TOWARDS A COMPLETE SCENARIO OF THE EARLY UNIVERSE

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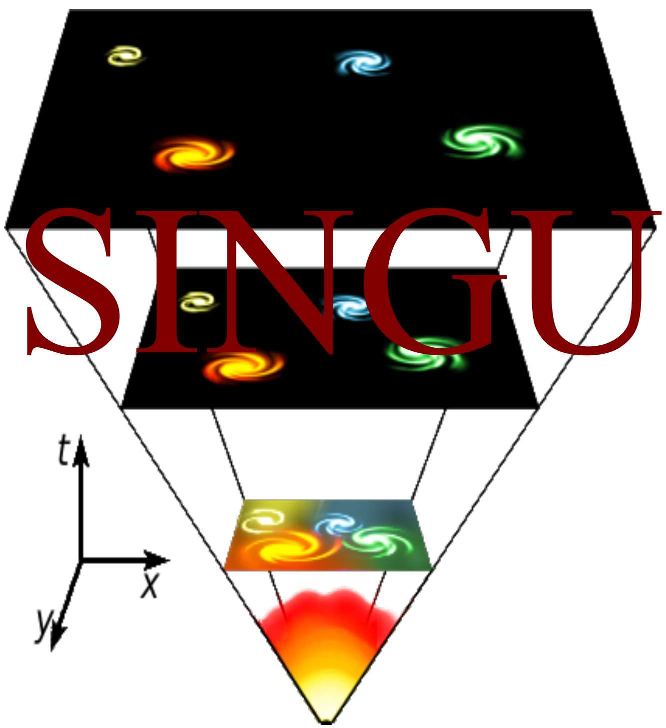
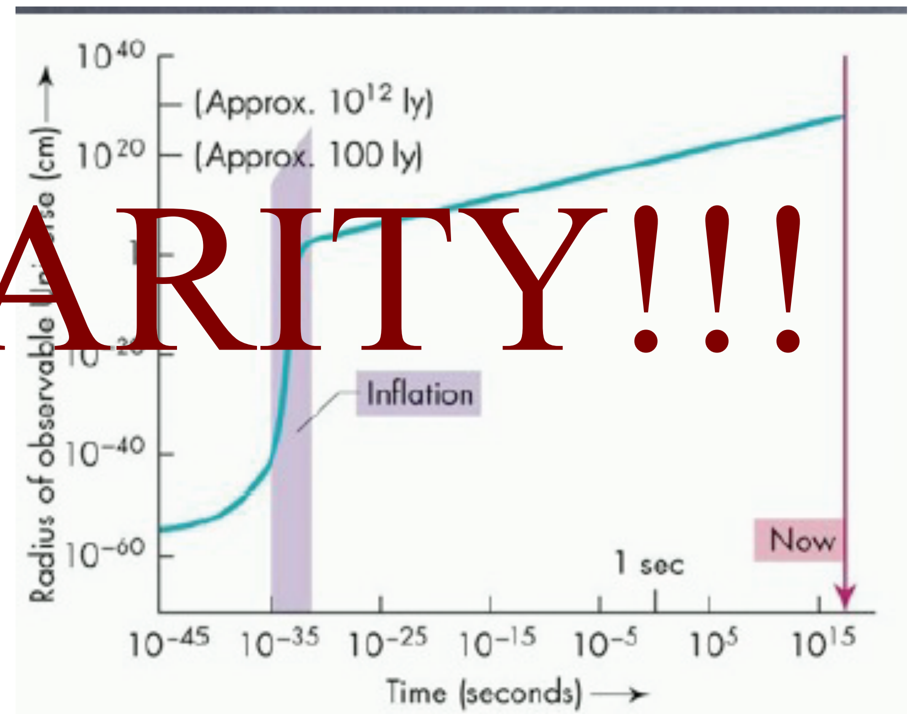
Standard Cosmology

Big-Bang Scenario | Inflation Scenario





SINGULARITY!!!

(Alpher/Bethe/Gamow 1948)

A. H. Guth/A. D. Linde/A. Starobinsky/…

(1980’s)

The Singularity Problem

Singularity Theorem:

The universe will meet a singularity when

(1) it is described by General Relativity;

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + L_m\right]$$

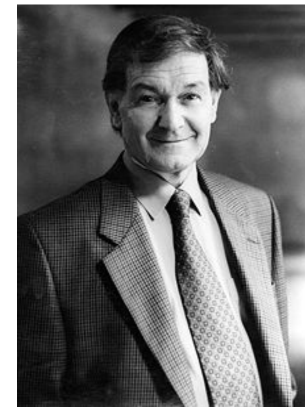
(2) it satisfies Null Energy Condition;

$$T_{\mu\nu}n^\mu n^\nu = [(\rho+P)u_\mu u_\nu + g_{\mu\nu}P]n^\mu n^\nu$$

$$= (\rho+P)(u_\mu n^\mu)^2 + Pn_\mu n^\mu$$

$$= (\rho+P) \geq 0$$

S. Hawking



R. Penrose

for any null vector n^μ: $u_\mu n^\mu = 1$, $n_\mu n^\mu = 0$

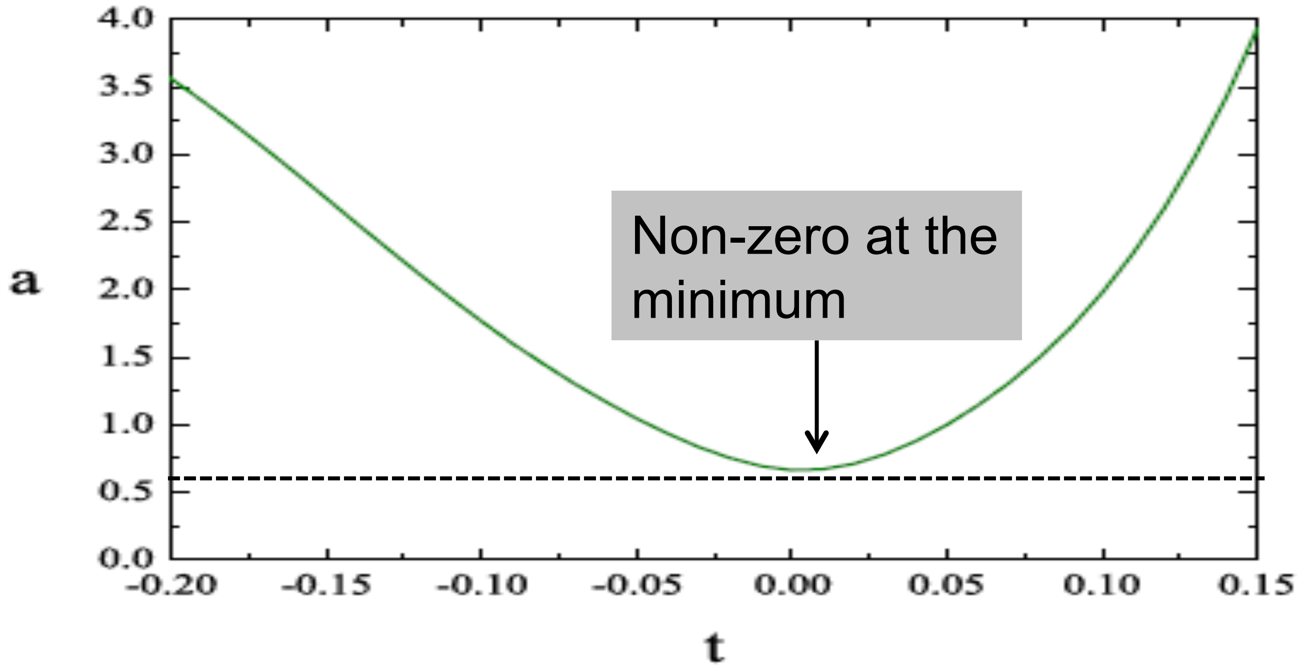
Where at finite time point

$$a_u(\mathrm{t}) \rightarrow 0, \quad \rho_u(\mathrm{t}) \rightarrow \infty$$

S.W. Hawking, G.F.R. Ellis, **Cambridge University Press, Cambridge, 1973**;

Borde and Vilenkin, **Phys.Rev.Lett.72, 3305 (1994).**

One of the Solutions: Bounce Cosmology



Contraction: $H < 0$ Expansion: $H > 0$

Bouncing Point: $\dot{H} > 0$ $\rho + p < 0$

Violating the Null Energy Condition (NEC)!

Issues of Bounce Cosmology and its Solutions

BACKGROUND

Anisotropy Problem

PERTURBATIONS

Scale Invariance of Power Spectrum

Ghost Instability

Gradient Instability

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Cosmic Anisotropy

If the initial metric is not exact isotropic:

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1}^{3} e^{2\beta_i(t)} dx^{i2}$$

Friedmann Equation:

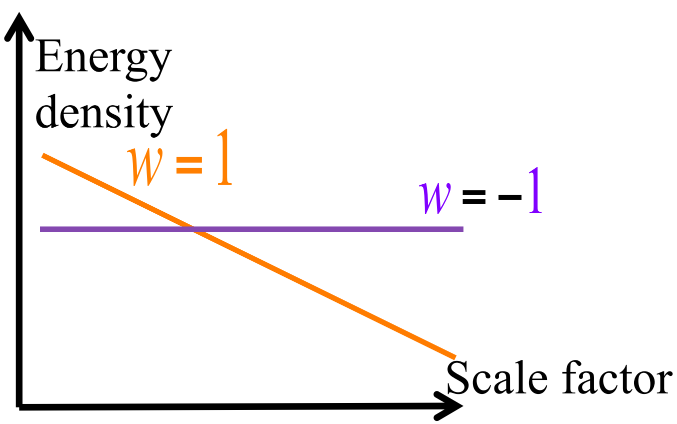
$$3H^2 = \rho_{bg} + \frac{1}{2}\sum_{i=1}^{3}\dot{\beta}_i^2$$

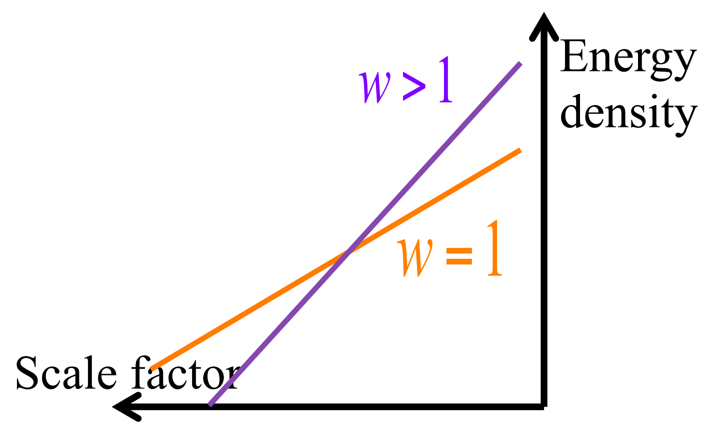
↑ Matter ↑ Anisotropy

Equation of motion for anisotropy:

$$\ddot{\beta}_i + 3H\dot{\beta}_i = 0 \Rightarrow w_{ani} = 1$$

$$\rho_{ani} \propto a^{-3(1+w)} = a^{-6}$$

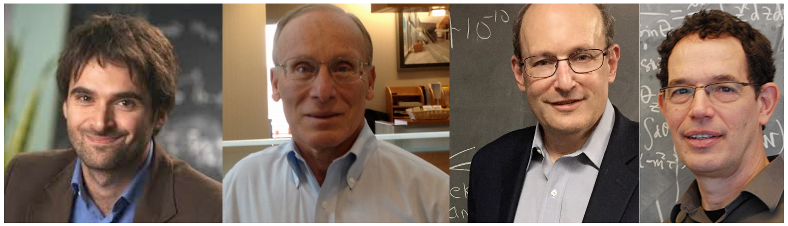




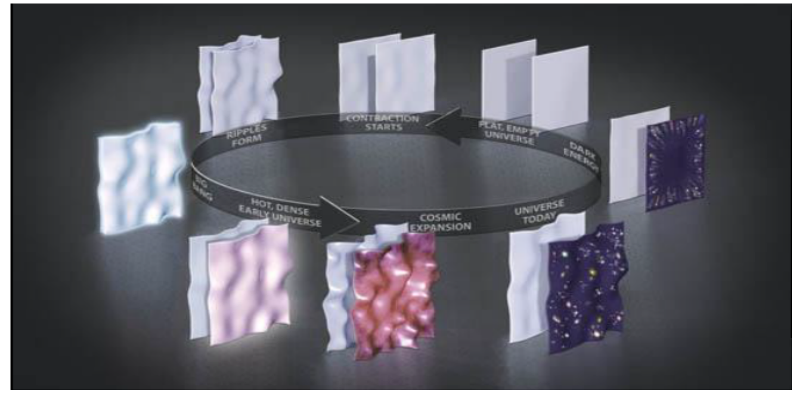
So we need contracting phase with w>1!

J. Erickson, D. Wesley, P. Steinhardt, N. Turok, **Phys.Rev. D69 (2004) 063514.**

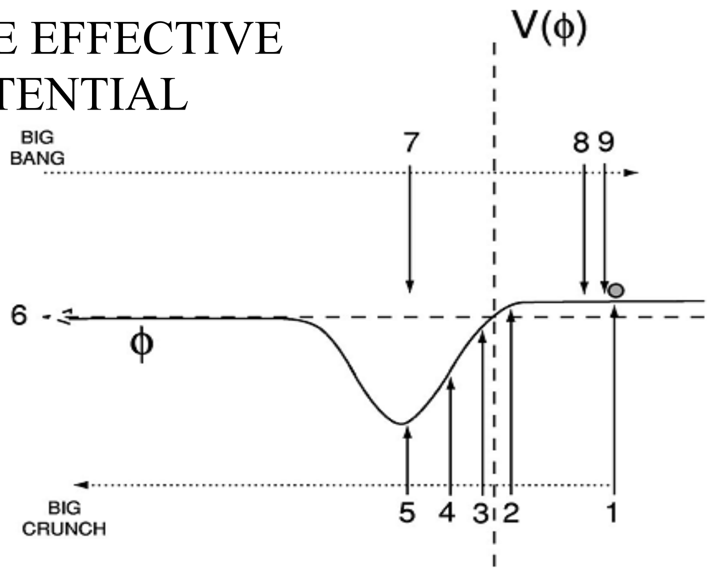
One of the Solutions: Ekpyrosis



Khoury, Ovrut, Steinhardt & Turok, PRD64:123522, 2001



THE EFFECTIVE POTENTIAL



The effective potential (Ekpyrotic phase):

$$V(\phi) = -V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{M_p}\right) < 0$$

In order to make $w_{eff} = \dfrac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V} > 1$

1 DE domination; 2 decelerated expansion; 3 turnaround; 4 Ekpyrotic contraction; 5 before big crunch; 6 a singular bounce in 4D; 7 after big bang; 8 radiation domination; 9 matter domination

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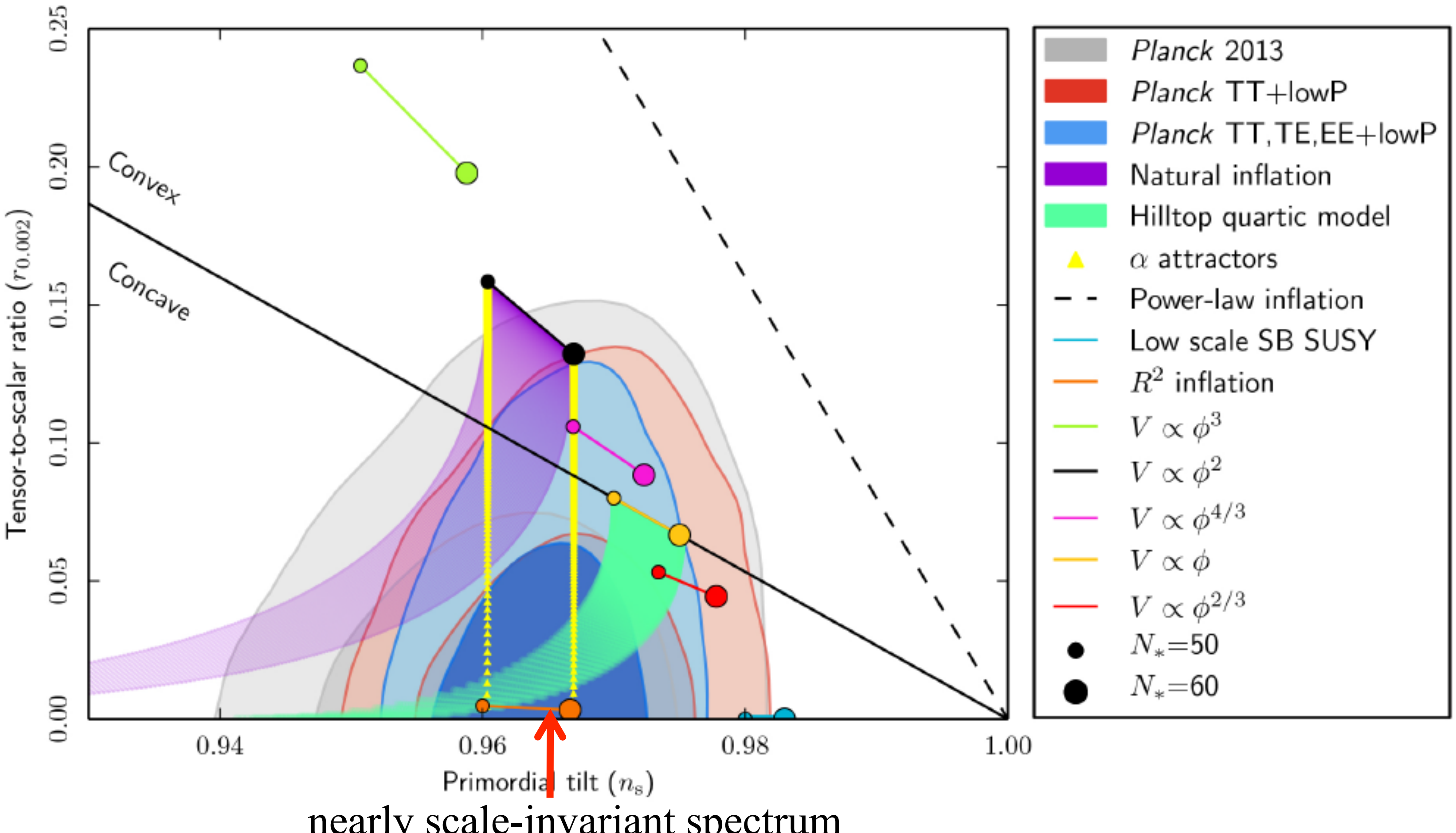
Scale Invariance of Power Spectrum

Observational data (Planck 2015):



P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 2016, cited 1000+

Scale Invariance of Power Spectrum

The perturbation equation:

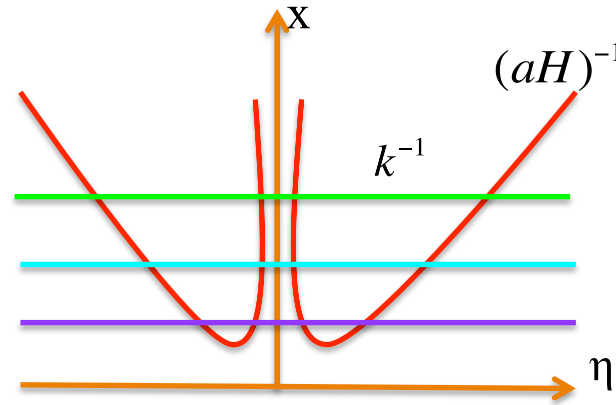
$$(z\zeta)'' + \left(c_s^2 k^2 - \frac{z''}{z}\right)(z\zeta) = 0$$

Solution:

$$\zeta \sim (c_s k)^{\frac{3}{2}\frac{1-w}{1+3w}} \eta^0, \quad (c_s k)^{-\frac{3}{2}\frac{1-w}{1+3w}} \eta^{-\frac{3}{2}\frac{1-w}{1+3w}}$$

constant

growing for viable bounce models



sketch plot of perturbations during the bounce

Power spectrum:

$$P_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta|^2 \sim k^{3-\frac{3(1-w)}{1+3w}} \eta^{-\frac{1-w}{1+3w}}$$

$$n_s - 1 = 3 - \frac{3(1-w)}{1+3w} \approx 0 \quad \Rightarrow \quad w \approx 0$$

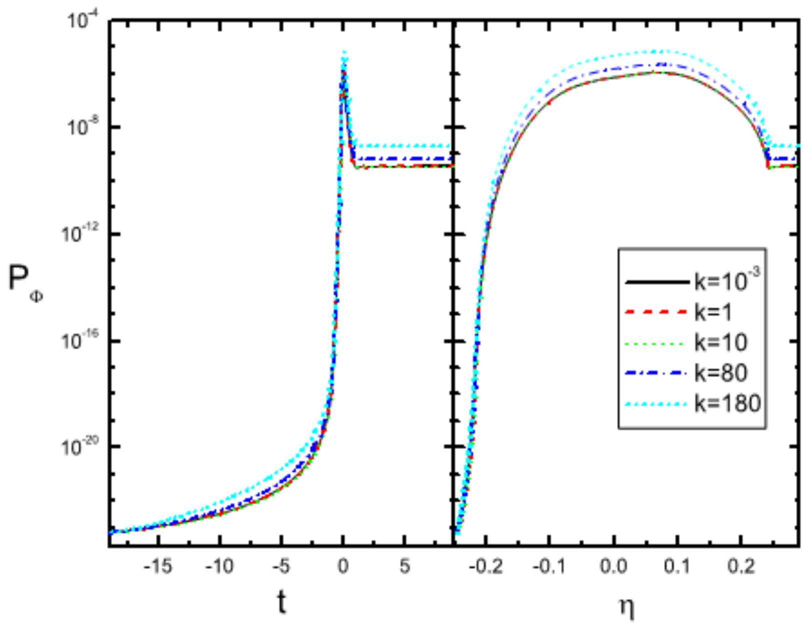
D. Wands, PRD 1999,
F. Finelli and R. Brandenberger, PRD 2002.

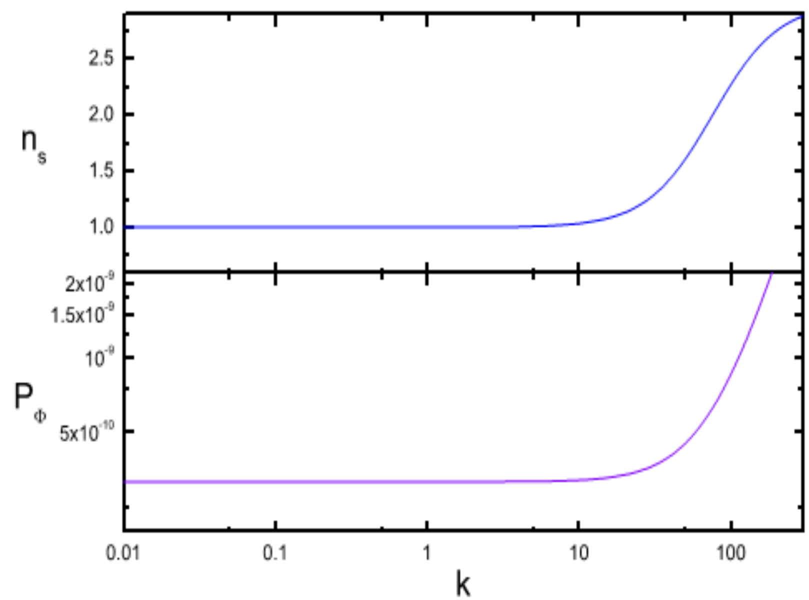
David Wands

Robert Brandenberger

Scale Invariance of Power Spectrum

The numerical plots of the power spectrum and spectral index:





Y. F. Cai, T. Qiu, R. Brandenberger, X. M. Zhang, PRD 2009

To Be Large or Not To Be Large? Is it a Problem?

Isotropy: w>1

Scale Invariance: w=0

POSSIBLE SOLUTIONS:

1) To have another field in contracting phase to generate power spectrum (Entropic Mechanism);

F. Finelli, PLB 2002;

K. Koyama and D. Wands, JCAP 2007;

K. Koyama, S. Mizuno, D. Wands, CQG 2007;

T. Qiu, X. Gao and E. N. Saridakis, PRD 2014… …

2) To have inflationary period following the bounce (Bounce Inflation).

Y. S. Piao, B. Feng and X. M. Zhang, PRD 2004;

Z. G. Liu, Z. Guo and Y. S. Piao, PRD 2013;

J. Q. Xia, Y. F. Cai, H. Li and X. M. Zhang, PRL 2014;

T. Qiu and Y. T. Wang, JHEP 2015……

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Ghost Instability

NEC violation will generally cause ghost mode!

Example: "Phantom" Energy

Lagrangian:

$$L_{phantom} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

↑ **Ghost mode!**

Hamiltonian (density):

$$H_{phantom} = \Pi\dot{\phi} - L = -\omega\int d^3k(a_k^\dagger a_k + \frac{1}{2})$$

unbounded energy!

S. Carroll, M. Hoffman, M. Trodden, **Phys.Rev. D68 (2003) 023509**;
J. Cline, S. Jeon, G. Moore, **Phys.Rev. D70 (2004) 043543**.

Solution: Galileon Theories

Galileon/Horndeski theories (2008/1974)

$$L_2 = K(\phi, X)$$

$$L_3 = -G_3(\phi, X)\Box\phi$$

$$L_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - G_{5,X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]/6$$

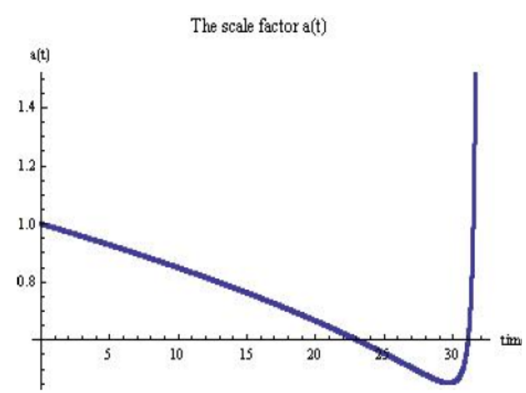
1) Higher derivative in lagrangian but 2nd order in equation of motion
2) Multi-degrees of freedom but only one is dynamical
3) violating NEC free of ghosts.

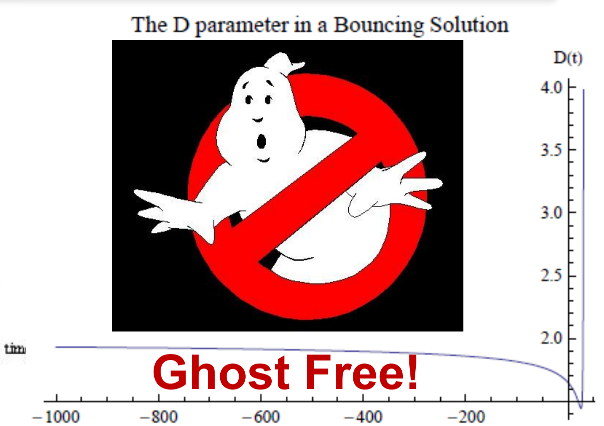
e. g. (T. Qiu et al., JHEP 2011)

$$L = F^2 e^{2\phi}(\partial\phi)^2 + \frac{F^3}{2M^3}(\partial\phi)^4 + \frac{F^3}{M^3}(\partial\phi)^2\Box\phi$$

Perturbation action:

$$\delta^{(2)}S = 3\int dtd^3xDM_P^2\left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial\zeta)^2\right]$$



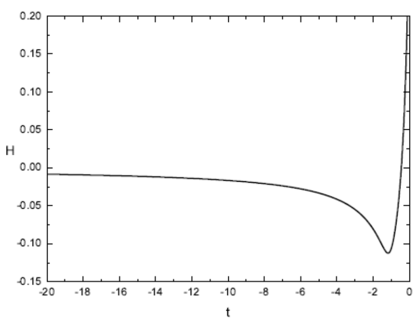


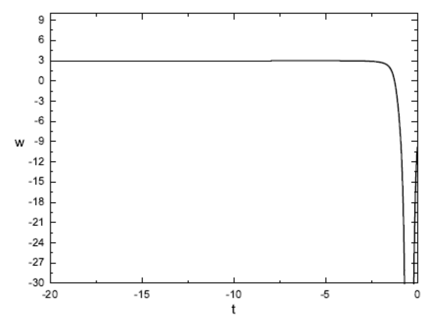
Solution: Galileon Theories

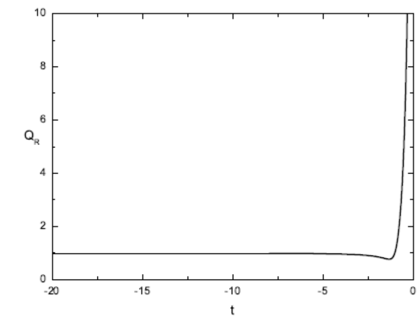
Taking into account the anisotropic and scale invariance issues, one can get more improved models:

1) multi-field bounce model (T. Qiu, X. Gao and E. N. Saridakis, PRD 2013):

$$S=\int d^4x\sqrt{-g}[\frac{R}{16\pi G}+X-V(\phi)-gX\Box\phi] \qquad V(\phi)=-V_0\,e^{c\phi}$$

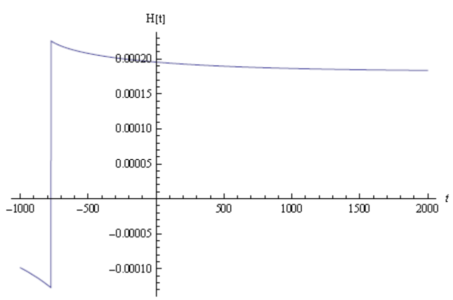


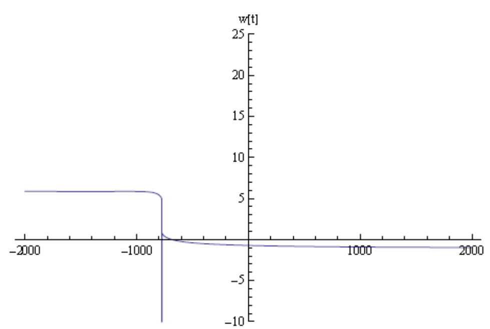


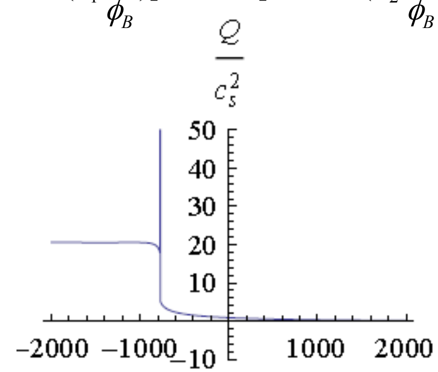


2) bounce inflation model (T. Qiu, Y. T. Wang, JHEP 2015):

$$S=\int d^4x\sqrt{-g}[\frac{R}{16\pi G}+k(\phi)X+t(\phi)X^2-V(\phi)-G(\phi,X)\Box\phi] \qquad V(\phi)=-V_0[1-\tanh(\lambda_1\frac{\phi}{\phi_B})]e^{c\phi}+\Lambda^4[1+\tanh(\lambda_2\frac{\phi}{\phi_B})](1-\frac{\phi^2}{v^2})^2$$







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Gradient Instability

In our model building, we found that it is difficult to also have the sound speed squared to be positive all the time.

The perturbation EoM:

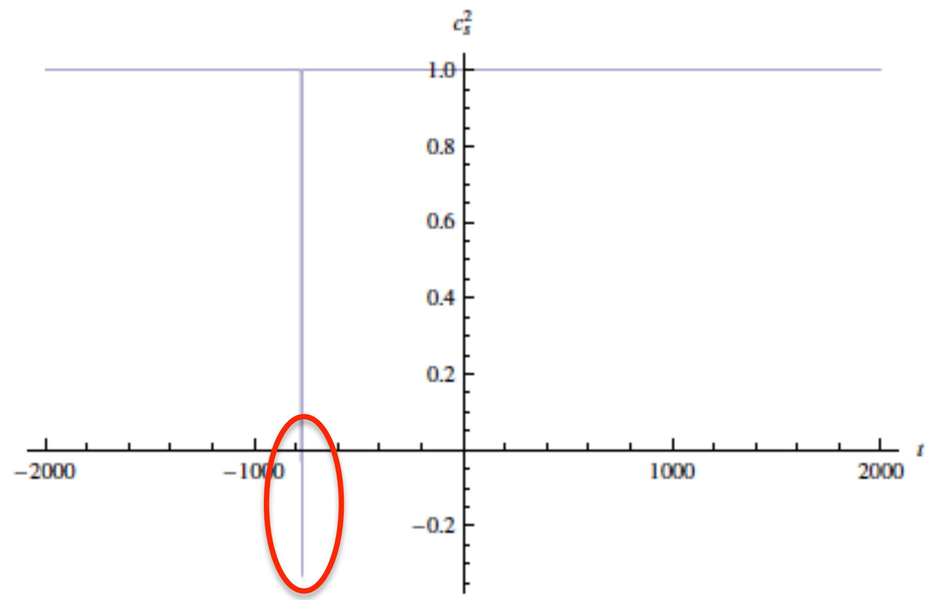
$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0$$

sound speed squared

For $c_s^2 < 0$

$$u_k \sim e^{i\sqrt{c_s^2} k\eta} \sim e^{\pm |c_s| k\eta}$$

for large k modes.



Gradient Instability

T. Qiu, J. Evslin, Y. F. Cai, M. Z. Li, X. M. Zhang, JHEP 2011/Y. F. Cai, D. A. Easson, R. Brandenberger, JCAP 2012/T. Qiu, X. Gao, E. Saridakis, PRD 2013/M. Koehn, J. L. Lehners and B. A. Ovrut, PRD 2014/L. Battara, M. Koehn, J. L. Lehners and B. A. Ovrut, JCAP 2014/T. Qiu and Y. T. Wang, JHEP 2015……

No-go Theorem

It has been proved that gradient instability is inevitable in cubic Galileon theories!

Abstract. We study spatially flat bouncing cosmologies and models with the early-time Genesis epoch in a popular class of generalized Galileon theories. We ask whether there exist solutions of these types which are free of gradient and ghost instabilities. We find that irrespectively of the forms of the Lagrangian functions, the bouncing models either are plagued with these instabilities or have singularities. The same result holds for the original Genesis model and its variants in which the scale factor tends to a constant as $t \to -\infty$. The result remains valid in theories with additional matter that obeys the Null Energy Condition and interacts with the Galileon only gravitationally. We propose a modified Genesis model which evades our no-go argument and give an explicit example of healthy cosmology that connects the modified Genesis epoch with kination (the epoch still driven by the Galileon field, which is a conventional massless scalar field at that stage).

It is this set of issues we address in this paper. We consider the simplest and best studied generalized Galileon theory interacting with gravity. The Lagrangian is (mostly negative signature; $\kappa = 8\pi G$)

$$L = -\frac{1}{2\kappa}R + F(\pi, X) + K(\pi, X)\Box\pi \ , \qquad (1.1)$$

Contents

Although linear perturbation theory suggests that, for some constructions, cubic Galileon theories can avoid pathologies during a period of NEC violation, it has been unclear until now whether this is possible when the NEC violating period includes a non-singular bounce. In fact, the recent arguments suggest that either the speed of sound of co-moving curvature modes becomes imaginary (*i.e.*, ghost or gradient instability) for some wavelengths during the NEC violating phase [6, 7] or the evolution must reach a singularity [8].

M. Libanov, S. Mironov, V. Rubakov, JCAP 1608 (2016) no.08, 037;
Anna Ijjas, Paul J. Steinhardt, Phys. Rev. Lett. 117 (2016) no.12, 121304.

Solution: one must consider theories beyond cubic Galileon!

Effective Field Theory Description

The Effective field theory lagrangian (Cheung et al., 2007; Gleyzes et al., 2013; Kase et al., 2014)

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[& \frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} \\ & + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \\ & - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + ... \\ & - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + ...\Big], \end{aligned}$$

Cubic Galileon: $f = 1 \qquad m_4^2 = \tilde{m}_4^2 = \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

Horndeski: $m_4^2 = \tilde{m}_4^2 \qquad \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

Beyond Horndeski (high order space derivative): every coefficient can be non-zero!

Eliminating The Gradient Instability

According to the No-Go Theorem proved using EFT approach (Y. Cai, Y. Wan, H. Li, T. Qiu, Y. S. Piao, JHEP (2017); Y. Cai, H. Li, T. Qiu, Y. S. Piao, EPJC (2017))

	$\gamma_i < 0$	$\gamma_i > 0$
Cubic Galileon	No way	No way
Beyond cubic galileon (in EFT language)	$Q_T = 0$: $\gamma \sim (t-t_\gamma)^p, Q_T \sim (t-t_\gamma)^n$, $n \geq 2p$	$Q_T = 0$: $Q_T \sim (-t)^{-p}, \gamma \sim (-t)^{-n}$, $p > n > 1$
	$Q_{\tilde{m}_4} = 0$	$Q_{\tilde{m}_4} = 0$

where $\gamma = HQ_T - \frac{m_3^3}{2M_p^2} + \frac{1}{2}\dot{f}$ $\quad Q_{\tilde{m}_4} = f + \frac{2\tilde{m}_4^2}{M_p^2}$

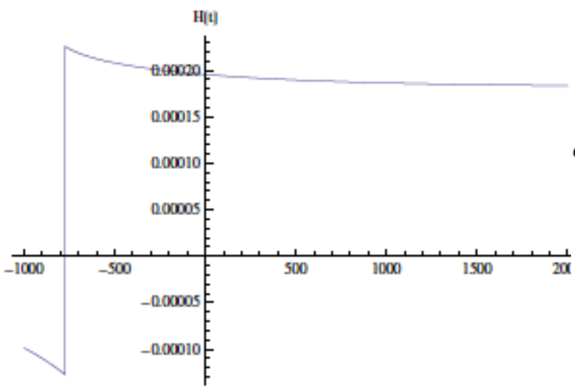
Q_T is the coefficient in front of kinetic term of tensor perturbation action.

Model Construction

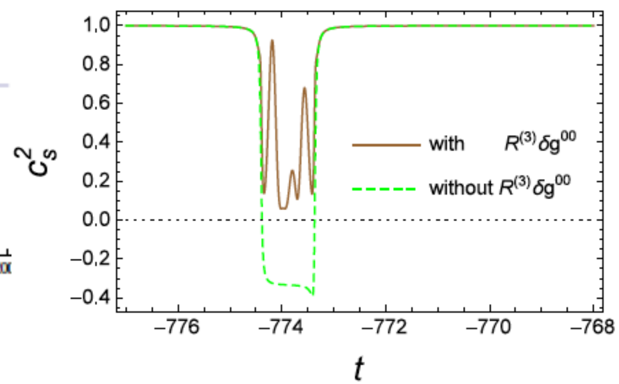
According to this conclusion, we can construct models free of gradient instability!

Action of a New Bounce Inflation Model:

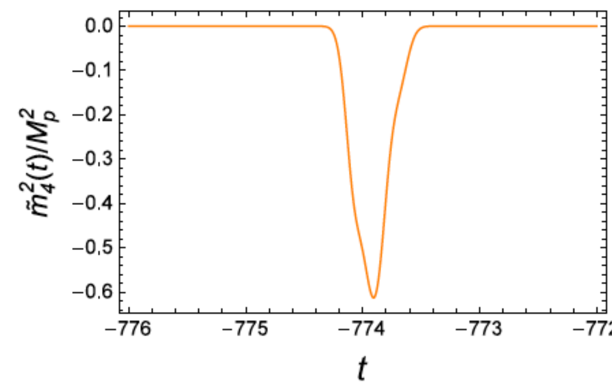
$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \mathcal{K}(\phi)X - G_3(\phi, X)\Box\phi + T(\phi)X^2 - V(\phi) + \underline{\frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}}\right]$$



Background



c_s^2 with/without the $\frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}$ term (green/brown)



Evolution of $\tilde{m}_4^2(t)$

For covariant models, see Y. Cai and Y. S. Piao, JHEP 2017;
Y. Cai, Y. T. Wang, J. Y. Zhao and Y. S. Piao, 1709.07464;
Y. F. Cai, X. Gao, T. Qiu, et al., in preparation.

Conclusion

- Singularity problem can be solved by a bounce scenario, but there are new problems.
- Anisotropy problem---Ekpyrotic phase contraction;
- Scale invariance---Matter contraction/Multifield contraction/ Bounce Inflation;
- Ghost instability---Galileon/Horndeski/Beyond H;
- Gradient instability---EFT approach;
- ***will there be other problems?***
- ***Maybe we can also consider the modified gravity approach…***

Thanks For Attention!