Kazunori Nakayama, JGRG 22(2012)111209

"Probing dark radiation with inflationary gravitational waves"

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Probing dark radiation with inflationary gravitational waves

Kazunori Nakayama (The University of Tokyo)

R.Jinno, T.Moroi, KN, arXiv:1208.0184

JGRG22 @ University of Tokyo (2012/11/12)

Contents

- Observational evidence of dark radiation
- Effects of dark radiation on inflationary gravitational waves

Dark radiation

- Radiation energy density $$\rho_{\rm rad} = \left[1 + N_{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

$N_{\rm eff} = 3.04$ in the standard model

- Helium abundance $N_{\rm eff} = 3.68^{+0.80}_{-0.70}\ (2\sigma)$

  Izotov, Thuan , 1001.4440

- WMAP+ACT+BAO $N_{\rm eff} = 4.56 \pm 0.75\ (68\%)$

  Dunkley et al., 1009.0866

- WMAP+SPT+BAO $N_{\rm eff} = 3.86 \pm 0.42\ (68\%)$

  Keiser et al., 1105.3182

- WMAP+ACT+SPT+BAO $N_{\rm eff} = 4.08^{+0.71}_{-0.68}\ (95\%)$

  Archidiacono, Calabrese, Melchiorri, 1109.2767

Dark radiation

$\Delta N_{\rm eff} \simeq 1$ → Dark radiation ?

- Dark radiation (X) should satisfy :
  - X interaction is negligibly small
  - X is relativistic at the CMB epoch
- Many models are proposed so far...

Ichikawa, Kawasaki, KN, Senami, Takahashi (2007), KN, Takahashi, Yanagida (2010),
Fischler, Meyers (2011), Kawasaki, Kitajima, KN (2011), Hasenkamp (2011)
Menestrina, Scherrer (2011), Jeong, Takahashi (2012), K.Choi et al (2012) and many others

What is unique signature of dark radiation ?

Inflationary GWs

- Inflation generates primordial GWs as quantum tensor fluctuations in de-Sitter spacetime

$$ds^2 = a^2(t)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$h_{ij} = \frac{1}{M_P} \sum_{\lambda=+,-} \int \frac{d^3k}{(2\pi)^{3/2}} h_k^\lambda(t) e^{i\mathbf{kx}} e_{ij}^\lambda$$

Quantization →

$$\langle h_k^\lambda h_{k'}^{\lambda'} \rangle = \frac{H_{\rm inf}^2}{2k^3} \delta^3(k-k') \delta^{\lambda\lambda'}$$

- Dimensionless power spectrum almost scale invariant

$$\Delta_h^2(k) = \left(\frac{H_{\rm inf}}{2\pi M_P} \right)^2$$

Evolution of GW

- Eq.of.m of GW (without dark radiation)

$$\ddot{h}_\lambda + 3H\dot{h}_\lambda + (k/a)^2 h_\lambda = 0 \Rightarrow \begin{array}{l} h_\lambda \sim \text{const} \quad \text{for} \quad k \ll aH \\ h_\lambda \propto a(t)^{-1} \quad \text{for} \quad k \gg aH \end{array}$$

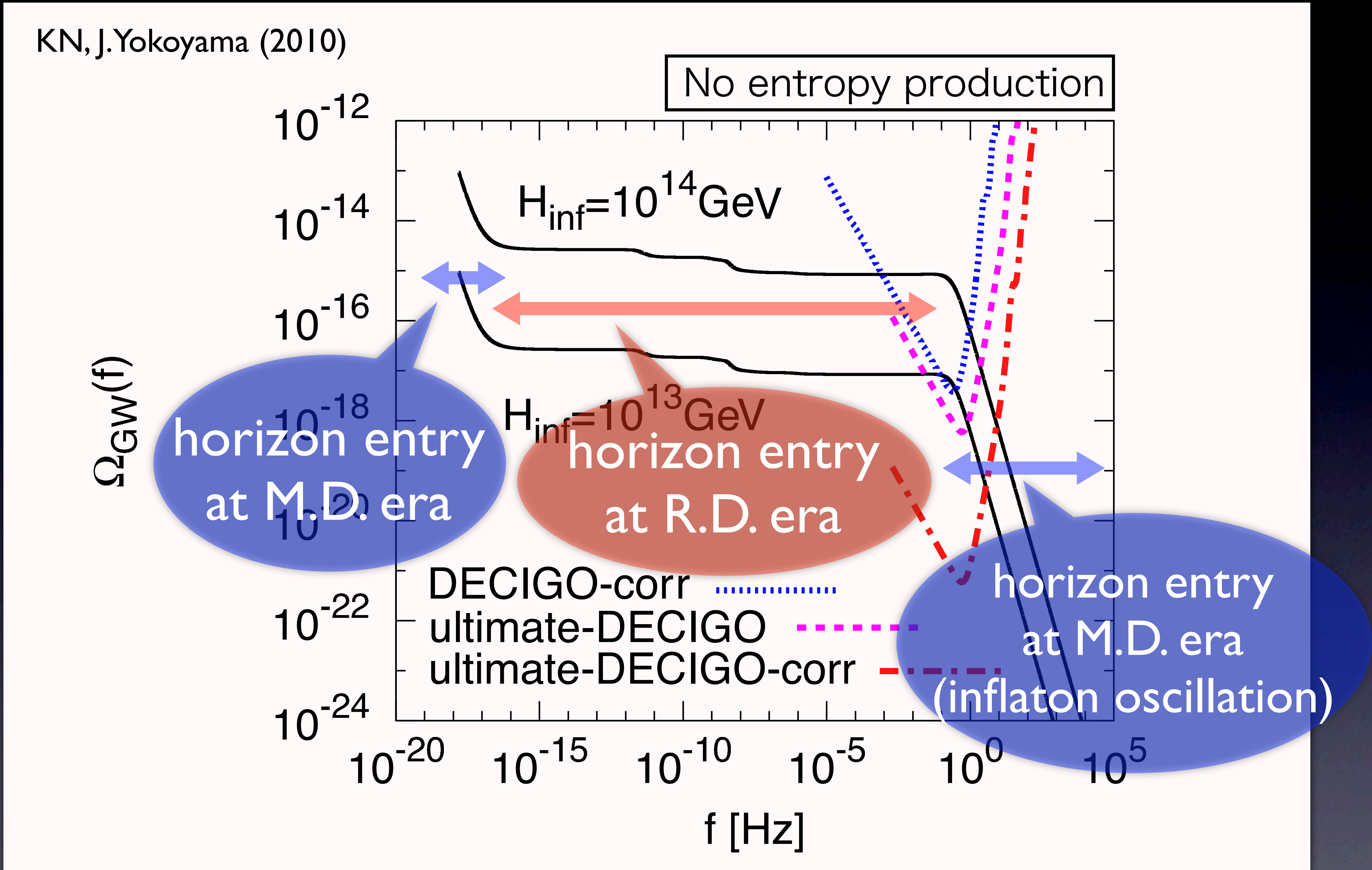
- GW energy density at horizon entry

$$\rho_{\rm GW}(k) \sim M_P^2 \Delta_h^2(k)(k/a)^2 \sim M_P^2 H_{\rm in}(k)^2 \Delta_h^2(k)$$

$$\rho_{\rm tot} \sim M_P^2 H_{\rm in}(k)^2$$

$$\Rightarrow \quad \Omega_{\rm GW}(k) = \frac{\rho_{\rm GW}(k)}{\rho_{\rm tot}} \sim \Delta_h^2(k) \sim \text{const at horizon entry}$$

$$\Rightarrow \quad \boxed{\Omega^0_{\rm GW}(k) \sim \Omega^0_{\rm rad}\Delta_h^2(k) \quad \text{at present for } k \gg k_{\rm eq}}$$



GW spectrum traces thermal history of the Universe !

N.Seto, J.Yokoyama (2003), Boyle, Steinhardt (2005), KN, Saito, Suwa, Yokoyama (2008)

Dark radiation and GW

- Dark radiation affects GW spectrum in two ways

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + (k/a)^2 h_{ij} = 16\pi G \Pi_{ij}$$

Modified expansion rate

Anisotropic stress of X

cf) For standard neutinos, see

S.Weinberg (2003), Y.Watanabe, E.Komatsu (2005)

- Modified expansion rate by parent field of X

→ Modification on GW spectrum at high frequency

- Anisotropic stress is turned on after X production

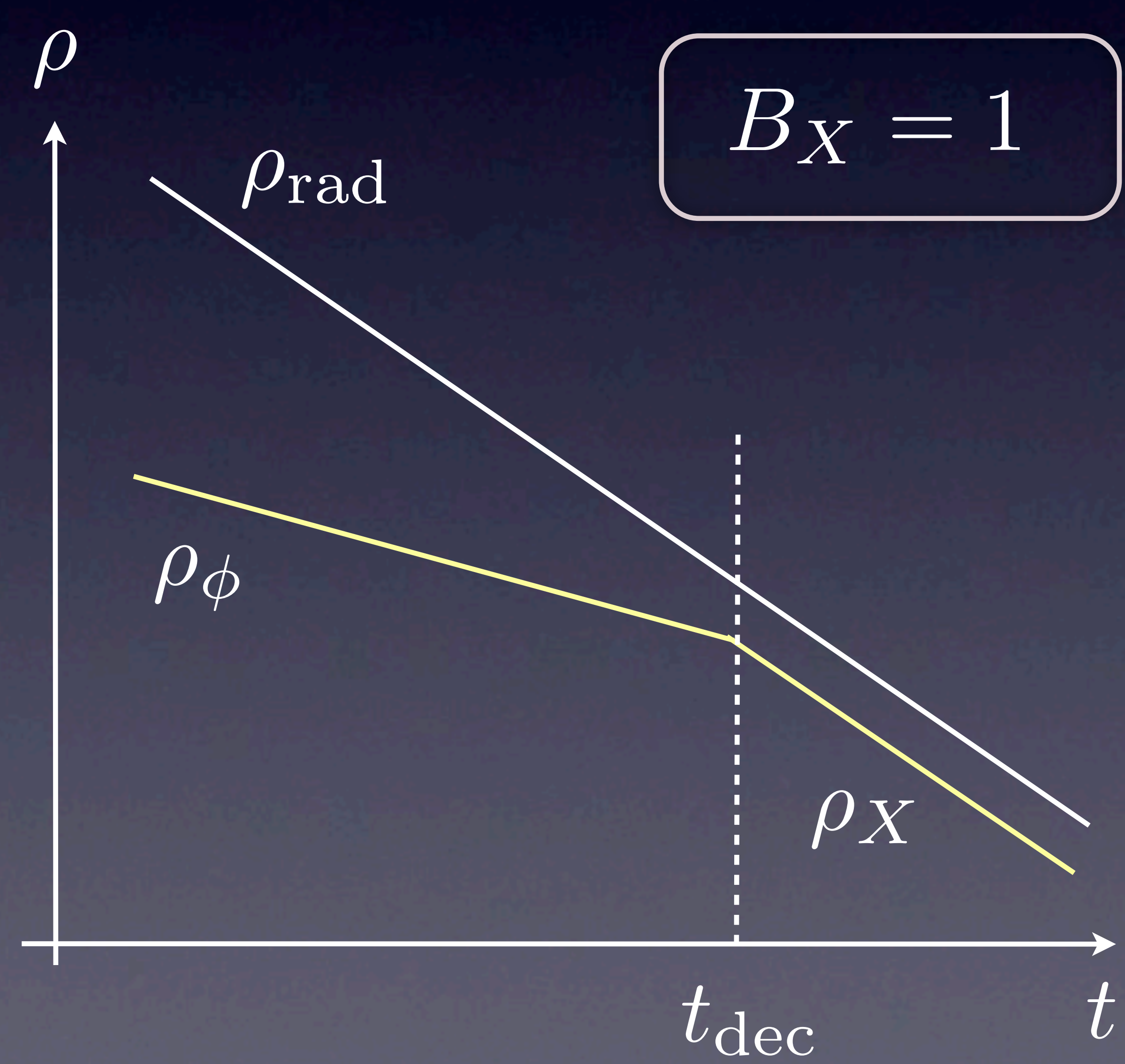
→ Modification on GW spectrum at low frequency

A model

- A scalar field ϕ decays into X at $H \sim \Gamma_\phi$ with branching ratio B_X
- Background evolution :

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi,$$
$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} = \Gamma_\phi (1 - B_X)\rho_\phi,$$
$$\dot{\rho}_X + 4H\rho_X = \Gamma_\phi B_X \rho_\phi,$$

- ϕ nearly dominate at decay for $\Delta N_{\rm eff} \simeq 1$
- Example) ϕ : saxion
  X : axion



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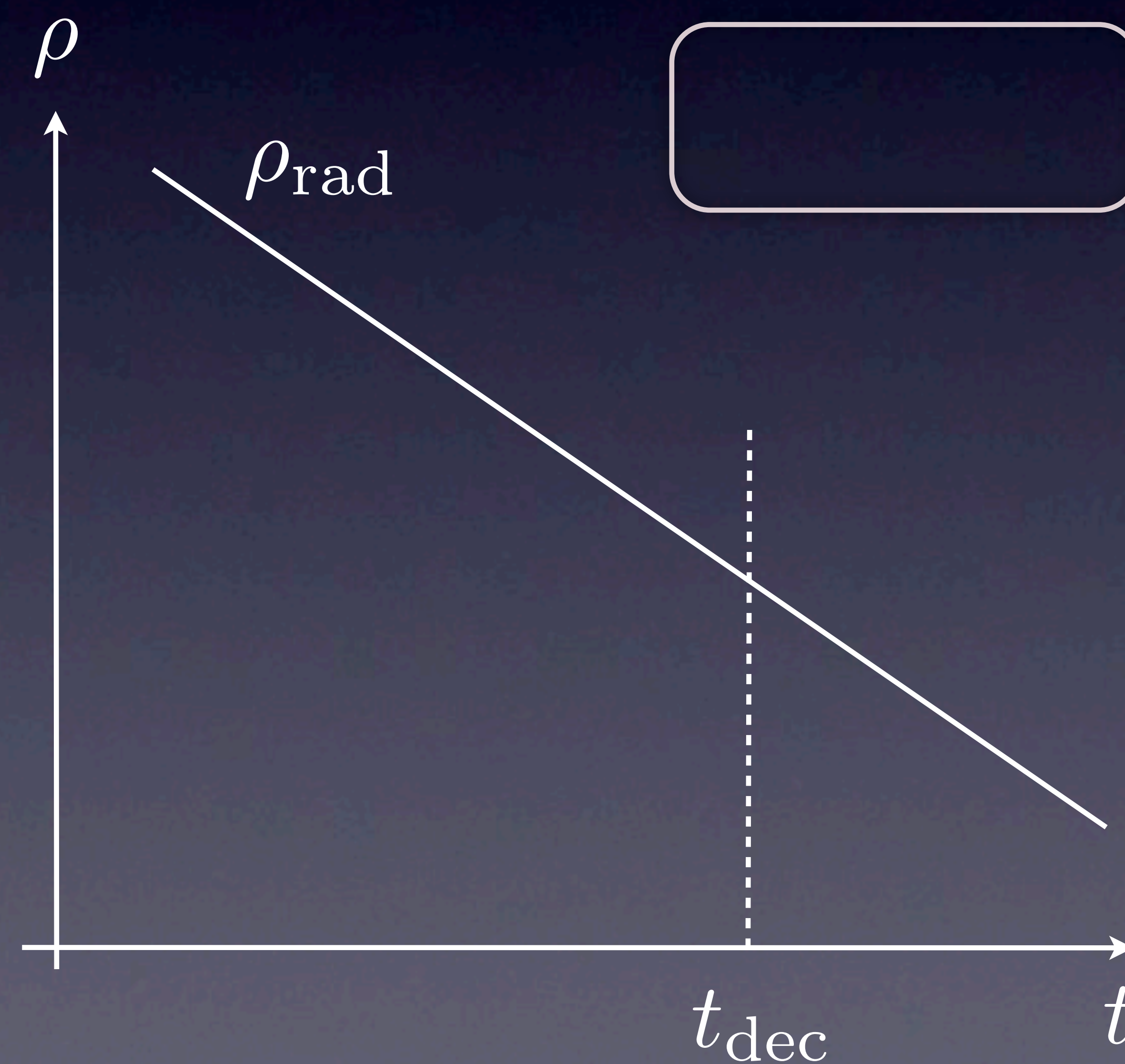

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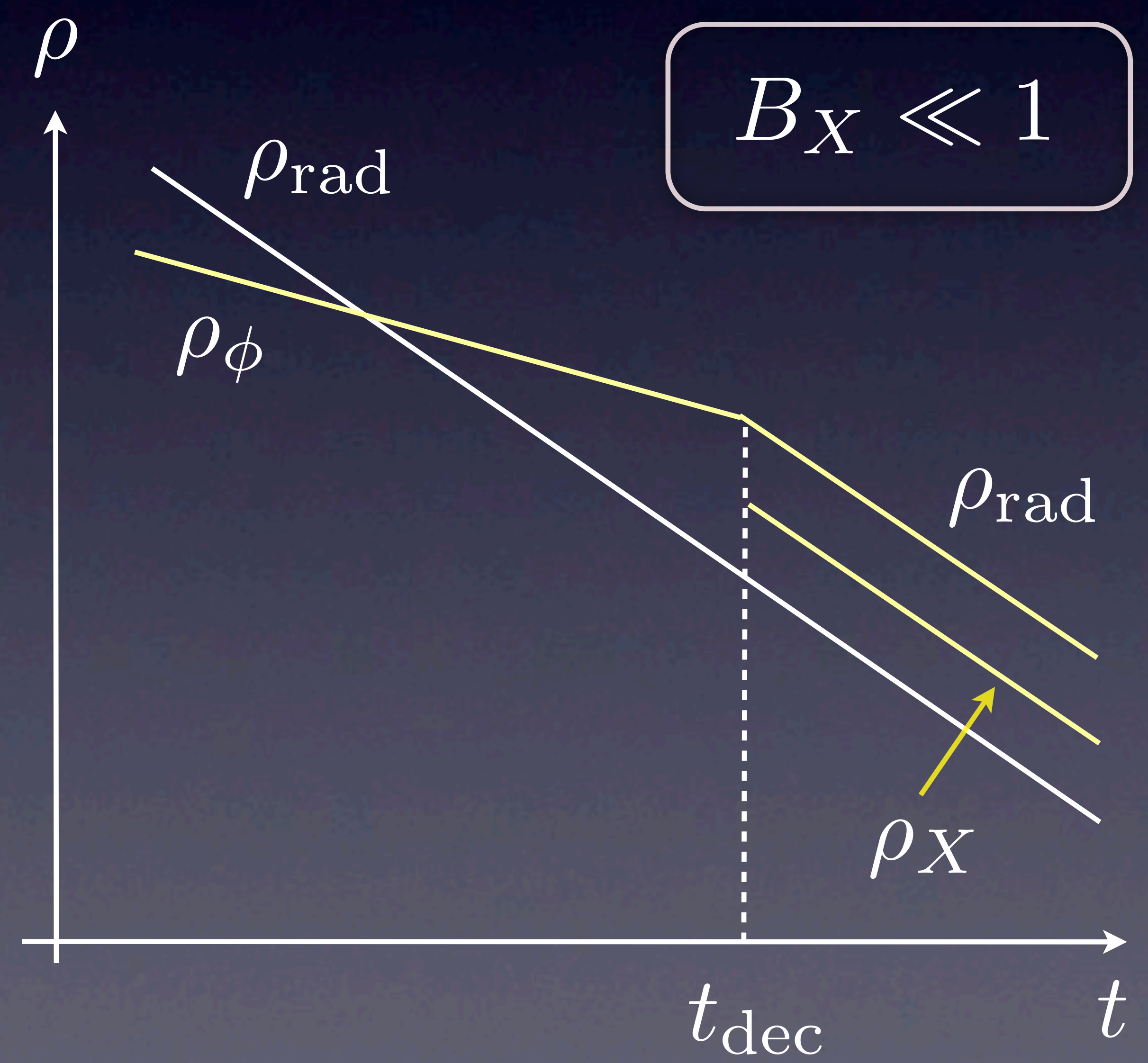

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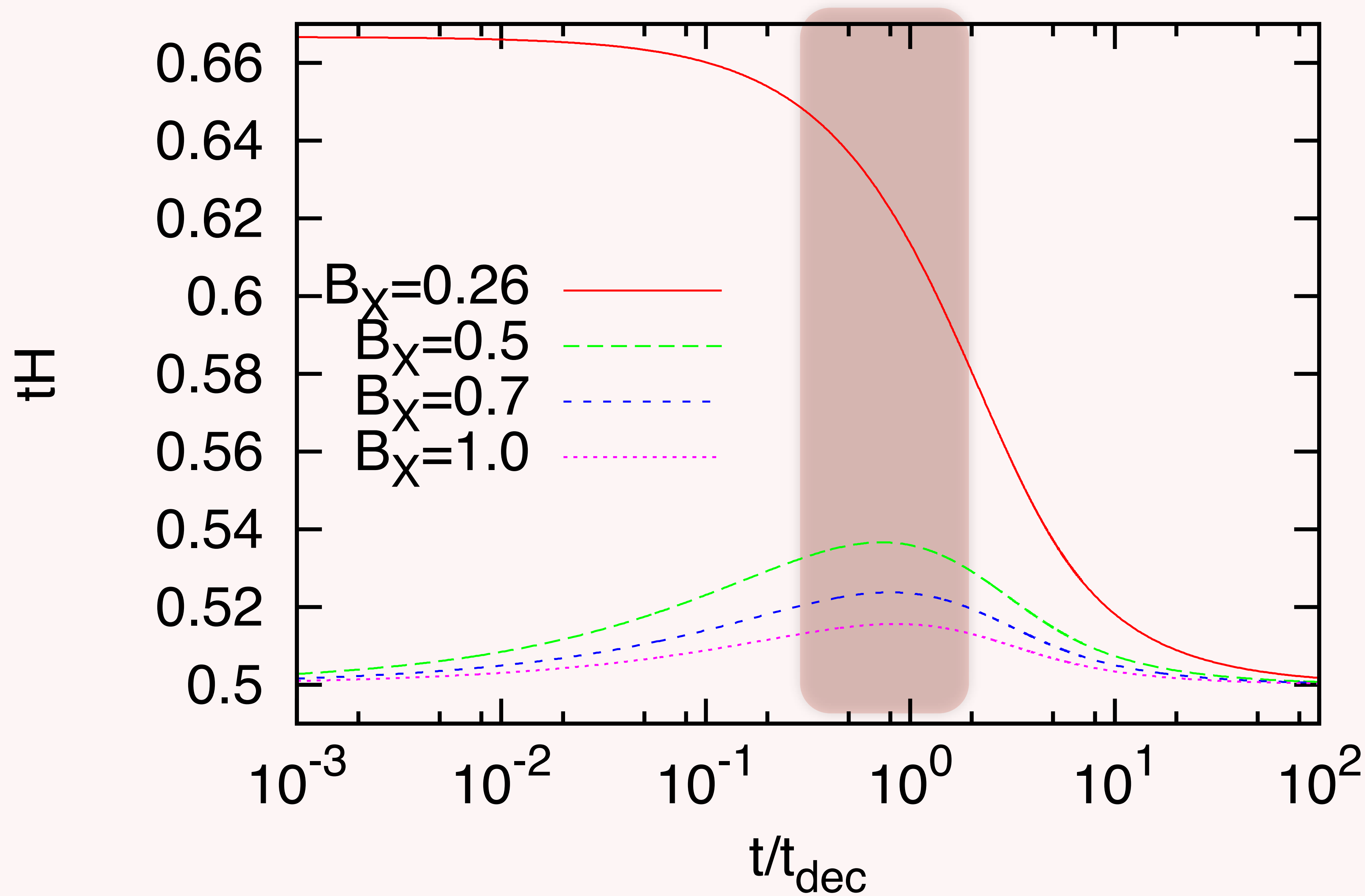
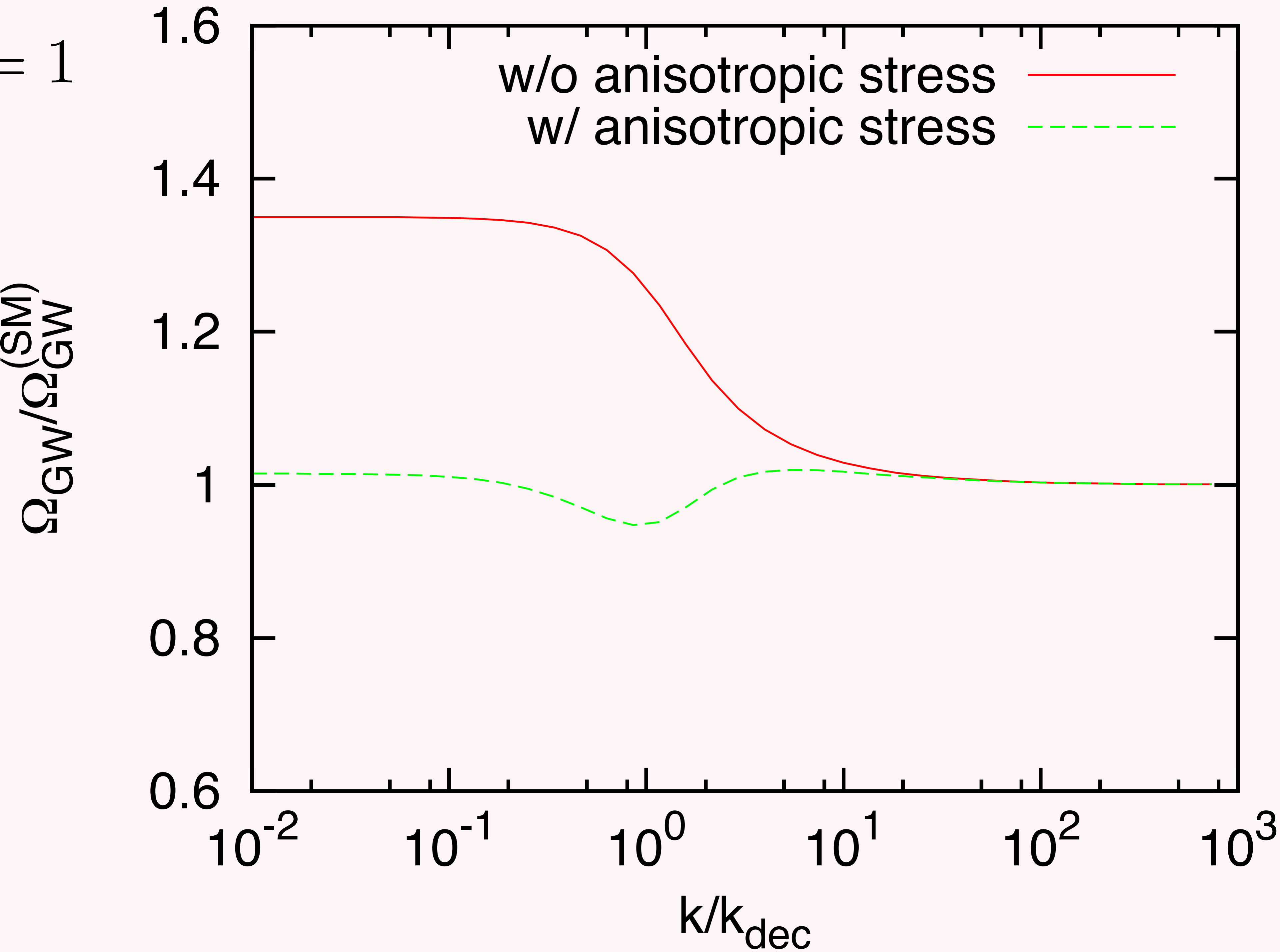


Deviation from R.D., tH=0.5, around Φdecay

Numerical result

$B_X = 1$



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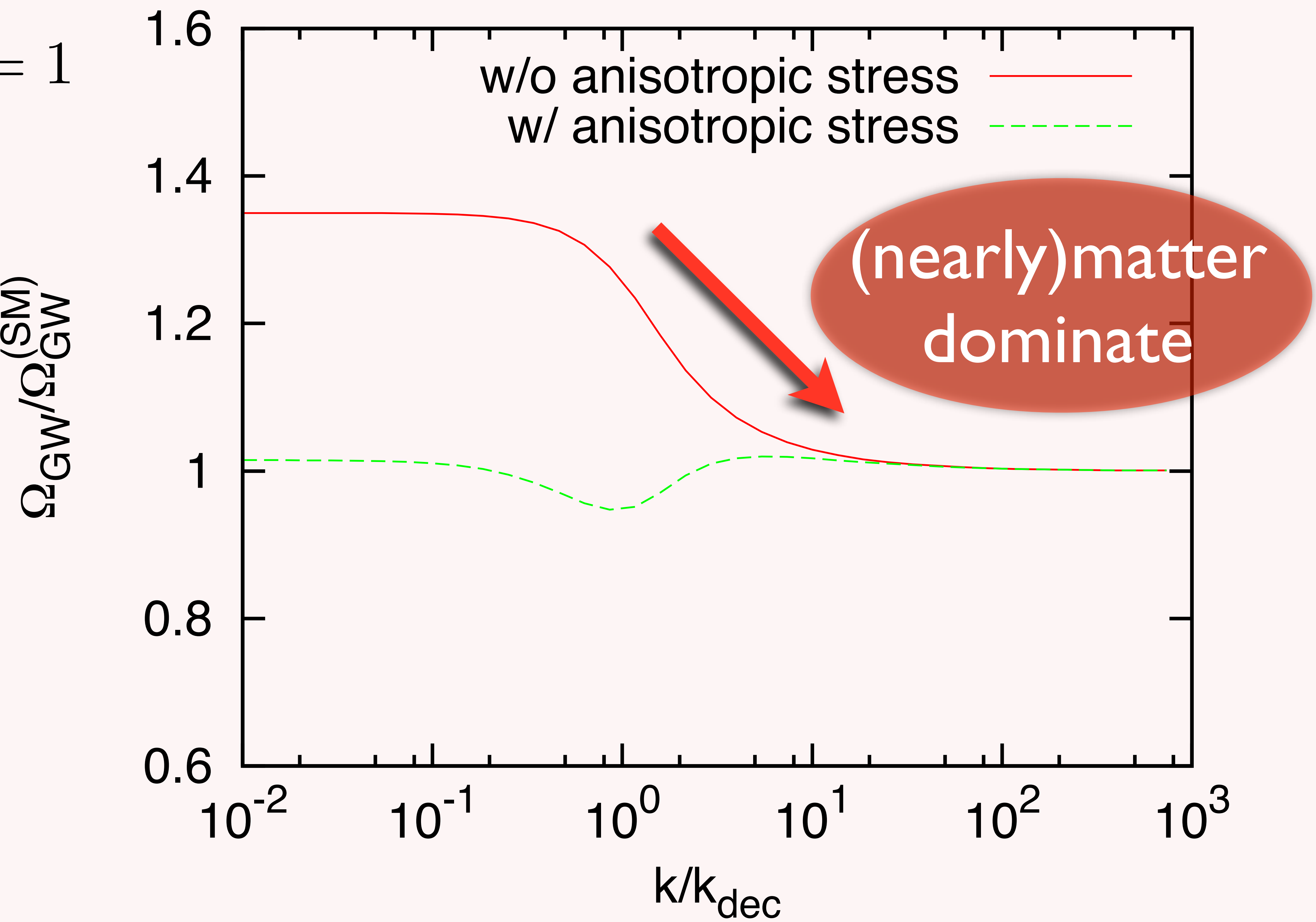

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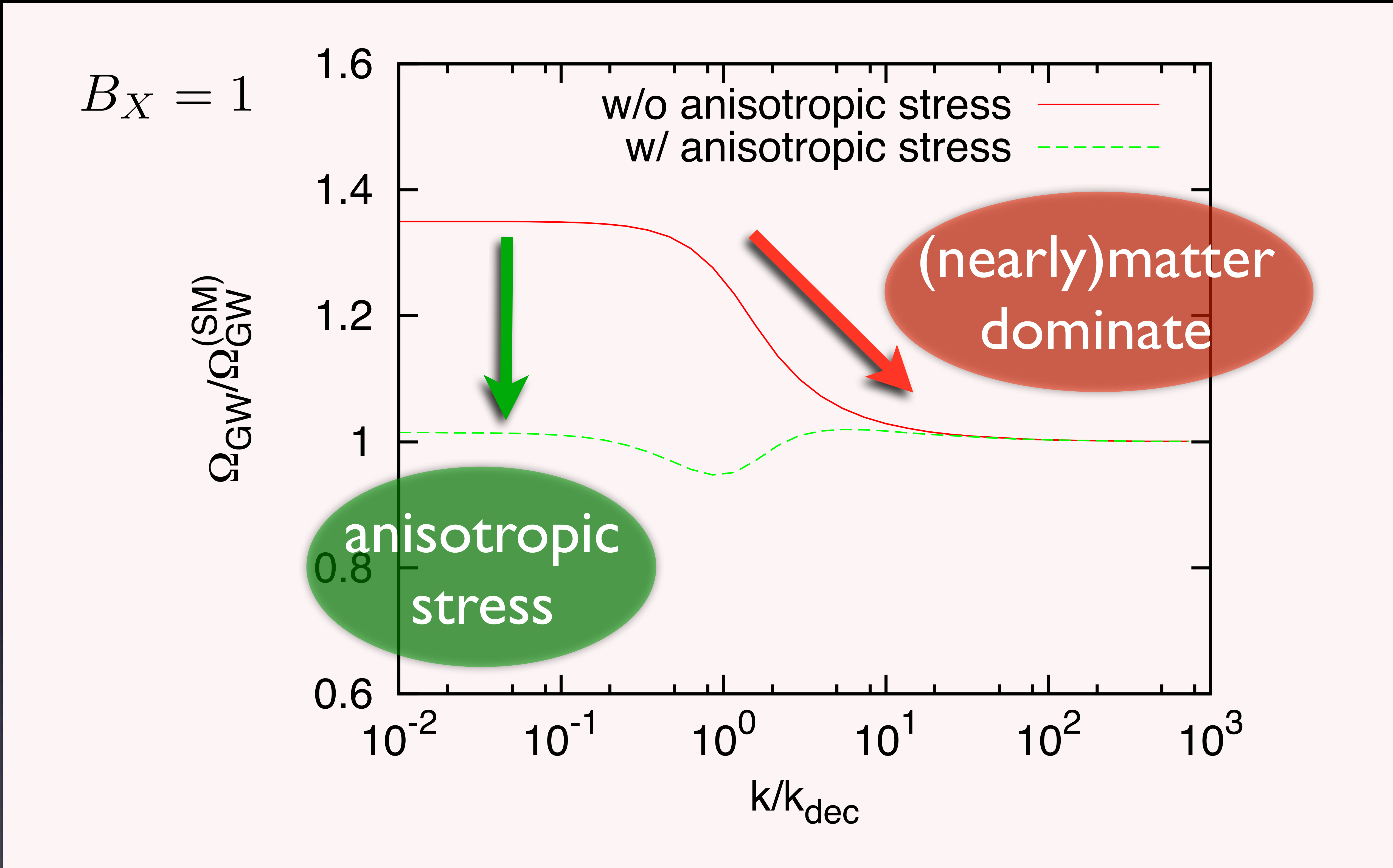
Numerical result



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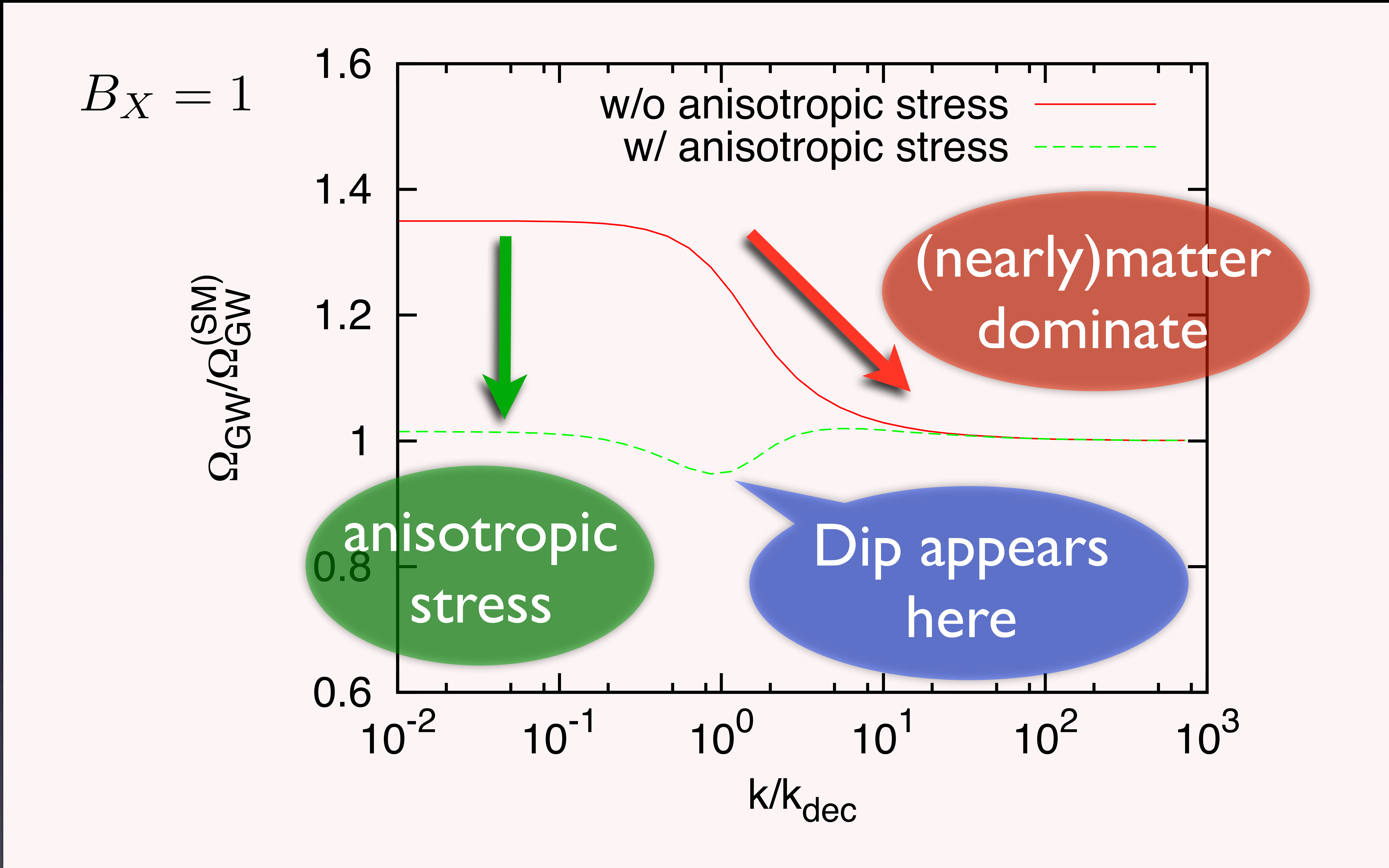

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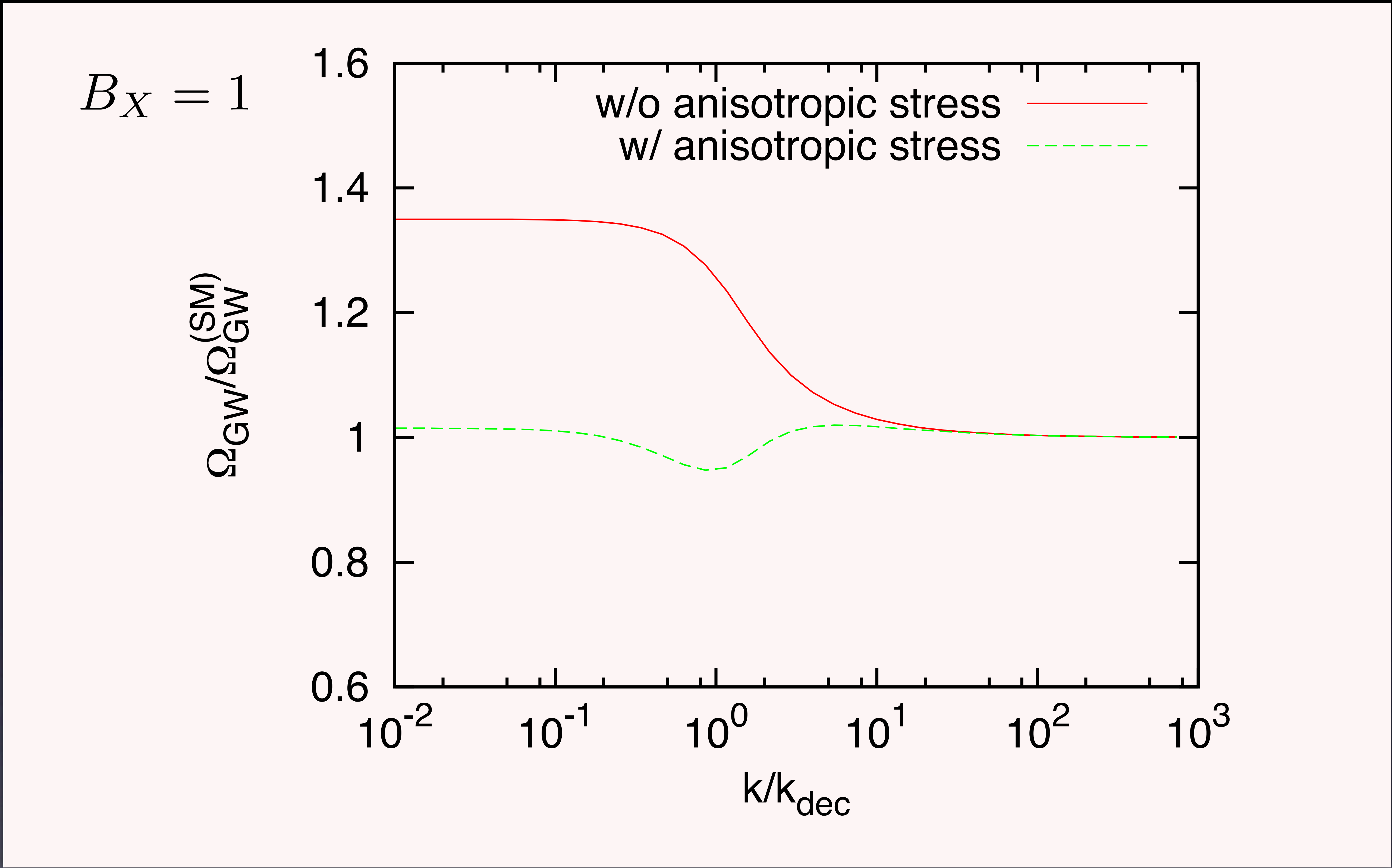

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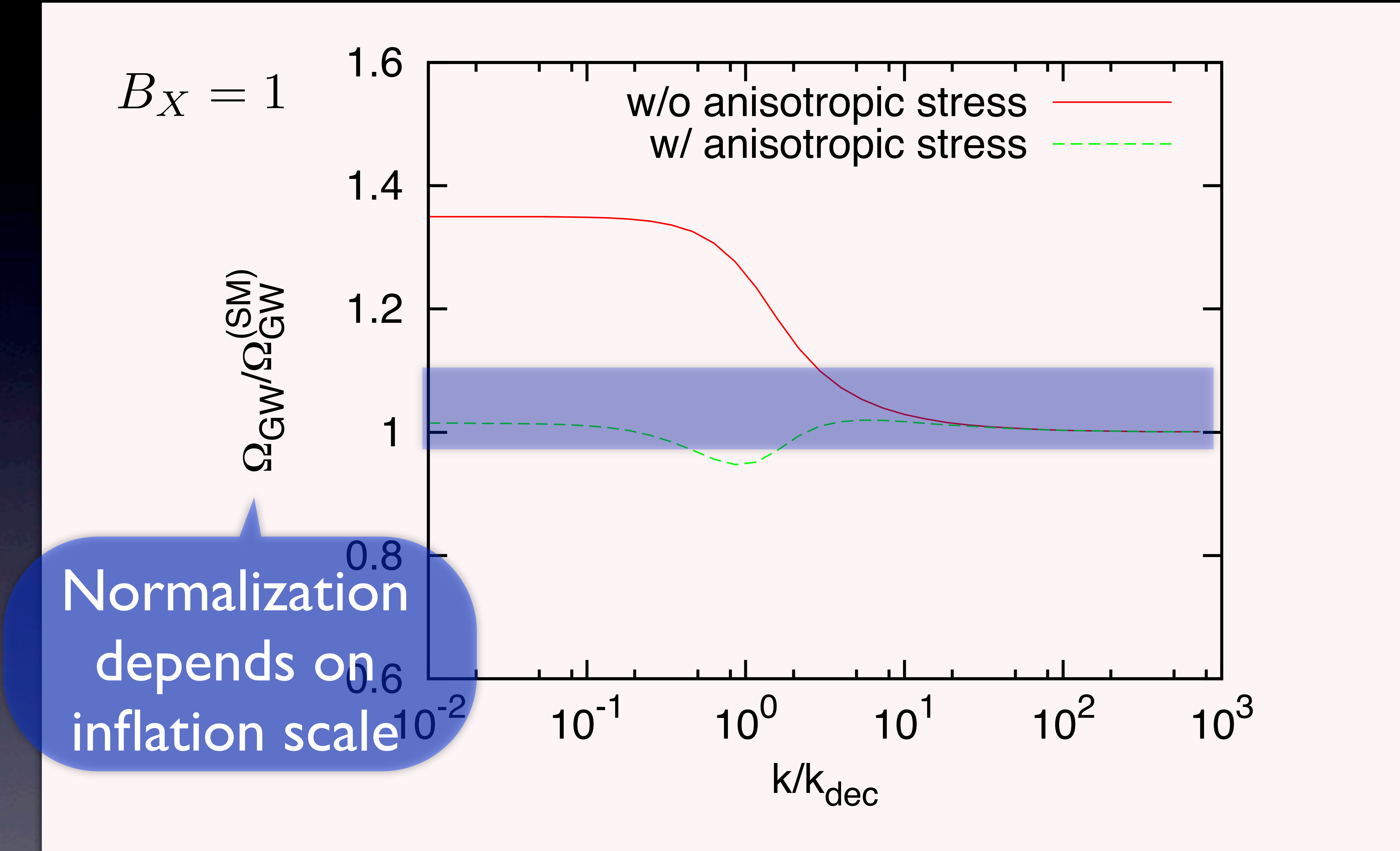

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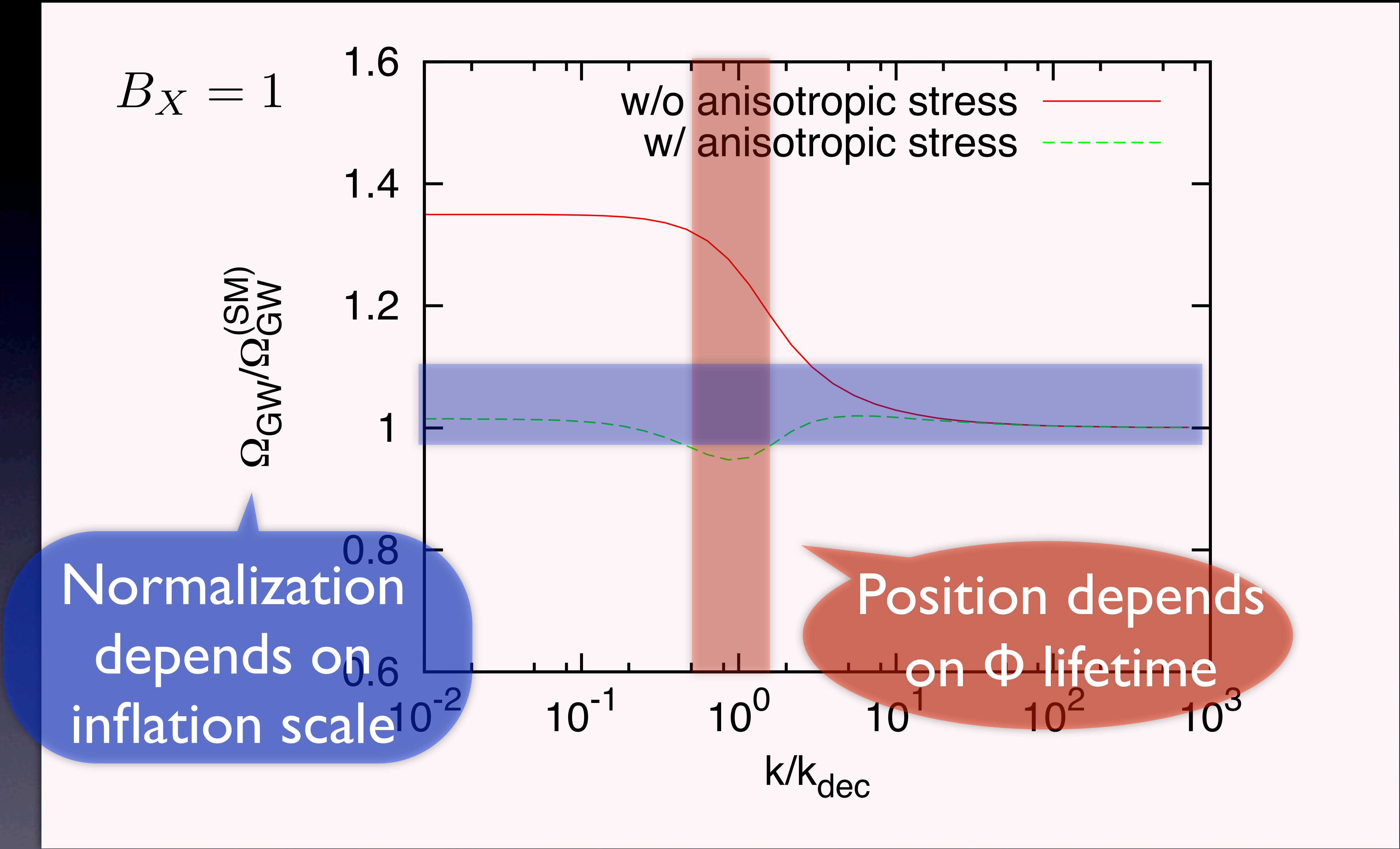

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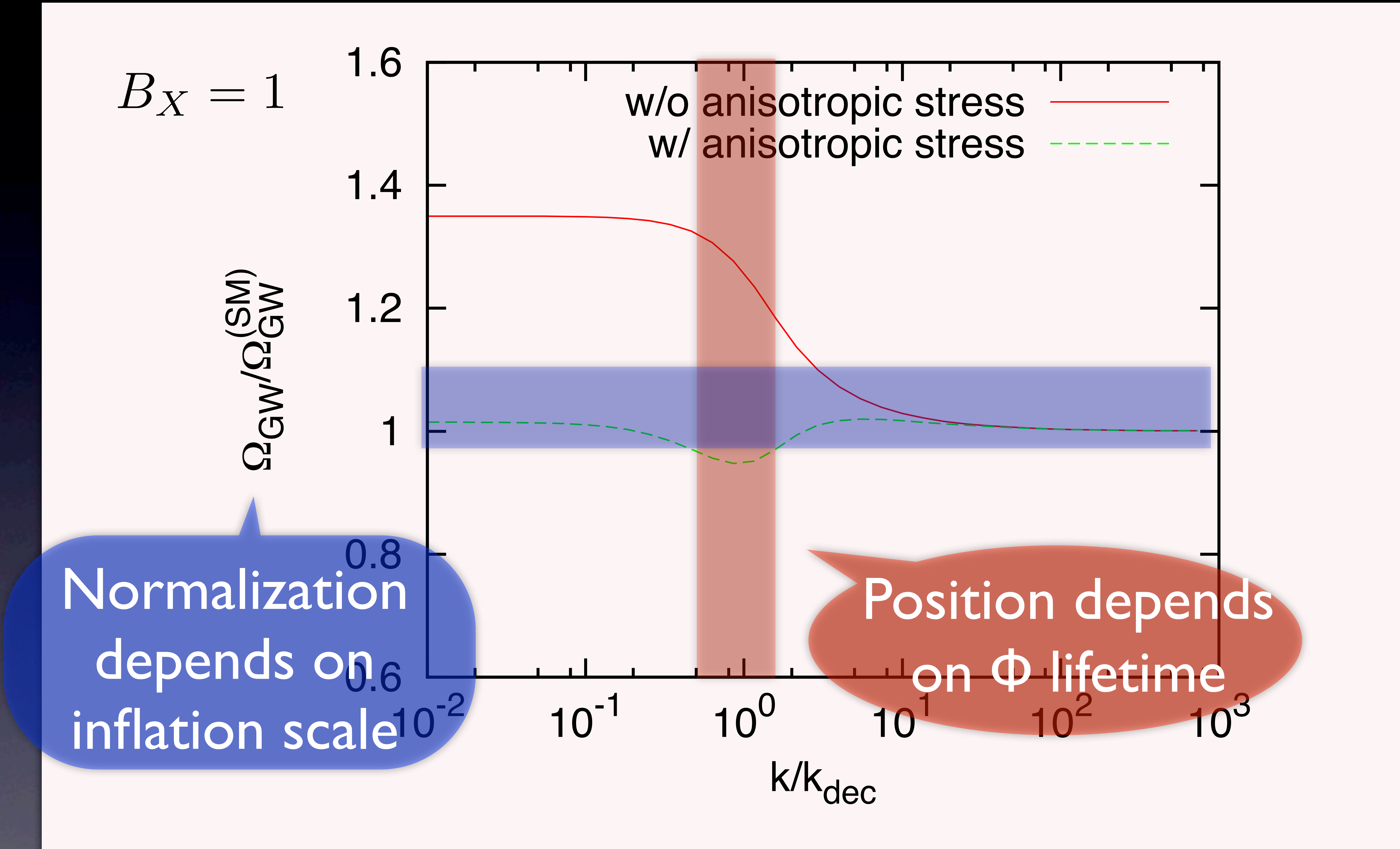
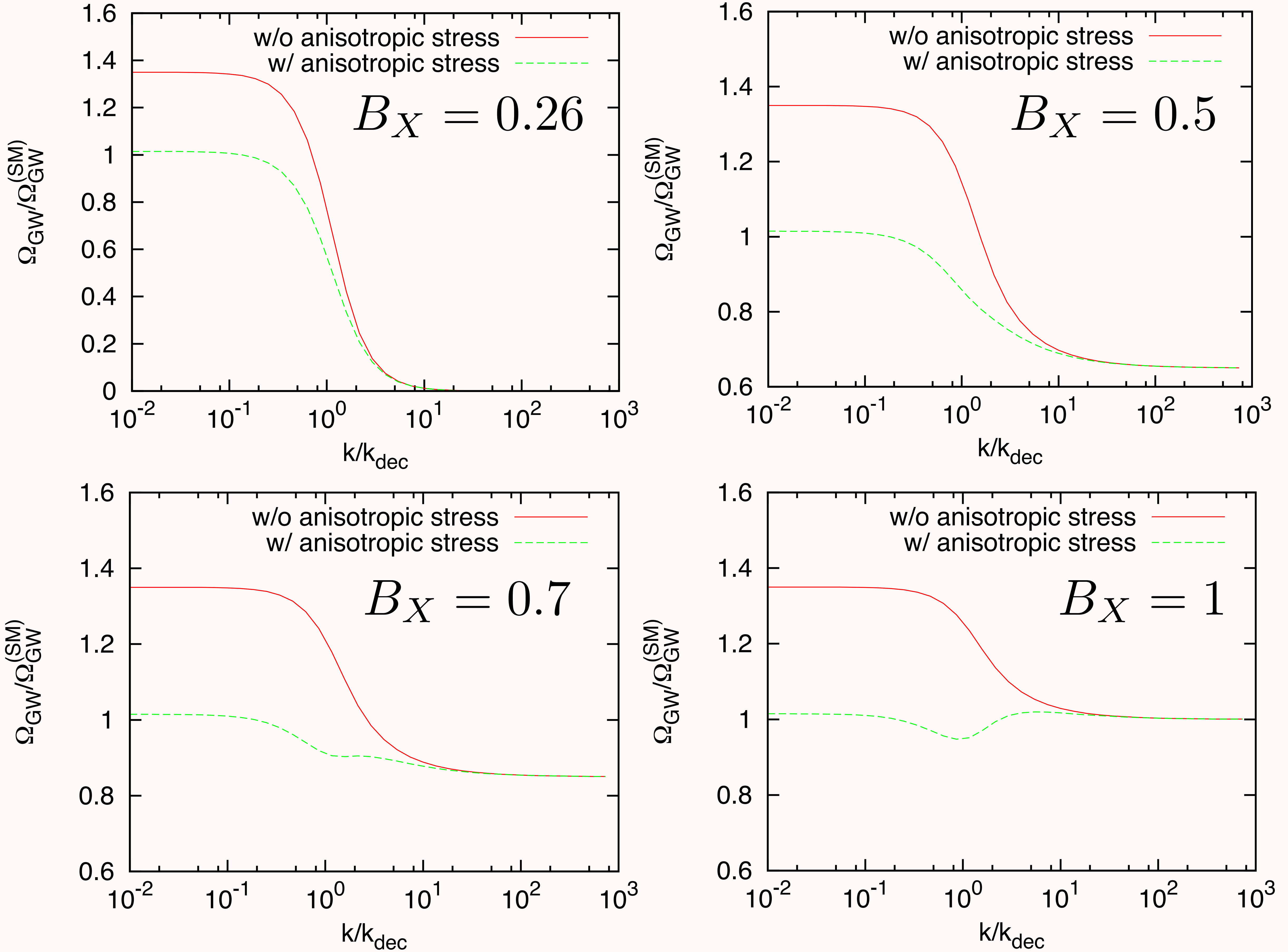


Detectable at DECIGO for $r \gtrsim 10^{-3}$ and $T_\phi \sim 10^7\,\mathrm{GeV}$


w/o anisotropic stress
w/ anisotropic stress
$B_X = 0.26$
$B_X = 0.5$
$B_X = 0.7$
$B_X = 1$
$\Omega_{GW}/\Omega_{GW}^{(SM)}$
k/k_{dec}
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Summary

- Recent observation suggest extra light species : dark radiation
- Dark radiation leaves characteristic signature in primordial GW spectrum
- It also contains information on the production mechanism of dark radiation.

Backup Slides

GW normalization

- Standard model
  - GW spectrum at horizon entry

$$\Omega_{\rm GW}(k=aH)=\frac{\Delta_h^2(k)}{24} \qquad \Delta_h^2(k)\equiv\frac{8}{M_P^2}\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{k}{k_0}\right)^{n_t}$$

  - GW spectrum at present $(k \gg k_{\rm eq})$

$$\Omega_{\rm GW}^{\rm (SM)}(k)=\gamma^{\rm (SM)}\Omega_{\rm rad}^{\rm (SM)}\times\Omega_{\rm GW}(k=aH),$$

Expansion history : $\gamma^{\rm (SM)}=\left[\frac{g_*(T_{\rm in}(k))}{g_{*0}^{\rm (SM)}}\right]\left[\frac{g_{*s0}^{\rm (SM)}}{g_{*s}(T_{\rm in}(k))}\right]^{4/3},$

GW normalization

- Standard model plus dark radiation
  - GW spectrum at present $(k \gg k_{\rm eq})$

$$\Omega_{\rm GW}(k) = \gamma \Omega_{\rm rad} \times \Omega_{\rm GW}(k = aH),$$

Expansion history modified by X :

$$\gamma = \frac{1 + \frac{7}{43}\left(\frac{g_{*s}(T_\phi)}{10.75}\right)^{1/3} \Delta N_{\rm eff}}{1/\gamma^{(\rm SM)} + \frac{7}{43}\left(\frac{g_{*s}(T_\phi)}{10.75}\right)^{1/3} \Delta N_{\rm eff}},$$

Radiation density :

$$\Omega_{\rm rad} = \Omega_{\rm rad}^{(\rm SM)} \times (g_{*0}/g_{*0}^{(\rm SM)}) \qquad g_{*0} = 2\left[1 + N_{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right]$$

Overall normalization is affected

GW normalization

- Parameterize normalization

$$\frac{\Omega_{\rm GW}(k)}{\Omega_{\rm GW}^{\rm (SM)}(k)} = C_1 \times C_2$$

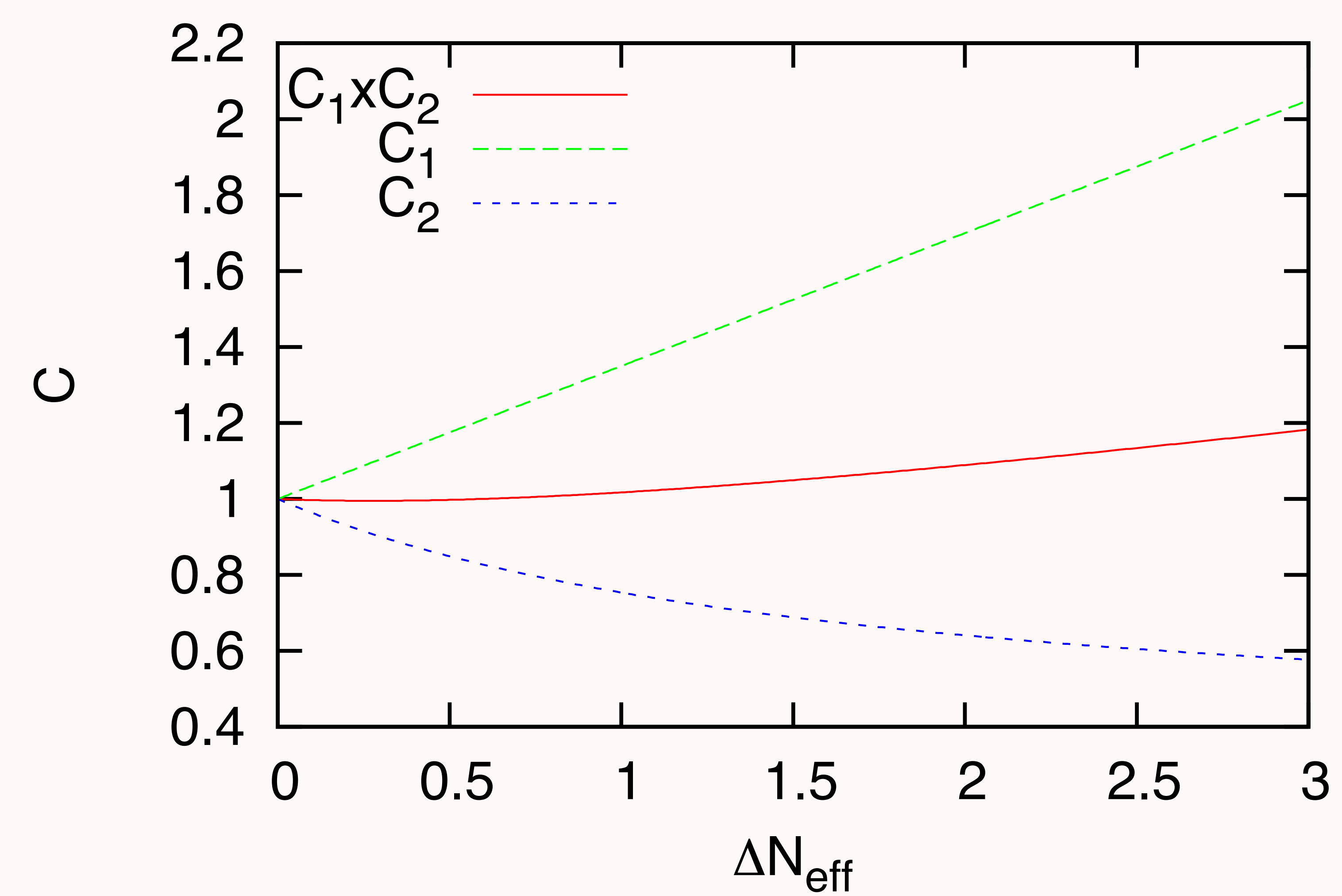
Modified BG by X :

$$C_1 \equiv \frac{\gamma}{\gamma^{\rm (SM)}} \frac{g_{*0}}{g_{*0}^{\rm (SM)}}$$

Anisotropic stress X :

C_2 analytically derived in

Dicus, Repko (2004)



C1xC2 accidentally close to unity

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Anisotropic stress

- Boltzmann eq. for X

$$\frac{dF}{dt} = \frac{B_X}{4\pi (p^0)^3}\Gamma_\phi \rho_\phi \delta\left(p^0 - \frac{m_\phi}{2}\right)$$

F : distribution function of X

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{p^i}{p^0}\frac{\partial F}{\partial x^i} + \frac{1}{2} g_{ij,k}\frac{p^i p^j}{p^0}\frac{\partial F}{\partial p_k}$$

cf) Geodesic eq.

$$\frac{dp_i}{dt} = \frac{1}{2} g_{jk,i}\frac{p^j p^k}{p^0}.$$

GW effect here

- Perturbed :

$$\frac{\partial(\delta F_1 + \delta F_2)}{\partial t} + \frac{\bar{p}^i}{\bar{p}^0}\frac{\partial(\delta F_1 + \delta F_2)}{\partial x^i} + \frac{1}{2}(\delta g_{jk})_{,i}\frac{\bar{p}^j \bar{p}^k}{\bar{p}^0}\frac{\partial \bar{F}}{\partial p_i} = a\frac{\partial^2 \bar{F}}{\partial p \partial t}\delta p^0$$

$$\delta F_1(t, x^i, p_i) \equiv \bar{F}(t, (g^{ij} p_i p_j)^{1/2}/a) - \bar{F}(t, p) \qquad \delta F_2(t, x^i, p_i) \equiv F - \bar{F} - \delta F_1.$$

$$\rightarrow \quad \frac{\partial \delta F_2}{\partial t} + \frac{\hat{p}_i}{a}\frac{\partial \delta F_2}{\partial x^i} = \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\frac{\partial \bar{F}}{\partial p} p \hat{p}_i \hat{p}_j.$$

Contributes to anisotropic stress

Anisotropic stress

- EM tensor of X

$$\delta T_{ij}^{(X)} = \frac{1}{a^3}\int d^3p\left[(\delta F_1+\delta F_2)\frac{p_ip_j}{\bar{p}^0}+\bar{F}p_ip_j\delta\left(\frac{1}{p^0}\right)\right] .= \frac{1}{a^2}\int d^3p\delta F_2 p\hat{p}_i\hat{p}_j + \frac{1}{3}a^2h_{ij}\rho_X$$

Anisotropic stress $a^2\Pi_{ij}$

- From Boltzmann eq :

$$\delta F_2 = \int_0^\tau d\tau' \frac{1}{2}\frac{\partial h_{ij}}{\partial\tau}(\tau')\frac{\partial\bar{F}}{\partial p}(\tau')p\hat{p}_i\hat{p}_j e^{-ik\mu(\tau-\tau')}$$

- Eq.of.m of GW (with dark radiation)

$$\ddot{h}^{(\lambda)}+3H\dot{h}^{(\lambda)}+\frac{k^2}{a^2}h^{(\lambda)} = -24H^2\frac{1}{a^4(t)\rho_{\rm tot}(t)}\times\int_0^t a^4(t')\rho_X(t')K\left(k\int_{t'}^t\frac{dt''}{a(t'')}\right)\dot{h}^{(\lambda)}(t',\mathbf{k})dt',$$

Anisotropic stress of X induced by GWs