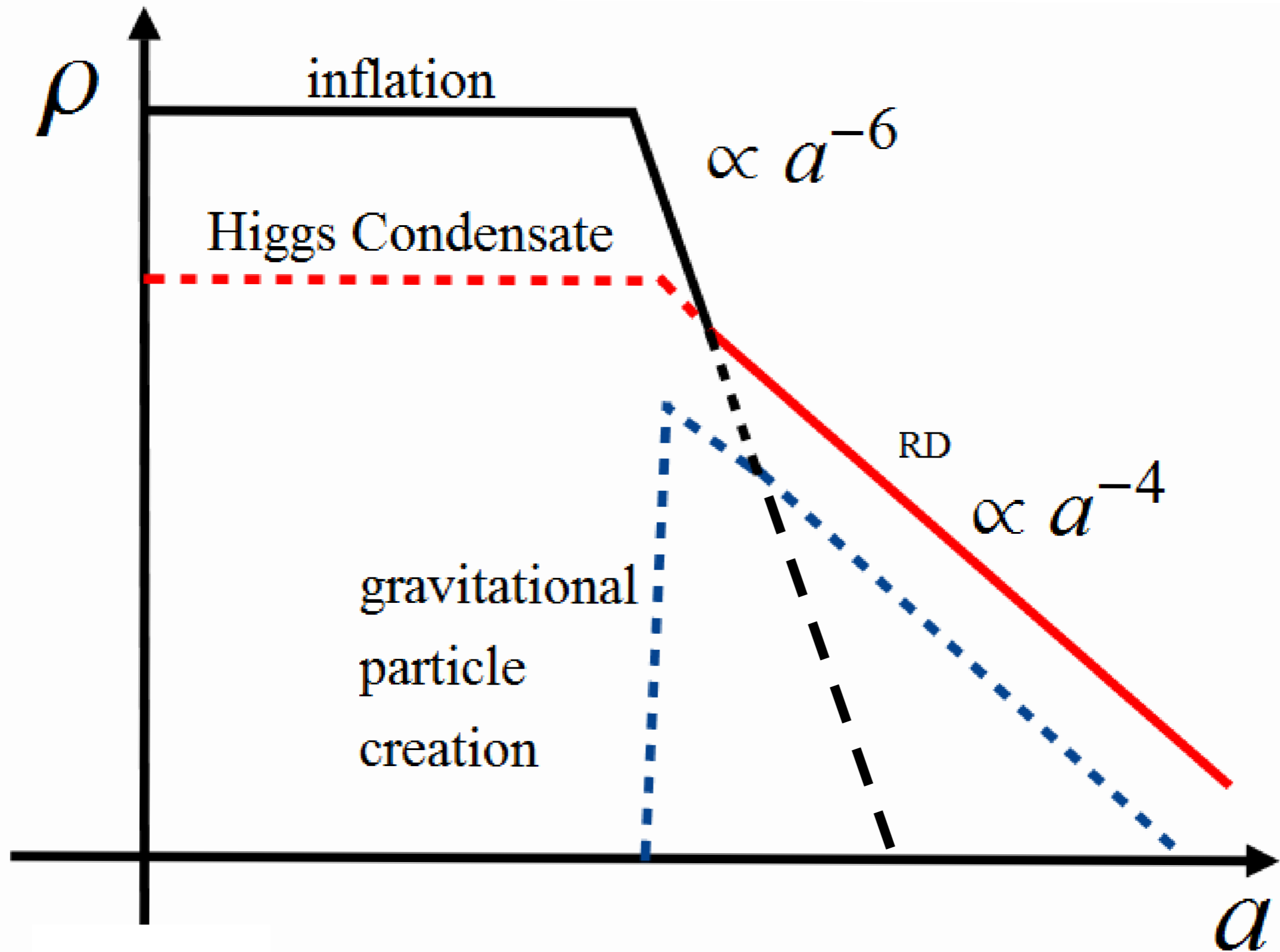
# Model : k-inflation

Armendariz-Picon, et al. (1999)





# k-inflation with Higgs





# Primordial Fluctuations

# Fluctuations

We want

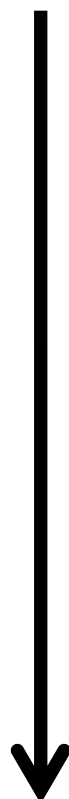
$$\mathcal{P}(k) \sim 10^{-9}$$

# Fluctuations


$$\begin{aligned} \mathbb{E}_{\Delta_h}(r) &\equiv \left\langle \frac{\delta \rho_h(r)}{\rho_h} \frac{\delta \rho_h(0)}{\rho_h} \right\rangle \\ &\simeq 4(Har)^{-0.178\sqrt{\lambda}} \end{aligned}$$

# Fluctuations

Fourier  
Transform


$$P_V(k) \simeq 14\sqrt{\lambda}k^{-3}$$

$$\mathcal{P}_V(k) \equiv \frac{4\pi k^3}{(2\pi)^3} P_V(k) \sim \sqrt{\lambda} \sim 0.1$$


$$\lambda \sim 0.01$$

# Fluctuations

We get

$$\mathcal{P}(k) \sim \mathcal{P}_V(k)$$

$$\sim 0.1$$

**Too Large!**

# Summary

Higgs condensation

→ Dominates the Universe in k-inflation

→ We Showed that the fluctuations are too large





**backups**

# Fokker-Planck equation

$$\frac{\partial \rho_1[\varphi(\mathbf{x}, t)]}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \varphi} \{V'[\varphi(\mathbf{x}, t)]\rho_1[\varphi(\mathbf{x}, t)]\} + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho_1[\varphi(\mathbf{x}, t)]}{\partial \varphi^2}$$

# Probability Distribution Function

→ An equilibrium state exists

Starobinsky, Yokoyama (1994)

for  $V[\varphi] = \frac{\lambda}{4}\varphi^4$

$$\rho_{1\text{eq}}(\varphi) = \left(\frac{32\pi^2\lambda}{3}\right)^{\frac{1}{4}} \frac{1}{\Gamma(\frac{1}{4})H} \exp\left(-\frac{3\pi^2\lambda\varphi^4}{3H^4}\right)$$

# Correlation time/length

$$t_c \simeq 76.2 H^{-1}$$

$$a(t)r_c \simeq e^{37.4} H^{-1}$$

# During inflation

$$\varphi \sim H_{\text{inf}}$$

$$\rho_{\text{inf}} \sim H_{\text{inf}}^2 M_P^2$$

$$\rho_h \sim \lambda H_{\text{inf}}^4$$

# When does Oscillation start?

$$\rho_h \sim \lambda H_{\text{inf}}^4$$

$$\rho_{\text{inf}} \sim H_*^2 M_P^2 \leftarrow H_*^2 \sim \lambda H_{\text{inf}}^2$$

$$\rightarrow \frac{\rho_h}{\rho_{\text{inf}}} \sim \frac{H_{\text{inf}}^2}{M_P^2} \sim \mathcal{P}_t < 10^{-9}$$

# Gravitational particle production

$$\rho_r = \frac{9H_{\text{inf}}^4}{32\pi^2 a^4} \ln \left( \frac{1}{H_{\text{inf}} \Delta t} \right)$$

$$\ln \left( \frac{1}{H_{\text{inf}} \Delta t} \right) \sim 1$$

# Energy density

$$\rho_r = \frac{9NH_{\text{inf}}^4}{32\pi^2} \left( \frac{m_{\text{eff}}}{H_{\text{inf}}} \right)^{\frac{4}{3}} \simeq 1.59 \times 10^{-3} NH_{\text{inf}}^4$$

$$\rho_{\text{cond}} = \frac{3H_{\text{inf}}^4}{32\pi^2} \simeq 9.50 \times 10^{-3} H_{\text{inf}}^4$$