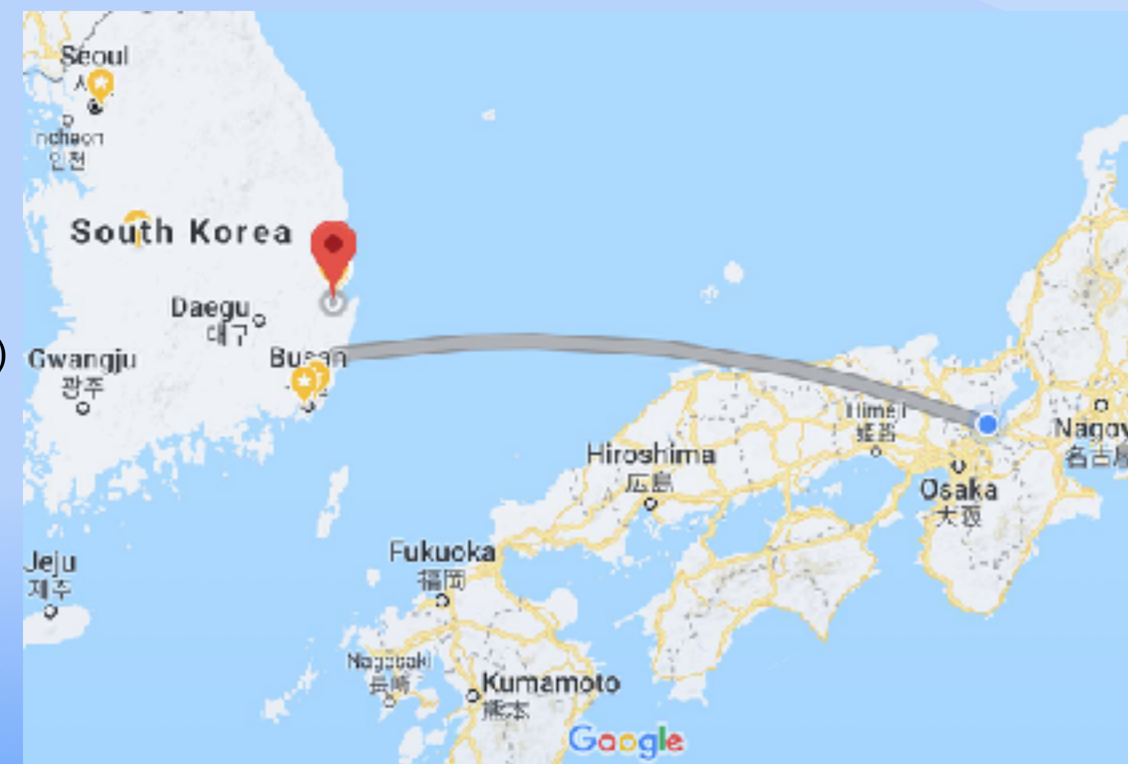
Towards **Dark Sectors from the Holographic Gravity**

by Yun-Long Zhang

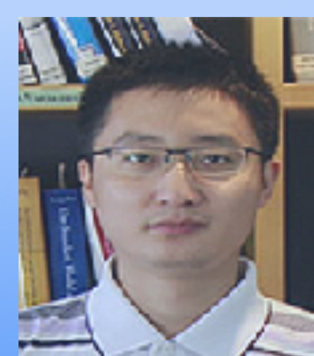
Asia Pacific Center for Theoretical Physics
@Pohang, Korea

Rindler Fluid
1705.05078

Sunly Khimphun, **Bum-Hoon Lee** (Sogang U.), **Chanyong Park** (APCTP)



de-Sitter Fluid
to appear



Rong-Gen Cai (ITP-CAS), **Sichun Sun** (NTU), **Yun-Long Zhang** (APCTP)

Cospa2017@YITP, Dec.14, 2017

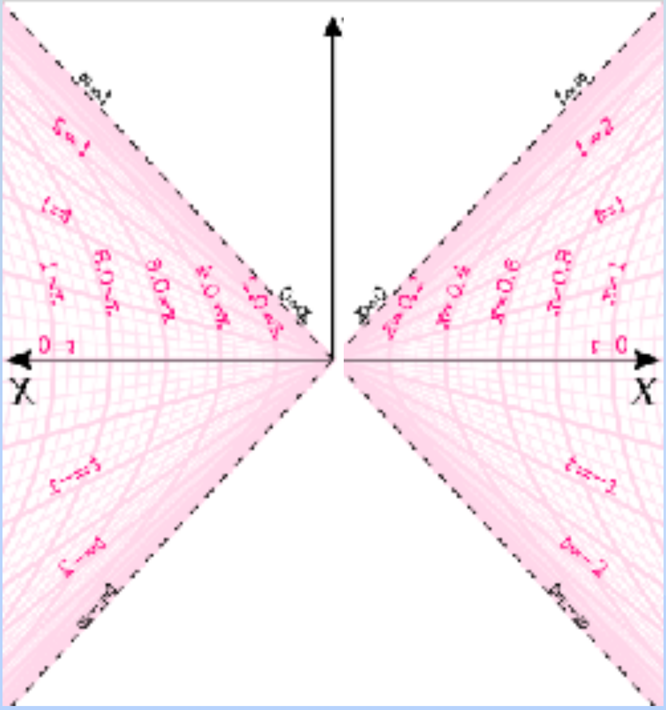
Holographic Screens in Flat Spacetime

— Rindler Screen & de-Sitter Screen

I. **Holographic Rindler Fluid**

— Accelerating Screen

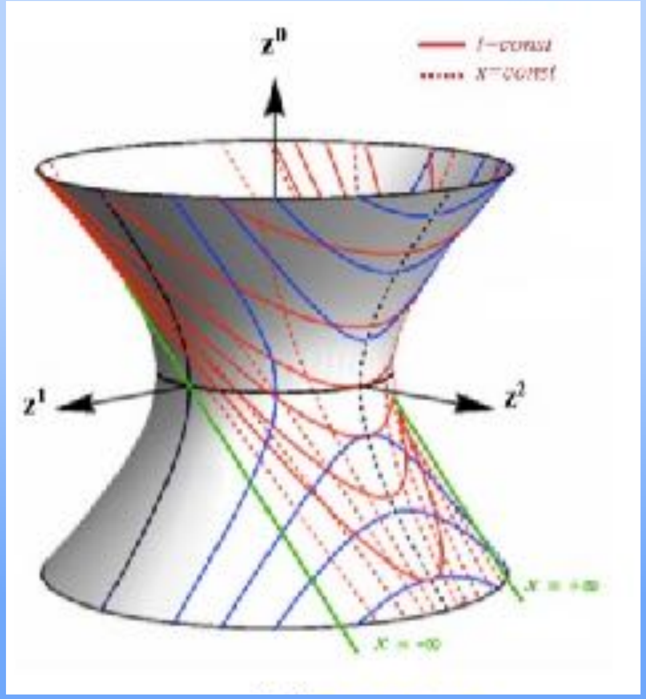
— Relation to AdS/CFT



II. **Holographic de-Sitter Fluid**

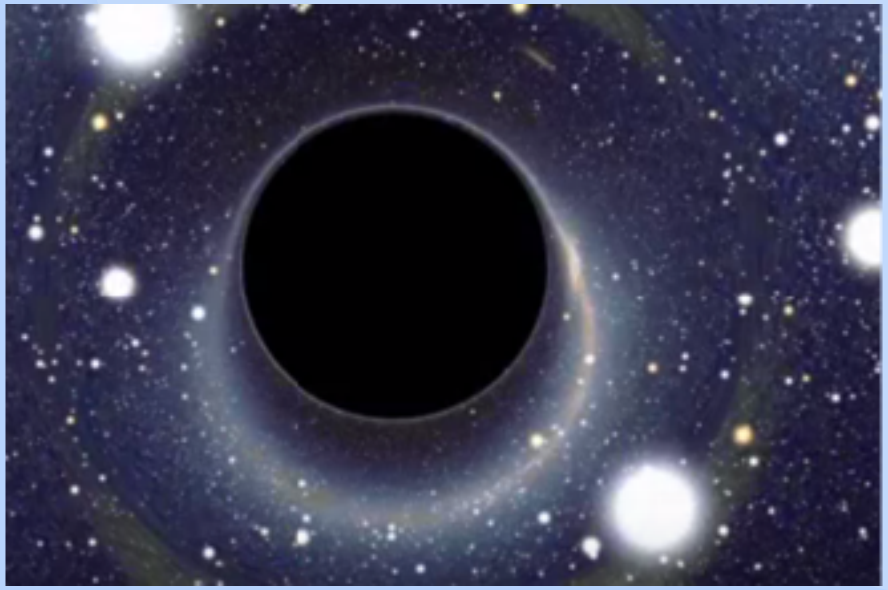
— de-Sitter&FRW Screen

— Relation to DGP brane



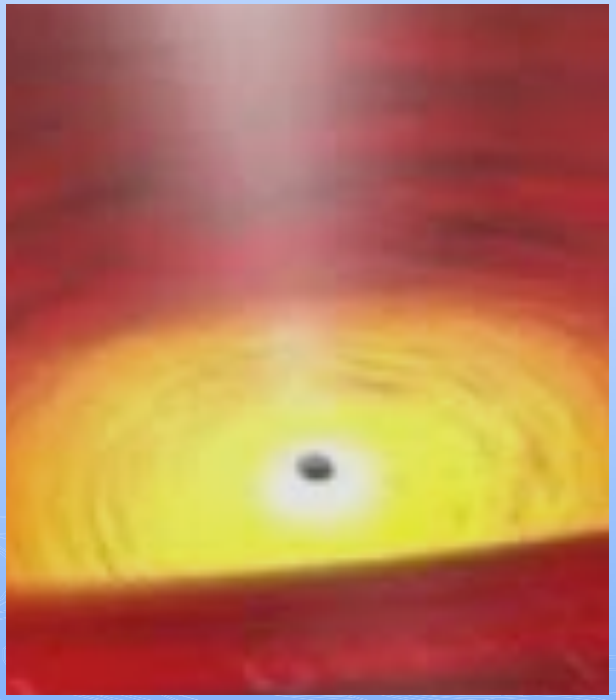
Thermodynamics (1970s): Hawking Radiation

Bekenstein & Hawking, …



Hawking Temperature $T_H = \frac{\hbar c^3}{8\pi G M k_B} = \frac{\kappa}{2\pi},$

Bekenstein-Hawking Entropy $S_{\rm BH} = \frac{kA}{4\ell_{\rm P}^2}$



0th Law: constant surface gravity

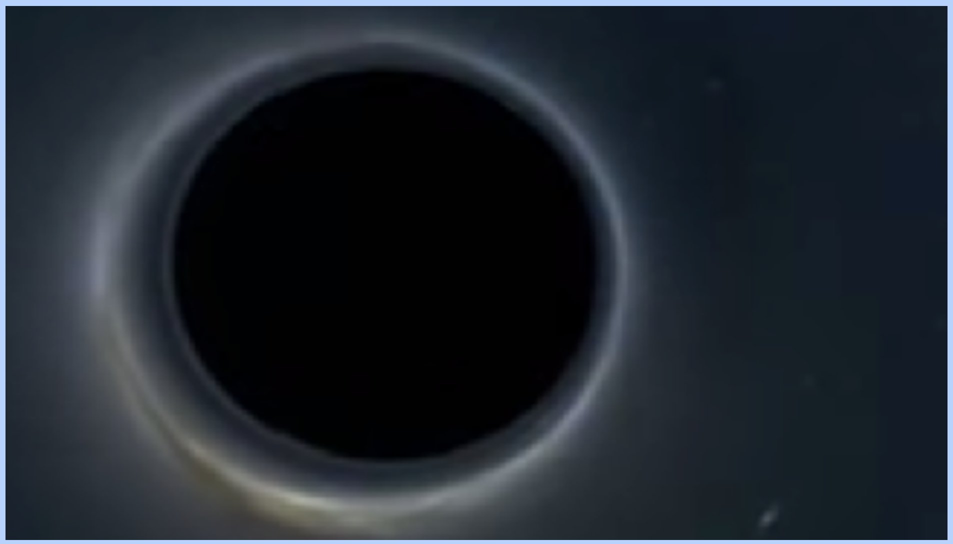
1st Law: $dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ,$

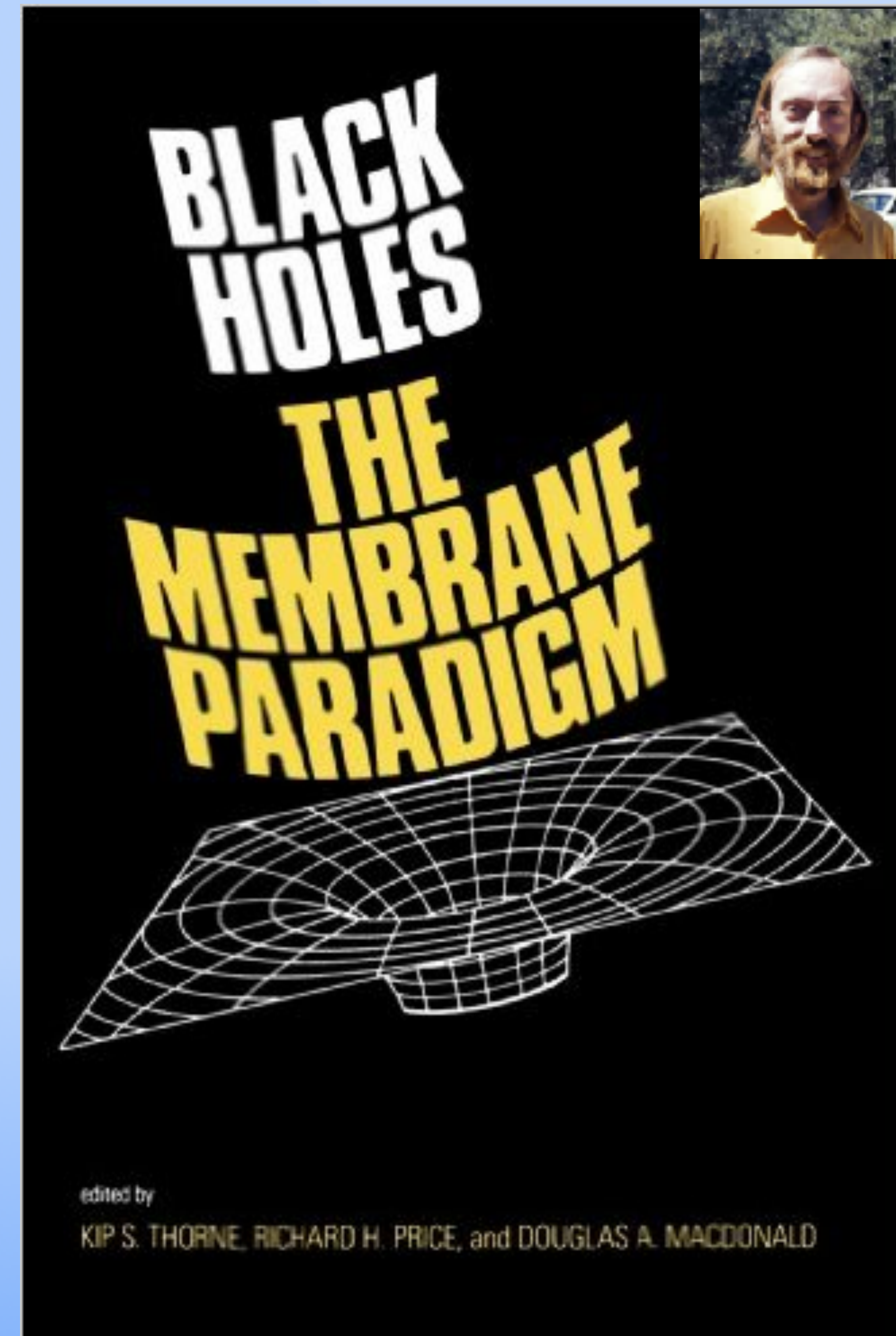
2nd Law: non-decreasing of entropy

3rd Law: extremal black hole is not possible

Membrane paradigm(1980s): Effective Fluid

Doumer & Thorne, ···





Effective Description

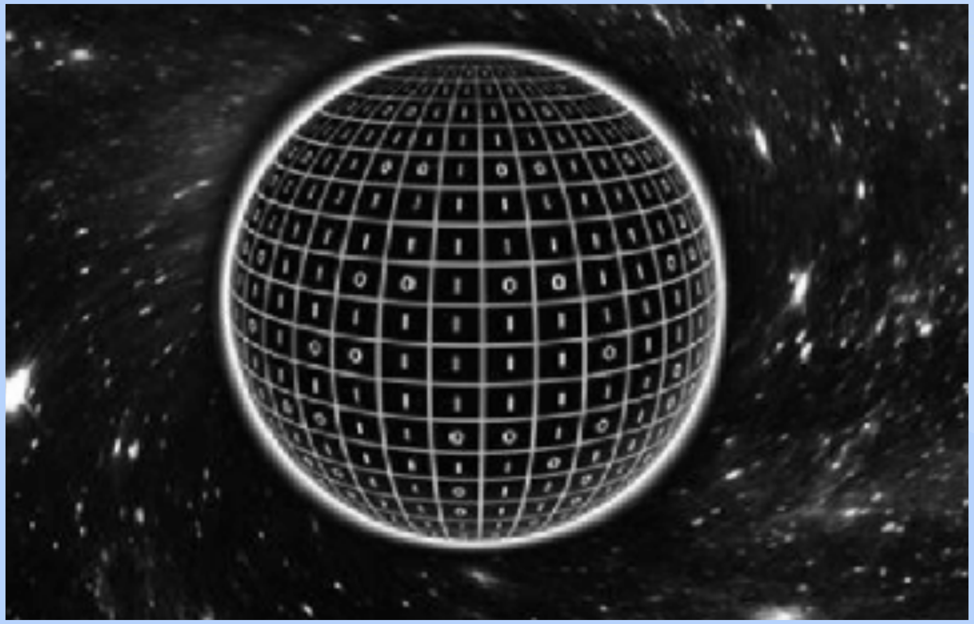
$$T_{ab} = \frac{1}{8\pi G}\left(\gamma_{ab}K - K_{ab} + C\gamma_{ab}\right),$$

Stretched horizon

Conductivity & Viscosity

Holographic Principle (1990s): Horizon encoding

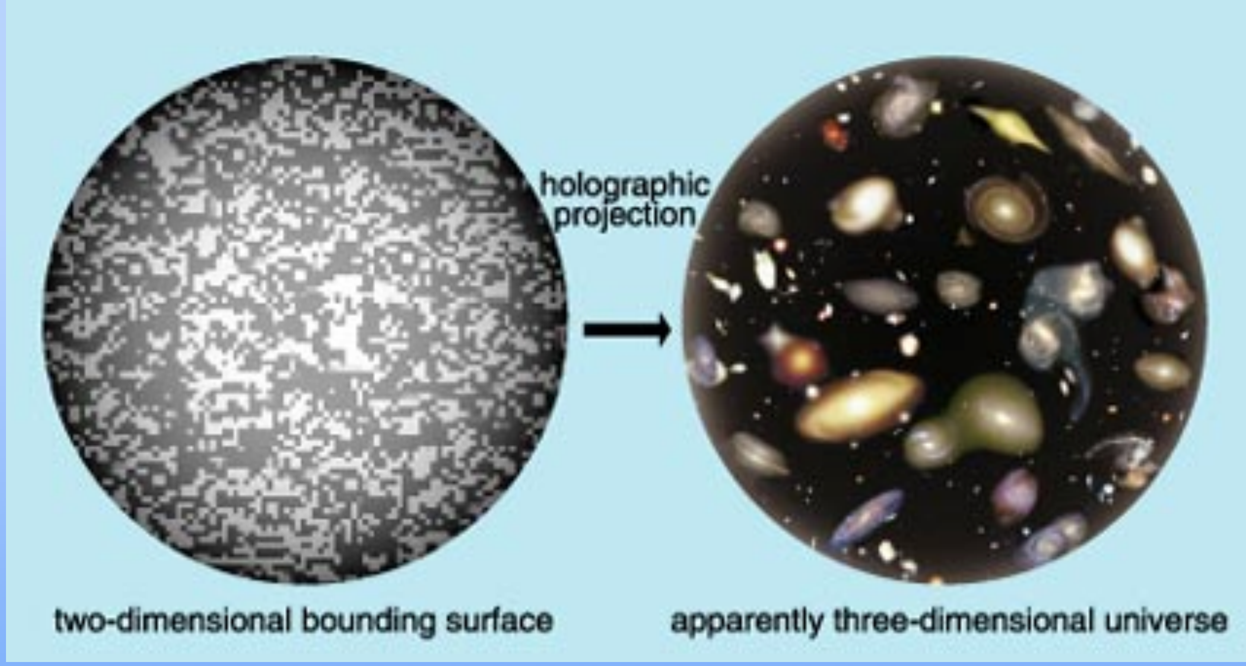
Susskind & 't Hooft, …



Black hole Horizon

Gravity in the Bulk=

Theory on the light-like boundary



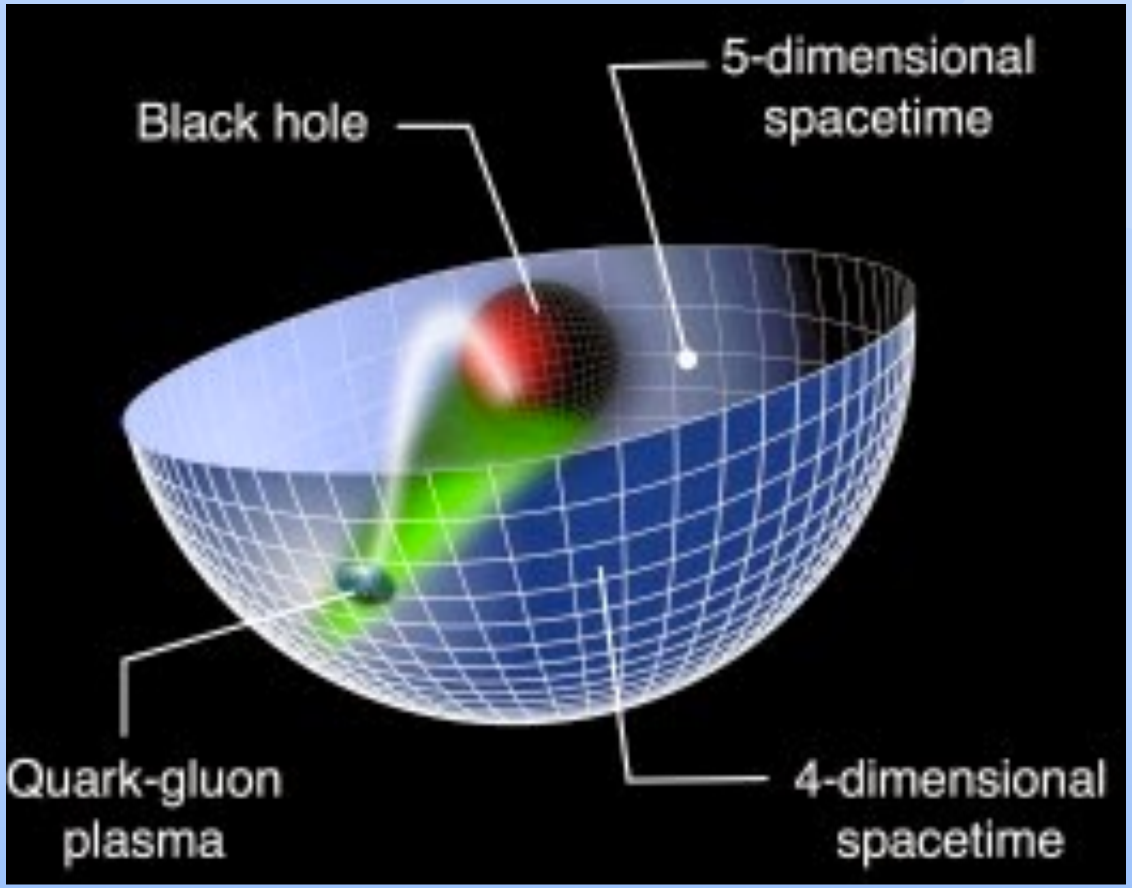
Cosmological Horizon

AdS/CFT Duality (2000s): Maldacena & Gubser & Witten,et al

AdS/CMT Correspondence Black Hole in a natural Box

Shear Viscosity $\frac{\eta}{s} \approx \frac{\hbar}{4\pi k}$

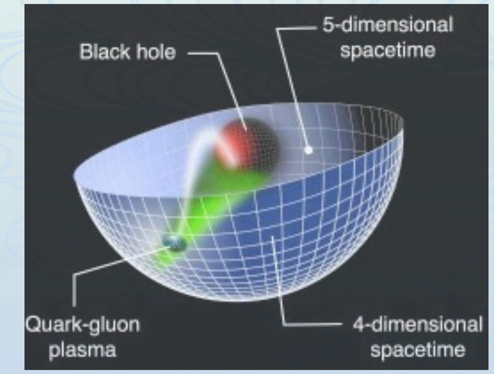
Conductivity



Holographic Superconductor

Holographic Non-Fermi Liquid

Motivations for the Accelerating Screen



$AdS_2/CFT_1 \times R_p$ & Non-Fermi Liquid ← Near Horizon ← **Cutoff AdS / Effective CMT?** → Near Boundary → **AdS /CFT&CMT**

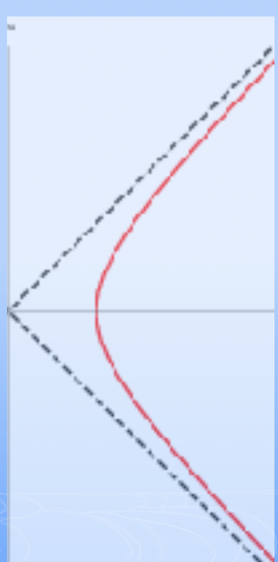
Extremal Charged BH

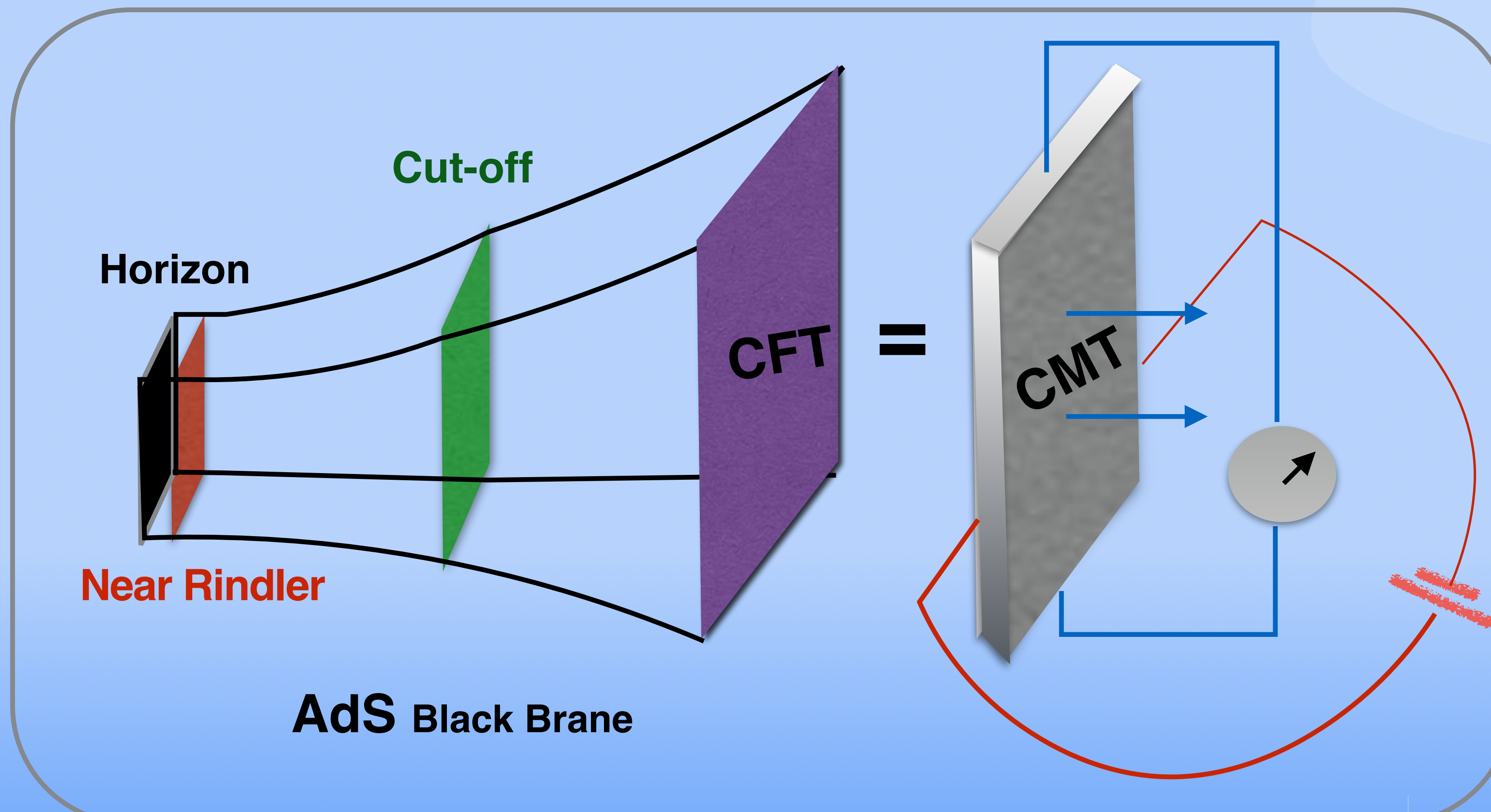
Finite Temperature

Rindler Space/ Special CMT

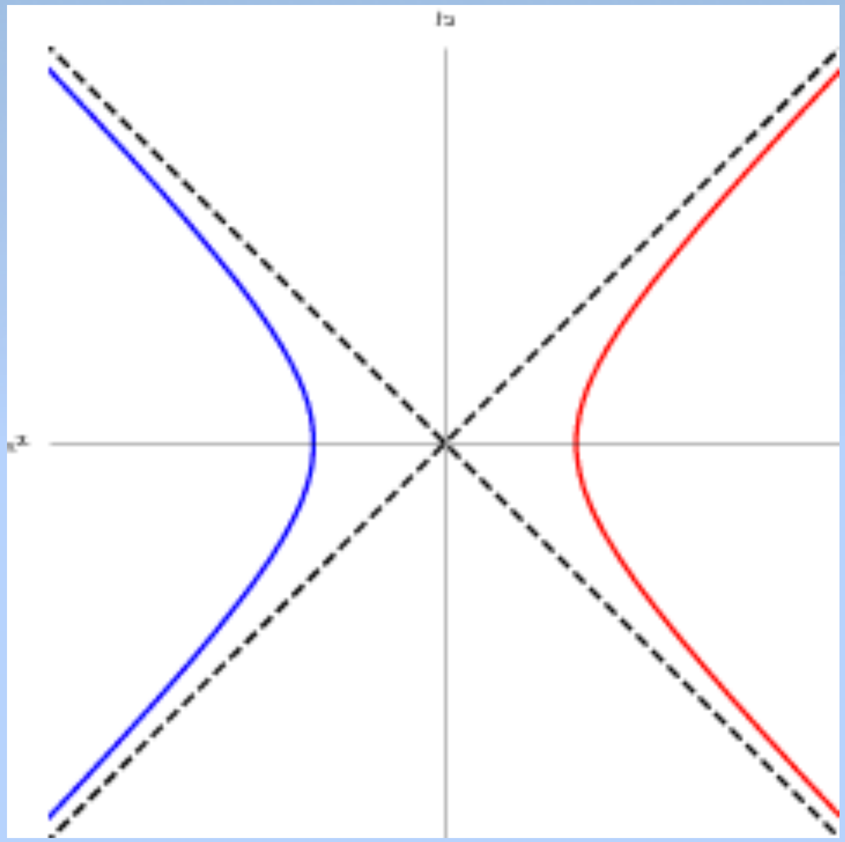
⋮

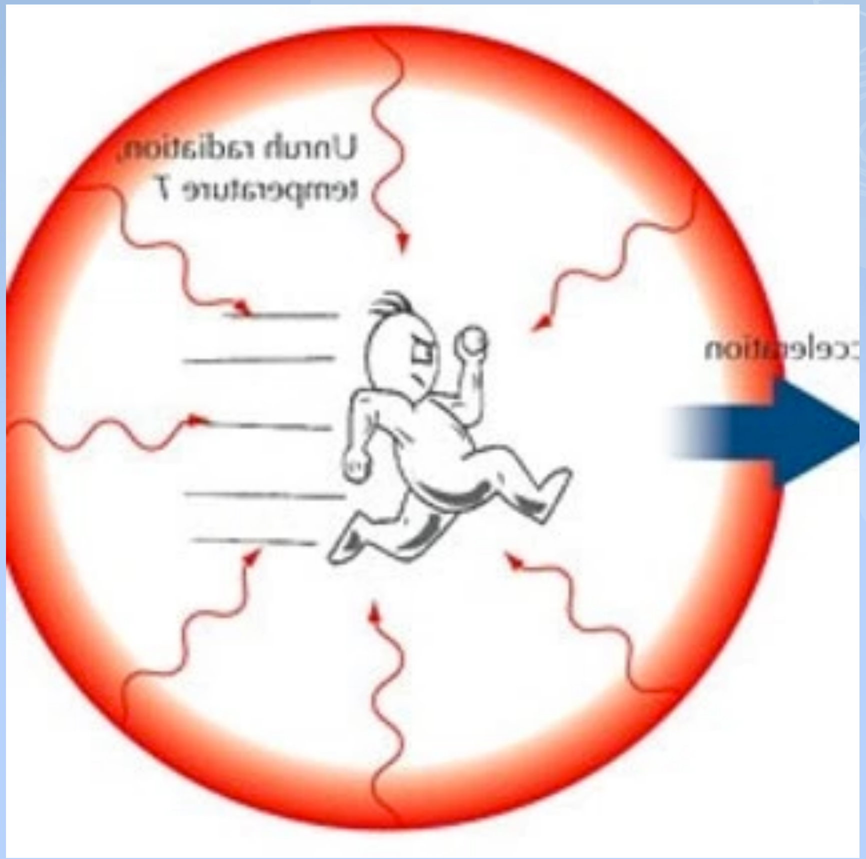
Membrane Black Holes





I. What is Rindler Fluid?





Credit: Physics Napkins

$$T_{ab} = 2(K\gamma_{ab} - K_{ab}).$$

$$ds^2_{p+1} = \gamma_{ab}dx_a dx^b = -r_c d\tau^2 + dx_i dx^i.$$

Relation with Membrane Paradigm

Black Holes <=> Lower Dimensional FLuid

- The horizon responds like a viscous fluid

Stress Tensor: $T_{ab} = 2(K\gamma_{ab} - K_{ab})$

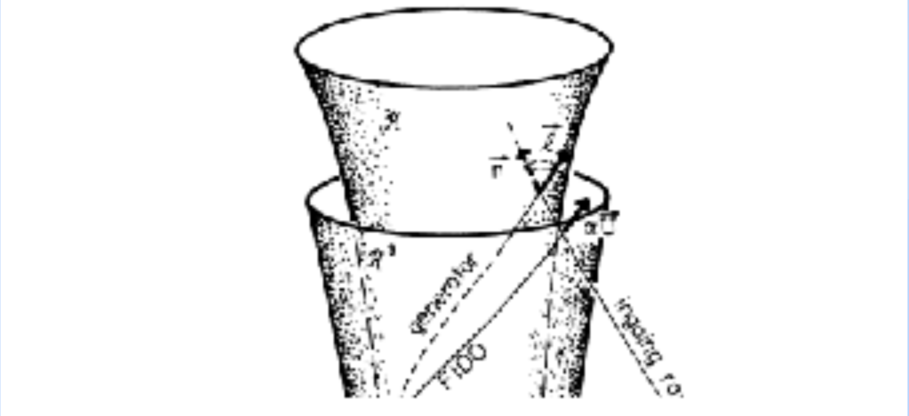
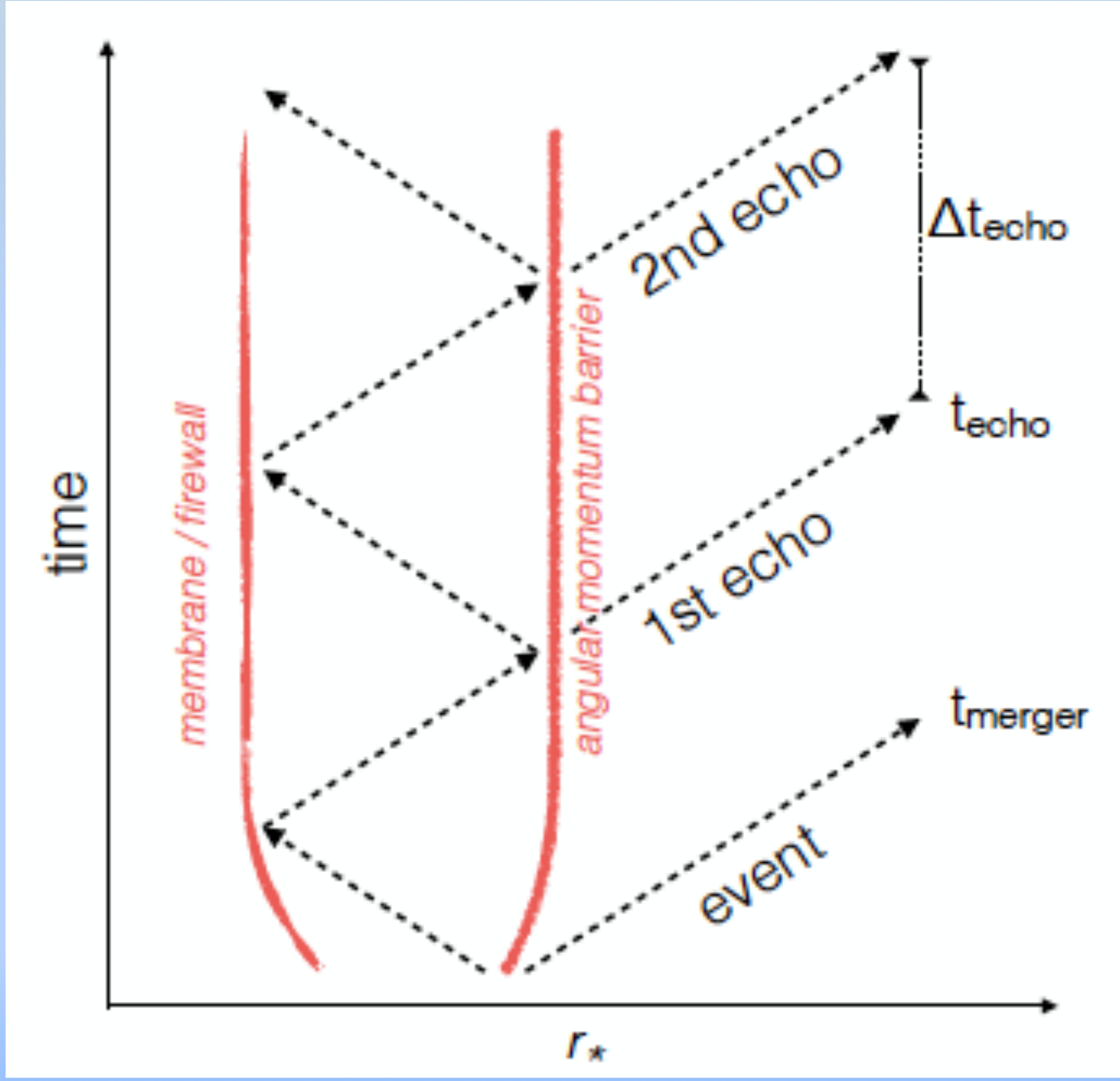


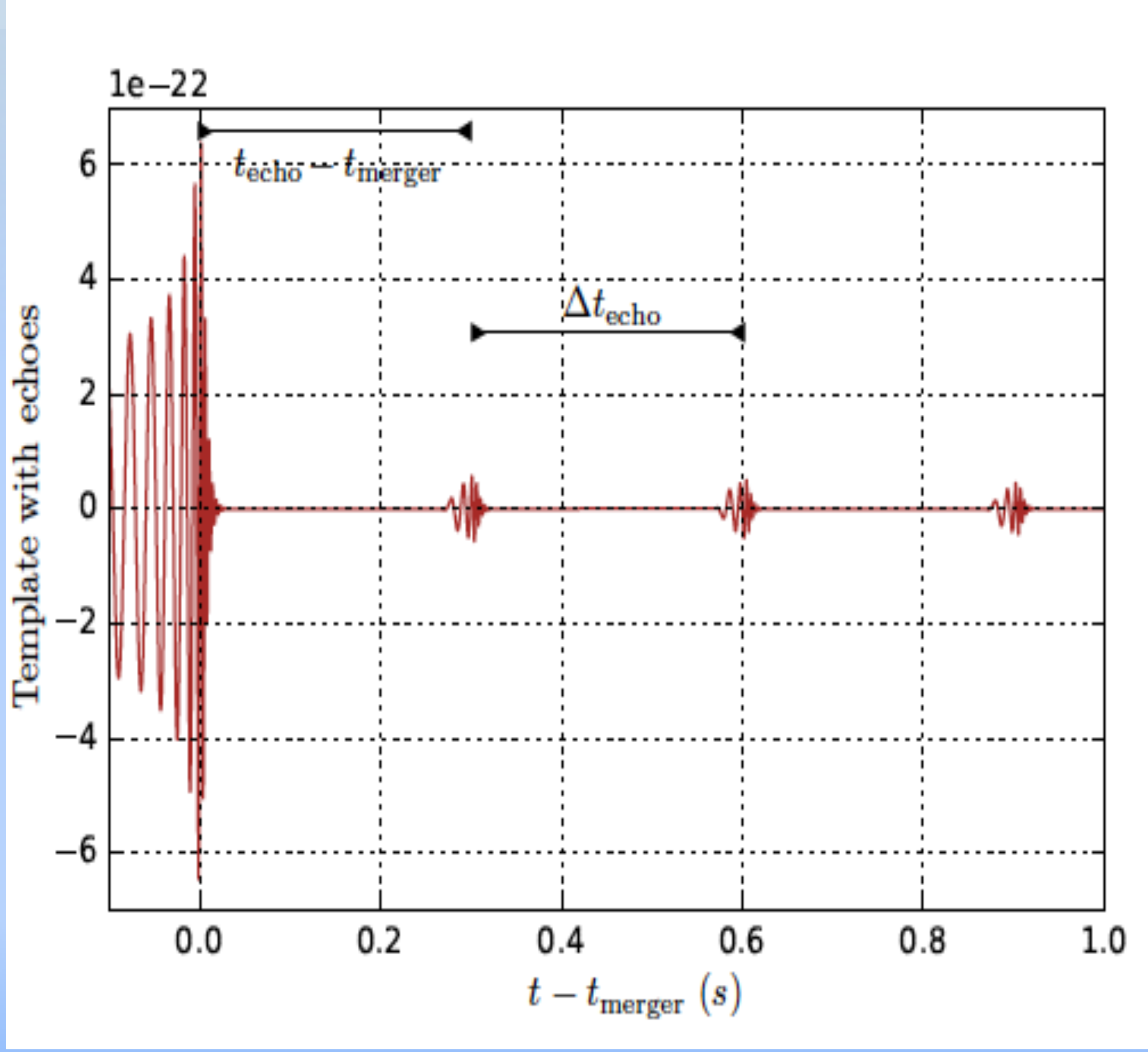
Figure: R. H. Price and K. S. Thorne, Phys. Rev. D 33, 915 (1986)

- Near horizon limit ➔ Rindler metric

$$ds^2_{p+2} = -r\mathrm{d}\tau^2 + 2\mathrm{d}\tau\mathrm{d}r + \mathrm{d}x_i\mathrm{d}x^i,$$

Near Horizon properties of Black Hole





aiXiv: 1612.00266 Echoes from the Abyss

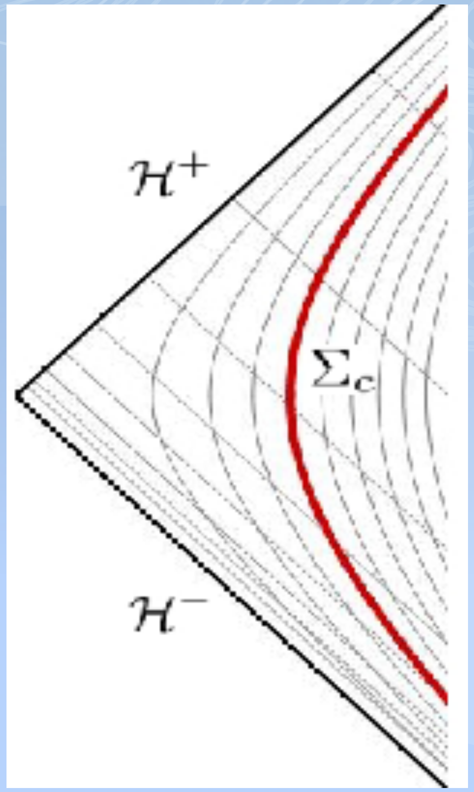
Rindler Hydrodynamics

- Induced metric

$$ds^2_{p+1} = \gamma_{ab} dx_a dx^b = -r_c d\tau^2 + dx_i dx^i.$$

- Dual Fluid:

$$T_{ab} = 2(K\gamma_{ab} - K_{ab}).$$



- Constraint equations

$$2G_{\mu b} n^\mu |_{\Sigma_c} = 2(\partial^a K_{ab} - \partial_b K) = 0 \Longrightarrow \partial^a T_{ab} = 0,$$

$$2G_{\mu\nu} n^\mu n^\nu |_{\Sigma_c} = (K^2 - K_{ab}K^{ab}) = 0 \Longrightarrow T^2 - p\, T_{ab}T^{ab} = 0,$$

Bredberg, Keeler, Lysov, Strominger (JHEP 07 (2012) 146)

Derivative Expansion:

$$T_{ab} = T_{ab}^{(0)} + T_{ab}^{(1)} + T_{ab}^{(2)} + O(\partial^3),$$

$$T_{ab}^{(0)} = \mathbb{p} h_{ab},$$

$$T_{ab}^{(1)} = \zeta' (u^c \partial_c \ln \mathbb{p}) u_a u_b - 2\eta \mathcal{K}_{ab},$$

$$T_{ab}^{(2)} = \mathbb{p}^{-1} \Big\{ \big[d_1 \mathcal{K}_{ab} \mathcal{K}^{ab} + d_2 \Omega_{ab} \Omega^{ab} + d_3 (u^c \partial_c \ln \mathbb{p})^2$$
$$+ d_4 u^c \partial_c (u^d \partial_d \ln \mathbb{p}) + d_5 h^{cd} (\partial_c \ln \mathbb{p})(\partial_d \ln \mathbb{p}) \big] u_a u_b$$
$$+ \big[c_1 \mathcal{K}_{ac} \mathcal{K}^c{}_b + c_2 \mathcal{K}_{c(a} \Omega^c{}_{b)} + c_3 \Omega_{ac} \Omega^c{}_b + c_4 h_a^c h_b^d \partial_c \partial_d \ln \mathbb{p}$$
$$+ c_5 \mathcal{K}_{ab} (u^c \partial_c \ln \mathbb{p}) + c_6 (h_a^c \partial_c \ln \mathbb{p})(h_b^d \partial_d \ln \mathbb{p}) \big] \Big\}.$$

Rindler Fluid

Transport coefficients

$$\zeta' = 0, \qquad \eta = 1,$$
$$d_1 = -2, \quad d_2 = d_3 = d_4 = d_5 = 0,$$
$$c_1 = -2, \quad c_2 = c_3 = c_4 = c_5 = -c_6 = -4\,.$$

G. Compere, et al. (JHEP 03 (2012) 076)

A Simple & Recursive Relation

- **A recursive relation between different orders?**
  - Gravity ⇔ A special Fluid
  - Gravity ⇔ Riemannian Geometry
- **Petrov type I condition!**

$$C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta} = 0$$

$$m_i = \partial_i, \quad \sqrt{2}\ell = \partial_0 - n, \quad \sqrt{2}k = -\partial_0 - n$$

Lysov & Strominger(1104.5502)

  - No more gravitational field equations

Petrov type I condition ➔ Rindler Fluid

- Give the 0th order

$$\mathbb{e}^{(0)} = 0,\ \Pi^{(0)}_{ab} = \mathbb{p} h_{ab}.$$

  - 1st order

$$\mathbb{H}^{(1)} = 0 \Rightarrow \mathbb{e}^{(1)} = 0,$$
$$\mathbb{P}^{(1)}_{ab} = 0 \Rightarrow \Pi^{(1)}_{ab} = -2\mathcal{K}_{ab},$$

  - 2nd order

$$\mathbb{H}^{(2)} = 0 \Rightarrow \mathbb{e}^{(2)} = -2\mathbb{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab},$$
$$\mathbb{P}^{(2)}_{ab} = 0 \Rightarrow \Pi^{(2)}_{ab} = \mathbb{p}^{-1}\Big[-2\mathcal{K}_{ac}\mathcal{K}^{c}_{\ b} - 4\mathcal{K}_{c(a}\Omega^{c}_{\ b)} - 4\Omega_{ac}\Omega^{c}_{\ b} - 4h^{c}_{a}h^{d}_{b}\partial_c\partial_d\ln\mathbb{p} - 4\mathcal{K}_{ab}(u^c\partial_c\ln\mathbb{p}) + 4(h^{c}_{a}\partial_c\ln\mathbb{p})(h^{d}_{b}\partial_d\ln\mathbb{p})\Big],$$

The Stress Tensor:

$$\hat{T}_{ab} = \mathbb{e}^{(2)}u_a u_b + \mathbb{p}h_{ab} + \Pi^{(1)}_{ab} + \Pi^{(2)}_{ab}.$$

Petrov type I condition

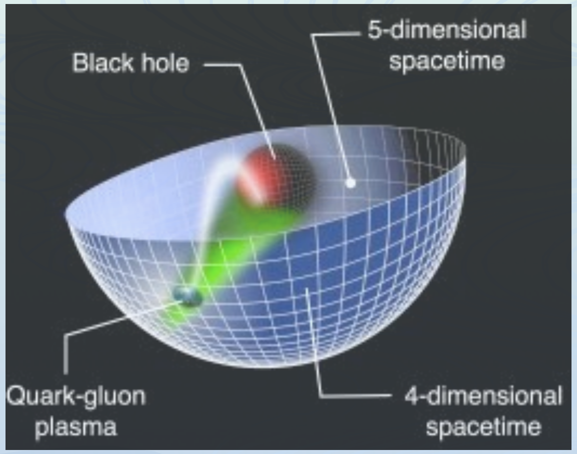
Obtain a recurrence relation $\mathbb{P}_{ab} \equiv \boldsymbol{n}^r h_a^c \boldsymbol{n}^r h_b^d C_{rcrd}$

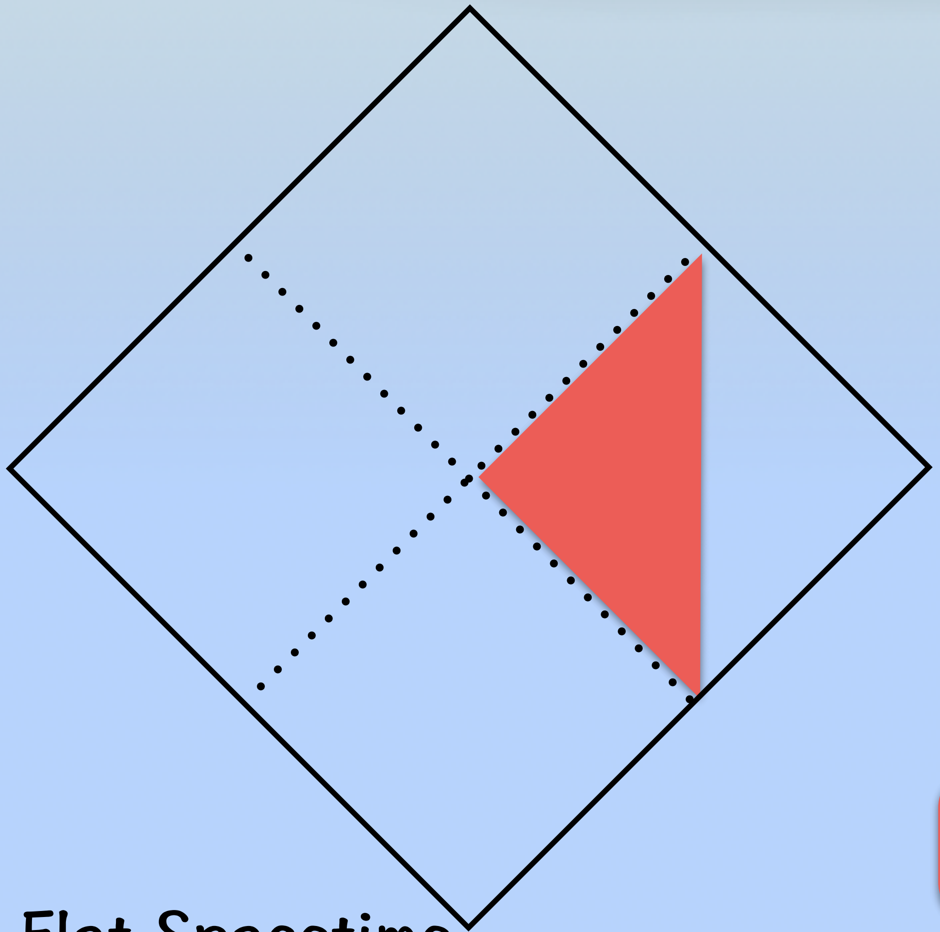
$$T_{ab}^{(0)} = \mathcal{E} u_a u_b + \mathcal{P} h_{ab} \stackrel{\mathbb{P}_{ab}=0}{\Longrightarrow} T_{ab}^{(1)} = -2\eta\sigma_{ab} + \ldots \stackrel{\mathbb{P}_{ab}=0}{\Longrightarrow} T_{ab}^{(2)} = \ldots$$

Rindler-Fluid	→	AdS Cutoff-Fluid	→	AdS-CFT Fluid
Up to 2nd order **How about Higher orders?** Cai *et al.* 1401.7792		**Up to to 0th order** **Modified Condition?** Y. Ling *et al.* 1306.5633		**Up to to 0th order** **AdS/Rindler correspondence?**

JHEP 1304, 118 (2013) , Phys. Rev. D 90, no. 4, 041901 (2014)

From AdS/CFT to Holographic Rindler Fluid
— with an Accelerating Screen



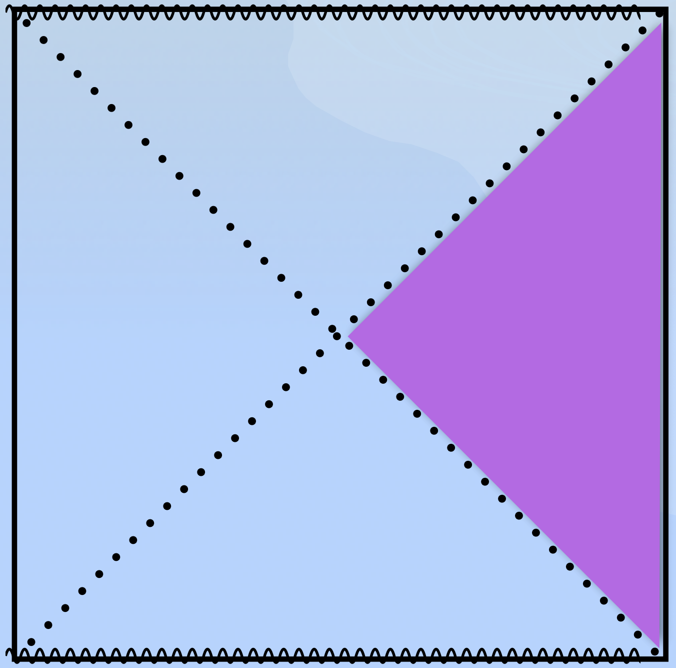


Flat Spacetime

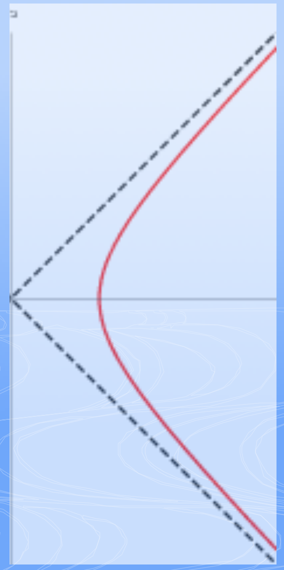
Holographic Screen
the Time-like boundary

What is Rindler Fluid?

Fluid dual to Rindler spacetime

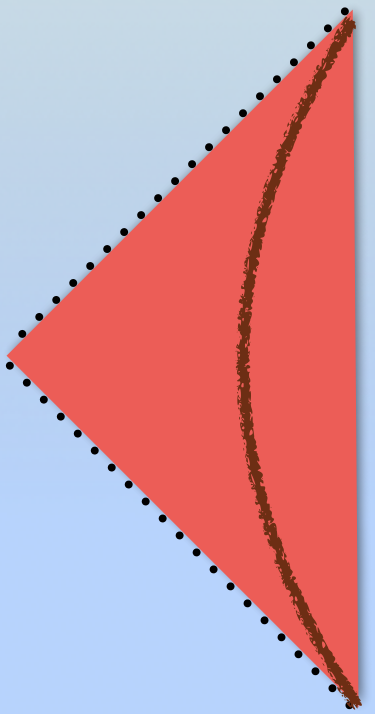


AdS Spacetime



Navier-Stokes Equations: Bredberg, Keeler, Lysov, Strominger [10',11']
Fluid/Gravity Expansion: Compere, McFadden, Skenderis, Taylor [11',12']
Entropy Current and Constraint: Chirco, Eling, Liberati, Meyer, Oz [12',13']
AdS/Rindler Correspondence: Caldarelli, Camps, Goutéraux, Skenderis [12',13']
Comparison with AdS/Fluid: Matsuo, Natsuume, Ohta, Okamura [12',13']
Recurrence Relation and Petrov type Cai, Li, Yang, Zhang [13',14']

Rindler Fluid with Weak Momentum Relaxation



$$S_0 = \frac{1}{16\pi G_{p+2}} \int d^{p+2}x\sqrt{-g}\left[R - \frac{1}{2}\sum_{\mathcal{I}=1}^{p}(\partial\phi_{\mathcal{I}})^2\right] - \frac{1}{8\pi G_{p+2}}\int d^{p+1}x\sqrt{-\gamma}K.$$

$$\begin{aligned} ds^2_{p+2} &= -2\kappa_0(r-r_0)dt^2 + 2dtdr + \delta_{ij}dx^i dx^j \\ &\quad - \frac{p}{4}(r-r_0)(r-r_c)k^2dt^2 - \frac{(r-r_c)}{2\kappa_0}k^2\delta_{ij}dx^idx^j + O(k^4), \end{aligned}$$

$$\phi_{\mathcal{I}} = kx_{\mathcal{I}}, \quad x_{\mathcal{I}} = x_i = x_1, x_2, ..., x_p\,.$$

Ward Identity $\quad \partial_t\langle T^t{}_i\rangle + \partial_i \mathbb{p}_k = -\bar{\tau}_0^{-1}\langle T^t{}_i\rangle - (\ell_0)k^2\partial_t v_i + \cdots\,. \quad \bar{\tau}_0^{-1} = \frac{k^2 s_k}{4\pi(\mathbb{e}_k + \mathbb{p}_k)}.$

Thermal Conductivity $\quad \bar{\kappa}_\omega = \frac{1}{1 - \mathbb{i}\omega\tau_0}\frac{4\pi s_k T_k}{k^2}, \qquad \tau_0^{-1} = \frac{k^2}{4\pi T_k}\left[1 - \frac{(\ell_0)T_0}{s_0}\frac{k^2}{T_0^2}\right] + O(k^6)$

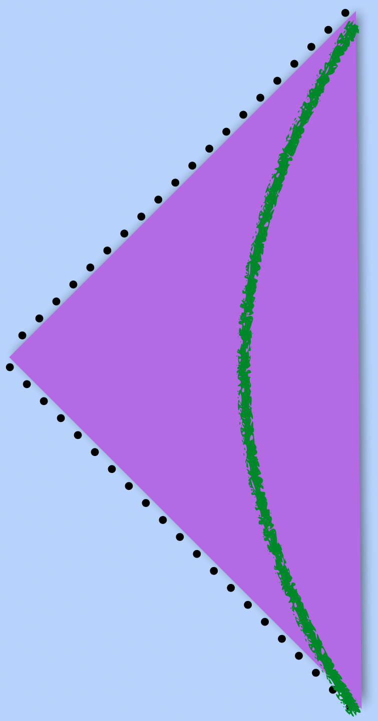
Correction to Relaxation Rate $\quad \ell_0 = -\frac{2}{\mathbb{p}} - \delta\ell_0 = -\frac{1}{\mathbb{p}}, \qquad \xi_0 \equiv \frac{\ell_0 T_0}{s_0} = -1$

"Momentum relaxation from the fluid/gravity correspondence" Blake [15']

"hydrodynamic of transports with momentum relaxation" Hartnoll, Kovtun, Muller, Sachdev[07']

Cutoff AdS Fluid with Momentum Relaxation

Rindler Fluid ← Near Horizon — **Cutoff AdS/Fluid** — Near Boundary → **AdS/CFT Fluid**



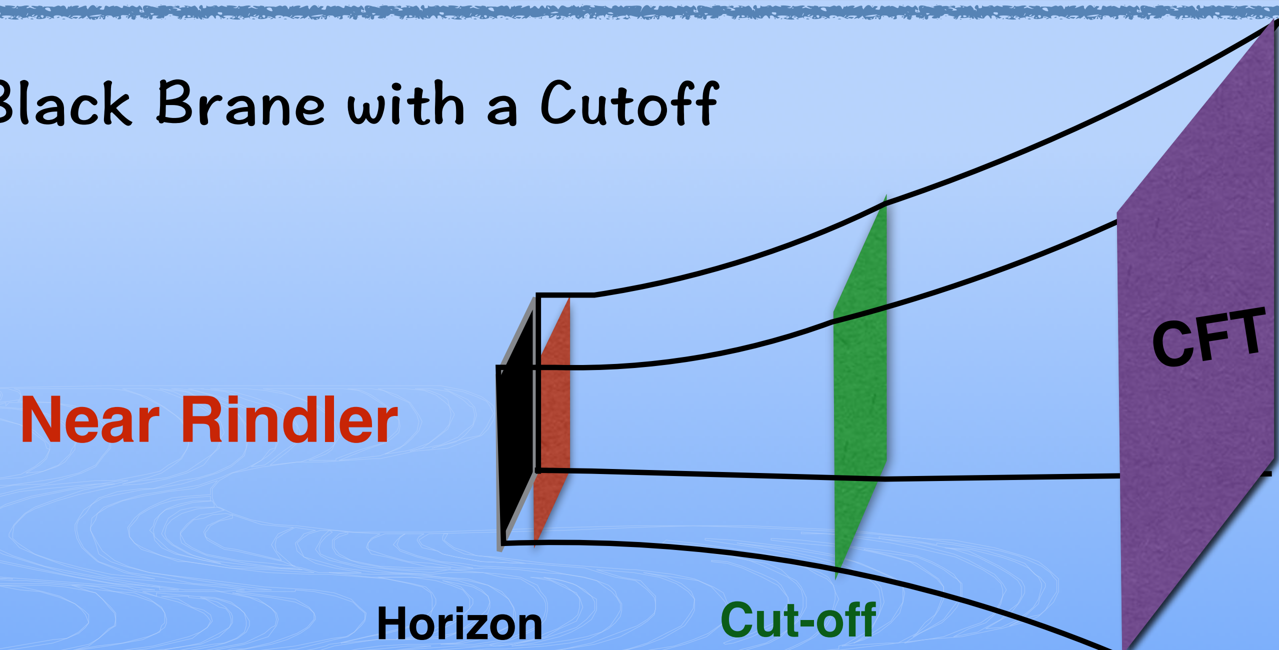
Ward Identity

$$\partial_t \langle \mathcal{T}^t{}_i \rangle + \partial_i \tilde{\mathcal{P}} = -\bar{\tau}_c^{-1} \langle \mathcal{T}^t{}_i \rangle - \ell_c k^2 \partial_t v_i + \cdots , \qquad \bar{\tau}_c^{-1} = \frac{k^2 \tilde{s}_c}{4\pi(\tilde{\mathcal{E}} + \tilde{\mathcal{P}})}.$$

Thermal Conductivity & Relaxation Rate

$$\tilde{\kappa}_\omega = \frac{1}{1 - \mathrm{i}\omega\tau_c} \frac{4\pi \tilde{s}_c \tilde{T}_c}{k^2}, \qquad \tau_c^{-1} = \frac{k^2}{4\pi \tilde{T}_c} \left[1 - \frac{\ell_c T_c}{s_c} \frac{k^2}{T_c^2} \right] + O(k^6).$$

AdS Black Brane with a Cutoff



Sub-Leading Correction

$$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1) \left[\tilde{\xi}_p(r_c) - \frac{r_c \tilde{\xi}'_p(r_c)}{(p-1)} \right],$$

Running From Conformal Fluid to Rindler Fluid

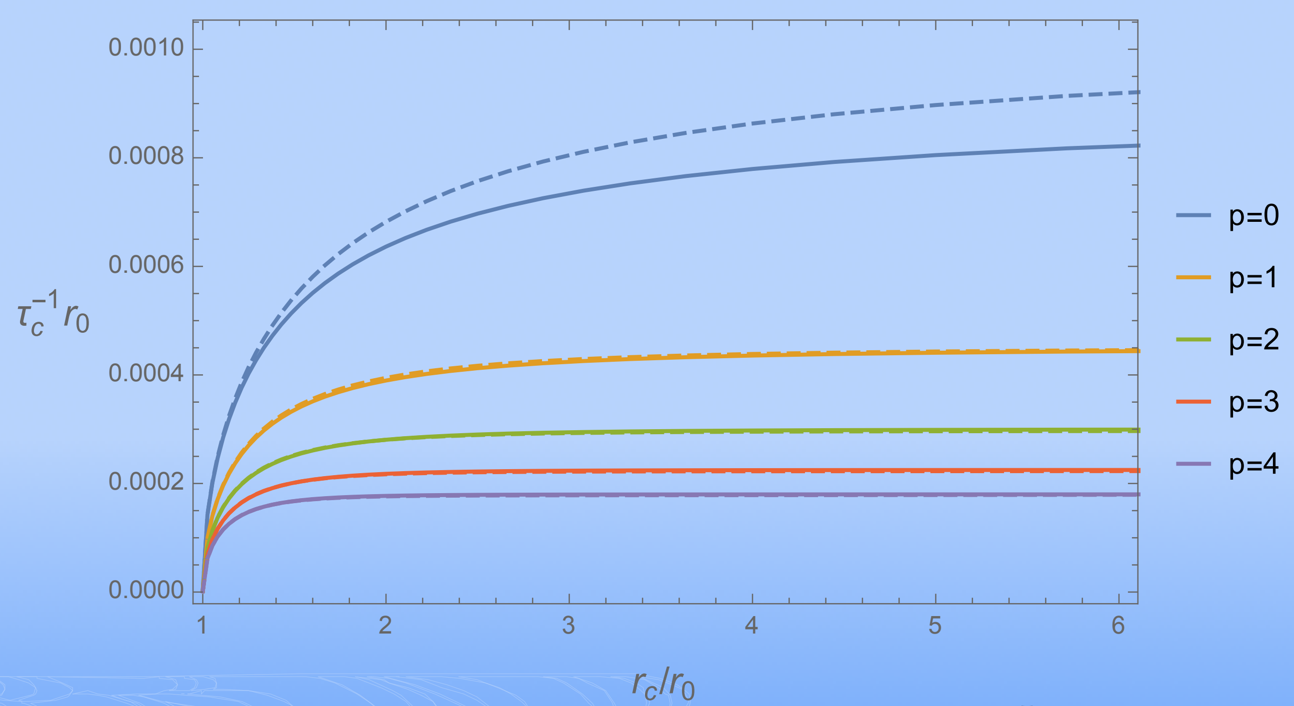
Rindler Fluid ← Near Horizon — **Cutoff AdS/Fluid** — Near Boundary → **AdS/CFT Fluid**

$$\lim_{r_c \to r_0} \xi_c = \xi_0 = -1,$$

$$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1)\left[\tilde{\xi}_p(r_c) - \frac{r_c \tilde{\xi}_p'(r_c)}{(p-1)}\right],$$

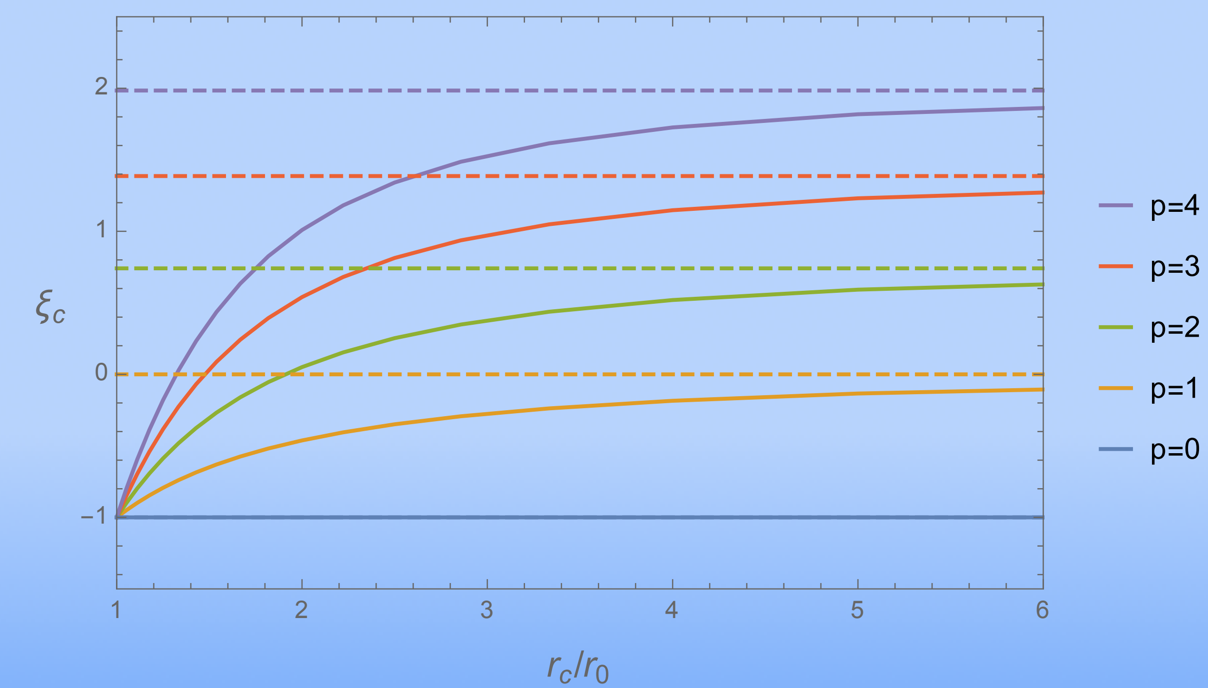
$$\lim_{r_c \to \infty} \xi_c = \xi_\infty = (p+1)\tilde{\xi}_p(\infty).$$

Momentum Relaxation Rate



$$\tau_c^{-1} = \frac{k^2}{4\pi \tilde{T}_c}\left(1 - \xi_c \frac{k^2}{\tilde{T}_c^2}\right), \qquad \xi_c = \frac{\ell_c T_c}{s_c}.$$

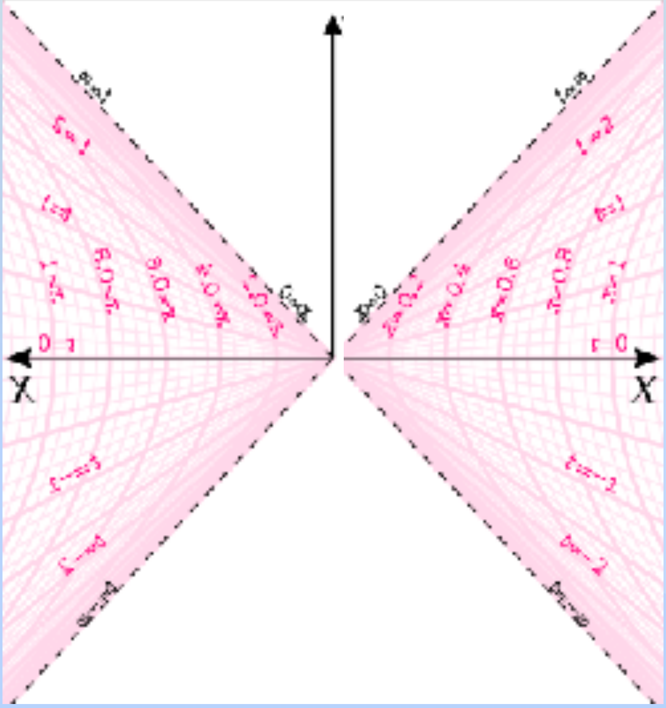
Sub-leading Corrections



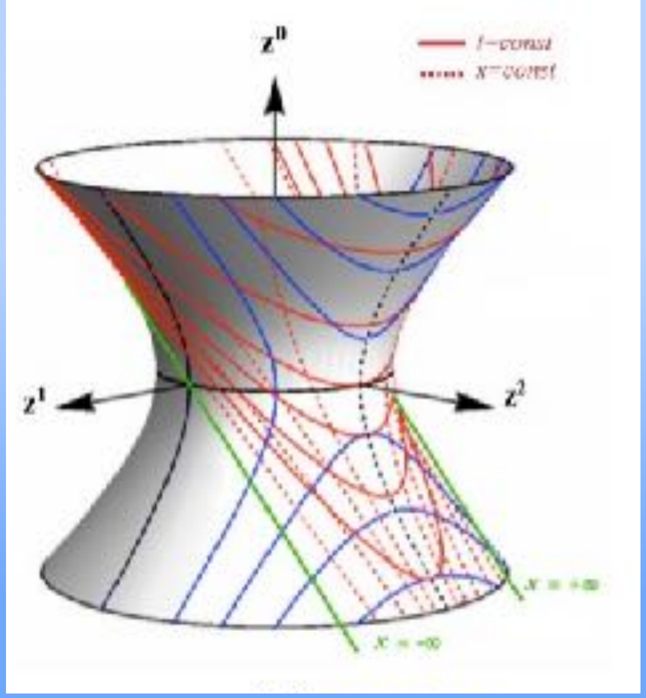
$$\tilde{\xi}_p(r) \equiv \int_{r_0}^{r} \frac{d\tilde{r}\, r_0^2}{\tilde{r}^3 f(\tilde{r})}\left(1 - \frac{r_0^{p-1}}{\tilde{r}^{p-1}}\right).$$

Holographic Screens in Flat Spacetime
— Rindler Screen & de-Sitter Screen

I. Holographic Rindler Fluid
— Accelerating Screen
— Relation to AdS/CFT



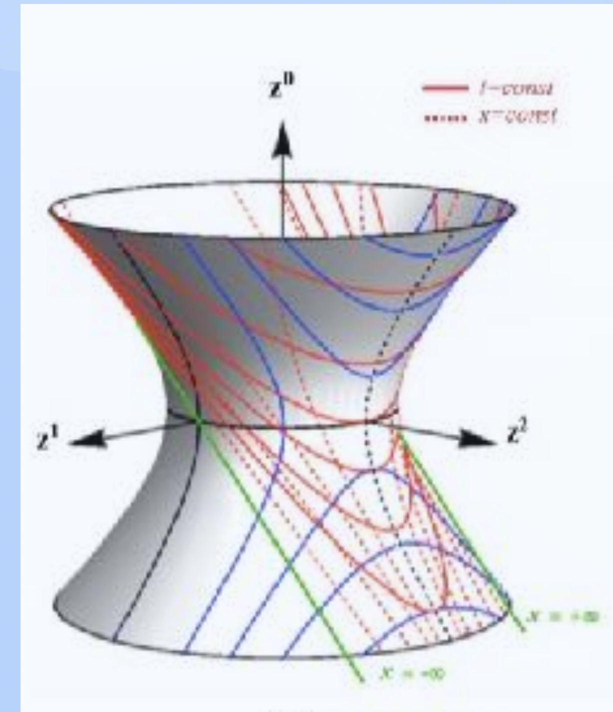
II. Holographic dS Fluid
— de-Sitter&FRW Screen
— Relation to DGP brane?



II. Holographic dS Universe? — de-Sitter Screen

1) Holographic Stress Tensor — Dark Sectors

$$\mathcal{T}_{\mu\nu} \equiv -\frac{H_0 c^3}{8\pi G}\left(\mathcal{K} g_{\mu\nu} - \mathcal{K}_{\mu\nu}\right).$$



Modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{H_0}{c}\left(\mathcal{K} g_{\mu\nu} - \mathcal{K}_{\mu\nu}\right) = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

Hamiltonian constraints

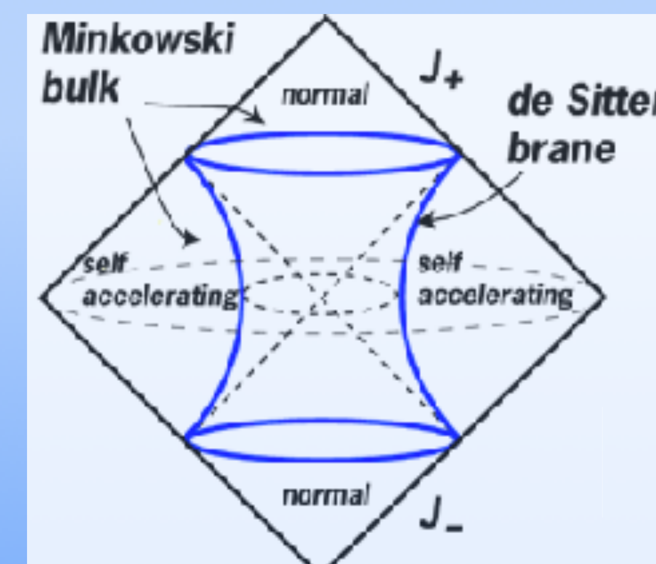
$$\mathcal{K}^2 - \mathcal{K}_{\mu\nu}\mathcal{K}^{\mu\nu} = R + 2\, G^{(d+1)}_{MN} \mathcal{N}^M \mathcal{N}^N,$$

2) Embedding in higher dimensions — Brane World (DGPs)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \mathcal{T}^{\mathcal{M}}_{\mu\nu} + T^{B}_{\mu\nu},$$

$$\mathcal{T}^{\mathcal{M}}_{\mu\nu} \equiv (\mathcal{K} g_{\mu\sigma} - \mathcal{K}_{\mu\sigma})\mathcal{K}^{\sigma}{}_{\nu} + \mathcal{M}_{\mu\nu} - \frac{1}{2}\left(\mathcal{K}^2 - \mathcal{K}_{\rho\sigma}\mathcal{K}^{\rho\sigma}\right) g_{\mu\nu},$$

$$\mathcal{M}_{\mu\nu} \equiv g_{\mu}{}^{M} g_{\nu}{}^{N} R^{(d+1)}_{MN} - g_{\mu}{}^{M} \mathcal{N}^P g_{\nu}{}^{N} \mathcal{N}^Q R^{(d+1)}_{MPNQ}.$$



Ref: 1106.2476

Compare with Verlinde's Emergent Universe

Gravitational quantity		Elastic quantity		Correspondence
Newtonian potential	Φ	displacement field	u_i	$u_i = \Phi n_i / a_0$
gravitational acceleration	g_i	strain tensor	ε_{ij}	$\varepsilon_{ij} n_j = -g_i / a_0$
surface mass density	Σ_i	stress tensor	σ_{ij}	$\sigma_{ij} n_j = \Sigma_i a_0$
mass density	ρ	body force	b_i	$b_i = -\rho\, a_0 n_i$
point mass	m	point force	f_i	$f_i = -m\, a_0 n_i$

Holographic Universe vs. Emergent Universe?

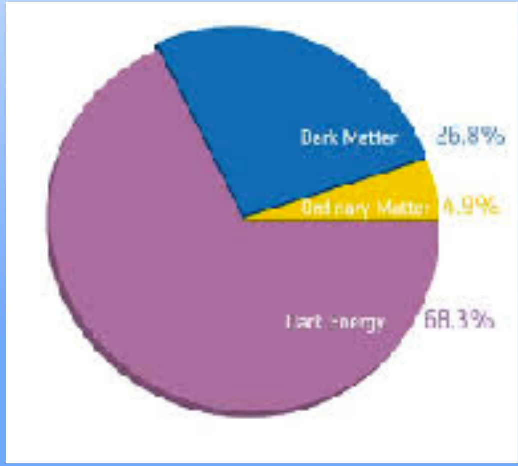
$$\frac{\mathcal{T}^2}{d-1} - \mathcal{T}_{\mu\nu}\mathcal{T}^{\mu\nu} = -\frac{\rho_\Lambda c^2}{d-1}(T + \mathcal{T}).$$

Constrain Equations

$$\Delta_V \equiv \Omega_D^2 - \frac{4}{3}\Omega_B \simeq 0.36\%\,,$$

$$\Delta_{CSZ} \equiv \Omega_D^2 - \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B) \simeq -0.34\%\,.$$

Ref: R.G. Cai, S. Sun, Y.L. Zhang, to appear



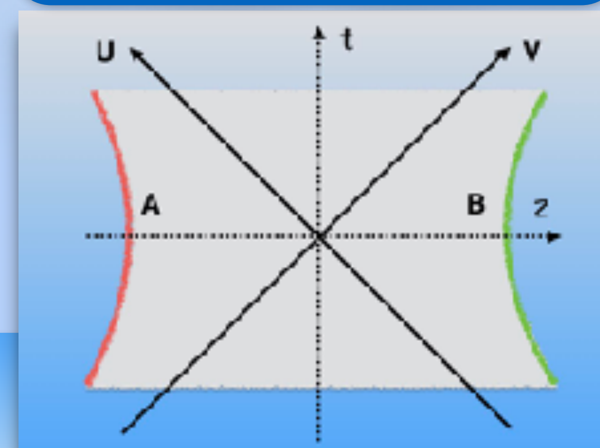
LCDM Universe? $H(a)^2 = H_0^2\left[\Omega_\Lambda + (\Omega_D + \Omega_B)a^{-3} + \Omega_R a^{-4}\right]$

Relevant Topics of Rindler Horizon

Charged Rindler Fluid
Chaos & Butterfly Effects?

Khimphun, Lee, Park, Zhang, to appear

Holography in Flat Spacetime

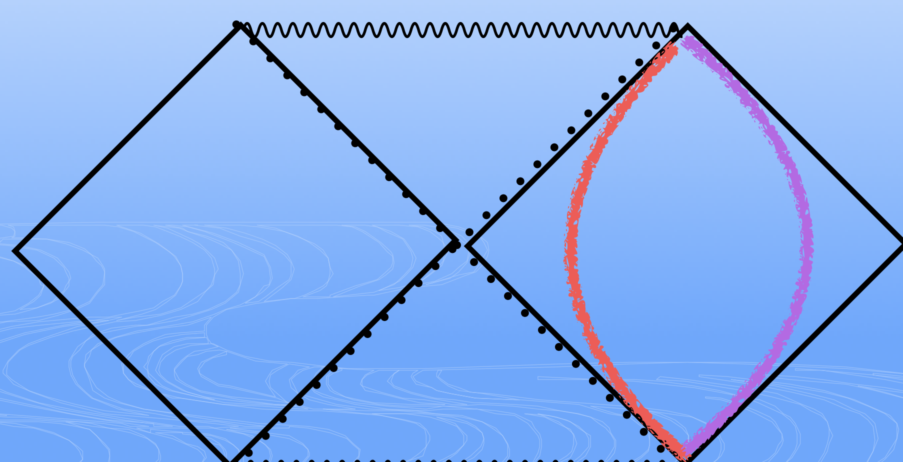


NAdS_2/SYK Models
WorldSheet Horizon?

Cai, Ruan, Yang, Zhang, 1709.06297

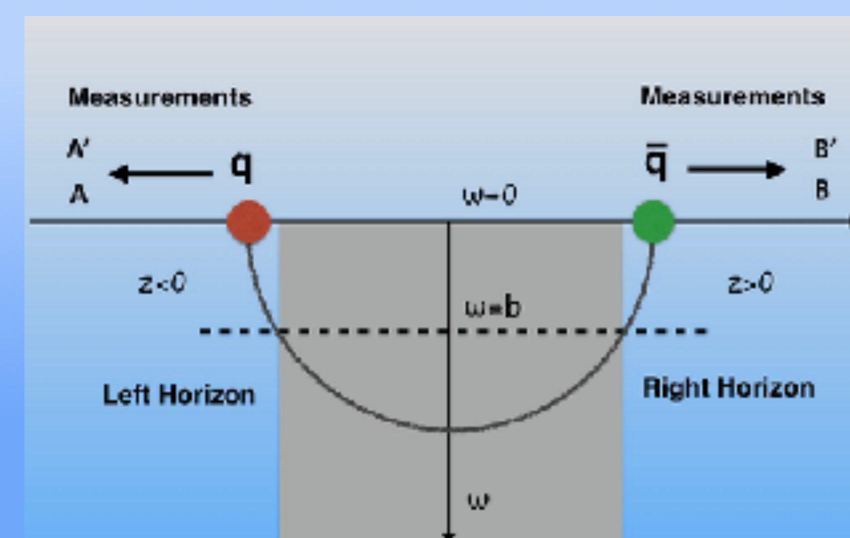
Schwarzschild Black Hole
Membranes/Soft Hairs?

Cai, Ruan, Zhang, 1609.01056



Accelerating EPR Pair
Bell's Measurement?

Chen, Sun, Zhang, 1612.09513

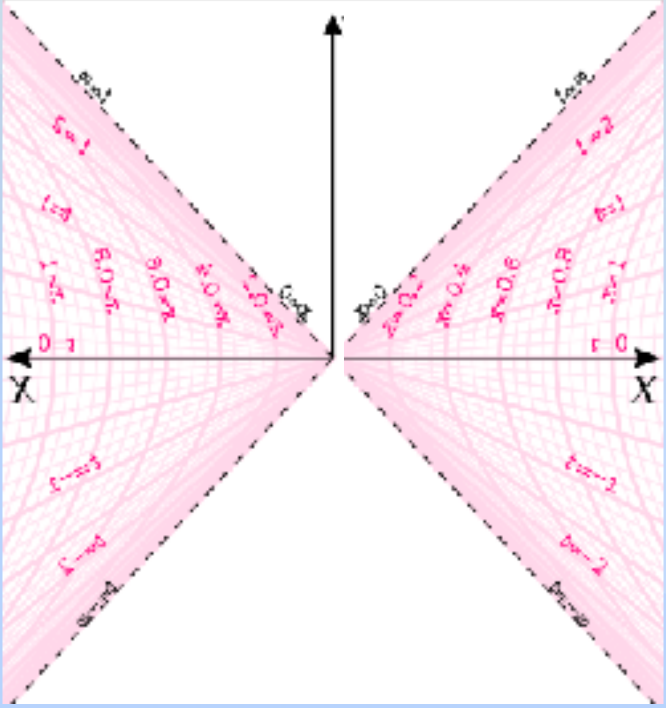


Summary and Discussions

Holographic Rindler Fluid

Accelerating Screen in Flat Spacetime

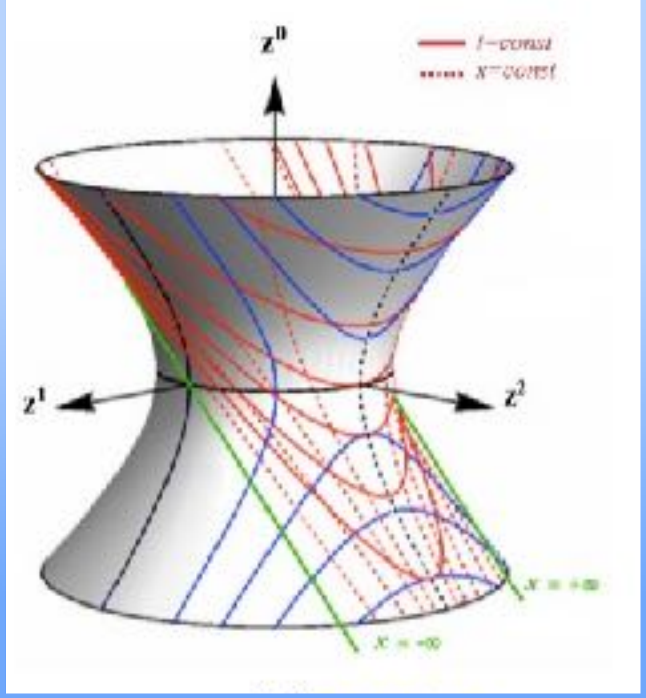
From Conformal Fluid to Rindler Fluid



Holographic de-Sitter Fluid

de-Sitter & FRW Screen

Relation to DGP brane world Models



Thanks for All Your Attention!