Shinji Mukohyama, JGRG 22(2012)111202

"Nonlinear massive gravity and cosmology"

RESCEU SYMPOSIUM ON

GENERAL RELATIVITY AND GRAVITATION

JGRG 22

November 12-16 2012

Koshiba Hall, The University of Tokyo, Hongo, Tokyo, Japan

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Nonlinear massive gravity and Cosmology

Shinji Mukohyama
(Kavli IPMU, U of Tokyo)

Based on collaboration with
Antonio DeFelice, Emir Gumrukcuoglu, Chunshan Lin

Happy Birthdays!

- I would like to congratulate Kodama-san, Sasaki-san and Futamase-san on their 60th birthdays.

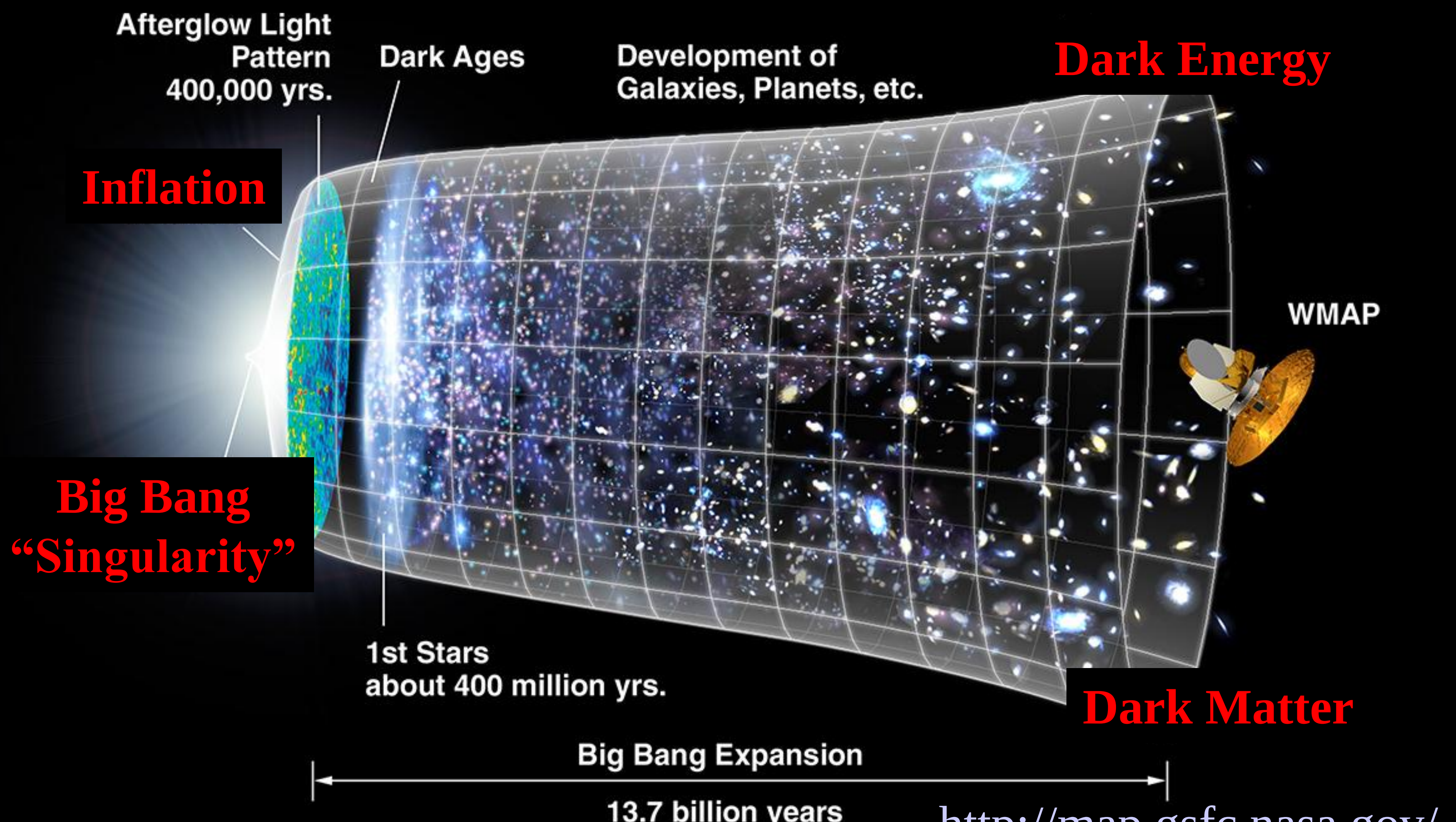
KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Nonlinear massive gravity and Cosmology

Shinji Mukohyama
(Kavli IPMU, U of Tokyo)

Based on collaboration with
Antonio DeFelice, Emir Gumrukcuoglu, Chunshan Lin

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
   e.g. no ghost instability
2. Experimentally viable
   solar system / table top experiments
3. Predictable
   e.g. protected by symmetry

Some examples

I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter

II. Nonlinear massive gravity
IR modification of gravity
motivation: “Can graviton have mass?”

III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity

IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

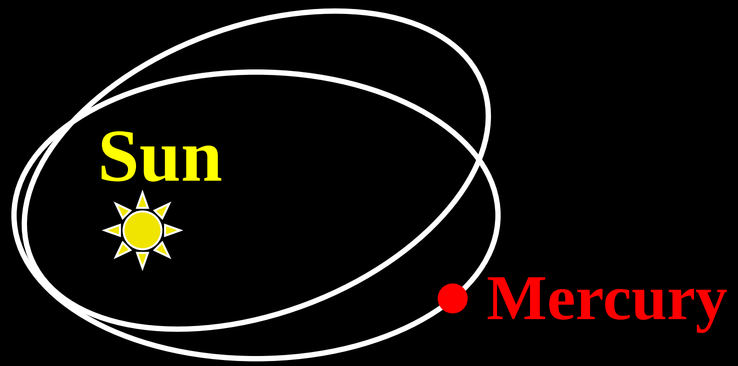
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

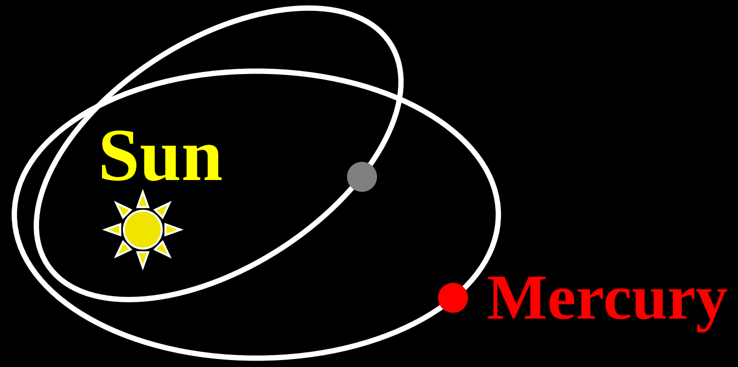
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's…



which people tried to
explain with a "dark
planet", Vulcan,



But the right answer wasn't "dark planet", it was
"change gravity" from Newton to GR.

Can we change gravity in IR?

- **Change Theory?**
  Massive gravity **Fierz-Pauli 1939**
  DGP model **Dvali-Gabadadze-Porrati 2000**

- **Change State?**
  Higgs phase of gravity
  The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

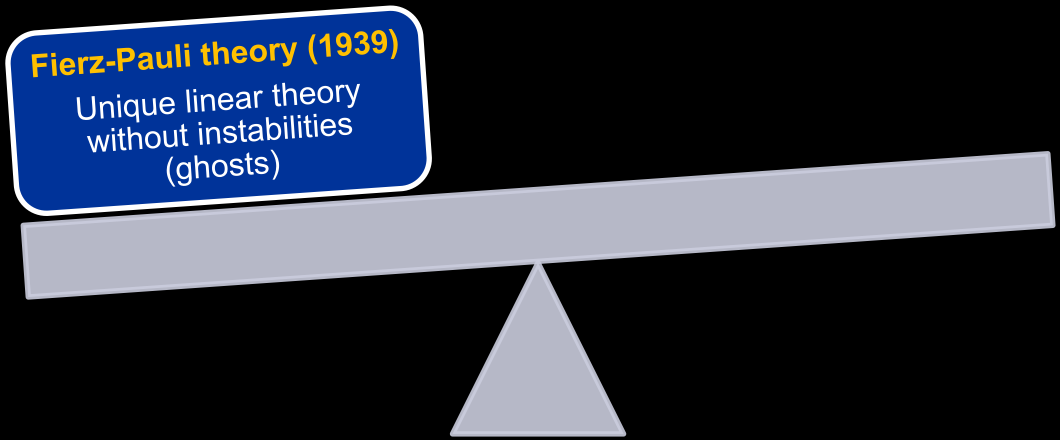
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

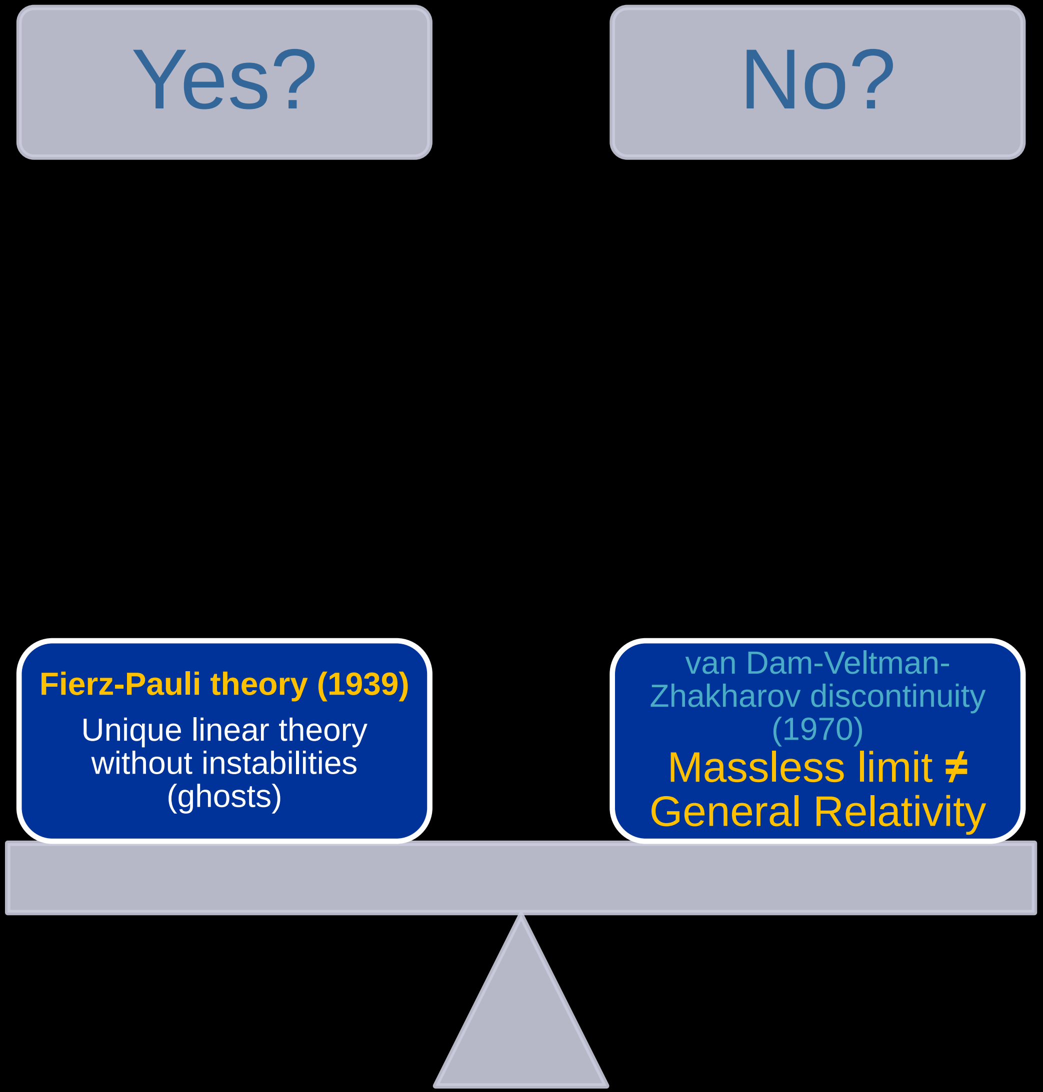
No?



Massive gravity: history

Simple question: Can graviton have mass?

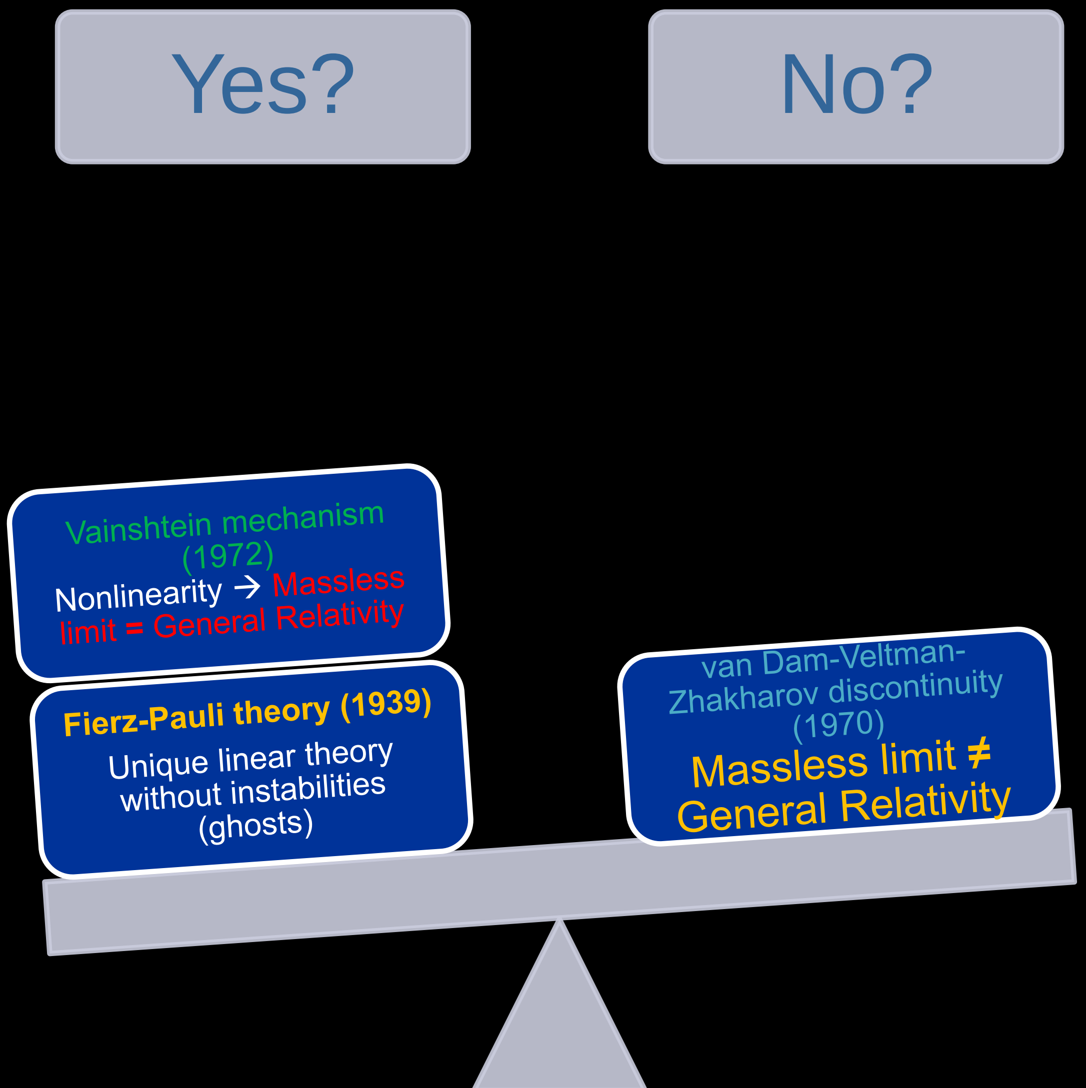
May lead to acceleration without dark energy



Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?	No?
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a (a=0,1,2,3)**
- **Poincare symmetry in the field space:**

$$\phi^a \to \phi^a + c^a, \quad \phi^a \to \Lambda^a_b \phi^b$$

→ $$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

Pullback of Minkowski metric in field space to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$f_{\mu\nu} \equiv \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu$$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$[\mathcal{A}] \equiv Tr\mathcal{A}$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

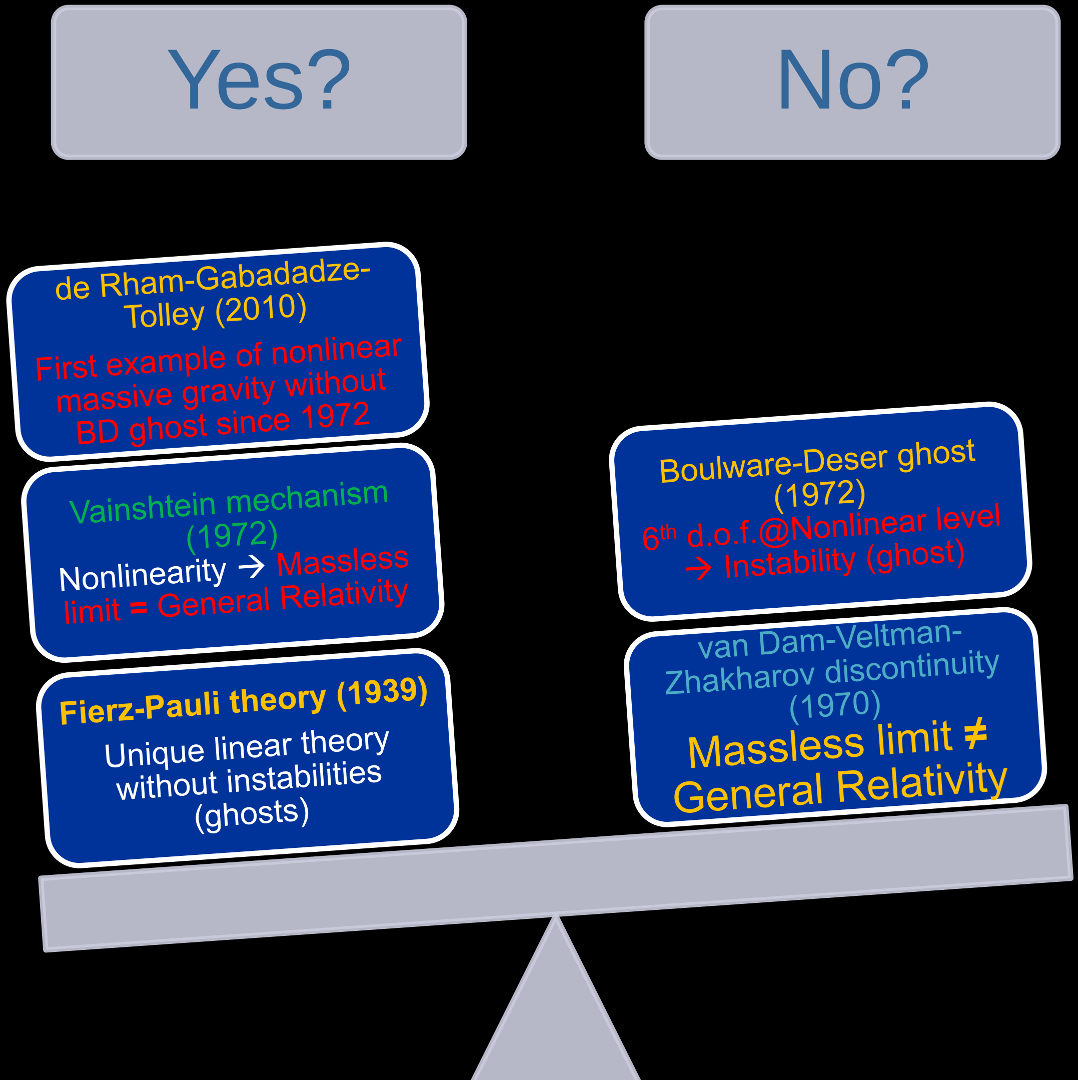
No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \;\Rightarrow\; \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy



No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in "Unitary gauge"
  $g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)(dx^2+dy^2+dz^2)$
  $\phi^a = x^a \Rightarrow f_{\mu\nu} = \eta_{\mu\nu}$
- Bianchi "identity" → a(t) = const.
  c.f. $$\nabla^\mu \left(\frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \right) = \frac{1}{\sqrt{-g}} \frac{\delta I_g}{\delta \phi^a} \partial_\nu \phi^a$$
  → no non-trivial flat FLRW cosmology
- "Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space"

Massive gravity: history

Simple question: Can graviton have mass?
May lead to acceleration without dark energy

Yes?	No?
de Rham, Gabadadze, Tolley (2010) First example of nonlinear massive gravity without BD ghost since 1972	D'Amico et al. (2011) Non-existence of flat FRW (homogeneous isotropic) universe!
Vainshtein mechanism (1972) Nonlinearity → Massless limit = General Relativity	Boulware-Deser ghost (1972) 6^{th} d.o.f.@Nonlinear level → Instability (ghost)
Fierz-Pauli theory (1939) Unique linear theory without instabilities (ghosts)	van Dam-Veltman-Zhakharov discontinuity (1970) Massless limit ≠ General Relativity

Consistent Theory found in 2010 but

No Viable Cosmology?

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\upsilon}$ spontaneously breaks diffeo.
- Both $g_{\mu\upsilon}$ and $f_{\mu\upsilon}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\upsilon}$
- No closed FLRW chart
- Open FLRW ansatz

$$
\begin{aligned}
\phi^0 &= f(t)\sqrt{1+|K|(x^2+y^2+z^2)},\\
\phi^1 &= \sqrt{|K|}f(t)x,\\
\phi^2 &= \sqrt{|K|}f(t)y,\\
\phi^3 &= \sqrt{|K|}f(t)z.
\end{aligned}
$$

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2\, dt^2 + |K|\,(f(t))^2\,\Omega_{ij}(x^k)dx^i dx^j$$

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^i dx^j\,,\\
\Omega_{ij}dx^i dx^j &= dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}\,,
\end{aligned}
$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a (a=0,1,2,3)

$$(\dot{a}-\sqrt{|K|N})\left[\left(3-\frac{2\sqrt{|K|}f}{a}\right)+\alpha_3\left(3-\frac{\sqrt{|K|}f}{a}\right)\left(1-\frac{\sqrt{|K|}f}{a}\right)+\alpha_4\left(1-\frac{\sqrt{|K|}f}{a}\right)^2\right]=0$$

- The first sol $\dot{a}=\sqrt{|K|N}$ implies $g_{\mu\upsilon}$ is Minkowski

  → we consider other solutions

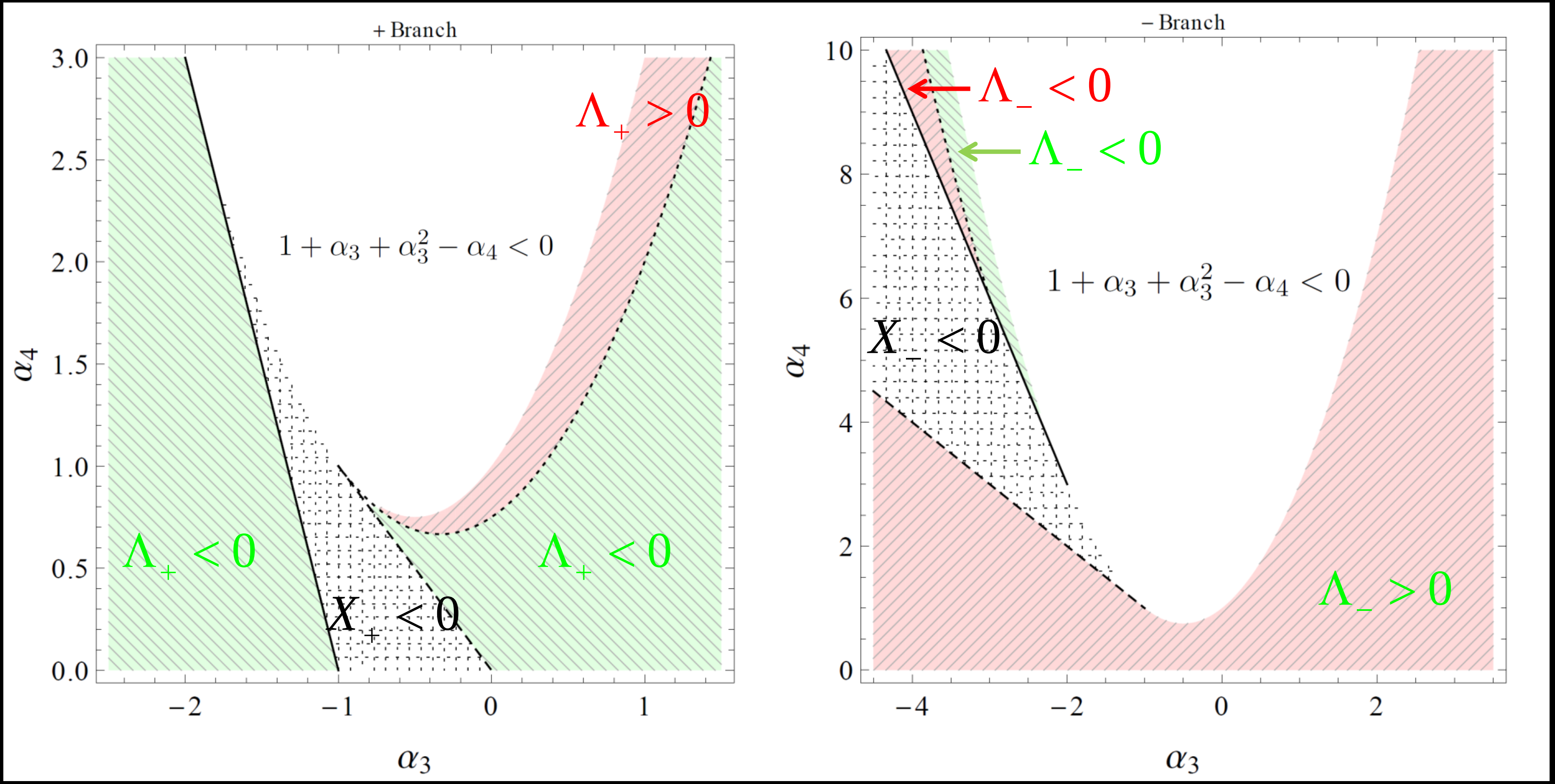
$$f=\frac{a}{\sqrt{|K|}}X_\pm,\quad X_\pm\equiv\frac{1+2\alpha_3+\alpha_4\pm\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}}{\alpha_3+\alpha_4}$$

- Latter solutions do not exist if K=0
- Metric EOM → self-acceleration

$$3H^2+\frac{3K}{a^2}=\Lambda_\pm+\frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm\equiv-\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\alpha_3^2-3\alpha_4\right)\pm2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_\pm, \quad X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

General fiducial metric

Appendix of Gumrukcuoglu, Lin, Mukohyama, arXiv: 1111.4107 [hep-th]

- **Poincare symmetry in the field space**
  → $f_{\mu\nu} = (Minkowski)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$
- **de Sitter symmetry in the field space**
  → $f_{\mu\nu} = (deSitter)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$
- **FRW symmetry in the field space**
  → $f_{\mu\nu} = (FLRW)_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$

Flat/closed/open FLRW cosmology allowed if "fiducial metric" $f_{\mu\nu}$ is de Sitter (or FRW)

→ **Friedmann equation with the same effective cc**

$$3\,H^2 + \frac{3\,K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Cosmological perturbation with any matter

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1111.4107 [hep-th]

$$I^{(2)} = \tilde{I}^{(2)}[Q_I, \Phi, \Psi, B_i, \boxed{\gamma_{ij}}] + \tilde{I}^{(2)}_{mass}[\psi^\pi,\ E^\pi,\ F^\pi_i, \boxed{\gamma_{ij}}]$$

$$\tilde{I}[g_{\mu\nu}, \sigma_I] \equiv I_{EH,\tilde{\Lambda}}[g_{\mu\nu}] + I_{matter}[g_{\mu\nu}, \sigma_I] \qquad \tilde{\Lambda} \equiv \Lambda + \Lambda_\pm$$

$$M_{GW}^2 \equiv \pm(r-1)m_g^2 X_\pm^2 \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4},$$

$$r \equiv \frac{na}{N\alpha} = \frac{1}{X_\pm}\frac{H}{H_f}, \qquad H \equiv \frac{\dot{a}}{Na}, \qquad H_f \equiv \frac{\dot{\alpha}}{n\alpha}$$

$$\tilde{I}^{(2)}_{mass} = M_{Pl}^2 \int d^4x N a^3 \sqrt{\Omega}\, M_{GW}^2 \times \left[3(\psi^\pi)^2 - \frac{1}{12}E^\pi \triangle(\triangle + 3K)E^\pi + \frac{1}{16}F^i_\pi(\triangle + 2K)F^\pi_i - \frac{1}{8}\gamma^{ij}\gamma_{ij}\right]$$

- **GR&matter part + graviton mass term**
- Separately gauge-invariant
  Common ingredient is γ_{ij} only
- Integrate out ψ^π, E^π and F^π_i → $I^{(2)}_{s,v} = I^{(2)}_{GR\ s,v}$
- Difference from GR is in the tensor sector only

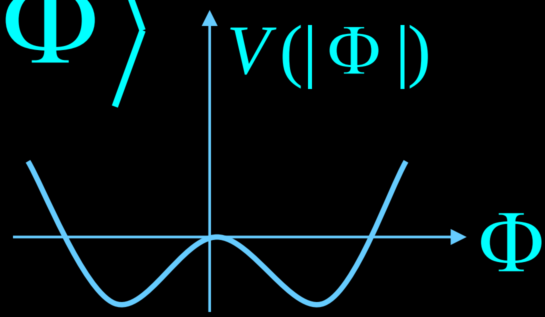
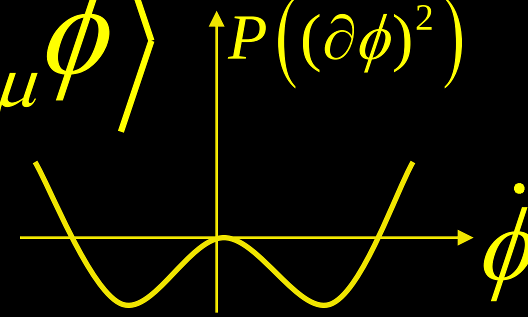
Summary so far

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc → FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
  → nonlinear perturbation around flat FLRW
- Odd-sector:
  1 healthy mode + 1 healthy or ghosty mode
- Even-sector:
  2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

	Higgs mechanism	***Ghost condensate***
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.

→ Anisotropic FLRW universe!

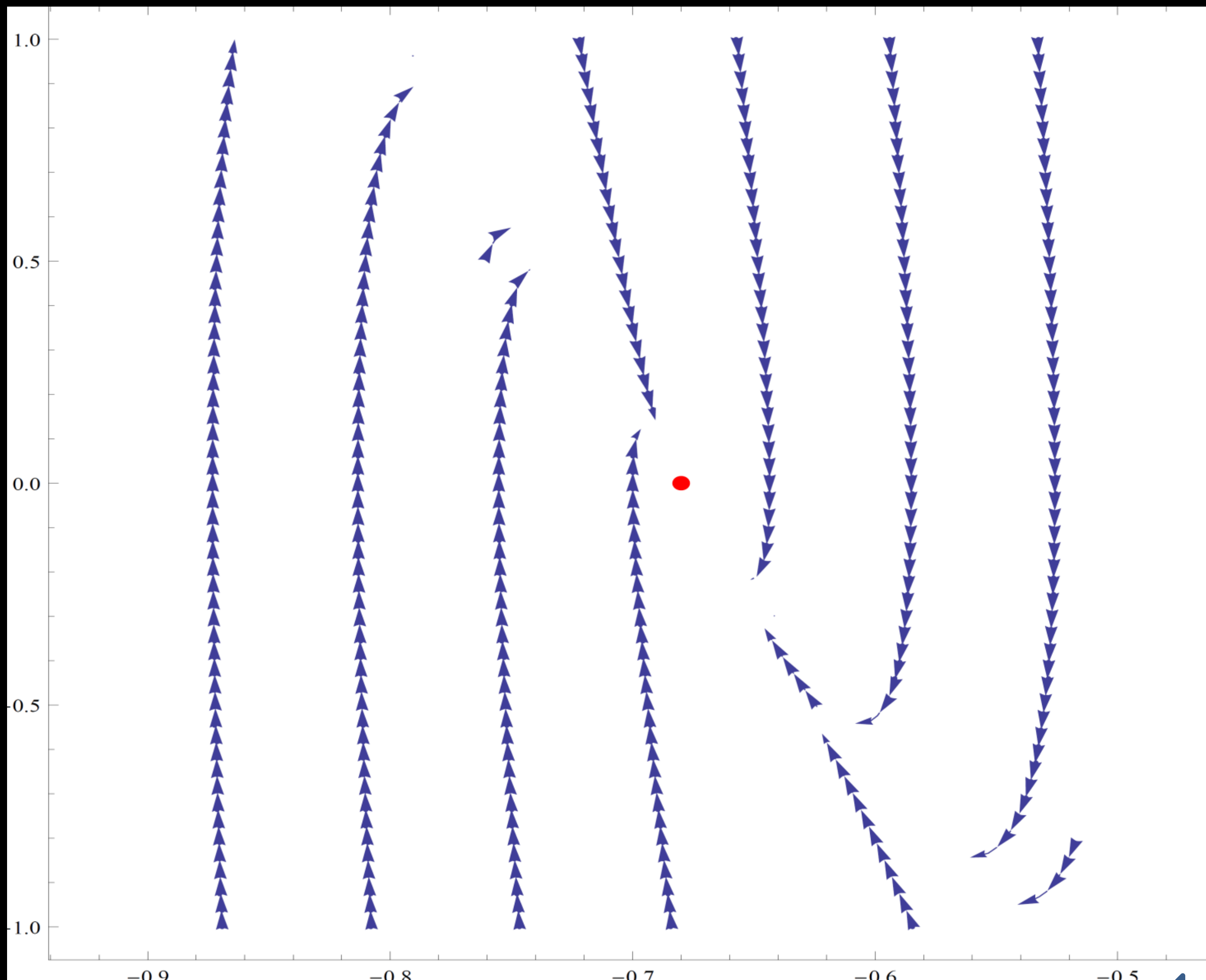
statistical anisotropy expected

(suppressed by small m_g^2)

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

Anisotropy in Expansion

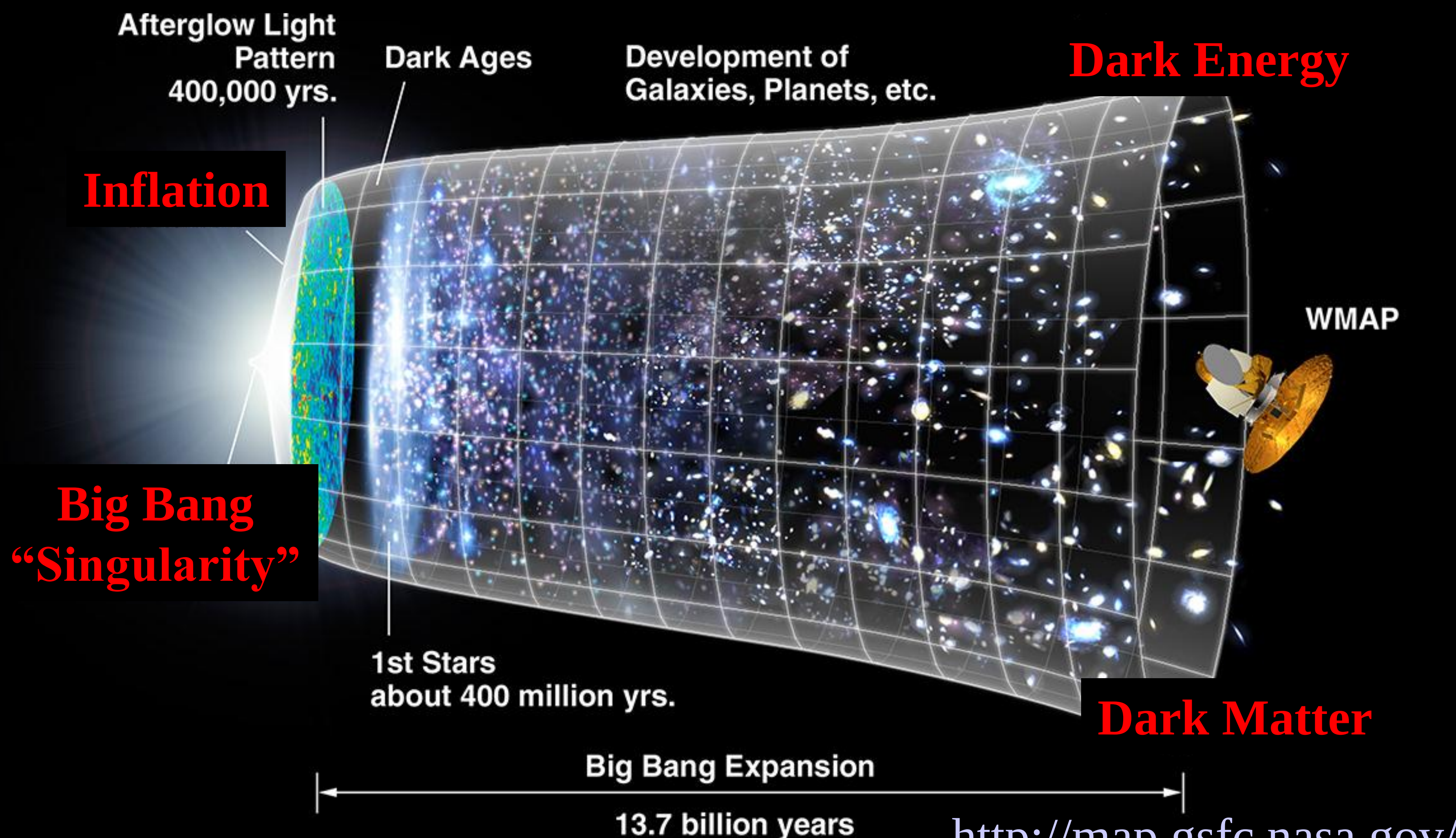


Anisotropy in fiducial metric

Summary

- Nonlinear massive gravity
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
  Friedmann eq does not depend on $f_{\mu\upsilon}$
- Cosmological linear perturbations
  Scalar/vector sectors → same as in GR
  Tensor sector → time-dependent mass
- All homogeneous and isotropic FLRW solutions have ghost
- New class of cosmological solution:
  anisotropic FLRW → statistical anisotropy (suppressed by small m_g^2)
- Analogue of Ghost Condensate!

Why alternative gravity theories?



http://map.gsfc.nasa.gov/

BACKUP SLIDES

Linear massive gravity (Fierz-Pauli 1939)

- Simple question: Can spin-2 field have mass?
- $L = L_{EH}[h] + \mathbf{m_g^2[\eta^{\mu\rho}\eta^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma}-(\eta^{\mu\nu}h_{\mu\nu})^2]}$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Unique linear theory without ghosts
- Broken diffeomorphism
  → no momentum constraint
  → 5 d.o.f. (2 tensor + 2 vector + 1 scalar)

vDVZ vs Vainshtein

- van Dam-Veltman-Zhakharov (1970)
  Massless limit ≠ Massless theory = GR
  5 d.o.f remain → PPN parameter $\gamma = \frac{1}{2} \neq 1$
- Vainshtein (1972)
  Linear theory breaks down in the limit.
  Nonlinear analysis shows continuity and GR is recovered @ $r < r_V = (r_g/m_g^4)^{1/5}$.
  Continuity is not uniform w.r.t. distance.

Naïve nonlinear theory and BD ghost

- FP theory with $\eta^{\mu\nu} \rightarrow g^{\mu\nu}$
  $L = L_{EH}[h] + m_g{}^2[g^{\mu\rho}g^{\nu\sigma}h_{\mu\nu}h_{\rho\sigma} - (g^{\mu\nu}h_{\mu\nu})^2]$
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Vainshtein effect (1972)
- Boulware-Deser ghost (1972)
  No Hamiltonian constraint @ nonlinear level
  → 6 d.o.f. = 5 d.o.f. of massive spin-2 + 1 ghost

Stuckelberg fields & Decoupling limit

Arkani-Hamed, Georgi & Schwarz (2003)

- Stuckelberg scalar fields ϕ^a (a=0,1,2,3)

  $$g_{\mu\nu} = \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b + H_{\mu\nu} \qquad \phi^a = x^a + \pi^a$$

  $H_{\mu\upsilon}$: covariant version of $h_{\mu\upsilon} = g_{\mu\nu} - \eta_{\mu\nu}$
- Decoupling limit

  $m_g \to 0$, $M_{Pl} \to \infty$ with $\Lambda_5 = (m_g{}^4 M_{Pl})^{1/5}$ fixed
- Helicity-0 part π: $\eta_{ab}\pi^b = \partial_a \pi$

  sufficient for analysis of would-be BD ghost

Would-be BD ghost vs fine-tuning

Creminelli, Nicolis, Papucci & Trincherini 2005
de Rham, Gabadadze 2010

$$H_{\mu\nu} = -2\partial_\mu\partial_\nu\pi - \partial_\mu\partial^\rho\pi\partial_\rho\partial_\nu\pi \Leftarrow h_{\mu\nu} = 0, \eta_{ab}\pi^b = \partial_a\pi$$

Decoupling limit | Helicity-0 part

- Fierz-Pauli theory
  $H_{\mu\upsilon}^2 - H^2$
  no ghost
- 3rd order
  $c_1H_{\mu\upsilon}^3 + c_2HH_{\mu\upsilon}^2 + c_3H^3$
  no ghost if fine-tuned
- …
- any order
  no ghost if fine-tuned