YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Nonminimally Coupled Scalar Field in R^2-inflation

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Based on Collaboration with
Yingli Zhang, Qing-guo Huang, and Misao Sasaki
(in preparation)

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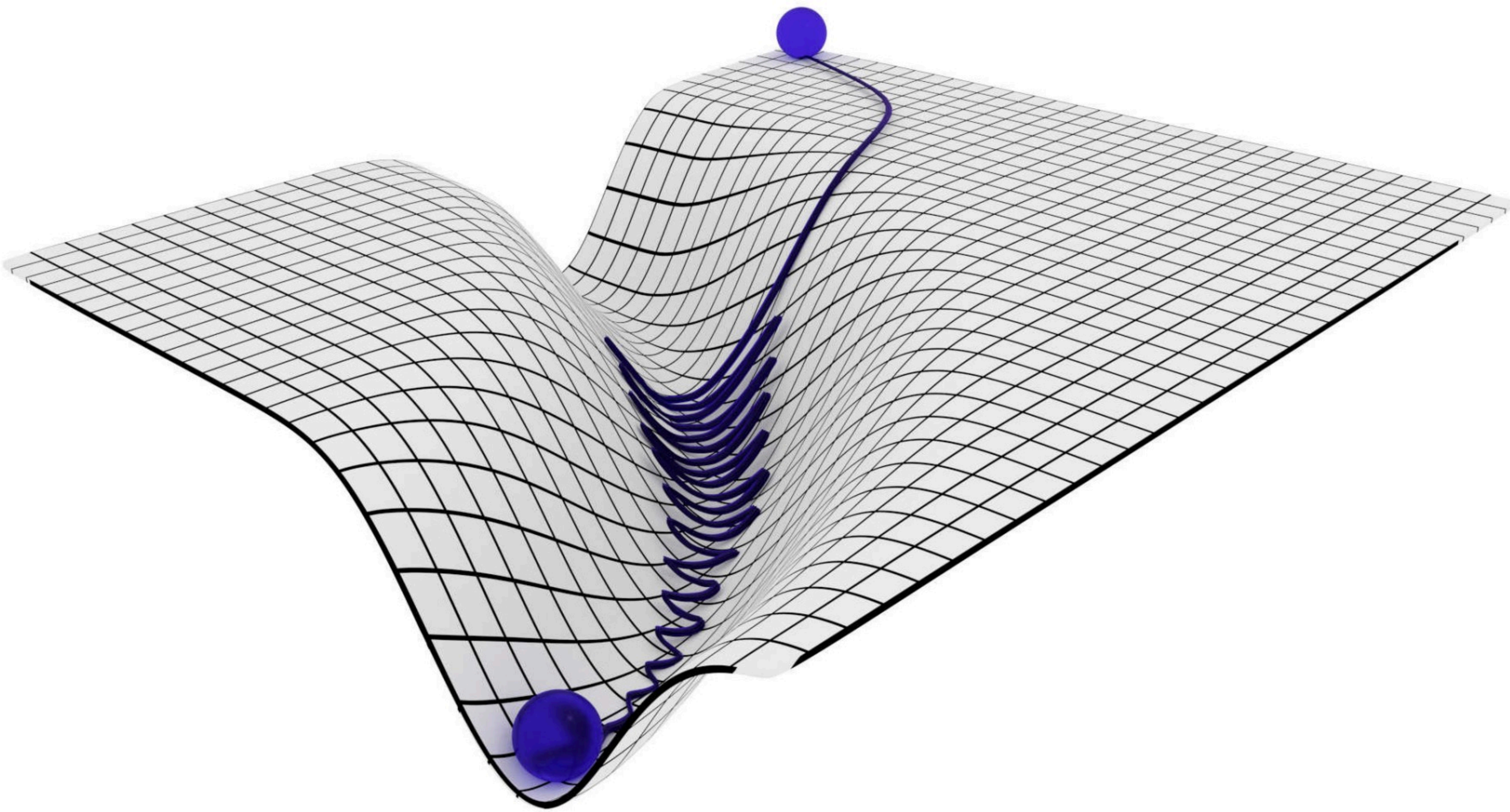
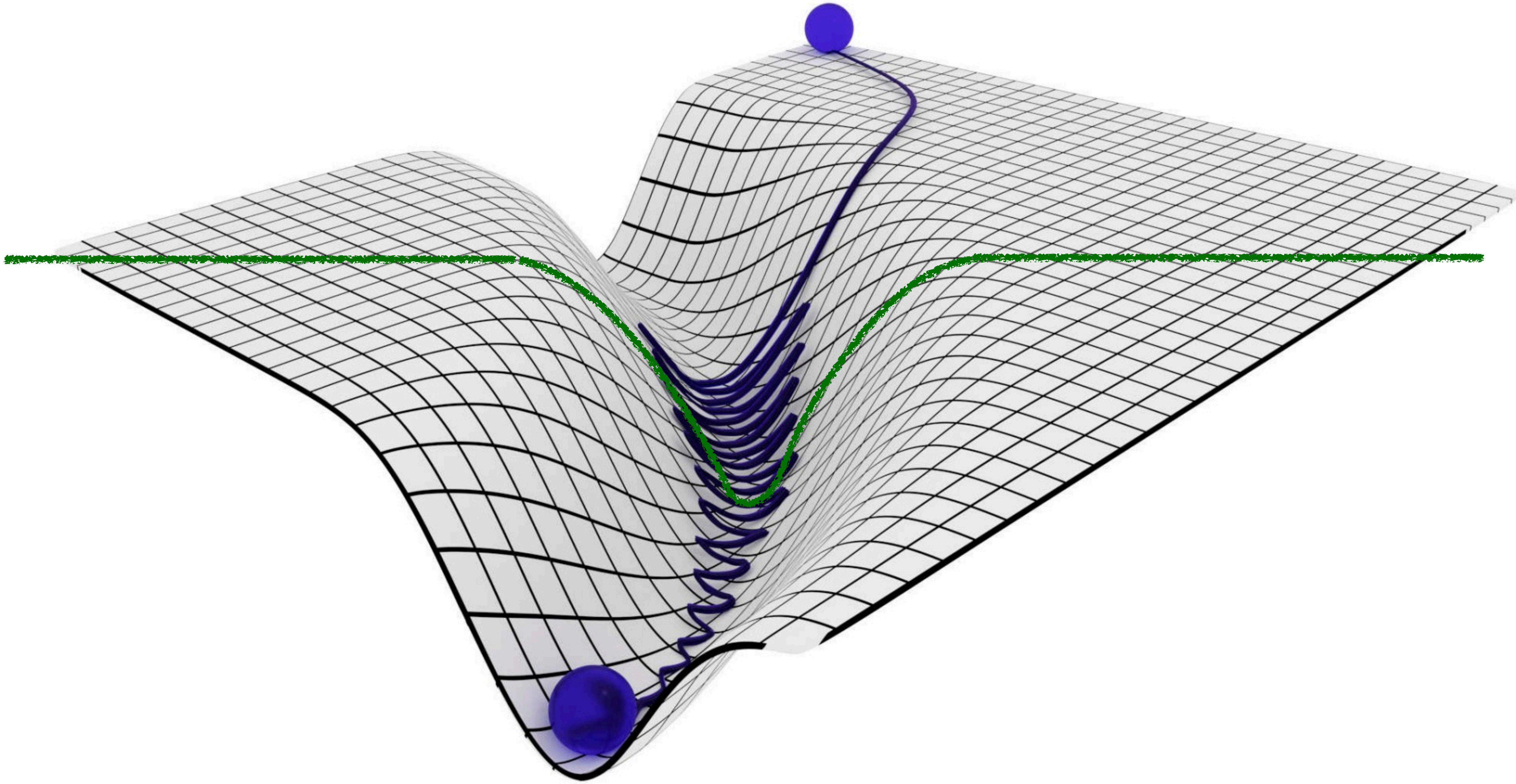
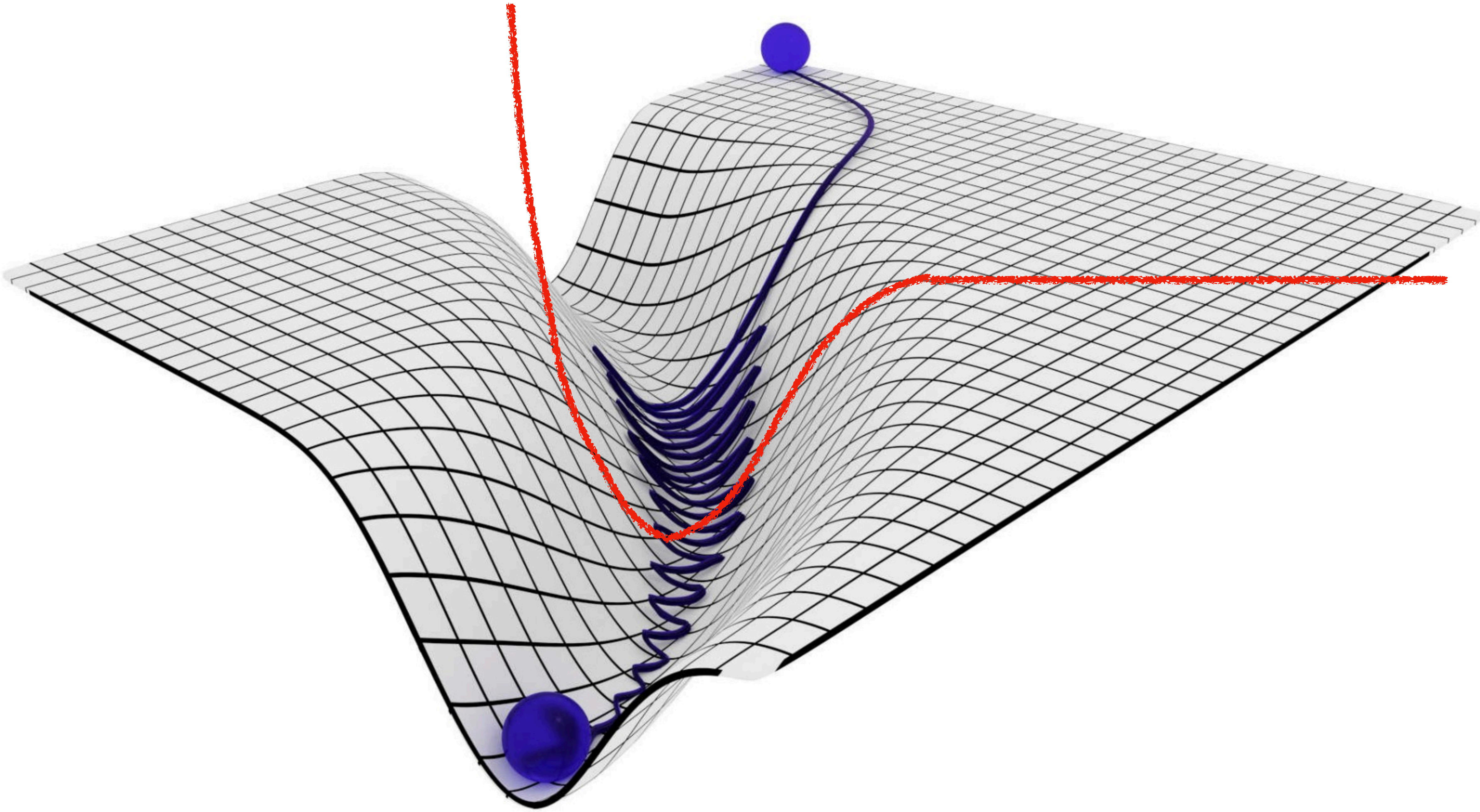


- Typical potential for ‘inflation + massive field’, where oscillatetions will be generated. (Borrowed from Chen, Namjoo & Wang 2015, see also Yi Wang’s talk)



- Chen, Namjoo & Wang 2015:

$$\sim (1 - e^{-\phi^2/M^2})$$

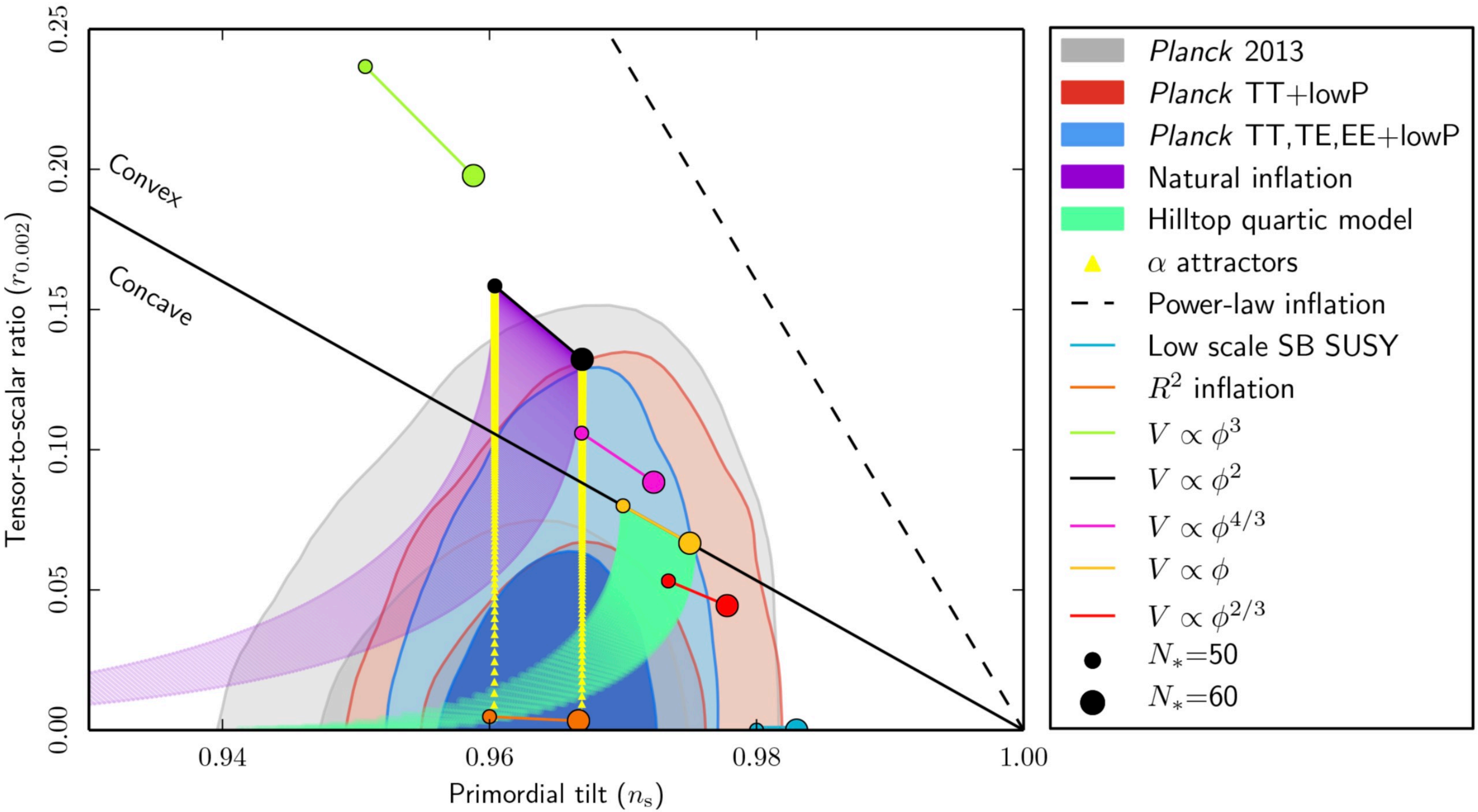


- Our model: (see also Minxi He's talk; Mori, Kohri & White 2017)

$$\sim \left(1 - e^{-\phi/M_{\rm Pl}}\right)^2$$

Motivation

- A natural way to realize it is just R^2 gravity plus inflaton.
- R^2 gravity itself can generate inflation. (Starobinsky 1980)
- Also, it is the best-fit inflation model. (Planck 2015)
- It is equivalent to study the scalar field(s) in Starobinsky model.



0.25
0.20
0.15
0.10
0.05
0.00
Tensor-to-scalar ratio ($r_{0.002}$)
0.94
0.96
0.98
1.00
Primordial tilt (n_s)
Convex
Concave
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Motivation

- A natural way to realize it is just R^2 gravity plus inflaton.
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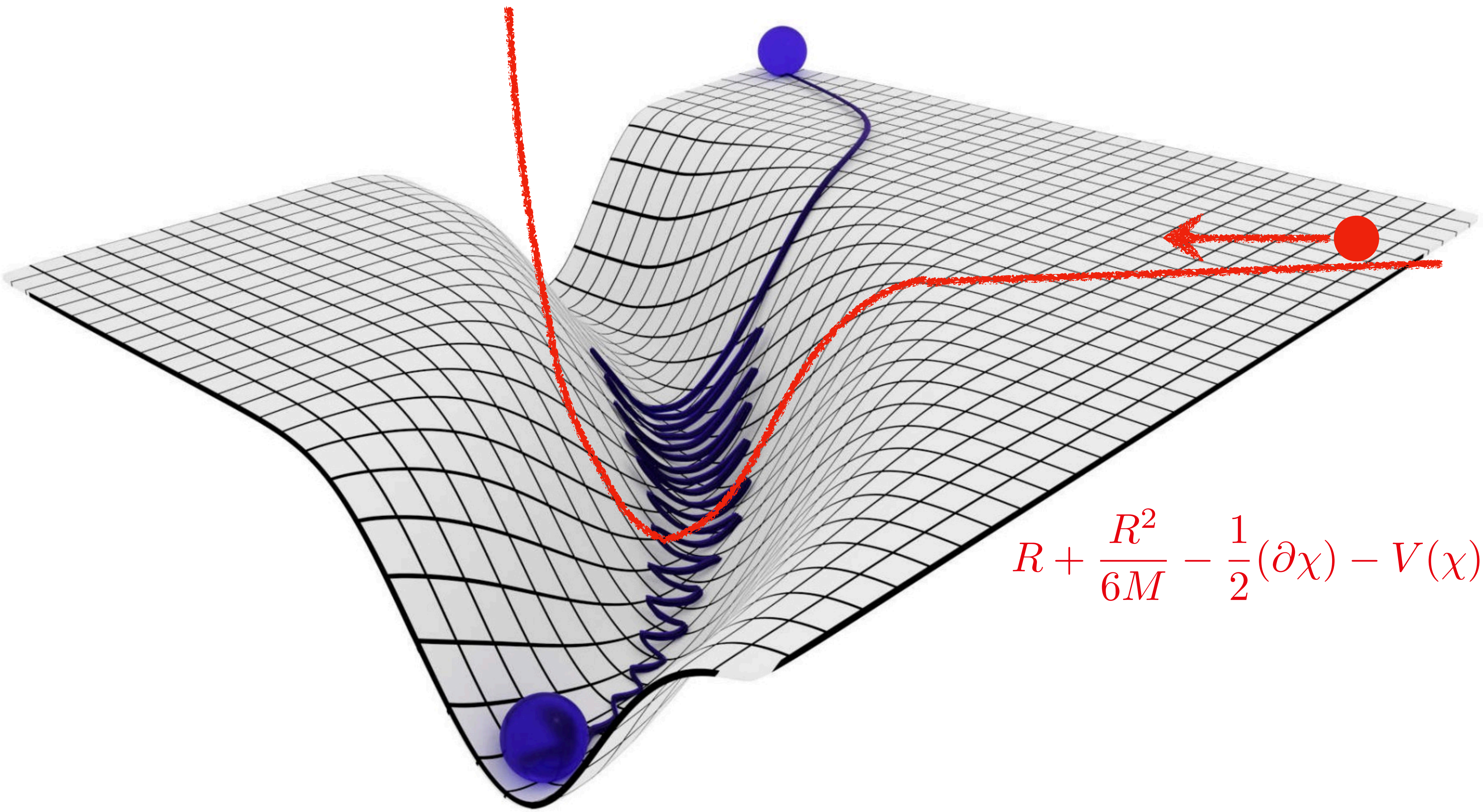
- It is equivalent to study the scalar field(s) in Starobinsky model.


$R + \frac{R^2}{6M} - \frac{1}{2}(\partial\chi) - V(\chi)$

Contents

- Introduction: Motivations
- Set up and calculation
- Results and applications
- Conclusion

Setup

- We propose the Lagrangian as the Starobinsky R^2 gravity plus a scalar field χ, nonminimally coupled to gravity

$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)-\frac{1}{2}\xi R\chi^2\right\}.$$

- Recall Minxi He's talk if χ is explained as Higgs boson.

- $V(\chi)$ is potential for χ, which we pick for the small-field form: $V(\chi)=V_0-(1/2)m^2\chi^2+\ldots$. (Recall Yipeng Wu's talk for a similar potential from SSB)

- ξ-term is the non-minimally coupled term to solve the initial condition problem. Another version of SSB in χ direction.

Setup

- It has been proved that the action with R^2 is equivalent to Einstein-Hilbert action plus one scalar field (scalaron). (Whitt 1984, Maeda 1988)
- After transferred to Einstein frame, our model becomes Hilbert Einstein action with two scalar fields: scalaron ϕ + light field χ, with nontrivial metric in field space (Mizuno's talk)

$$S_E = \int d^4x\sqrt{-\tilde{g}} \cdot \left\{ \frac{M_{\rm Pl}^2}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi \right.$$
$$\left. - \frac{3}{4}M^2M_{\rm Pl}^2\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}} + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V(\chi)\right\}.$$

EoM

- The equations of motion:

$$3M_{\rm Pl}^2H^2=\frac{1}{2}\dot{\phi}^2+F^{-1}\frac{1}{2}\dot{\chi}^2+\frac{3}{4}M^2M_{\rm Pl}^2\left[1-F^{-1}\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]^2+F^{-2}V(\chi),$$

$$\ddot{\phi}+3H\dot{\phi}+\sqrt{\frac{3}{2}}M^2M_{\rm Pl}F^{-1}\left\{1-\xi\frac{\chi^2}{M_{\rm Pl}^2}+\frac{\dot{\chi}^2}{3M^2M_{\rm Pl}^2}-F^{-1}\left[\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)^2+\frac{4V}{3M^2M_{\rm Pl}^2}\right]\right\}=0,$$

$$\ddot{\chi}+\left(3H-\sqrt{\frac{2}{3}}\frac{\dot{\phi}}{M_{\rm Pl}}\right)\dot{\chi}+3M^2\left[1-F^{-1}\left(1-\xi\frac{\chi^2}{M_{\rm Pl}^2}\right)\right]\xi\chi+F^{-1}V'(\chi)=0,$$

- with $F=\exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)$ and $V(\chi)=V_0-\frac{1}{2}m^2\chi^2+\cdots\ ,$

Slow-roll EoM

- The slow-roll version of equations of motion:

$$3H\dot{\phi} = -\sqrt{\frac{3}{2}}e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(1 + \frac{4}{\mu^2}\right)\right)M^2 M_{\rm Pl},$$

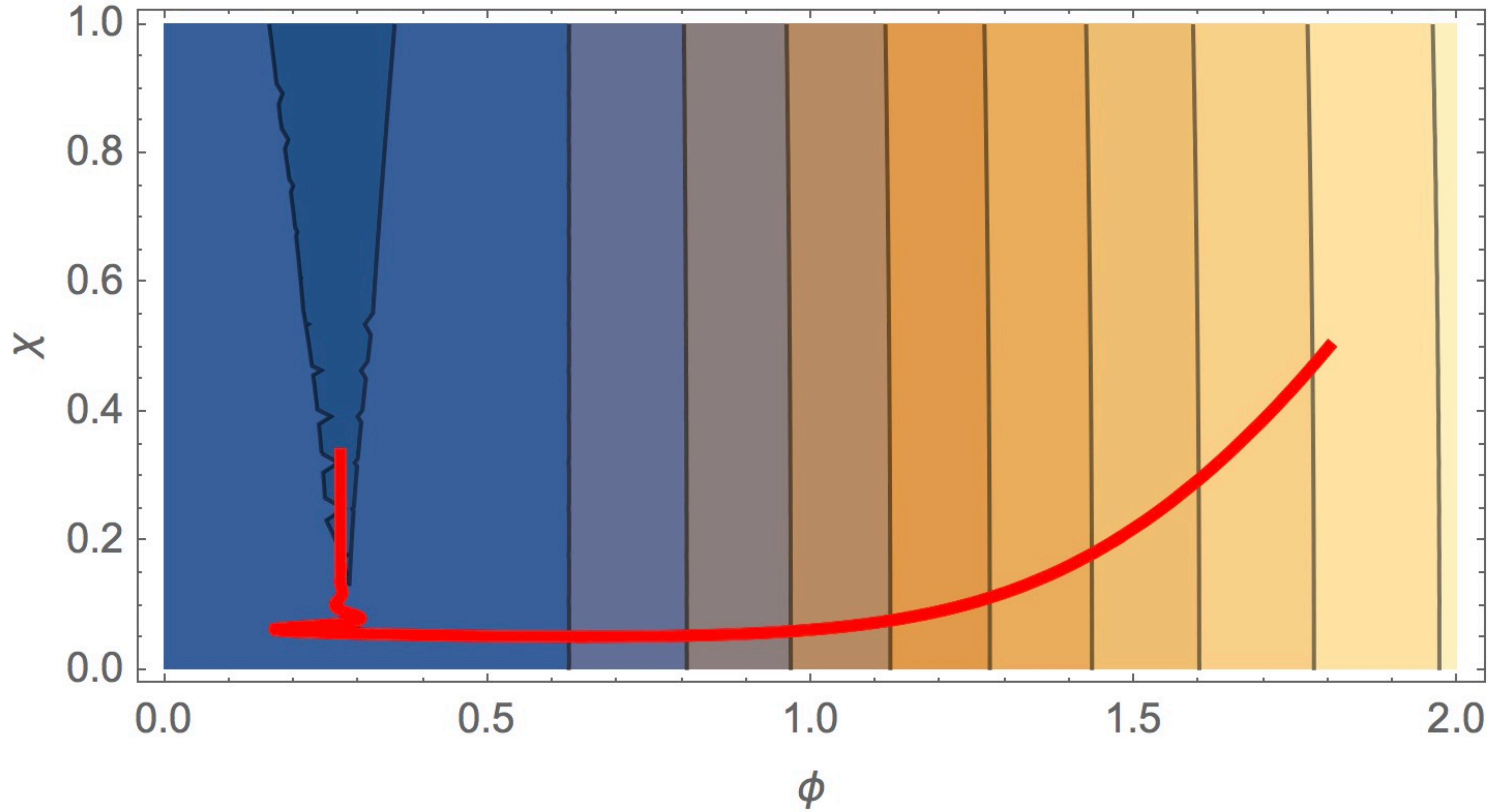
$$\left(3H - \sqrt{\frac{2}{3}}\frac{\dot{\phi}}{M_{\rm Pl}}\right)\dot{\chi} + 3M^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left[\xi\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}} - 1\right) - \frac{m^2}{3M^2}\right]\chi = 0.$$

- We have defined an important mass parameter

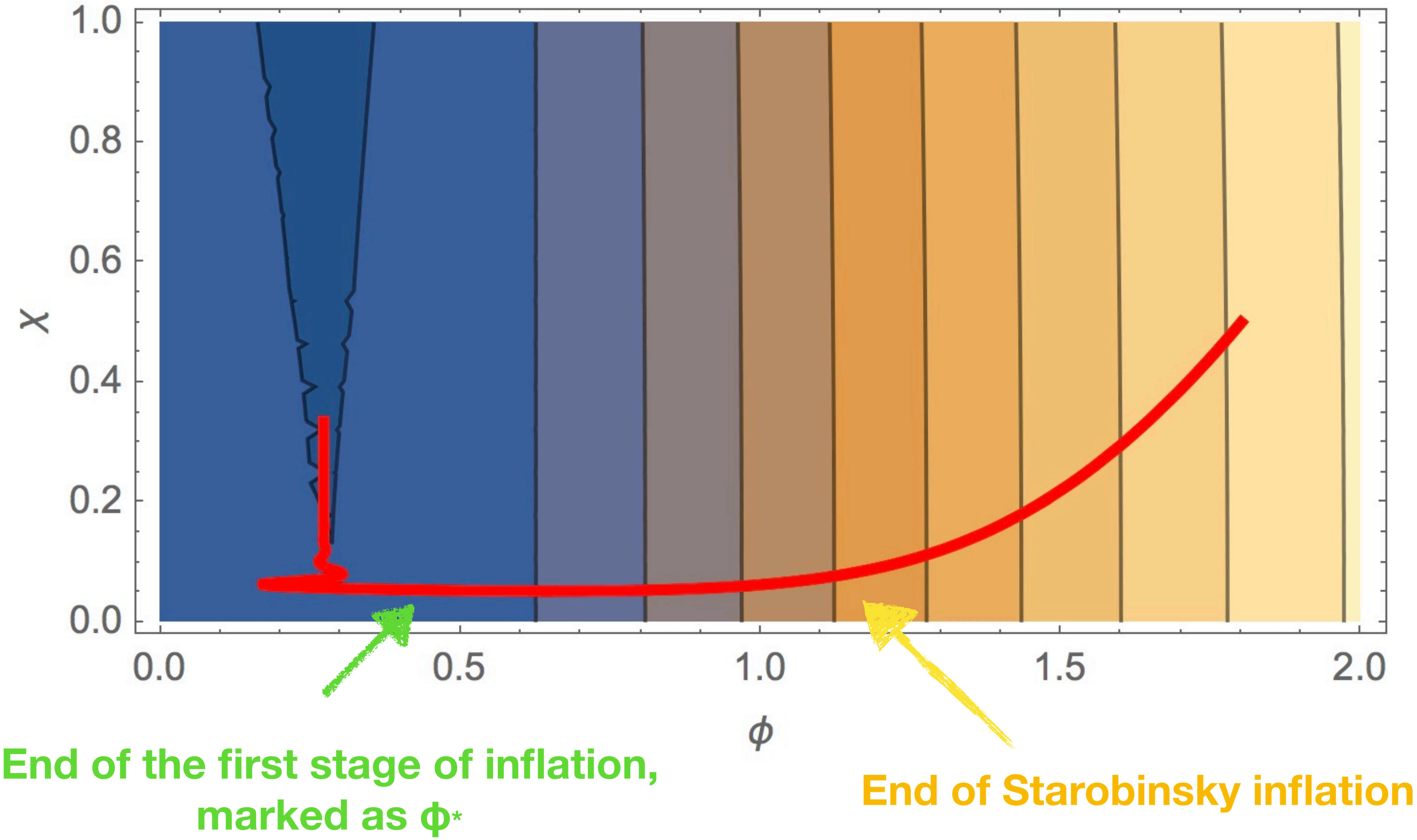
$$\mu^2 \equiv \frac{3M^2 M_{\rm Pl}^2}{V_0},$$

- which is the mass M measured by H at ϕ =0.

- The relations in ϕ go back to Starobinsky model for $\mu \to \infty$



1.0
0.8
0.6
0.4
0.2
0.0
χ
0.0
0.5
1.0
1.5
2.0
ϕ



- We have the ``later end of inflation”, for V_0 can become important at the end of the first stage of inflation.

Some Conditions

- Late end of first stage: $F_* = 1.18 + \frac{1.92}{\mu}$ (Starobinsky model $F_* \approx 2.6$)
- If μ is not too large, the transition between two stages does not violate the inflation. (see Polarski & Starobinsky 1992)
- We will focus on this range, for $2<\mu<8.95$.
- To solve the initial conditions, we require ξ to be positive and small: $\xi < m^2/M^2$.
- If ξ is too small, the initial condition for the small field inflation for χ will again arise. So in our model we require ξ is not too smaller than $O(m^2/M^2)$.
- In the first stage, ϕ dominates inflation, and the curvature perturbation can be calculated by δN formalism. (Sasaki & Stewart 1995)

Power Spectrum in the First Stage

- We use δN formalism to calculate the power spectrum in the first stage

$$P_\zeta = N_{,\phi}^2 \langle \delta\phi^2 \rangle = \frac{3H^2}{32\pi^2 M_{\rm Pl}^2} \left(\frac{1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)}{F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)} \right)^2$$

$$= \frac{3M^2}{128\pi^2 M_{\rm Pl}^4} \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

$$= \frac{V_0}{24\pi^2 M_{\rm Pl}^4} \left(\frac{3}{16}\mu^2 \right) \frac{\left(1 - 2F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^3}{\left(F^{-1} + F^{-2}\left(1 + 4/\mu^2\right)\right)^2},$$

- With the slow-roll parameters

$$\epsilon_H^{(1)} = \frac{8}{3} \frac{\left(F - \left(1 + 4/\mu^2\right)\right)^2}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2},$$

$$\eta_H^{(1)} = \frac{8}{3} \frac{F\left(F^2 - 2F\left(1 + 4/\mu^2\right) + \left(1 + 4/\mu^2\right)\right)}{\left(F^2 - 2F + \left(1 + 4/\mu^2\right)\right)^2}$$

Transition to the Second Stage

- After ϕ stops slow-rolling, it becomes a ``heavy field", and we can use the EFT method to integrate it out. (Tolley & Wyman 2009, Achucarro, Gong, Hardeman, Palma & Patel 2010.)

- The ϕ field goes to a ``gelaton" solution

$$\frac{\phi_g}{M_{\rm Pl}} = \sqrt{\frac{3}{2}} \ln \frac{\left(1 - \xi \frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V(\chi)}{3M^2 M_{\rm Pl}^2}}{1 - \xi \frac{\chi^2}{M_{\rm Pl}^2} + \frac{2X}{3M^2 M_{\rm Pl}^2}}.$$

- And the effective action at this trajectory is

$$S_g = \int d^4x \sqrt{-g} \left\{ \frac{M_{\rm Pl}^2}{2} R + \frac{X\left(1 - \xi \frac{\chi^2}{M_{\rm Pl}^2}\right) + \frac{X^2}{3M^2 M_{\rm Pl}^2} - V(\chi)}{\left(1 - \xi \frac{\chi^2}{M_{\rm Pl}^2}\right)^2 + \frac{4V(\chi)}{3M^2 M_{\rm Pl}^2}} \right\}.$$

Second Stage

- The inflation is now dominated by the scalar field χ, and the background evolution can be easily solved as

$$\chi = \chi_* \exp\left[-\frac{m^2 M_{\rm Pl}^2}{V_0}(N - N_*)\right] = \chi_* e^{-\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

$$\epsilon_H^{(2)} \equiv -\frac{\dot{H}_g}{H_g^2} = \frac{1}{2M_{\rm Pl}^2}\left(\frac{\partial \chi}{\partial N}\right)^2 \frac{1}{1+4/\mu^2} = \frac{m^4 M_{\rm Pl}^2}{2V_0^2}\frac{\chi^2}{1+4/\mu^2} = \frac{\eta_H^{(2)\,2}}{8}\frac{(\chi/M_{\rm Pl})^2}{1+4/\mu^2}.$$

$$\eta_H^{(2)} \equiv \frac{\dot{\epsilon}_H^{(2)}}{H_g \epsilon_H^{(2)}} = -\frac{\partial \ln \epsilon_H^{(2)}}{\partial N} = \frac{2m^2 M_{\rm Pl}^2}{V_0}.$$

Second Stage

- But we know that ϕ does not lie on ϕ_g from the beginning: it rolls down to it from the Starobinsky plateau.

- The evolution of ϕ is just a classical perturbation to the ``gelaton" trajectory ϕ_g: $\phi = \phi_g + \Delta\phi$

- And the oscillation of ϕ can be solved as perturbations to the EFT solution.

$$\ddot{\Delta\phi} + 3H\dot{\Delta\phi} + \sqrt{\frac{3}{2}} M^2 M_{\rm Pl} \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}}\right) e^{-\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}} \frac{\left(1 + \frac{2X}{3M^2 M_{\rm Pl}^2}\right)^2}{1 + \frac{4V}{3M^2 M_{\rm Pl}^2}} = 0.$$

- After that we can linearize it and find its solution.

Second Stage

- The solution is

$$\frac{\Delta\phi}{M_{\rm Pl}} = e^{\frac{3}{2}(N-N_*)}\sqrt{\frac{3}{2}}\ln\frac{F_*(\mu)}{1+\frac{4}{\mu^2}}\sqrt{1+\Upsilon^2}\cos\left[\left(\frac{\mu^2}{1+4/\mu^2}-\frac{9}{4}\right)^{\frac{1}{2}}(N-N_*)+\arctan\Upsilon\right],$$

$$\Upsilon = \left(\frac{\mu^2}{1+4/\mu^2}-\frac{9}{4}\right)^{-\frac{1}{2}}\left[\frac{3}{2}-\frac{4}{3}\frac{F_*-1-4/\mu^2}{F_*^2-2F_*+1+4/\mu^2}\left(\ln\frac{F_*}{1+4/\mu^2}\right)^{-1}\right]$$

- There is oscillation only for $\mu \gtrsim 2.08$

- There is also an upper bound for not violate inflation during the transition: $\mu \lesssim 8.95$

Power Spectrum In the Second Stage

- Since ϕ is a heavy field, its perturbations are exponentially suppressed.

- We can use δN formalism to calculate the power spectrum in the second stage, mainly contributed by the quantum fluctuations of χ.

- The dependence of e-folding number can be calculated by its slow-roll EoM, which is dynamically coupled to $\Delta\phi$.

$$3H\left(1-\frac{1}{3}\sqrt{\frac{2}{3}}\frac{\dot{\phi}}{HM_{\rm Pl}}\right)\dot{\chi}+e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}V'(\chi)=0.$$

Power Spectrum In the Second Stage

$$\frac{\partial N}{\partial \chi} = \frac{2}{\eta_H^{(2)}\chi}\left(1+\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}+\frac{1}{3}\sqrt{\frac{2}{3}}\frac{\partial}{\partial N}\frac{\Delta\phi}{M_{\rm Pl}}\right).$$

$$\langle\delta\chi\delta\chi\rangle = F\langle\delta\hat{\chi}\delta\hat{\chi}\rangle = e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\left(\frac{H}{2\pi}\right)^2 \approx \left(1+\frac{4}{\mu^2}\right)\left(1+\sqrt{\frac{2}{3}}\frac{\Delta\phi}{M_{\rm Pl}}\right)\left(\frac{H}{2\pi}\right)^2.$$

$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \left(\frac{\partial N}{\partial\chi}\right)^2\langle\delta\chi\delta\chi\rangle,\\ &= \mathcal{P}_{\mathcal{R}}^{(0)}\left\{1-\frac{2}{3}e^{\frac{3}{2}(N-N_*)}\ln\frac{F_*}{1+\frac{4}{\mu^2}}\sqrt{1+\Upsilon^2}\Big[\omega\sin\left(\omega(N-N_*)+\tan^{-1}\Upsilon\right)\right.\\ &\left.-\ 6\cos\left(\omega(N-N_*)+\tan^{-1}\Upsilon\right)\Big]\right\}^2,\end{aligned}$$

Power Spectrum In the Second Stage

$$\mathcal{P}_{\mathcal{R}}^{(0)} \equiv \frac{H^2}{8\pi\epsilon_H^{(2)} M_{\rm Pl}^2} \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^2} \left(\frac{M_{\rm Pl}}{\chi_*}\right)^2 \frac{8}{\eta_H^{(2)2}} e^{\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

$$= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2} \Big[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) - 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right)\Big]\right\}^2,$$

Power Spectrum In the Second Stage

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Large Enhancement

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

$$= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{ 1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2} \Big[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) - 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) \Big] \right\}^2,$$

Power Spectrum In the Second Stage

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Large Enhancement

$$\omega \equiv \sqrt{\frac{\mu^2}{1+4/\mu^2} - \frac{9}{4}}$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

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Power Spectrum
In the Second Stage

$$\mathcal{P}_{\mathcal{R}}^{(0)} \equiv \frac{H^2}{8\pi\epsilon_H^{(2)} M_{\rm Pl}^2} \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^2}\left(\frac{M_{\rm Pl}}{\chi_*}\right)^2 \frac{8}{\eta_H^{(2)2}} e^{\frac{\eta_H^{(2)}}{2}(N-N_*)},$$

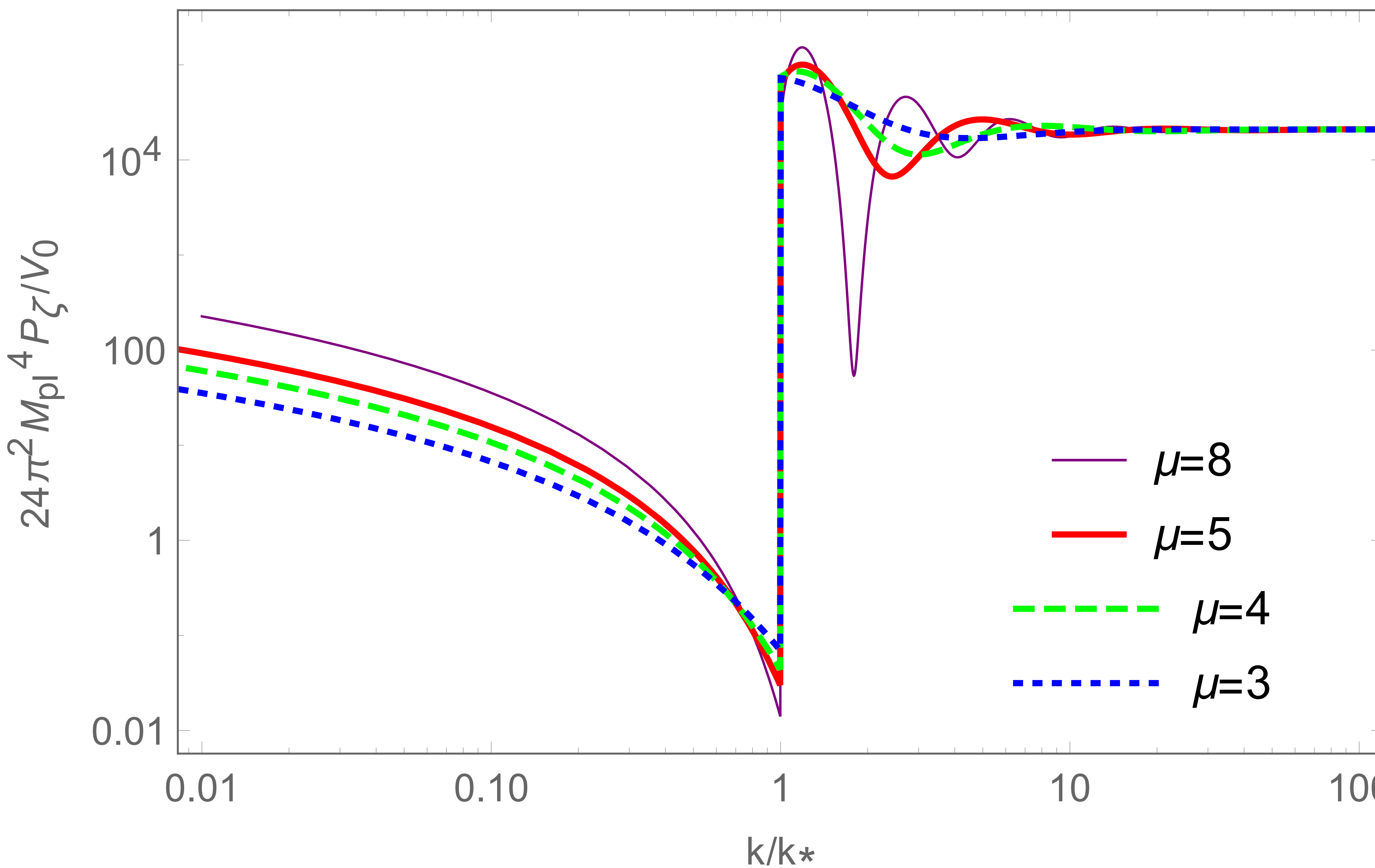
Large Enhancement

$$\omega \equiv \sqrt{\frac{\mu^2}{1+4/\mu^2} - \frac{9}{4}}$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{\partial N}{\partial \chi}\right)^2 \langle \delta\chi\delta\chi \rangle,$$

$$= \mathcal{P}_{\mathcal{R}}^{(0)} \left\{1 - \frac{2}{3} e^{\frac{3}{2}(N-N_*)} \ln \frac{F_*}{1+\frac{4}{\mu^2}} \sqrt{1+\Upsilon^2}\Big[\omega \sin\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right) - 6\cos\left(\omega(N-N_*) + \tan^{-1}\Upsilon\right)\Big]\right\}^2,$$

Order 1 pre-factor

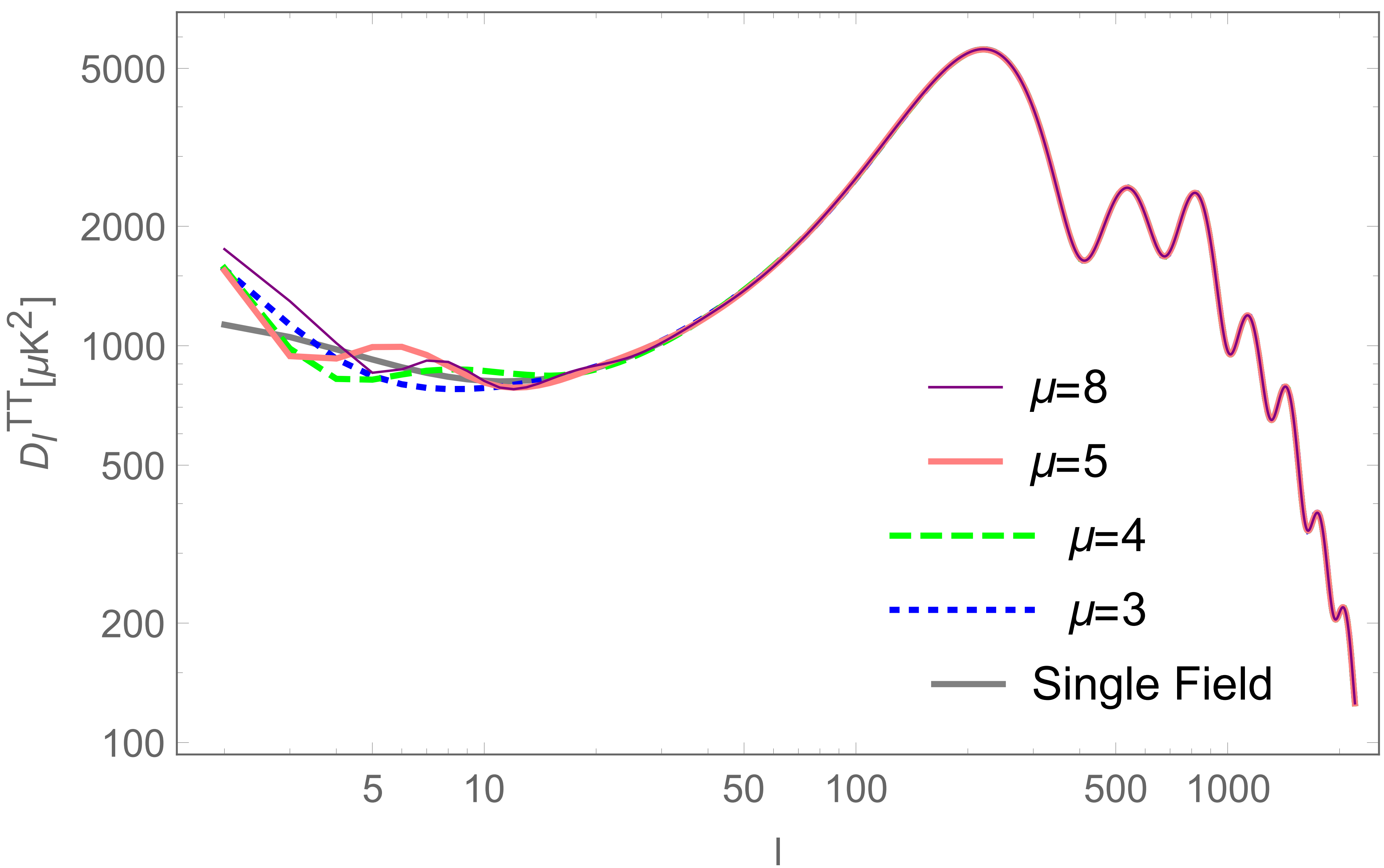


$\eta_H = 0.02, \;\; \chi_*/M_{\rm Pl} = 0.1,$

$\mu = 2, 3, 5, 8$

Contents

Application at large scales: Large scale anomaly



CMB anisotropy for $k_*=(10^5\ \mathrm{Mpc})^{-1}$.

Application at small scales: PBH as dark matter

- If the peak of density spectrum has exceeded some critical value δ_c(~0.4), there will be PBH formation when the mode re-enters the horizon. (Carr, Kühnel, Sandstad 2016)
- Initial mass fraction:

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_H))d\delta(M_H)$$

- For a Gaussian probability distribution:

$$\beta(M) = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_H)}\right)$$

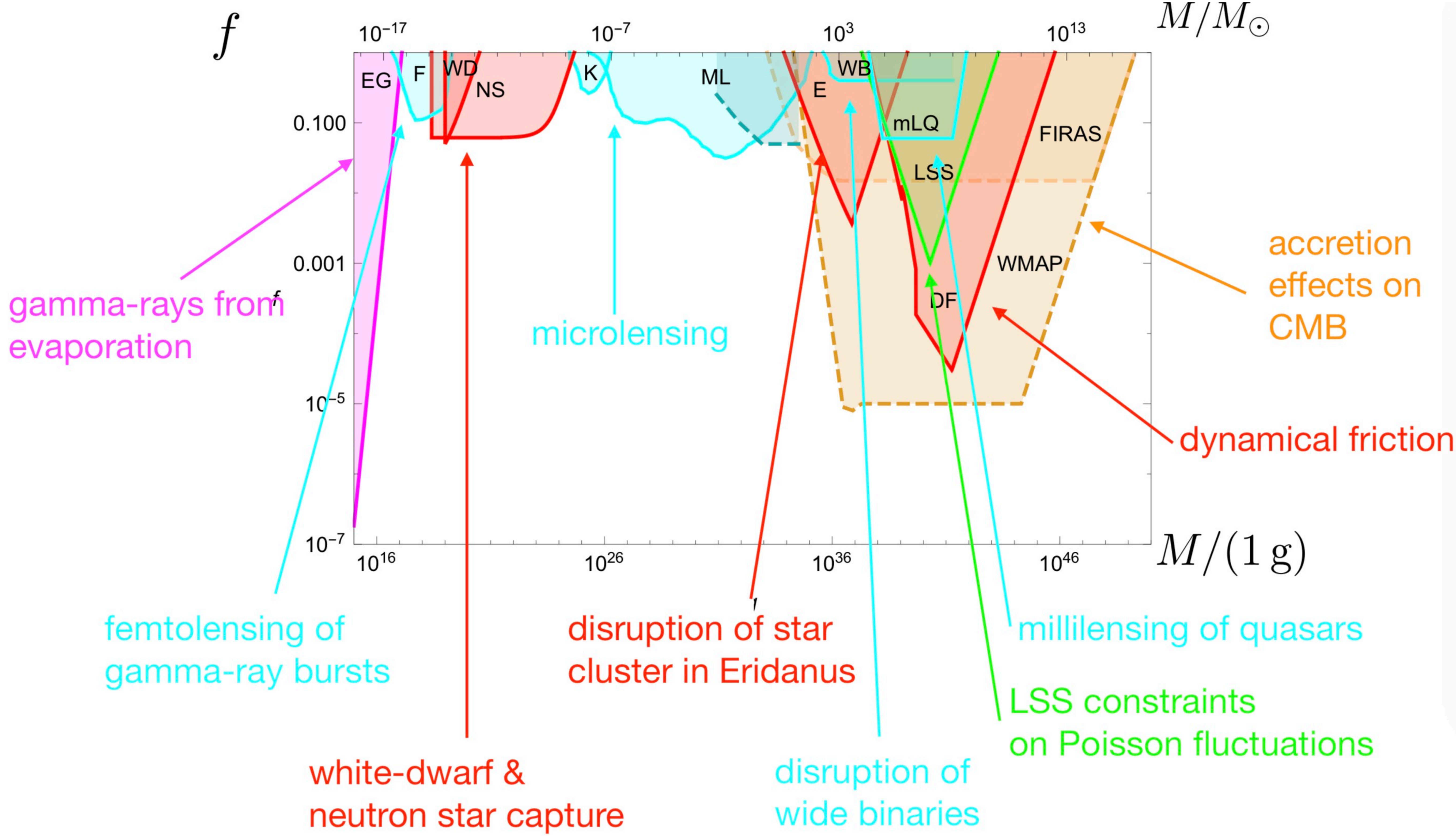
- M_H is the horizon mass at re-entry, and $\sigma(M_H)$ is the variance of its PDF.
- And β can be transferred to the mass spectrum today by

$$f \sim 10^9 \left(\frac{M}{M_\odot}\right)^{1/2} \beta(M)$$

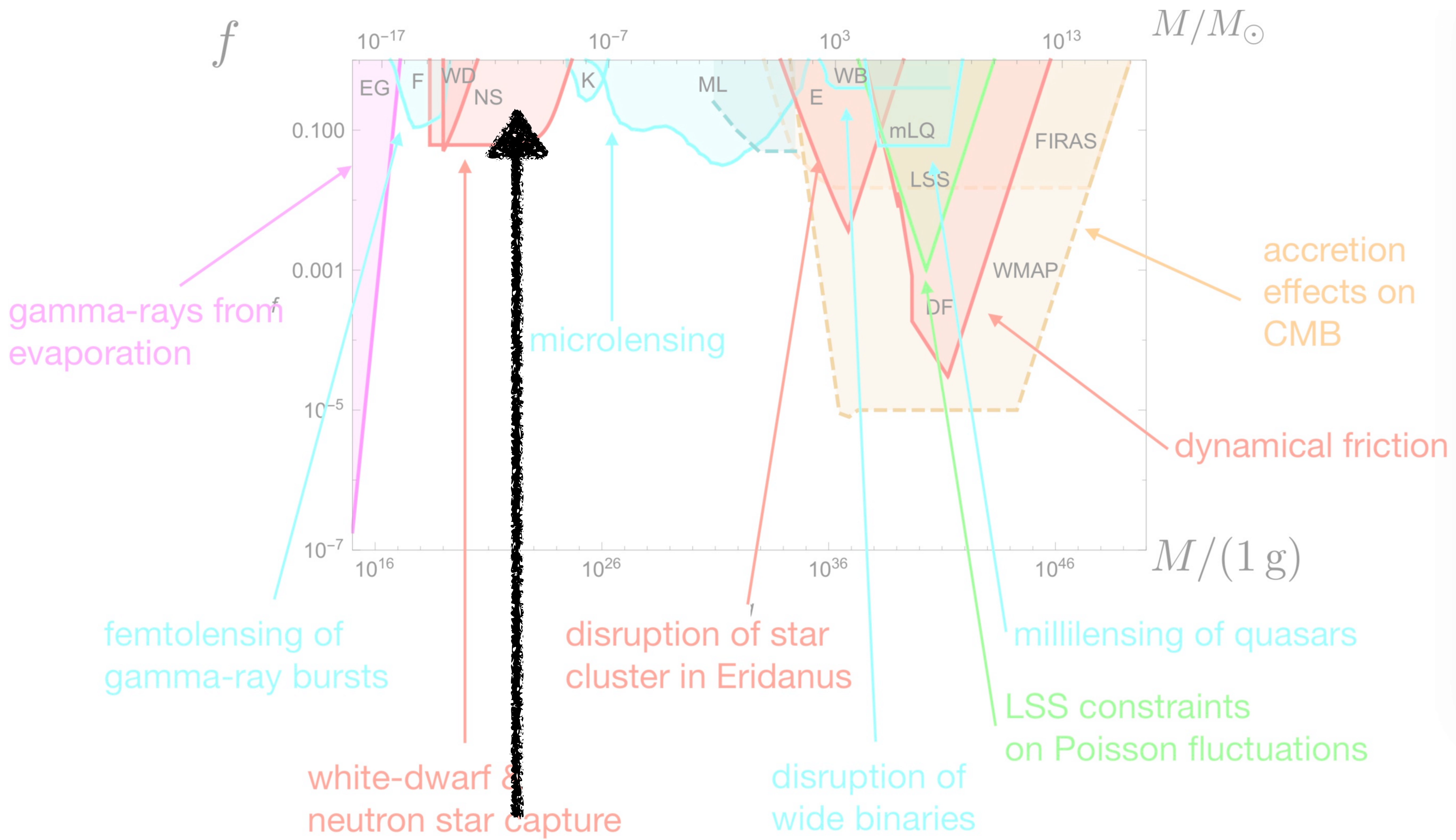
Application at small scales: PBH as dark matter

- On CMB scales $\sigma(M_H) \approx 10^{-5}$, and β is exponentially suppressed.

- At transition there is a huge enhancement of the power spectrum, where $\sigma(M_H)$ may be around $\sigma(M_H) \approx 10^{-2}$. And a significant amount of PBHs may be produced at the re-entry.

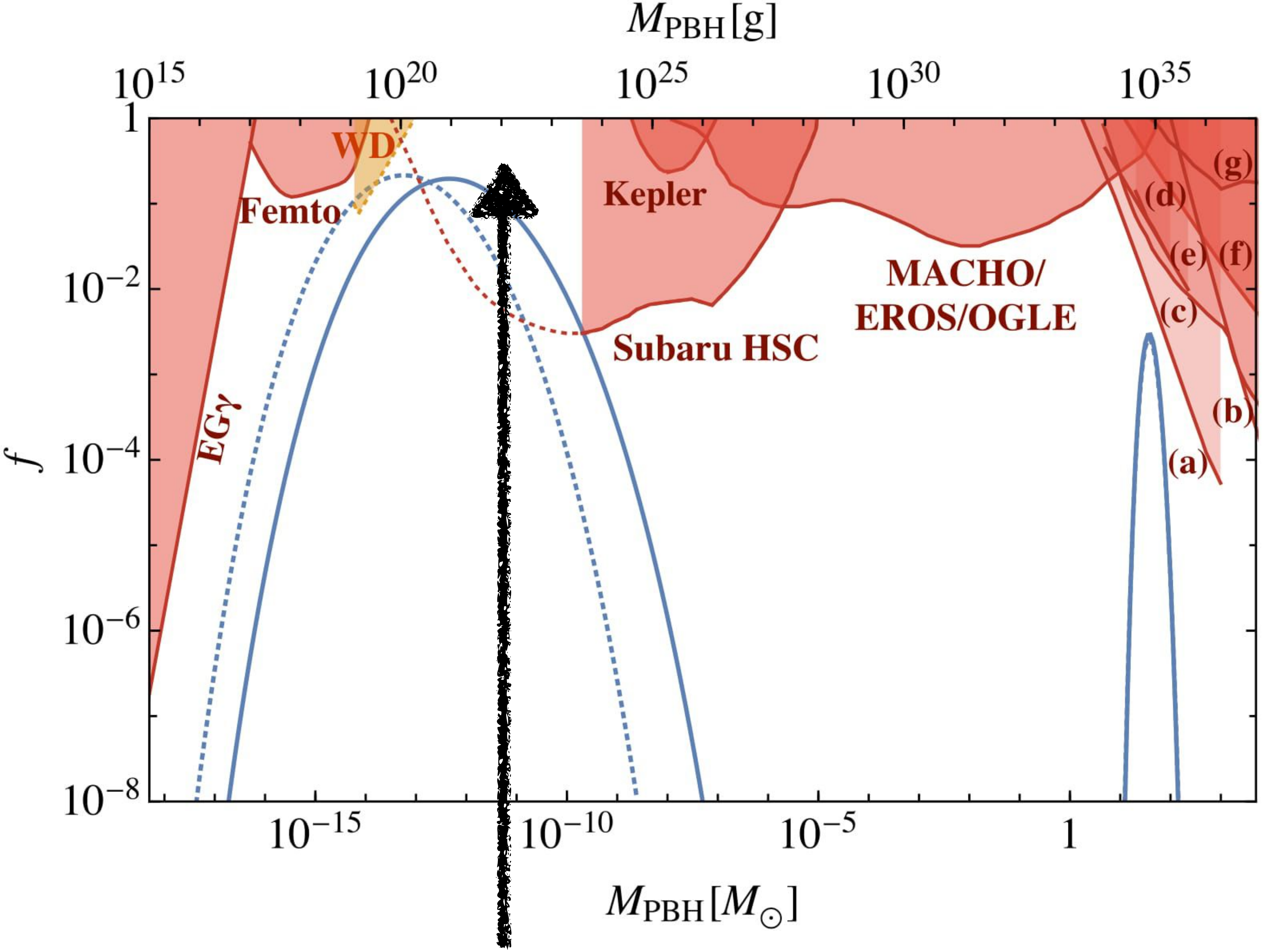


Carr, Kühnel &Sandstad 2016. Borrowed from Anne Green's slides.



However, the wave effect may weaken the constraints at 10^{20}~10^{24} g,

Takada, talk@IPMU, see also Inomata et. al. 2017



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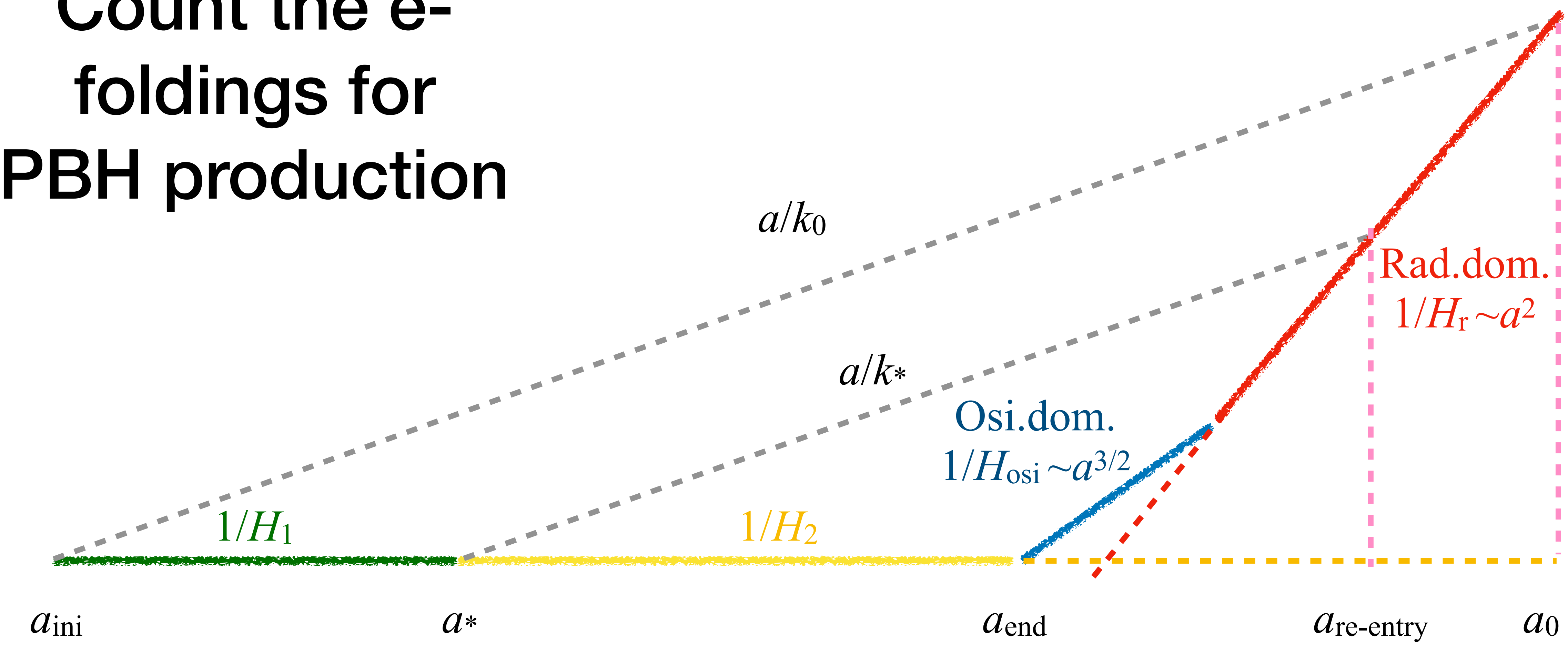
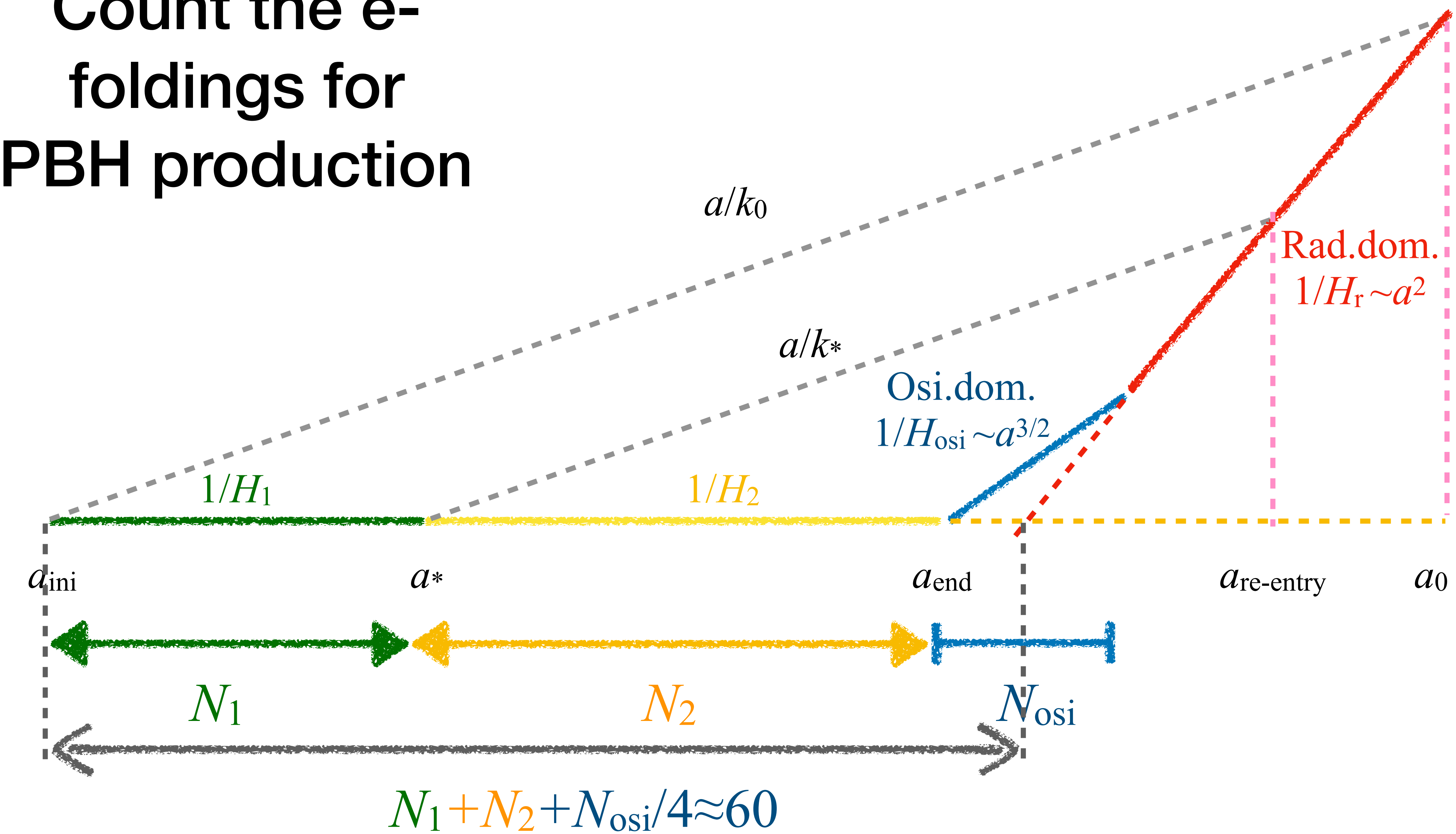
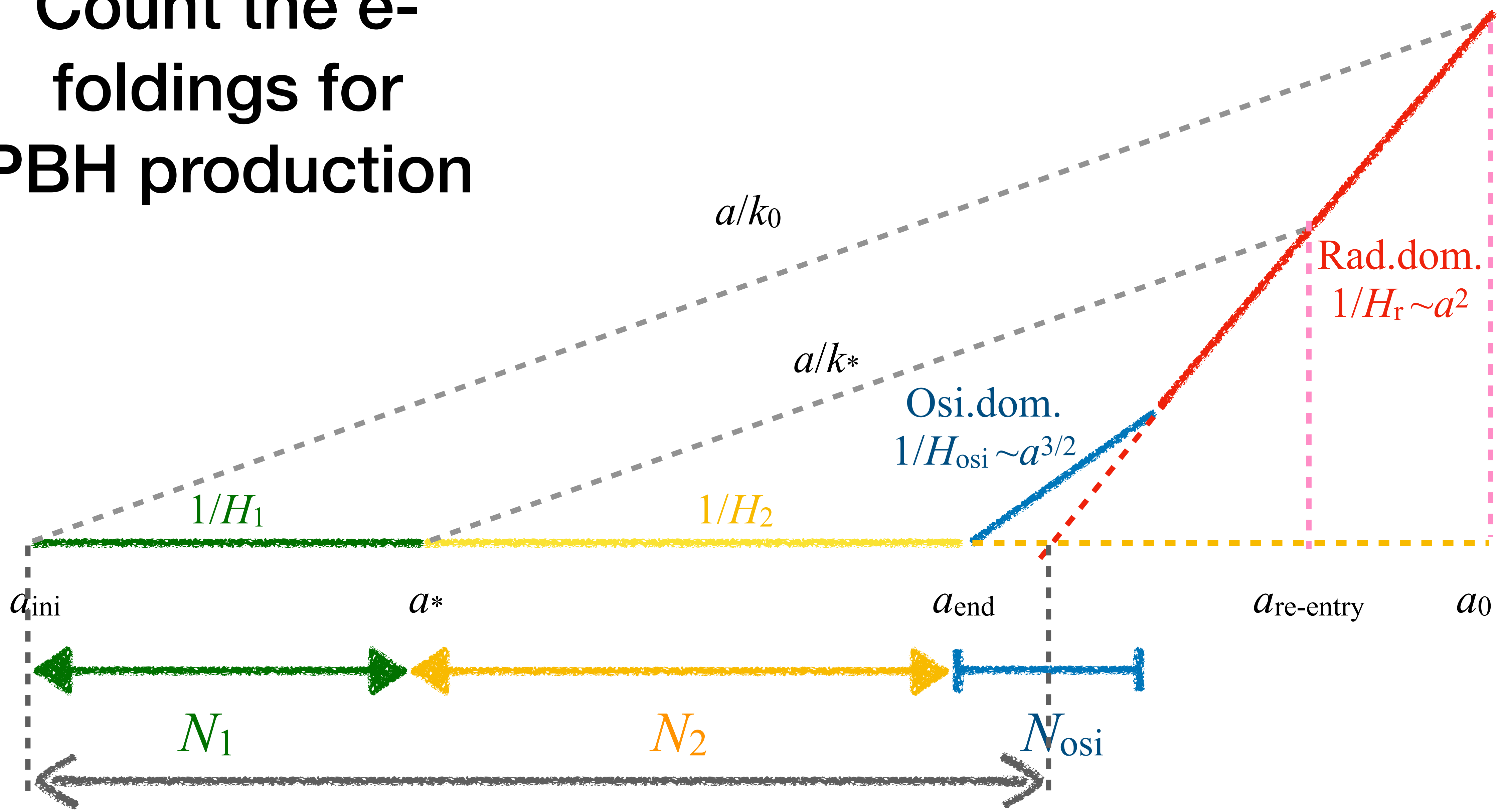
Count the e-foldings for PBH production



Count the e-foldings for PBH production



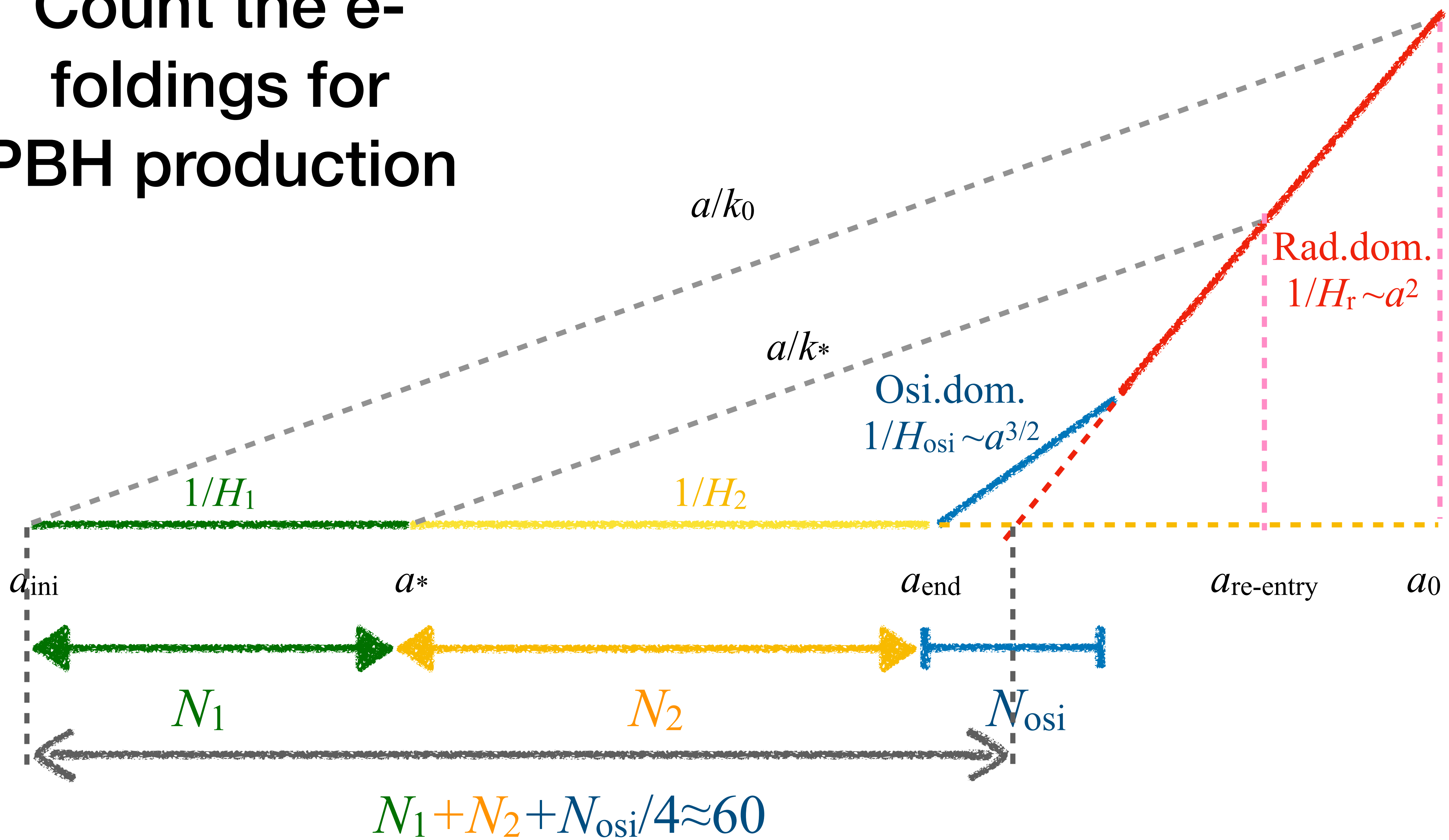
Count the e-foldings for PBH production



$N_1+N_2+N_{osi}/4 \approx 60$

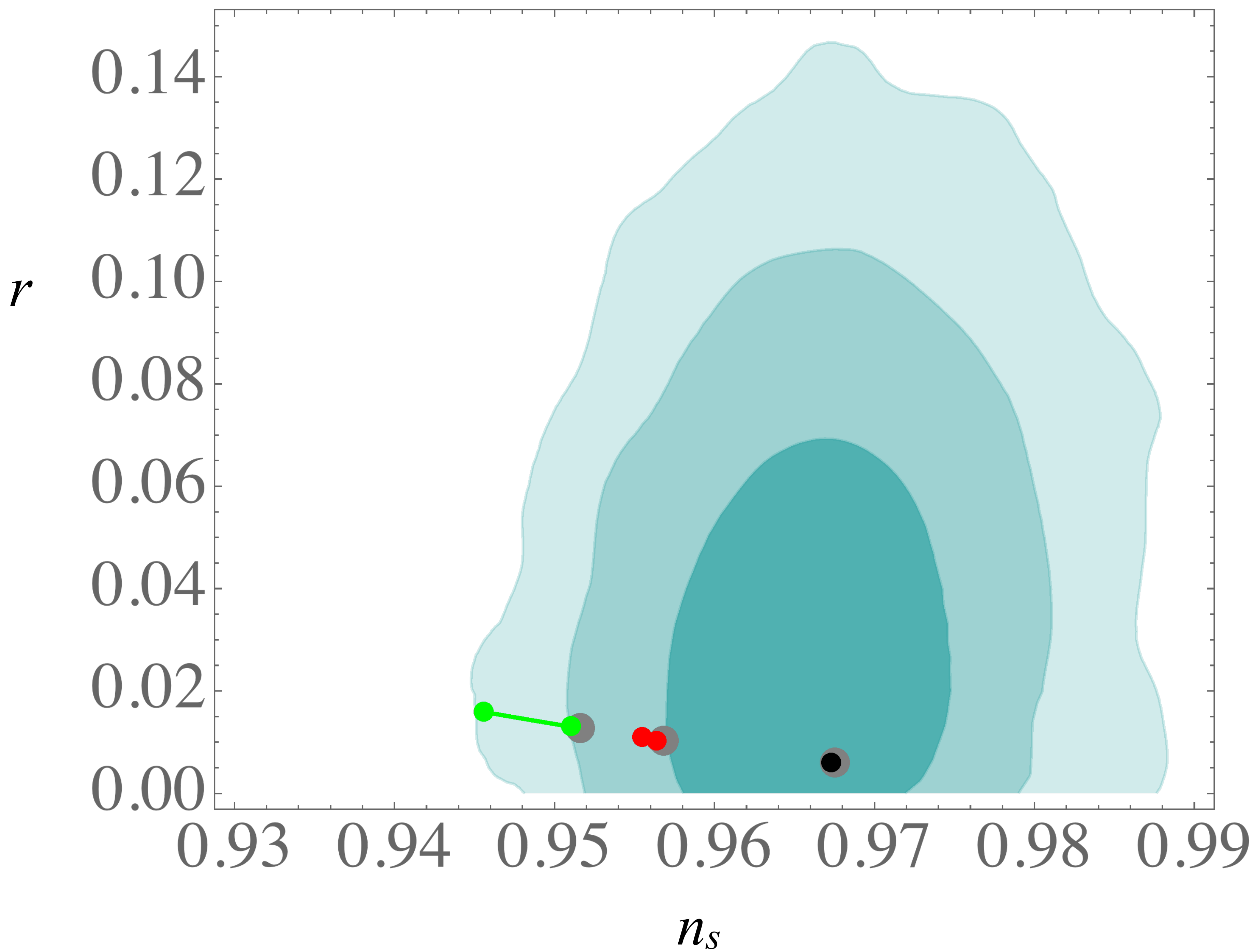
PBH mass: $M_{PBH} \sim M_H \sim \dfrac{M_{Pl}^2}{H_*} e^{2(N_2+N_{osi}/4)} = \dfrac{M_{Pl}^2}{H_*} e^{2(60-N_1)}$

Count the e-foldings for PBH production

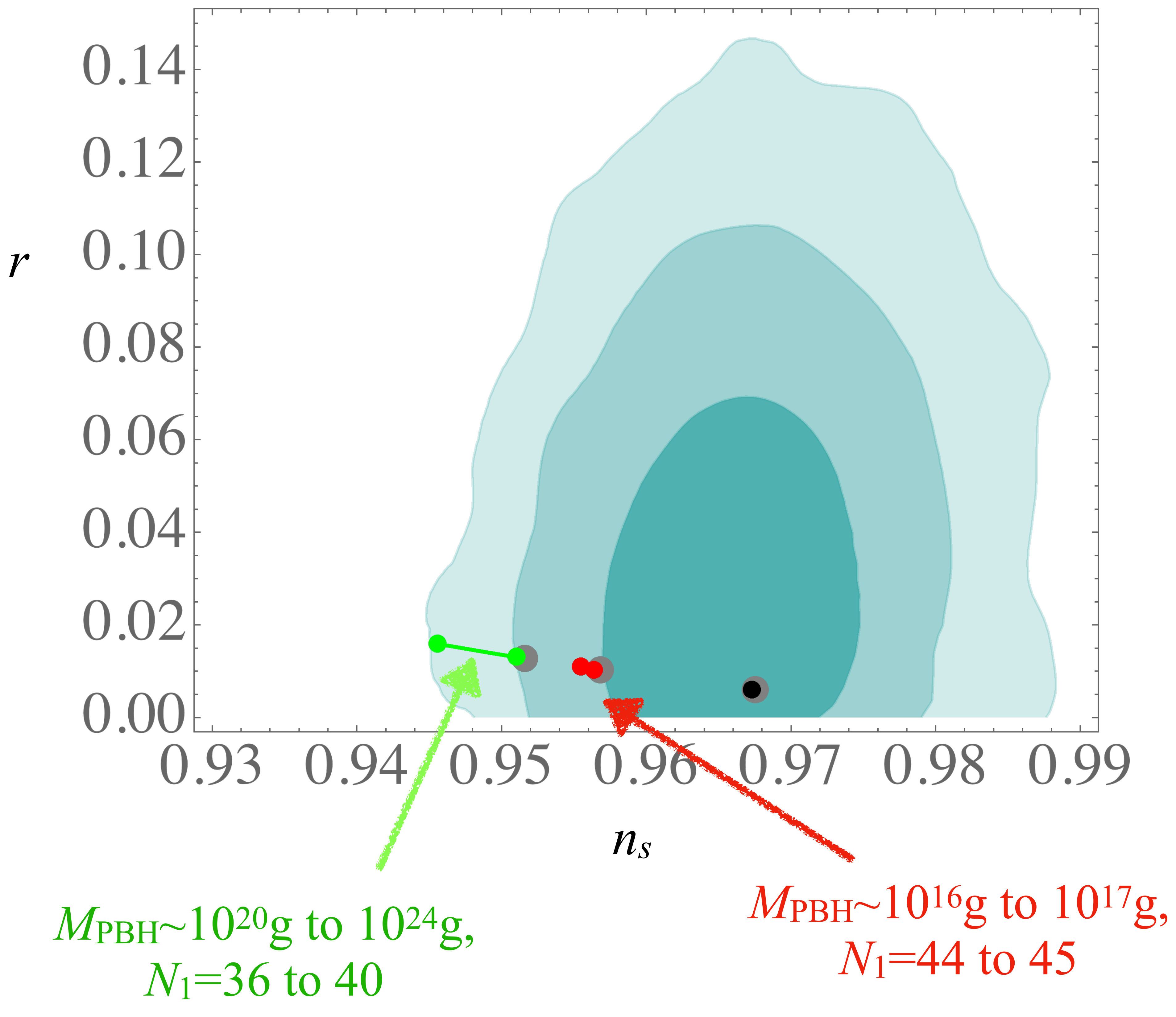


PBH mass: $M_{\rm PBH} \sim M_H \sim \frac{M_{\rm Pl}^2}{H_*} e^{2(N_2 + N_{\rm osi}/4)} = \frac{M_{\rm Pl}^2}{H_*} e^{2(60 - N_1)}$

Inverse relation: $N_1 = 44.4 - \frac{1}{2} \ln \left(\frac{M_{\rm PBH}}{10^{16}\ {\rm g}} \right).$



0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
r
0.93
0.94
0.95
0.96
0.97
0.98
0.99
n_s



0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
r
0.93
0.94
0.95
0.96
0.97
0.98
0.99
n_s
$M_{\rm PBH}$~10^{20}g to 10^{24}g,
N_1=36 to 40
$M_{\rm PBH}$~10^{16}g to 10^{17}g,
N_1=44 to 45

Summary

- R^2+scalar field ≡ two-field with non-trivial field metric.
- Scalar field may provide a second stage inflation after the end of Starobinsky-stage.
- The transition of two stages may give enhanced features on the power spectrum.
- This enhanced ``feature” can be used to produce PBHs as dark matter.


Thank you!