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"Curvature perturbation spectrum in two-field inflation with a turning trajectory"

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Curvature Perturbation Spectrum in Two-field Inflation with a Turning Trajectory

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Collaborate with Misao Sasaki,

based on arXiv:1205.0161,

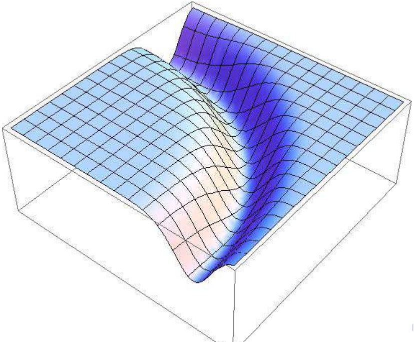
JGRG 2012, RESCUE, University of Tokyo.

Outline

Primary Parameters

Define the parameters

- Slow-roll parameter along the trajectory ϵ and η.
- Angular speed of rotation in field space $\dot{\theta} \sim V_s$.
- Effective mass perpendicular to the trajectory $M_{\text{eff}} = V_{ss} + 3\dot{\theta}$.



Classification

The ordinary 2-field inflation can be classified by these parameters in the slow-roll region as

1. $\dot{\theta} \ll H$, $M_{\rm eff} \ll H$: 2-field inflation with a negligible coupling between adiabatic and curvature perturbations inside the horizon. Gordon 2001.
2. $\dot{\theta} \ll H$, $M_{\rm eff} \sim H$: Quasi-single field inflation in the original form. Chen 2010.
3. $\dot{\theta} \ll H$, $M_{\rm eff} \gg H$: After integrating the heavy field out, one can get an effective single field with a corrective speed of sound. Achucarro 2011,2012. Cespedes 2012.

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We are suppose to connect 2 and 3.

"Massless" Slowball

Quasi-single Panda

Coaster with "Large Isocurvaton Mass".

EFT result

In EFT, after integrating out the heavy field (σ in our case), one have an effective single field inflation with an effective speed of sound c_s which is

$$c_s^{-2} = 1 + \frac{4H^2}{\tilde{M}_{\rm eff}^2}\left(\frac{\dot{\theta}}{H}\right)^2, \tag{1}$$

Finally we got via EFT that

$$\delta\mathcal{P}_{\mathcal{R}} \propto c_s^{-1} - 1 \sim 2\left(\frac{\dot{\theta}}{\tilde{M}_{\rm eff}}\right)^2.$$

Our main task is to verify this relation by in-in formulism.

Outline

Lagrangian

The action for the fields can be decomposed into

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\tilde{R}+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right],$$

where

- $R\theta$(tangent field) and σ(radial field),
- $V_{\rm sr}(\theta)$ is a slow-roll potential along the valley,
- $V(\sigma)$ is a potential that forms the valley and traps the isocurvaton at $\sigma = \sigma_0$,
- $\tilde{R}$ denotes the radius of the minima valley,
- $R = \tilde{R} + \sigma_0$ is the constant radius where the trajectory is trapped with the centripetal force under consideration.

EOM

The Hubble equations and equations of motion are

$$
\begin{aligned}
3M_p^2 H^2 &= \frac{1}{2} R^2 \dot{\theta}_0^2 + V + V_{\rm sr}, \\
-2M_p^2 \dot{H} &= R^2 \dot{\theta}_0^2, \\
0 &= R^2 \ddot{\theta}_0 + 3R^2 H \dot{\theta}_0 + V'_{\rm sr}, \\
0 &= \ddot{\sigma}_0 + 3H\dot{\sigma}_0 + V' - R^2 \dot{\theta}_0^2 \,,
\end{aligned}
$$

We can see in the tangent direction of the trajectory, field $R\theta$ obeys the ordinary equation of motion for single-field inflation.

Perturbative Hamiltonian

Hamiltionian density in interaction picture (spatially flat gauge)

$$
\begin{aligned}
\mathcal{H}_0 &= a^3\left[\frac{1}{2}R^2\dot{\delta\theta}^2+\frac{R^2}{2a^2}(\partial_i\delta\theta)^2+\frac{1}{2}\dot{\delta\sigma}^2+\frac{1}{2a^2}(\partial_i\delta\sigma)^2+\frac{1}{2}M_{\rm eff}^2\delta\sigma^2\right],\\
\mathcal{H}_2^I &= -c_2a^3\delta\sigma\dot{\delta\theta}, \qquad c_2=2R\dot{\theta},\\
\mathcal{H}_3^I &= -a^3R\delta\sigma\dot{\delta\theta}^2-a^3\dot{\theta}\dot{\delta\theta}\delta\sigma^2+aR\delta\sigma\left(\partial_i\delta\theta\right)^2+\frac{a^3}{6}V'''\delta\sigma^3,\\
M_{\rm eff}^2 &= V''+3\dot{\theta}^2,
\end{aligned}
$$

Our method is valid when

$$
\left(\frac{\dot{\theta}}{H}\right)^2\ll 1, \quad \frac{|V'''|}{H}\ll 1. \tag{2}
$$

Perturbative Hamiltonian

Hamiltonian density in interaction picture (spatially flat gauge)

$$
\begin{aligned}
\mathcal{H}_0 &= a^3\left[\frac{1}{2}R^2\dot{\delta\theta}^2 + \frac{R^2}{2a^2}(\partial_i\delta\theta)^2 + \frac{1}{2}\dot{\delta\sigma}^2 + \frac{1}{2a^2}(\partial_i\delta\sigma)^2 + \frac{1}{2}M_{\text{eff}}^2\delta\sigma^2\right], \\
\mathcal{H}_2^I &= -c_2 a^3\delta\sigma\dot{\delta\theta}, \qquad c_2 = 2R\dot{\theta} = \text{constant}, \\
\mathcal{H}_3^I &= -a^3 R\delta\sigma\dot{\delta\theta}^2 - a^3\dot{\theta}\dot{\delta\theta}\delta\sigma^2 + aR\delta\sigma\left(\partial_i\delta\theta\right)^2 + \frac{a^3}{6}V'''\delta\sigma^3, \\
M_{\text{eff}}^2 &= V'' + 3\dot{\theta}^2 = \text{constant},
\end{aligned}
$$

In a constant turn case!

Illustrative Explanation

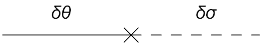


Figure: The second order interacting vertex $\mathcal{H}_2 = -c_2 a^3 \delta\sigma\dot{\theta}$, while $c_2 = 2R\dot{\theta}$.

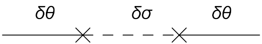


Figure: The 2-pt func with a heavy isocurvaton mediation.

And the curvature perturbation $\mathcal{R}$ is connected to θ via

$$\mathcal{R} = -\frac{H}{\dot{\theta}}\delta\theta.$$

Quantization

- Quantize the Fourier components

$$\delta\theta^I_{\mathbf{k}} = u_{\mathbf{k}} a_{\mathbf{k}} + u^*_{-\mathbf{k}} a^\dagger_{-\mathbf{k}},$$
$$\delta\sigma^I_{\mathbf{k}} = v_{\mathbf{k}} b_{\mathbf{k}} + v^*_{-\mathbf{k}} b^\dagger_{-\mathbf{k}}.$$

- The commutators

$$[a_{\mathbf{k}}, a^\dagger_{-\mathbf{k}'}] = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}'), \quad [b_{\mathbf{k}}, b^\dagger_{-\mathbf{k}'}] = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}').$$

Quantization

The equation for mode functions,

$$\begin{aligned} u''_{\mathbf{k}} - \frac{2}{\tau}u'_{\mathbf{k}} + k^2 u_{\mathbf{k}} &= 0, \\ v''_{\mathbf{k}} - \frac{2}{\tau}v'_{\mathbf{k}} + k^2 v_{\mathbf{k}} + \frac{M_{\text{eff}}^2}{H^2\tau^2}v_{\mathbf{k}} &= 0. \end{aligned}$$

Solve The EOMs by setting the initial conditions

$$Ru_{\mathbf{k}}\,, \quad v_{\mathbf{k}} \to i\frac{H}{\sqrt{2k}}\tau e^{-ik\tau},$$

when $k \gg Ha$.

Solution

The solution is

$$u_{\mathbf{k}} = \frac{H}{R\sqrt{2k^3}}(1 + ik\tau)e^{-ik\tau},$$

and

$$v_{\mathbf{k}} = -ie^{i(\nu+\frac{1}{2})\frac{\pi}{2}}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{\nu}^{(1)}(-k\tau), \quad \text{for } M_{\text{eff}}^2/H^2 \leq 9/4,$$

where $\nu = \sqrt{9/4 - M_{\text{eff}}^2/H^2}$, or

$$v_{\mathbf{k}} = -ie^{-\frac{\pi}{2}\mu+i\frac{\pi}{4}}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{i\mu}^{(1)}(-k\tau), \quad \text{for } M_{\text{eff}}^2/H^2 > 9/4,$$

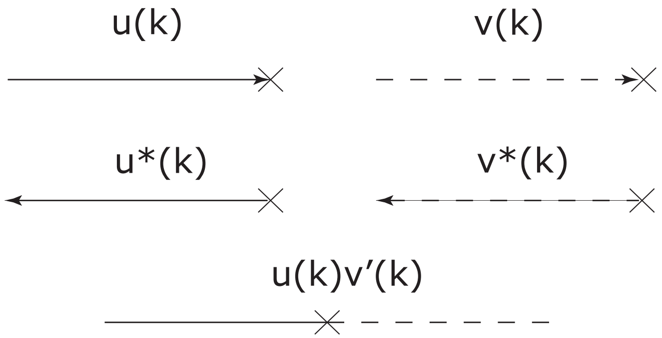
where $\mu = \sqrt{M_{\text{eff}}^2/H^2 - 9/4}$.

2-point function

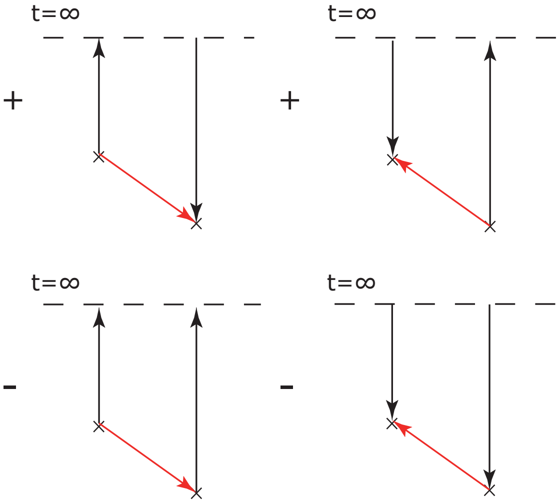
We use in-in formulism to calculate the 2-point function of $\delta\theta^2$

$$\begin{aligned}
\langle\delta\theta^2\rangle &\equiv \langle 0| \left[\bar{T}\exp\left(i\int_{t_0}^{t} dt' H_I(t')\right)\right] \delta\theta_I^2(t) \left[T\exp\left(-i\int_{t_0}^{t} dt' H_I(t')\right)\right] |0\rangle \\
&\sim {\mathcal{P}_{\mathcal{R}}}^{(0)} + \delta\mathcal{P}_{\mathcal{R}} \\
&= \frac{H^4}{4\pi^2 R^2 \dot{\theta}^2}\left[1 + \frac{\delta\mathcal{P}_{\mathcal{R}}}{{\mathcal{P}_{\mathcal{R}}}^{(0)}}\right].
\end{aligned}$$

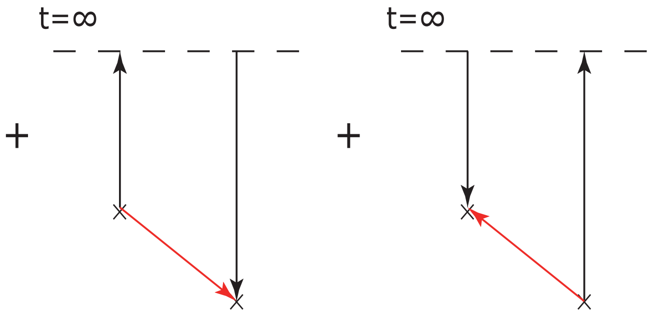
Feynman Rules



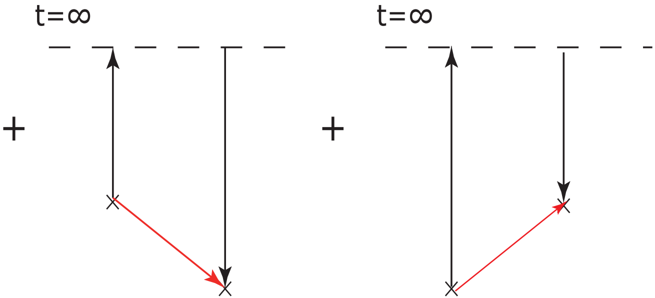
Correction to Power Spectrum



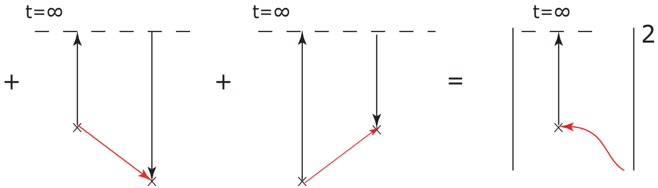
Calculating α



Interchange the momenta



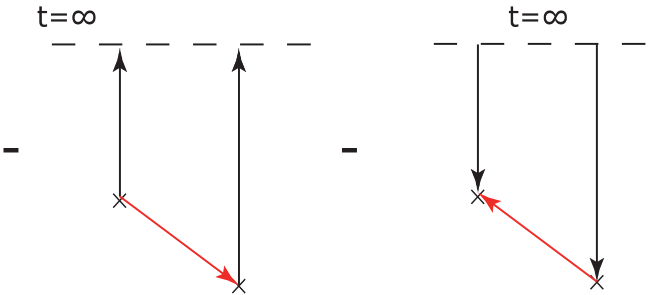
"Split" the integral



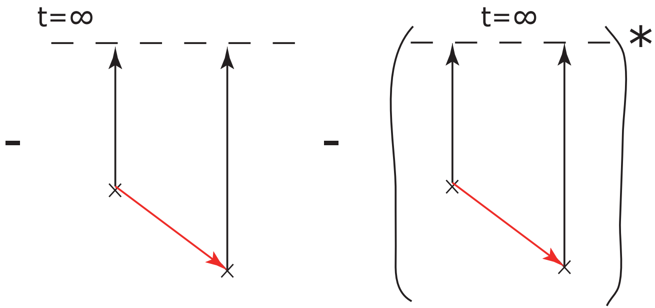
The Cut-in-the-Middle integral α is

$$\alpha = \left| \int_0^\infty dx\, x^{-1/2} H_{i\mu}^{(1)}(x) e^{ix} \right|^2 .$$

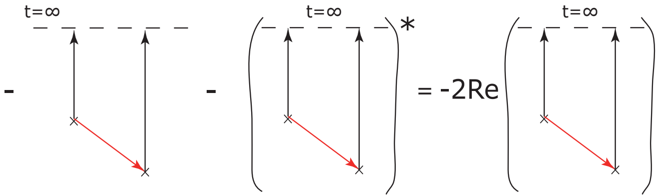
Calculating β



Take the Conjugate



Sum the Integral

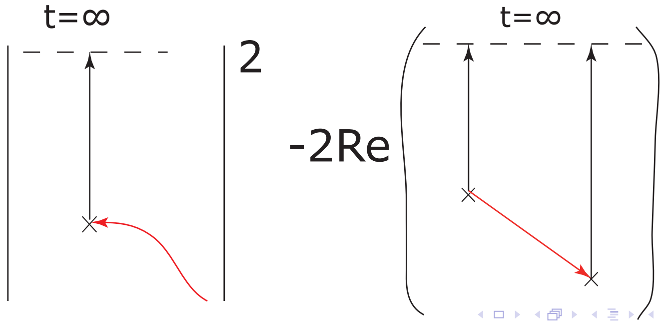


The Cut-in-the-Side integral β is

$$\beta = 2\mathrm{Re}\int_0^\infty dx_1\, x_1^{-1/2} H_{i\mu}^{(1)}(x_1) e^{-ix_1} \int_{x_1}^\infty dx_2\, x_2^{-1/2} (H_{i\mu}^{(1)}(x_2))^* e^{-ix_2} .$$

The Correction to Power Spectrum

$$\frac{\delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}^{(0)}} = \pi \left(\frac{\dot{\theta}}{H} \right)^2 e^{-\mu\pi} (\alpha - \beta),$$
$$\alpha - \beta =$$



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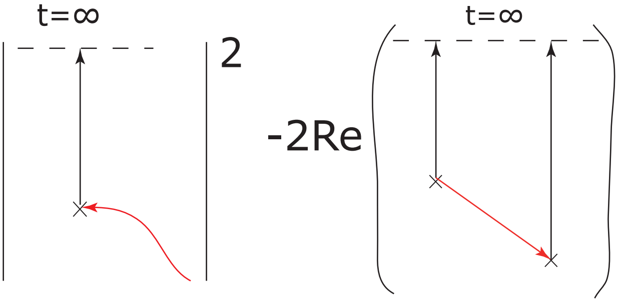

Calculating α

- α can be directly integrated,

$$\begin{aligned}\alpha &= \frac{1}{\pi}\left|\frac{e^{\mu\pi/2}}{2}-\frac{\sqrt{2}}{\sinh\mu\pi}+i\left(\frac{e^{-\mu\pi}}{2}+\sqrt{2}\coth\mu\pi\right)\right|^2 \\ &\to 1,\end{aligned}$$

  when $\mu \to \infty$.

- CIM is exponentially suppressed!

Calculating β

Use the asymptotic formula of Hankel function when $x \ll \mu$:

$$H_{i\mu}^{(1)} \to \frac{1}{e^{i\mu(\ln\mu - 1)}} \sqrt{2\frac{e^{\pi\mu}}{\mu}} \exp\left[-\frac{x^2}{4\mu} e^{-i\frac{\pi}{4}}\right] \left(\frac{x}{2}\right)^{i\mu} .$$

The main contribution to β comes from infrared $x \ll 1$. The result is

$$\beta = -2\frac{e^{\mu\pi}}{\pi\mu^2}\left[1 + \mathcal{O}\left(\frac{1}{\mu^2}\right)\right] .$$

The Power Spectrum

- We have the final result (SP & Sasaki 2012, Chen & Wang 2012, Noumi et. al. 2012)

$$\begin{aligned} \mathcal{C}(\mu) &\approx \frac{1}{4\mu^2}, \\ \mathcal{P}_\mathcal{R} &\approx \mathcal{P}_\mathcal{R}^{(0)} \left[1 + 2\frac{H^2}{M_{\rm eff}^2} \left(\frac{\dot\theta}{H} \right)^2 \right]. \end{aligned}$$

- This result coincide with that from Effective Single Field Approach. (Tolley 2010, Achucarro 2011 & 2012, Sebastian 2012)

Outline

Bad News

- There are $\mathcal{O}(10)$ terms of 3-p vertices.
- There are 10 integrals for each vertex (with 6 momenta permutations).
- There is an integral of quadruple product of Hankel functions.

Good News

- There are $\mathcal{O}(10)$ terms of 3-p vertices. But the only vertex that is possible to generate large Non-Gaussianity is V'''.
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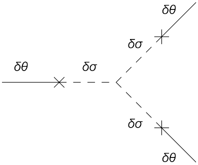
- There are 10 integrals for each vertex (with 6 momenta permutations). But the integrals have similar structures.
- There is an integral of quadruple product of Hankel functions. But we are free to use the asymptotic forms.

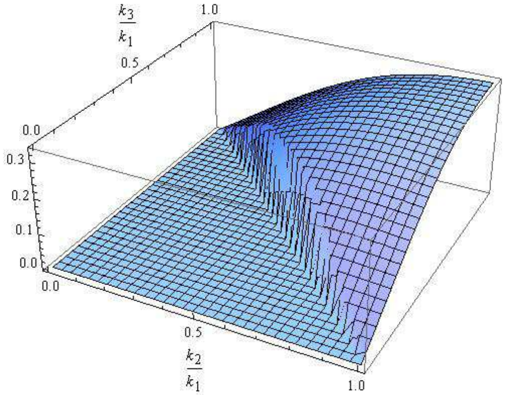


This is the only vertex that can generate large non-Gaussianity. And we calculate one integral

$$\begin{aligned}\langle\delta\theta^3\rangle \quad &\supseteq \quad -12u_{p1}u_{p2}u_{p3}(0)c_2^3c_3 \\ &\times \quad \mathrm{Re}\left[\int_{-\infty}^{0} d\tau\ a^4 v_{p1}v_{p2}v_{p3}(\tau)\int_{-\infty}^{\tau} d\tau_1\ a^3 v_{p1}^* u_{p1}'^*(\tau_1)\right. \\ &\qquad \left.\int_{-\infty}^{\tau_1} d\tau_2\ a^3 v_{p2}^* u_{p2}'^*(\tau_2)\int_{-\infty}^{\tau_2} d\tau_3\ a^3 v_{p3}^* u_{p3}'^*(\tau_3)\right] \\ &\times \quad (2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\ \text{perms.}\end{aligned}$$

The result is

$$\langle \delta\theta^3 \rangle \supseteq -\frac{1}{\sqrt{2}} \frac{\dot{\theta}^3 V'''}{H R^3 \mu^4} \frac{k_1 + k_2 + k_3}{k_1 k_2 k_3 \left(k_1^2 + k_2^2 + k_3^2\right)^2}.$$



Outline

Our Conclusion

Effective Single Field Approach $\equiv$ In-in Formulism

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Effective Single Field Approach $\equiv$ In-in Formulism

(But it seems only hold for 2-point function and at leading order...)

Comments

1. Non-constant turn case.
2. Non-adiabatic turn. Shiu 2011, Gao2012.
3. To embed the QSI into a segment of inflationary trajectory.
4. Loop corrections. Chen 2012.
5. Effective field theory of QSI. Noumi 2012.
6. Non-Gaussianities with (1)large mass limit and (2)small mass limit.

Figure: "New star near Antares", record of a possible supernova in Shang Dynasty, 1600-1046 B.C.

Thank you!

GBKsong